Dynamic enfranchisement

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Abstract

Why would an enfranchised elite voluntarily dilute its power by expanding the franchise? The central intuition behind our analysis is that the dilution of power by an enfranchised elite is equivalent to the delegation of power by one member of the elite—a pivotal voter—to another citizen, who in turn becomes the pivotal voter in the new (expanded) elite. Such delegation might be useful if it allows the current pivotal voter to credibly commit to future policy choices. The current pivotal voter realizes that the agent to whom authority is delegated will face similar incentives to subsequently transfer power, and this effect tempers the extent to which the franchise is extended. We develop a recursive, infinite horizon model that generates the possibility of gradual franchise expansion. We show that, in equilibrium, expansion occurs if and only if the private decisions of the citizenry have a net positive spillover to the dynamic payoff of the current pivotal voter. The class of games we study can accommodate a number of proposed explanations for franchise extension, including the threat of insurrection, and ideological or class conflict within the elite.
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1. Introduction

“There is no more invariable rule in the history of society. The further electoral rights are extended, the greater the need for extending them; for after each concession, the strength of the democracy increases, and its demands increase with its strength.” Alexis de Tocqueville

Voluntary expansion of political rights by a ruling elite is at first glance paradoxical. The elite, after all, dilutes its power when it extends these rights to others. Yet, significant extensions of the voting franchise took place in Europe throughout the 19th and early 20th centuries. Instances of franchise extensions date back, in fact, much further. The constitutional reforms of Cleisthenes in 508 BC in Athens was arguably an early form of franchise extension. Another early instance occurred in 494 BC, when the patricians in the early Roman Republic conceded the right of the plebs (the “commoners”) to participate in the election of magistrates.

This paper examines the determinants of franchise extension—that is, the reasons an enfranchised elite might voluntarily dilute its power. The central intuition behind our analysis is that the dilution of power by an enfranchised elite is equivalent to the delegation of power by one member of the elite—a pivotal voter—to another citizen, who in turn becomes the pivotal voter in the new (expanded) elite.

In any dynamic context, economists are used to the idea that a decision maker might wish to relinquish power, or discretion—that is, to commit to future choices in advance. But commitment to future choices is often not credible. Nonetheless, such commitment could be effected if it were possible to identify another agent who, if given future decision-making authority would, acting in his own interests, make the choice to which the initial decision maker had wanted to commit. The delegation of decision-making authority might then be a credible, and useful, means of relinquishing discretion.

Naturally, the current decision maker should realize that the agent to whom authority is delegated will face similar incentives to subsequently transfer power. Indeed, delegating power to the “right” person might backfire, as he in turn transfers it further. Accounting for this, the current decision maker may temper the extent to which he transfers power initially. Such a recursive structure implies the delegation of authority, which corresponds to franchise expansion, may well evolve incrementally and gradually over time.

Of course, the value of commitment—and thereby the incentive to expand the franchise—depends on the nature of the interactions among agents. Giving up discretion is valuable only if it induces changes in the (equilibrium) behavior of other agents that have a net positive impact on the decision maker’s payoff. In the model we develop, each period agents take private actions that co-mingle with a government policy (chosen by the pivotal voter) to determine the value of an economic state variable in the subsequent period. These actions can affect the payoffs of others either directly, or through their effect on the state. In either case the current pivotal voter might wish to commit to certain future policies (e.g., low tax rates) in order to induce certain actions (e.g., labor supply, effort, higher savings).

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1 Alexis de Tocqueville (1886), Democracy in America, Vol. 1, ch. 4.
from the other agents. This could be achieved by delegating policy-making authority to an agent who would, ceteris paribus, prefer lower tax rates.

A currently enfranchised group votes over current policies and over whether and how far to expand the voting rights to a larger set of citizens in the subsequent period. To analyze this recursive group decision problem, we adopt the protocol of majority voting, but this presents some technical difficulties. In particular, ensuring the existence of well-defined majority winning outcomes—e.g., a median voter—in the context of endogenous dynamic game payoffs is non-trivial.

To formulate the decision problem we consider an alternative game in which all public decisions in a given period are made by a single agent—a dictator. This dictator chooses a current policy and the identity of the agent who will be next period’s dictator (possibly himself). In the given period these public decisions are made simultaneously with the choices of private actions by the citizenry. A complete description of the state at the beginning of a period includes the economic state variable imported from the previous period, and the “political state” variable—the identity of the dictator for this period, as prescribed by the previous dictator. An equilibrium of this dictator delegation game is a state-contingent profile of private actions, public policies, and delegation decisions that constitutes a Markov Perfect equilibrium.

We show that under certain conditions the equilibrium outcomes of this game correspond to the outcomes that would occur if public decisions were determined by a majority-vote. Under these conditions, the majority outcome of the restricted voting franchise corresponds to the preferred outcome of the median voter within that franchise. This median voter, in turn, designates a new median voter of a new (usually larger) voting franchise in the next period. Consequently, the outcomes correspond to a new franchise decision by the currently enfranchised each period.

Our model has three features that we believe are critical to the formulation of a credible theory that can account for the wide heterogeneity of historical enfranchisement (discussed in more detail in Section 2).

- First, political rights are explicitly chosen to solve a strategic delegation problem. Each period’s median voter delegates decision-making authority to a new median in the next period by changing the set of eligible voters. Universal suffrage need not result immediately, and the current median may choose to extend the vote to only a subset of the remaining citizens. Indeed we admit the possibility that the franchise could be contracted in equilibrium.
- Second, this strategic delegation is recursive. Franchise extension is not a once-and-for-all decision, and voting rights can be further extended. This induces the current pivotal voter to dampen the extent of franchise expansion below that of a once-and-for-all decision. This gradualism is an implication of the recursive (infinite horizon) formulation that cannot be captured in, for example, a two period model.
- Third, the franchise is instrumental rather than a fundamental objective for each voter. A model would explain little if rights were extended simply because exogenous costs or benefits of the franchise were inserted directly into preferences or technology. Instead, incentives to expand the franchise are derived from fundamental preferences about its affect on policies and private decisions of individuals.
There are other models, which we also review in Section 2, that satisfy one or possibly two of these features. However, we are not aware of other work that satisfies all three.

We show that equilibria in this model may exhibit partial, gradual, and possibly uneven franchise extensions. We provide a characterization of equilibrium in terms of the associated Euler equations, each corresponding to a participant’s decision problem. Using this characterization, our main result is that an extension of the franchise occurs in a given period if and only if the private decisions of the citizenry have a net positive (marginal) spillover to the dynamic payoff of the current median voter. The size of the extension depends on the size of the spillover.

Our framework highlights the idea that franchise expansion can occur only when the private decisions of citizens are important, simply because there is no value to commitment in environments with a single decision maker. Private decisions of the citizenry represent an implicit policy-relevant externality that the pivotal decision maker does not control. Because of the dynamic nature of the problem, current “policy-bribes” cannot induce the appropriate effort from the public since they do not guarantee favorable policies in the future.

The idea that franchise expansion is a commitment device has been explored in a seminal paper by Acemoglu and Robinson (2000). They posit a model in which a ruling elite can choose whether in any period to make a once-and-for-all, universal extension of voting rights to the rest of the population. The motive is to pre-empt a threat of uprising or to resolve a hold-up problem. We refer to this pre-emption motive as the “external conflict” explanation. The external conflict explanation contrasts with an “internal conflict” explanation, an example of which is a recent paper by Lizzeri and Persico (2003). According to the “internal conflict” explanation, rights are extended to gain support in an environment with ideological or class conflict among the elite.

Section 2 discusses similarities and differences between our approach and these and other models of franchise extension. Section 3 describes the basic framework. We show by means of several examples that the class of dynamic enfranchisement problems posited here is broad enough to accommodate both internal and external conflict explanations. The results therefore suggest a common causal mechanism that underlies both types of rationale. Section 4 contains our main results. There we characterize equilibria that admit a first order characterization, i.e., that satisfy and are fully characterized by interior solutions to Euler equations. Section 5 contains concluding remarks. Proofs of the main results are relegated to the Appendix.

2. Three observations and two types of models

Many of the franchise extensions observed throughout history have common characteristics. There are three qualitative characteristics of observed franchise extensions that the present framework should confront.

2.1. Most extensions are partial extensions

Historically, ruling elites have not had to choose exclusively between dictatorship and universal suffrage. More often, voting rights are offered to the “adjacent” group in the social
hierarchy. Often the restricted franchise was defined by wealth. Finer (1997, p. 336) writes of nascent democracy in the Greek city states:

“In the earliest forms of restricted participation, that is, in the oligarchies, a property qualification constituted the basis for full citizenship. Later, in some cities, all sources of wealth were put on equality with land, and citizens’ rights and duties were gradated according to one’s riches.”

In the 19th century, England partially expanded along lines of wealth or property ownership as well. However, in Italy, the franchise was granted to citizens who passed certain educational as well as financial criteria in 1849. Nineteenth century Prussia presents an interesting case: in 1849, voting rights were extended to most citizens, but these rights were accorded proportionately to the percentage of taxes paid. Finally, even today in most countries the franchise is usually restricted in some way.

2.2. Extensions are typically gradual processes, not one shot decisions

England’s history bears this out. A brief chronology of 19th and early 20th century franchise extensions in the U.K. indicates a gradual broadening of political rights.

1830: Voting franchise restricted to some 2% of population.
1832: Reform Act extends franchise to 3.5% of population.
1867: Second Reform Act extends to some 7.7% of population.
1884: Extension to 15% of population.
1918: Universal male (over 21) suffrage and female (over 30) suffrage.
1928: Universal suffrage (over 21).

Franchise extension in England had, in fact, a longer history whose beginnings predated these extensions. In a number of other European countries, gradual extensions corresponded to technological innovations such as those of the industrial revolution. In ancient Rome, extensions occurred as the state’s boundaries gradually expanded.

2.3. Extensions are often uneven

In many countries, large delays, lasting decades or longer have occurred between successive extensions. Again, England’s chronology is an example. In the Netherlands,
voting rights were extended in 1857 from 2% to 14% of the population. The next major expansion occurred in 1894 when rights were extended to all males. In Italy, universal male suffrage in 1912 was preceded by an extension in 1882 (14%) which, in turn, was preceded by the partial extension in 1849. In the ancient Roman Republic, various extensions not associated with territorial expansion occurred in 494 BC, 336 BC, and 287 BC.\(^7\)

Little is known about whether and what types of models can accommodate these three criteria. There is a sizable informal literature in which a number of rationales for the franchise—including the ones discussed here—have been proposed. For this we refer the reader to the useful surveys in Acemoglu and Robinson (AR) (2000) and Lizzeri and Persico (LP) (2003). We concentrate instead on the much sparser formal modeling that has been done, starting with Acemoglu and Robinson’s (2000, 2001) work, itself.\(^8\) The essential claim in Acemoglu and Robinson’s work is that the primary force behind, at least, the 19th century extensions was the desire by the elite to head off social unrest. AR postulate a dynamic game in which the timing of an all-or-nothing franchise extension is determined by the median voter of a ruling elite. A stochastically evolving state variable determines the likelihood of success of any popular revolt. In the absence of a franchise decision, the disenfranchised mob, acting as a unitary actor, revolts in certain states of the world, and refrains in others. A policy of redistribution to the disenfranchised is not a credible deterrent since it will only be used in threatening states of the world. By contrast, an extension of voting rights to the entire population puts the decision in the hands of the population median who chooses redistribution in all states. Extensions are then a credible way to buy-off the populace. Hence, franchise extensions pre-empt revolutions.

Similar motives for extending the franchise appear in models by Justman and Gradstein (1999) and Conley and Temimi (2001). They both examine games in which extension of voting rights occurs because of the potential for the disenfranchised group to impose costs on the elite through rioting, protest, or some other form of alienation if the franchise is not extended. These costs induce a trade-off similar to AR. Expansion entails a loss of decision-making power, but it also pre-empts the costly social unrest.\(^9\) In contrast to AR, the Conley and Temimi model is static and so it cannot address dynamic issues. On the other hand it can address the explicit free rider problems (unlike AR) in the decision to revolt. The Justman and Gradstein model operates in an overlapping generations environment and so it can address issues of gradual extension. However, they exogenously assume (rather than derive, as in AR) costs of disenfranchisement.

These “external conflict” models may be contrasted with an alternative “internal conflict” story in which political competition within the elite leads one or another faction to

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\(^7\) In 336 BC, one of the consulships became available for election by plebians. In 287 BC the Hortensian Law was introduced which gave resolutions in the plebian council the force of law. Again, see Finer (1997).

\(^8\) We limit our attention to models in which franchise decisions are explicitly endogenous. In particular, we acknowledge but do not discuss a large literature that examines the consequences of the expansion of rights. To name one example, Husted and Kenny (1997) examine the effect of extensions on the size of government expenditures.

\(^9\) Though the rationale for extension is slightly different, Fleck and Hanssen (2003) examine a simple hold-up problem between two actors with the same type of trade-off.
reach out to disenfranchised citizens. Lizzeri and Persico (2003) formulate a game with this type of internal conflict. They examine a static, random voter model of Downsian competition between two candidates who vie for votes among a restricted franchise. The competition creates an inefficiency when there are relatively few eligible voters. A franchise extension, determined by referendum, is shown to lead to a more efficient electoral process in terms of the allocation of expenditure between public goods and private transfers.

In contrast with the aforementioned models, Roberts (1998, 1999) and Barbera et al. (2001) examine long horizon dynamic game models with forward looking decision makers who can choose the decision maker(s) in the subsequent period. In this respect, our work is closest to theirs. Both papers examine dynamic games in which a country or an organization can “invite in” desirable outsiders to join the group from abroad. In Barbera et al., this decision is made unilaterally by any member of society. In Roberts, the decision is made by the median voter as a way to generate endogenous hysteresis in the size of the group. Another innovation of Roberts’ papers is that they derive well-defined majority voting outcomes each period in the dynamic game from single crossing properties on the primitive preferences. There are two main differences between these papers and ours. First, these are essentially models of immigration rather than of franchise extension (since outsiders are not members of society before they enter). Second, and more importantly, voters in these models have exogenous, rather than derived, preferences over the size or composition of the group.

In the subsequent section, we describe a class of dynamic games that can accommodate many of the key elements of these diverse models. Rather than focusing on one source of conflict (external threat) or another (internal political competition), or assuming exogenous preferences over the franchise, the present model derives such conflicts and preferences. At the same time, we require a rich enough class of environments that can produce dynamic paths of franchise extensions roughly consistent with the aforementioned facts.

3. Dynamic enfranchisement

We begin with a “road map” outlining our strategy for modeling franchise extension. First, Section 3.1 describes the economic fundamentals of the model. We show, with some examples, that the formulation is broad enough to encompass most of the internal and external conflict explanations of political rights examined in the formal literature. The remainder of this section embeds these economic fundamentals into a political environment. Specifically, in Section 3.2 we introduce a class of dynamic games in which policy decisions are made each period by a dictator, who may then delegate decision authority to another individual, who becomes the dictator in the subsequent period. We refer to these games as dictator delegation games (DDGs). Section 3.3 demonstrates that equilibrium outcomes of these games correspond to the decisions that

10 Because the stage game payoffs in Robert’s framework depend on group membership directly, these assumptions cannot be adapted to the present paper.
would be made in accordance with a simple majority rule in a restricted voting franchise. The delegation of a dictator corresponds to a decision by the currently enfranchised group to alter the franchise. Finally, Section 3.4 defines an equilibrium of a Dictator Delegation game.

3.1. The economic fundamentals

There are \( n \) citizens in a society, each labeled \( i = 1, \ldots, n \). Citizens are assumed to differ according to a taste, productivity, or income parameter. The population of all citizens is denoted \( N \). At each date \( t = 0, 1, 2, \ldots \) each citizen chooses some action \( e_i^t \) that describes a private decision taken by citizen \( i \) at date \( t \). We let \( E \) denote the set of feasible private decisions for each citizen, and denote the vector of private decisions by

\[
e_t = (e_1^t, \ldots, e_n^t).
\]

These decisions may capture any number of activities, including labor effort, savings, or investment activities. They may also include “non-economic” activities such as religious worship. To simplify language, we refer to the decision as simply the effort choice.

Also at time \( t \), a policy variable \( p_t \) is chosen by government at date \( t \) from some feasible set \( P \). Leaving aside, for the moment, the question of what is a “government” here, the variable \( p \) could correspond to any number of commonly observed policy instruments. It may, for instance, be a flat tax rate on income which generates revenue to produce a public good. It may also represent military expenditures, or clean water regulations, etc.

Effort and policy choices interact to influence a physical state variable denoted by \( x_t \). In most of the analysis the state is one-dimensional, i.e., \( \Omega \subset \mathbb{R} \). This state may represent a level of capital stock or a stock of natural resource. Alternatively, it could represent aggregate wealth or another moment of the distribution of income, or the strength of an overthrow threat. This physical state \( \omega_t \) is assumed to evolve according to a transition function \( Q \) where

\[
\omega_{t+1} = Q(\omega_t, e_t, p_t)
\]

and \( \omega_0 \) is given exogenously in order to begin the process.

The payoff to each individual, \( i \in N \), is a time separable function,

\[
\sum_{t=0}^{\infty} \delta^t u_i(\omega_t, e_t, p_t)
\]

where \( \delta \) is a common discount factor, and the stage payoff is \( u_i \). Note that since a citizen’s private decision can affect others, his decision may be subject to a “free rider” problem in the sense that under (or over) provision of \( e_i \), relative to some socially optimum benchmark, is likely.

This class of economic environments is broad enough to cover a large number of interesting political/policy examples including environments in which internal or external conflicts exist.
Example 1 (Internal conflicts over public goods). Each citizen holds wealth in the form of land. The land endowment, $y_i$, of citizen $i$ is exogenous, and it does not vary over time. Aggregate income is $Y = \sum_i y_i$. The policy $p_t$ in period $t$ is a flat tax on land, yielding revenue $p_t Y$ which is used to invest in an asset which, in turn, yields a public consumption good. Aggregate individual effort, $\sum_i e_{it}$, instead of augmenting personal incomes, increases the value of the public good next period. That is, at each date $t$,

$$\omega_{t+1} = f\left(p_t Y, \sum_i e_{it}, \omega_t\right)$$

where $\omega_{t+1}$ is the public good produced next period. Finally, citizen $i$ cares about after-tax wealth, about leisure, and about the public good. His payoffs in period $t$ is

$$u_i(\omega_t, e_t, p_t) = u(y_i(1 - p_t), e_{it}, \omega_t).$$

Here, citizens in the population could differ in at least two ways. First, they could differ according to a taste parameter $\alpha_i \in [0, 1]$. Citizens with higher values of $\alpha$ may place a higher value on the public good. Examples of this type of conflict include attitudes toward state-supported religion, or the support of certain social policies, such as opposition to scientific theories of evolution, the promotion of liberal attitudes towards race and sexual preference issues, and the enactment and enforcement of anti-abortion laws. One would expect in this case that preferred tax rates will differ across the population. We refer to cases of taste heterogeneity such as this as cases of ideological conflict.

Second, citizens may differ in the amount of land wealth, $y_i$, they have. We refer to cases of income or wealth heterogeneity as cases of class conflict. Class conflict of this type is common in public economics, and can be shown to induce differences in voting behavior regarding redistribution, public goods, and tax policies generally.

Example 2 (External threat of insurrection). According to the “external conflict” explanation, franchise expansion occurs to head off the threat of revolution, uprising, or insurrection among the dispossessed. Implicitly, such threats arise from the non-satisfaction of the preferences of the disenfranchised by the policies chosen by the elite, and franchise extension may be an effective means of reducing the incentives of agents to engage in uprising.

In this example, a class conflict coupled with the threat of insurrection is the driving force behind a franchise extension. Our example is constructed deliberately to be close in certain respects to Acemoglu and Robinson’s model of “threat of revolt” as an explanation for the 19th century extensions.

To simplify things there are two distinct groups, referred to concretely as the nobility (Group A) and the peasantry (Group B), respectively. There are $J$ peasants, and $n - J$
noblemen. The franchise belongs to a subset of the nobility. A nobleman with index $i$ has a quantity of land $y_i$ where, as before, $y_i$ is constant across time. Each period, a unit of land generates a unit of a consumption good. By contrast, peasants are completely disenfranchised and landless.

Each period $t$, there is a possibility that the peasants may successfully revolt and confiscate the nobility’s aggregate return, $Y = \sum_{i \in A} Y_i$. If the revolt is successful, the roles of nobility and peasants are subsequently reversed. Each peasant $j = 1, \ldots, J$, contributes $e_{jt}$ toward this effort, while each nobleman $i = J+1, \ldots, n$ contributes effort $e_{it}$ toward suppressing the revolt. As before, effort is costly to all citizens.

Let $E_{At} = \sum_{i \in A} e_{it}$ and $E_{Bt} = \sum_{j \in B} e_{jt}$ denote the aggregate effort by nobility and peasantry, respectively, in period $t$. The state variable, $\omega_t$, is the probability in period $t$ that the confiscation by the peasants is unsuccessful. Formally,

$$\omega_{t+1} = f(E_{At}, E_{Bt}, \omega_t)$$

so that the likelihood of failure depends on the aggregate effort of each group, presumably increasing in $E_{At}$ (less likely confiscation) and decreasing in $E_{Bt}$ (more likely confiscation). If a confiscation is successful, then the entire return $Y$ is expropriated by the peasantry who split it evenly. On the other hand, if the revolt is unsuccessful, then peasants receive a redistributive subsidy chosen by the pivotal decision maker in the restricted franchise before the revolt’s success is known. Roughly, the idea is that redistribution is used to “buy off” the peasants by inducing them to reduce their effort toward the uprising.

Each period $t$, the pivotal nobleman chooses a redistributive tax rate $p_t$ which produces revenue $p_tY$. However, the technology for redistribution is concave—implies that some of the revenue is lost in the redistributive process. Formally, revenue $p_tY$ produces $g(p_tY) < p_tY$ available to be equally distributed to all members of society if there is no confiscation.

All citizens have von Neumann–Morganstern utility $u$ defined on consumption and effort. Members of the nobility have expected utility in period $t$ of

$$u_a = \omega_t u((1 - p_t)y_i + g(p_tY)/n, e_{it}) + (1 - \omega_t)u(0, e_{it})$$

while members of the peasantry have utility

$$u_{jt} = \omega_t u(g(p_tY)/n, e_{jt}) + (1 - \omega_t)u(Y/J, e_{jt}).$$

To summarize, individuals in the nobility differ by income, and the policy instrument is a redistributive tax. Individuals can either be supportive of the current policy or they can undermine it. Their current efforts determine the likelihood that the currently enfranchised group remains in power.

### 3.2. Dictator delegation games

For now, assume that policies, whatever they happen to be, are chosen at date $t$ by a single individual, whom we refer to as a dictator. Let, $m_t \in N$ be the identity of the dictator at time $t$, with $m_0$ an exogenously given initial dictator. As well as choosing the policy
variable, in any period the current dictator can also choose the identity of the subsequent period’s dictator. That is, $m_t$ chooses $m_{t+1}$ (not necessarily different from $m_t$). Following the standard practice in modeling dynamic games, the choices of $e_t, p_t,$ and $m_{t+1},$ are made simultaneously and non-cooperatively at time $t$. A Dictator Delegation Game (DDG), $G$, is therefore summarized by the collection

$$G = ((u_i)_{i=1}^n, Q, \Omega, E, P, \omega_0, m_0; N).$$

Clearly, dictator delegation games are interesting in their own right. Delegation of authority from one monarch or dictator to another is more the rule than the exception in the history of governance.\(^\text{12}\)

Dictator delegation (and thus franchise extension) is a substitute for credible commitment to future policy. In the set-up we have described, commitment to a sequence of future policies is potentially valuable for two reasons. First, since the model is deterministic, commitment to policies in future periods has the same effect as if the dictator was somehow able to act as a Stackelberg leader in each future period. Thus commitment to future policies can enable the dictator to affect agents’ future effort choices. Second, by committing to future policies, the dictator can credibly influence the “value” of economic states in those periods (e.g., by promising a low tax rate in the future, it might increase the value of accrued savings), which can thereby affect agents’ current effort decisions, since these efforts in part determine future economic states. When commitment to future policies is not credible, dictator delegation may serve as a useful substitute. Acemoglu and Robinson’s (2000) paper is a specific example of this kind of mechanism.

Of course, we do not expect that, by delegating authority to an alternative dictator, the current dictator will be able to perfectly replicate the outcome he could attain with commitment. This is for two reasons: first, because he cannot instruct his choice of dictator how to choose future policies; and second, because he cannot stop her from delegating authority further. Nonetheless, the option of delegating authority may be valuable to the current dictator, and we show that such delegation can arise in equilibrium. The second factor limiting the dictator’s ability to replicate the first best outcome— inability to stop future dictators subsequently delegating authority to others—leads to gradual evolution of the franchise in equilibrium. This richer, recursive, feature differentiates our model from the once-and-for-all extensions assumed in Justman and Gradstein (1999) and Acemoglu and Robinson (2000).

We study DDGs in which $Q \subset \mathbb{R}$ and both $E$ and $P$ are compact, convex intervals in $\mathbb{R}^+$, and in which for each $i$, $u_i$ and $Q$ are twice continuously differentiable and strictly, jointly concave in all variables. This specification makes two key simplifying assumptions. First, we assume that all decisions are one-dimensional. This is clearly made for reasons of tractability. Second, we assume that the dynamic game is deterministic. This could easily be modified to allow for shocks and other stochastic

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\(^{12}\) Finer (1997) notes, further, that although the rule appeared to be dynastic, it often was not. In the Roman Empire, for example, the current emperor simply “adopted” the monarch to whom he planned to make the future emperor.
The deterministic assumption is made, not so much for tractability, but for ease of illustration. The basic ideas are expressed most directly in the deterministic case.

3.3. Dictator delegation as enfranchisement

There is a clear relationship between DDGs and the enfranchisement problem. In the latter, political rights are conferred upon a subset of the population $M_t \subseteq N$, whose preferences are aggregated through some political process (e.g., a voting mechanism). We do not model how these rights are enforced or preserved, although an enforcement technology could be included in the state variable. By their vote, enfranchised citizens have the right to choose current policies.

If $M_t$ is a strict subset of $N$, then the franchise is restricted. However, the currently enfranchised can also choose to extend these rights to others in the future. This may be done for a number of reasons, some of which were outlined in the Introduction. Each period, therefore, a currently enfranchised group can choose, along with the policy $p_t$, a group next period that will have the same rights in period $t + 1$. Specifically, citizens in $M_t$ can choose to enfranchise a group $M_{t+1}$ next period.

In DDGs the preferences of a single individual, whom we call the dictator, determine policy outcomes. However, this attribute is true of any aggregation mechanism characterized by a pivotal decision maker. In particular, a pivotal decision maker may be, under certain conditions, the median voter arising from a political process in which political rights are voting rights. If $m_t$ is the dictator in a DDG, then ordering the enfranchised citizenry such that $M_t = \{1, 2, \ldots, 2m_t - 1\}$, $m_t$ is also the median index in the enfranchised group, $M_t$.

If the aggregation procedure were to reflect the preferences of a median voter in a well-defined sense, then it would be natural to think of the current period’s median voter as choosing both current policy, and, by her choice of $M_{t+1} = \{1, 2, \ldots, 2m_{t+1} - 1\}$, the identity of the next period’s median voter, $m_{t+1}$. Indeed, since each period’s median voter (if well-defined) is pivotal, we might reasonably expect her choices of current policy and future median to coincide with the policy and delegation choices she would have made if she had been the dictator in the corresponding DDG.

The problem that we face, of course, is to ensure that such a median voter exists in each period and is well-defined. In what follows we establish conditions under which this is the case. In the following analysis, we refer to the pivotal decision maker, which covers both the dictator in a DDG, and a median voter when one exists.

3.4. Equilibria of dictator delegation games

Fix a dictator delegation game $G$. We assume that all citizens condition their behavior only on a payoff relevant state. The current state is a pair $(\omega_t, m_t)$, where $\omega_t$ is interpreted

\[\text{features.} \quad \text{13} \quad \text{The deterministic assumption is made, not so much for tractability, but for ease of illustration. The basic ideas are expressed most directly in the deterministic case.} \]

\[\text{3.3. Dictator delegation as enfranchisement} \]

\[\text{There is a clear relationship between DDGs and the enfranchisement problem. In the latter, political rights are conferred upon a subset of the population } M_t \subseteq N, \text{ whose preferences are aggregated through some political process (e.g., a voting mechanism). We do not model how these rights are enforced or preserved, although an enforcement technology could be included in the state variable. By their vote, enfranchised citizens have the right to choose current policies.} \]

\[\text{If } M_t \text{ is a strict subset of } N, \text{ then the franchise is restricted. However, the currently enfranchised can also choose to extend these rights to others in the future. This may be done for a number of reasons, some of which were outlined in the Introduction. Each period, therefore, a currently enfranchised group can choose, along with the policy } p_t, \text{ a group next period that will have the same rights in period } t + 1. \text{ Specifically, citizens in } M_t \text{ can choose to enfranchise a group } M_{t+1} \text{ next period.} \]

\[\text{In DDGs the preferences of a single individual, whom we call the dictator, determine policy outcomes. However, this attribute is true of any aggregation mechanism characterized by a pivotal decision maker. In particular, a pivotal decision maker may be, under certain conditions, the median voter arising from a political process in which political rights are voting rights. If } m_t \text{ is the dictator in a DDG, then ordering the enfranchised citizenry such that } M_t = \{1, 2, \ldots, 2m_t - 1\}, \text{ } m_t \text{ is also the median index in the enfranchised group, } M_t. \]

\[\text{If the aggregation procedure were to reflect the preferences of a median voter in a well-defined sense, then it would be natural to think of the current period’s median voter as choosing both current policy, and, by her choice of } M_{t+1} = \{1, 2, \ldots, 2m_{t+1} - 1\}, \text{ the identity of the next period’s median voter, } m_{t+1}. \text{ Indeed, since each period’s median voter (if well-defined) is pivotal, we might reasonably expect her choices of current policy and future median to coincide with the policy and delegation choices she would have made if she had been the dictator in the corresponding DDG.} \]

\[\text{The problem that we face, of course, is to ensure that such a median voter exists in each period and is well-defined. In what follows we establish conditions under which this is the case. In the following analysis, we refer to the pivotal decision maker, which covers both the dictator in a DDG, and a median voter when one exists.} \]

\[\text{3.4. Equilibria of dictator delegation games} \]

\[\text{Fix a dictator delegation game } G. \text{ We assume that all citizens condition their behavior only on a payoff relevant state. The current state is a pair } (\omega_t, m_t), \text{ where } \omega_t \text{ is interpreted} \]

\[\text{13 See, for example, Lagunoff (2005a,b).} \]

\[\text{14 Here we ignore integer problems by framing the problem as if } |M_t| \text{ were always odd. Note also that we restrict the analysis to (weak) franchise extensions although, in principle, franchise contraction could be permitted.} \]
as the “economic” state while \( m_t \) represents the “political” state. Strategies that condition only on the state are commonly referred to as Markov strategies. A citizen’s Markov strategy in effort is \( \sigma_i \) where \( \sigma_i(\omega, m_t) = e_i \) is the effort taken by \( i \) in state \((\omega, m_t)\). A profile of effort strategies of the citizenry is denoted \( \sigma = (\sigma_1, \ldots, \sigma_n) \). The current dictator’s Markov policy strategy is \( \psi \) where \( \psi(\omega, m_t) = p_t \) is the policy chosen by the current dictator. Finally, \( \mu \) is the dictator’s delegation strategy so that \( \mu(\omega, m_t) = m_{t+1} \) is next period’s dictator chosen by the current dictator \( m_t \) when the physical state is \( \omega_t \). All these objects are summarized by the profile,

\[
\Pi = (\sigma, \psi, \mu).
\]

A Markov profile \( \Pi \) ultimately determines the evolution of all behavior, public and private, in a society governed by an endogenously determined succession of dictators. The transition rule \( Q \) determines the economic state each period, and each Citizen \( i \) evaluates economic states and behavior according to his discounted sum of stage game payoffs. These payoffs are expressed recursively: Citizen \( i \)’s payoff in state \((\omega_t, m_t)\) given a Markov profile \( \Pi \) is

\[
V_i(\omega_t, m_t; \Pi) = u_i(\omega_t, \sigma(\omega_t, m_t), \psi(\omega_t, m_t)) + \delta V_i(\omega_{t+1}, m_{t+1}; \Pi)
\]

where

\[
\omega_{t+1} = Q(\omega_t, \sigma(\omega_t, m_t), \psi(\omega_t, m_t)).
\]

and

\[
m_{t+1} = \mu(\omega_t, m_t).
\]

**Definition 1.** An equilibrium of a DDG is a Markov profile, \( \Pi = (\sigma, \psi, \mu) \), consisting of state-contingent efforts, policies, and delegation choices such that at each date \( t = 0, 1, 2, \ldots \), and in any state \((\omega_t, m_t)\),

(i) Effort decisions are individually optimal: for each \( i \), and each \( \tilde{\sigma}_i \),

\[
V_i(\omega_t, m_t; \Pi) \geq V_i(\omega_t, m_t; \tilde{\sigma}_i, \sigma_{-i}, \psi, \mu),
\]

(ii) Policy, delegation, and effort decisions are optimal for the current dictator: for any \( \hat{e}_m, \hat{\mu} \), and \( \hat{m}_{t+1} \),

\[
V_m(\omega_t, m_t; \Pi) \geq u_m(\omega_t, \sigma_{-m}(\omega_t, m_t), \hat{e}_m, \hat{\mu}) + \delta V_m(\omega_{t+1}, m_{t+1}; \Pi)
\]

where \( \omega_{t+1} = Q(\omega_t, \sigma_{-m}(\omega_t, m_t), \hat{e}_m, \hat{\mu}). \)

This definition corresponds to a standard Markov Perfect equilibrium of the dictator delegation game. Each citizen chooses his own effort optimally given the state and his (correct) forecast of others’ effort rules and the policy/delegation rules. The dictator chooses policy, effort, and the identity of next period’s dictator optimally given the state and his (correct) forecast of the effort rules of the rest of the citizenry.
While we do not address the issue of equilibrium existence here, the question is taken up in a more general class of dynamic political games in a recent paper by Lagunoff (2005a). For parametric examples in which equilibria of the type described above are constructed explicitly, see Jack and Lagunoff (2005).

Now suppose \( \Pi = (\sigma, \psi, \mu) \) is an equilibrium of a DDG. Let \( \{m^*_t\} \) be the identities of the dictators and \( \{\omega^*_t\} \) the economic states reached along the equilibrium path induced by \( \Pi \). Our goal is to associate \( \{m^*_t\} \) and \( \{\omega^*_t\} \) with decisions regarding extensions of the voting franchise.

**Definition 2.** Given an ordering of the citizens (without loss of generality, \( i = 1, \ldots, n \)), a strategy profile is said to be rationalized by franchise extension at date \( t \) with respect to that ordering if \( (p_t, m_{t+1}^*) = (\psi(\omega^*_t, m^*_t), \mu(\omega^*_t, m^*_t)) \) is a Condorcet winner within the set \( M^*_t = \{1, 2, \ldots, 2m^*_t - 1\} \) with \( 2m^*_t - 1 \leq n \).

In other words, there does not exist another policy \( \hat{p}_t \) and choice of future policy maker \( \hat{m}_{t+1}^* \) that defeats \( (p_t, m_{t+1}^*) \) in a strict majority vote in \( M^*_t \).

Notice that the definition requires only that states along the equilibrium path produce pivotal decision makers that arise from a majority vote. The interpretation is as follows. Each period, an enfranchised group votes to alter the voting institution used in the future. One option among many is to expand the franchise to \( M_{t+1}^* \) in such a way as to produce a new median voter \( m_{t+1}^* \) next period. The strategy profile \( \Pi \) is rationalized precisely when this occurs. Our task then is to establish conditions under which equilibria of DDGs are rationalized by franchise extension—that is, conditions under which a median voter exists.

**Definition 3.** (Order Restrictedness) Fix a strategy profile \( \Pi \) and current state \((\omega, m)\). A citizen’s payoff function of policy and future dictator is given by

\[
W_i(p_t, m_{t+1}) = u_i(\omega_t, \sigma(\omega_t, m_t), p_t) + \delta V_i(\omega_{t+1}, m_{t+1}; \Pi)
\]

(suppressing the transition rule \( Q \) and the notational dependence of \( W_i \) on \( \Pi \) and \((\omega_t, m_t)\)). A profile \( W = (W_1, \ldots, W_n) \) of payoff functions satisfies Order Restrictedness if there exists an ordering of \( N \) (without loss of generality, \( i = 1, \ldots, n \)), such that for any two pairs \((p_t, m_{t+1})\) and \((\hat{p}_t, \hat{m}_{t+1})\), for each \( i \) for which

\[
W_i(p_t, m_{t+1}) - W_i(\hat{p}_t, \hat{m}_{t+1}) > 0
\]

it is the case that either

\[
W_j(p_t, m_{t+1}) - W_j(\hat{p}_t, \hat{m}_{t+1}) > 0 \quad \forall \ j > i
\]

or

\[
W_j(p_t, m_{t+1}) - W_j(\hat{p}_t, \hat{m}_{t+1}) > 0 \quad \forall \ j < i.
\]

This definition is due to Rothstein (1990), and is related to the single crossing property of Gans and Smart (1996). Order Restrictedness allows us to prove a median voter theorem when the objects over which voters vote are fundamentally multi-
dimensional, as is the case here (because voters choose the policy $p_t$ and the franchise extension $m_{t+1}$).\(^{15}\)

**Lemma 1. (Median Voter Theorem)** Let $M$ be a set of voters. Suppose $(W_i)_{i \in M}$ satisfies Order Restrictedness, and let $j$ be the identity of the individual with the median index in $M$. Let $(p_{j, m'})$ denote individual $j$’s most preferred voting outcome. Then $(p_{j, m'})$ is a Condorcet winner.

This lemma is an immediate consequence of a result by Rothstein (1990), in which the Order Restrictedness property on voter preferences implies existence of a Condorcet Winner that coincides with the individual with the median index, $m$.\(^{16}\) The lemma allows us to infer that if Order Restrictedness holds at each state along an equilibrium path, then the equilibrium is rationalized by franchise extension.

The following result shows that when individuals have stage game payoffs that admit an Intermediate Preference representation in the sense of Grandmont (1978), then equilibria of DDGs are rationalized by franchise extension.

**Proposition 1.** Suppose that in a DDG, stage game preferences can be expressed as

$$u_i(\omega, e, p) = h(\omega, e, p)f(i) + g(\omega, e, p)$$

where $f$ is monotone. Then any equilibrium $\Pi = (\sigma, \psi, \mu)$ is rationalized by franchise extension at date $t$ if $2m_{t-1} \leq n$.

The class of Intermediate Preferences is restrictive. On the other hand, this class admits a wide variety of standard parametric environments studied in the literature. Obviously, there may be other representations that admit franchise extension, but they may be harder to verify.

### 3.5. Finite number of agents versus the continuum

We find it necessary to make one further assumption which deserves comment. It will prove more tractable to treat the voter type as chosen from a continuum rather than from a discrete set $M$. Specifically, let $N \subset [0, 1]$. At the same time, we maintain the assumption that only finitely many citizens make effort choices.

Clearly, the continuum is a reasonable approximation of a finite set of voters in which the set is uniformly and densely distributed in the continuum. In that case, the resulting franchise choices constitute an approximation of the actual equilibrium, and outcomes of the DDG are approximately rationalized by majority voting.

Naturally, it is sensible to ask: why not posit a continuum of voters from the beginning? The reason is that the continuum presents a fundamental problem for the theory. In much of the history of voter enfranchisement, the effort choices of citizens correspond to voluntary decisions in a collective action problem such as volunteering to take part in a protest or public insurrection. With the continuum, free rider problems in these decisions

\(^{15}\) In Gans and Smart, although the choice space can be multi-dimensional, it is necessary that there be a strict ordering on that space. We place no such restriction on the space of policy and franchise decisions.

\(^{16}\) See also Roberts (1977) and Gans and Smart (1996) for related theorems.
are extreme. An individual in a continuum would never choose to riot or threaten the status quo, or alternatively, to defend the status quo. The finite agent assumption is therefore critical to prevent the unreasonable boundary solution $e_i = 0$ in effort choices of citizens. Indeed, we later show that for franchise extension to exist, these boundary solutions must not occur.

For this reason, franchise choices are characterized in the next sections as if the current median could choose the subsequent median from a continuum of types, but in the citizens’ private decisions, the finite agent assumption is maintained.

4. First order characterization of equilibria

In this section we characterize equilibria in which extension of the voting franchise can occur. Following standard conventions, we drop the time notation, $t$, and use primes, e.g., $\omega'$, to denote variables in the subsequent period $t+1$, and double primes, e.g., $\omega''$, to denote those two periods ahead $t+2$.

Let $\Pi = (\sigma, \psi, \mu)$ be an equilibrium of the dictator delegation game. Consider, first, a citizen’s effort decision. One can write the recursive payoff evaluated at an equilibrium as the functional equation:

$$V_i(\omega, m; \Pi) = \max_{e_i} \left[ u_i(\omega, e_i, \sigma_{-i}(\omega, m), \psi(\omega, m)) + \delta V_i(\omega', \mu(\omega, m); \Pi) \right]$$

subject to $\omega' = Q(\omega, e_i, \sigma_{-i}(\omega, m), \psi(\omega, m))$. If this value function is differentiable, then the (interior) Euler equation is

$$\frac{\partial u_i}{\partial e_i} + \delta \frac{\partial V_i}{\partial \omega'} \frac{\partial Q}{\partial e_i} = 0. \quad (9)$$

As for the pivotal decision maker’s problem, recall that he makes two choices (in addition to his choice of effort). He chooses a policy in the current period given the state $\omega$, and chooses next period’s pivotal decision maker by making a franchise decision in the current period. That is, a pivotal decision maker with index $m$ chooses next period’s pivotal decision maker, $m'$. The functional equation resulting from the choice of policy, franchise, and effort is

$$V_m(\omega, m; \Pi) = \max_{e_m, p, m'} \left[ u_m(\omega, e_m, \sigma_{-m}(\omega, m), p) + \delta V_m(\omega', m'; \Pi) \right]$$

subject to $\omega' = Q(\omega, e_m, \sigma_{-m}(\omega, m), p)$. Derived from this value function, the interior Euler equation for the policy decision, $p$ is

$$\frac{\partial u_m}{\partial p} + \delta \frac{\partial V_m}{\partial \omega'} \frac{\partial Q}{\partial p} = 0 \quad (10)$$

and, the interior Euler equation for franchise decision, $m'$, made by pivotal voter $m$ is

$$\delta \frac{\partial V_m}{\partial m'} = 0 \quad (11)$$
Definition 4. We will say that an equilibrium, \( \Pi = (\sigma, \psi, \mu) \), admits a first order characterization if for each Citizen \( i \) and each voter \( m \), in every state \( (\omega, m) \), (i) the profile \( \Pi = (\sigma, \psi, \mu) \) satisfies the Eqs. (9), (10), and (11); (ii) the expression in (9) is strictly decreasing in \( e_i \); and (iii) the matrix of second derivatives of the system formed by (the left-hand sides of) Eqs. (9), (10), and (11) is negative semi-definite.

Any equilibrium that admits a first order characterization is fully characterized by its Euler equations. Among them, Eq. (11), is the most relevant for understanding franchise expansion. Expressed in terms of a useful decomposition of marginal effects, Eq. (11) is given by,

\[
\left[ \frac{\partial u_m}{\partial e'_i} + \delta \frac{\partial V_m}{\partial p'} \frac{\partial Q}{\partial e'_i} \right] \frac{\partial \sigma_j}{\partial m'} + \frac{\delta}{\partial m''} \frac{\partial V_m}{\partial m''} \frac{\partial u}{\partial p} = 0.
\]

Clearly, a decisive voter \( m \) chooses to expand the current franchise only if Eq. (12) is satisfied at values \( \mu(\omega, m) > m \). The decomposition illustrates the various marginal effects that a change in the future pivotal voter has on the payoff of the current pivotal voter. This means that the current pivotal voter, \( m \), rationally anticipates his choice of \( m' \) on future effort choices of the citizenry, and future policies and franchise decisions of subsequent median voters (including himself, should he choose to retain political power). Among other things, the current median realizes that his choice of franchise expansion may not be the end of the process. Since next period’s pivotal voter, \( m' \), also satisfies his Euler equations, (10) and (11), if the current pivotal decision maker, \( m \), extends the franchise to \( m' > m \), then Order Restrictedness implies

\[
\left[ \frac{\partial u_m}{\partial e'_i} + \delta \frac{\partial V_m}{\partial p'} \frac{\partial Q}{\partial e'_i} \right] \frac{\partial \sigma_j}{\partial m'} + \frac{\delta}{\partial m''} \frac{\partial V_m}{\partial m''} \frac{\partial \mu}{\partial p} \leq 0.
\]

A franchise extension, therefore, implies that the marginal payoff from other citizens’ effort responses to the extension be non-negative, i.e.,

\[
\sum_{j \neq m} \left[ \frac{\partial u_m}{\partial e'_j} + \delta \frac{\partial V_m}{\partial p'} \frac{\partial Q}{\partial e'_j} \right] \frac{\partial \sigma_j}{\partial m'} \geq 0.
\]
Hence, an optimal enfranchisement decision for voter \( m \) balances the positive marginal effect from future effort choices (Eq. (14)) from the citizenry with the negative marginal effect of putting future policy and franchise decisions in the hands of other agents (Eq. (13)). This trade-off is illustrated by the two solid lines in Fig. 1. If the current median voter is \( m \), retaining the franchise results in no loss of control—that is, a zero marginal cost. On the other hand, extending the franchise to a median \( \tilde{m} \) generates maximal benefits associated with effort inducement, but imposes large costs in terms of future policy and franchise decisions. The index \( m' \) balances these two effects. In fact, the logic can be extended to obtain the following necessary and sufficient condition for franchise extension.

**Proposition 2.** In any equilibrium that admits a first order characterization, the franchise is extended in state \((\omega, m)\), i.e., \( \mu(\omega, m) = m' > m \), if and only if

\[
\sum_{j \neq m} \left[ \frac{\partial u_m}{\partial e_j} - \left( \frac{\partial u_m}{\partial e_m} \right) \frac{\partial Q}{\partial e_j} \right] \frac{\partial \sigma_j}{\partial m'} > 0
\]

holds at \( m = m' \).

Though the result is a straightforward application of the Envelope Theorem, we include the complete proof in the Appendix. Roughly, the idea is that franchise extension requires the spillover of effort choices of ordinary citizens, without which a current policy-maker would preserve his own power to make future policy decisions into perpetuity. This is true regardless of whether the effort choices are directed toward investment in public goods or in political upheaval.

This last point is worth emphasizing. Specifically, the same causal mechanism underlies both the so-called “internal conflict” and “external conflict” explanations for franchise extension. In the internal conflict story, disagreements within the elite over public goods create a motive for some to extend voting rights to “sympathetic outsiders.” The effort choice is, for instance, a private input needed to produce the controversial good. In the external conflict story, the threat of uprising or insurrection creates a “buy off”

![Fig. 1. The enfranchisement decision.](image-url)
motive for expansion of rights. The effort choice, in that case, is one’s contribution either
to the cause of overturning the current regime or defending it. In either case, the franchise
is extended if and only if the aggregate effect of these spillovers is positive. The larger the
spillover effect, the larger is the extension.

In the presence of spillovers, a franchise extension can accomplish what a policy
change cannot. Namely, the franchise extension is a credible commitment to future policy
changes. The pivotal voter cannot credibly use current policy instruments to change future
behavior except through (blunt) changes in the physical state. Since current policy changes
do not imply future policy changes, citizens with preferences that differ widely from those
of the pivotal voter expect the same median voter to continue to adopt poor policy choices
in the future from their point of view.

By contrast, an extension delegates authority to a different pivotal voter tomorrow. This
guarantees that future policies in subsequent periods are closer to those that the current
median voter would like to be able to commit to. Since this elicits a positive spillover in
their effort choices, the pivotal voter today is willing to sacrifice his power. In this sense,
the role of franchise extension is a familiar one in time-consistent models of policy. A
credible extension is desirable since it delegates policy-making authority to a median
whose decisions bring forth a more favorable response from the citizenry.

Notice, however, that while the enfranchisement option may improve things, it is not
generally a perfect substitute for the optimal, time inconsistent policy sequence. Given the
recursive environment, the initial voter cannot limit future franchise extensions. A future
median may delegate beyond the point at which the first median would choose if the first
median could make a once-and-for-all franchise decision. In turn, this possibility distorts the
current decision. To see this, consider an optimal once-and-for-all extension. A once-and-
for-all extension trades off the marginal benefits of extra effort against the marginal costs of
future policy changes (the dashed curve) as illustrated in Fig. 1. Since these costs do not
include the costs of future extensions, the new median is \( \tilde{n}' > \tilde{n} \). Since \( \tilde{n}' > \tilde{n} \) in Fig. 1, the
current median limits the extension of the franchise below that of a once-and-for-all decision.

An immediate corollary of the Proposition is: *absent spillovers in private decisions, the
level of voter enfranchisement remains fixed.* This statement has predictive content. Consider an example of a policy that subsidizes a particular “state religion.” Current
subsidies determine, say, the subsequent available stock of churches. Suppose an
individual’s church attendance does not affect others’ payoffs, and it does not affect the
technology for building churches. In this case, the current median voter will not delegate
authority to another. Though conflicts over state-funded religion may, in fact, create
serious social conflict, it would not then lead to broader political rights.

Proposition 2 provides a relatively simple way to check if an expansion of the franchise
occurs in equilibrium. To make full use of it, however, requires practical use of all the
Euler equations, since the Inequality (Eq. (15)) depends on knowing both values of the
equilibrium strategies, and their curvature. The working paper version of this paper (see
Jack and Lagunoff, 2005) reformulates the Euler Eqs. (9)–(11) in order to provide a
computationally tractable characterization in terms of “Generalized Euler equations.”

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17 See Klein et al. (2002) for developments on this. Related ideas are found in Basar and Olsder (1995,
Theorem 6.5).
turn, these Generalized Euler equations are used to compute equilibria in a number of concrete examples. We refer to reader to that version for details.

5. Summary

This paper introduces a class of games in which the voter franchise is an explicit voting decision of the currently enfranchised group. This decision is formulated as a fully recursive delegation decision, and preferences for enfranchisement options are derived rather than assumed. We know of no other model with these features.

We characterize equilibria of a related game in which a current dictator may designate a new dictator who is granted policy-making authority. We show that under certain conditions, the outcome path produced by this game may be rationalized by a well-defined majority voting rule operating on a limited voting franchise. This enfranchised group votes for a possibly larger voting franchise in the next period. The outcome of a vote in any period is shown to coincide with the preferred choice of a median voter from that group.

The current median voter is motivated by a desire to change the policy-responsive private decisions of ordinary citizens. The franchise extension is therefore used as a commitment device to change private behavior through irreversible expansions of the policy-making elite, which induce credible changes in future policy choices. This underlying causal mechanism is at the heart of both “internal” and “external” explanations of observed franchise extensions.

The assumption that political aggregation occurs via a simple majority vote clearly omits some important subtleties of actual political processes. It also requires restrictive conditions in the present multi-dimensional policy space. Nevertheless, its use in this context is as good, in our view, as any alternative. Consider, for example, the influential citizen-candidate model (see Besley and Coate, 1997 or Osborne and Slivinsky, 1996). While that model can be applied to multi-dimensional policy spaces, it typically requires burdensome mixed strategies in precisely those cases where majority voting is problematic. In either model, the fundamental mechanism for institutional change is the same.

Our framework is shown to cover a variety of policy environments. However, some caveats apply. The present environment is deterministic and assumes simple, single dimensional policies and private decisions. Naturally, the framework can be extended to include environments with higher dimensional policies and private decisions.

The framework can also be extended to stochastic games. The extension to stochastic environments is important because, it turns out, most existence results either do not apply to deterministic environments, or apply only when all sets of states, policies, and actions are finite. These issues and others are taken up in Lagunoff (2005a,b).

Future research might be directed toward computational methods for generating equilibria with franchise extension. It is hoped that a broader comprehension of the dynamic game model of enfranchisement leads to a deeper understanding of the mechanisms that sustain and extend democracy.

18 In which case existence of equilibrium is in mixed strategies.
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Appendix A. Proofs of the propositions

Proof of Proposition 1. If, for a given state \((x, m)\) and strategy profile \(P\), for each \(i\),

\[
W_i(p, m'; \omega, m, \Pi) = H(p, m'; \omega, m, \Pi)F(i) + G(p, m'; \omega, m, \Pi)
\]

for some monotone function \(F\), then it can be readily verified that \(W\) satisfies the Order Restrictedness, and Lemma 1 applies. Suppose then that the stage game utility function \(u_i\) satisfies Eq. (7). We now show that given a strategy profile \(\Pi = (\sigma, \psi, \mu)\), \(i\)’s dynamic preferences exhibits this decomposition. At time \(t\),

\[
W_i(p_t, m_{t+1}; \omega_t, m_t, \Pi) = u_i(\omega_t, \sigma(\omega_t, m_t), p_t) + \delta V_i(\omega_{t+1}, m_{t+1}; \Pi)
\]

\[
= h(\omega_t, \sigma(\omega_t, m_t), p_t)f(i) + g(\omega_t, \sigma(\omega_t, m_t), p_t)
\]

\[
+ \delta \sum_{t'=t+1}^\infty \delta^{t'-t+1}h(\omega_t, \sigma(\omega_{t'}, m_{t'}), \psi(\omega_{t'}, m_{t'}))f(i)
\]

\[
+ g(\omega_t, \sigma(\omega_{t'}, m_{t'}), \psi(\omega_{t'}, m_{t'}))]
\]

\[
=[h(\omega_t, \sigma(\omega_t, m_t), p_t)
\]

\[
+ \delta \sum_{t'=t+1}^\infty \delta^{t'-t+1}h(\omega_t, \sigma(\omega_{t'}, m_{t'}), \psi(\omega_{t'}, m_{t'}))f(i)
\]

\[
+ g(\omega_t, \sigma(\omega_t, m_t), p_t)
\]

\[
+ \delta \sum_{t'=t+1}^\infty \delta^{t'-t+1}g(\omega_{t'}, \sigma(\omega_t, m_t), \psi(\omega_t, m_t))]
\]

\[
= H(p_t, m_{t+1}; \omega_t, m_t, \Pi)F(i) + G(p_t, m_{t+1}; \omega_t, m_t, \Pi)
\]

where \(F(i) = f(i)\). This gives the desired result. □

Proof of Proposition 2. Let \(\Pi\) admit a first order characterization. If the current median voter, \(m\), chooses to keep the current franchise, i.e., if \(\mu(\omega, m) = m' = m\), then the Envelope Theorem implies:

\[
\delta \frac{\partial V_m}{\partial m'} = 0 \quad \text{and} \quad \frac{\partial u_m}{\partial p'} + \delta \frac{\partial V_m}{\partial \omega_m} \frac{\partial Q}{\partial p'} = 0.
\]
Next, since the citizen’s Euler equation, (9), must hold each period and in each state, we obtain
\[
\frac{\partial V_i}{\partial o_i} = -\frac{\partial u_i}{\partial e_i'} \frac{\partial e_i'}{\partial Q}.
\]  

Substituting these three equations in the franchise Euler equation, (12), if \( II \) admits a franchise extension then Eq. (15) must hold at \( m' = m \). To obtain the converse, observe that since \( II \) admits a first order characterization then Eq. (12) is decreasing, and so if Eq. (15) holds at \( m' = m \), then by the Envelope Theorem, the solution to Eq. (12) entails a choice \( m' > m \). □

References