

Practice problems for calculus II

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Abstract

Problems are given which require some basic techniques.

1 Integration by parts, and other techniques

1. Find the antiderivative of $x \ln(3x)$
2. Find $\int 2 \cdot (x^2 - 8x + 15)^{-1} dx$
3. Find $\int \sin^7(t) \cos^3(t) dt$.

2 Improper integrals

Determine if the following converge, and if they do, compute them.

4. $\int_1^\infty x^{-1} [\ln(x)]^{-a} dx < \infty$ if and only if $a > 1$
5. $\int_1^\infty x^{-1/2} dx < \infty$
6. $\int_4^\infty e^{-2x} dx$
7. $\int_{-1}^{+1} x^{-1} dx$

3 Volumes of rotation

8. Take the region \mathcal{R} bounded by the lines $x = 2$ and $x = 3$, and the curves $y = x$ and $y = x^2$. Set up the integral which gives the volume which would result from rotating the region \mathcal{R} about the line $y = 12$.

9. If you used the method of washers (annuli) to solve problem 5, which method should you use if the problem were changed to require rotation about the line $x = 0$?

4 Separable differential equations

10. Suppose $dy/dx = (1 + y^2)(e^x)$. Solve for y as a function of x subject to the condition that $y(0) = 1$.

11. If $f'(x) = f(x)$ and $f(x)$ is a constant function, what is the value of $f(1)$?

5 Sequences, series, and convergence

12. Does the sequence $1, 2, 1, 2, 1, 2, \dots$ converge?

13. If a sequence is bounded below by 3 and it is decreasing, what can you say about the convergence of the sequence?

14. If $\sum_{k=1}^{\infty} a_k = s < \infty$, describe explicitly what the two associated sequences converge to.

15. If a convergent series is added term-by-term to a divergent series, does the new sum of the two series converge or diverge?

6 Tests for convergence and divergence

Do the following converge or diverge? Why?

16. $\sum_{n=1}^{\infty} \frac{x^{1/2}}{4+x^2}$

17. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{3/2}}{14+x^2}$

18. $\sum_{n=3}^{\infty} \frac{1}{n(\ln(n)^4)}$ (hint: use problem 4 above)

7 Power series: radius and interval of convergence

19. What is the radius of convergence of $\sum_{n=1}^{\infty} n^2 + 17n^{-3}(x-5)^{3n}$?

20. What is the interval of convergence in the previous problem?

21. If $x-5$ is changed to $2x-5$, how do the last two answers change?

8 Taylor series

more later ...