Calculus I - Fall 2001 - Final Exam - 12/12/01

- 13 problems on 11 pages plus two scratch pages.
- 100 points, 2 hours 30 minutes.
- Write your answers directly on this sheet. Continue on back if necessary.
- Support your answers and show all your work.
- Closed notes, closed book, no calculators.
- Exact answers are required: For example, if the answer is π, then 3.14 is incorrect.

1. (5) Use the limit definition of the derivative to show that the derivative of
$f(t) = \dfrac{1}{t+3}$ is $f'(t) = \dfrac{-1}{(t+3)^2}$.

2. (8) Find the following limits:

$$\lim_{x\to 2} e^{x+1} \qquad\qquad \lim_{x\to 0} (1+x)^{\csc(x)}$$

$$\lim_{x\to 0} \frac{x^2}{\cos(x)-1} \qquad\qquad \lim_{x\to\infty} \frac{4x^7-2x^2+5}{11x^7+x^3-8x}$$

3. (15) Find

$$\frac{d}{dx}\left(x^8-x^4+7x^2-\frac{1}{\sqrt{x}}\right) \qquad\qquad \frac{d}{dx}\frac{\sin(x)}{x^2}$$

$$\frac{d}{d\theta}\left(\cos\left(\sqrt{\theta}\right)\right)^2 \qquad\qquad \frac{d}{dx}y \quad \text{where } y = x^{2x}$$

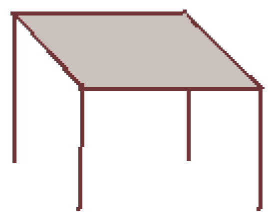
$f''(t)$ where $f(t) = A\cdot\cos(\omega\cdot t)$

4. (5) Find the equation of the tangent line to the graph of $y = x^2$ at the point (2,4).

5. (5) Use linear approximation to approximate $\sqrt[3]{8.01}$. Use the fact that $\sqrt[3]{8} = 2$.

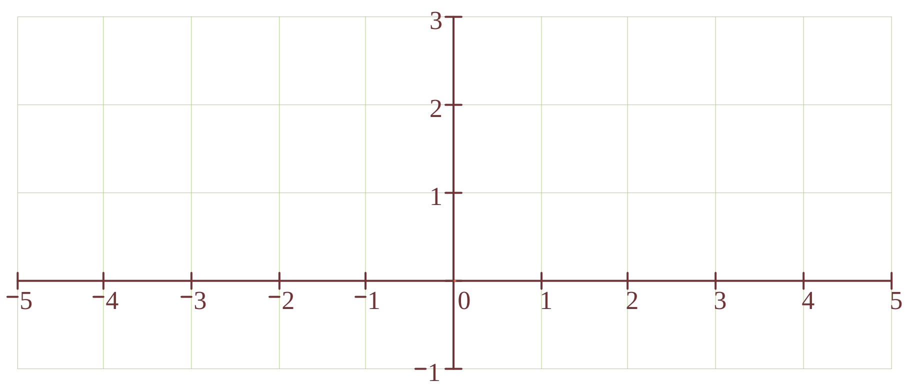
6. (10) A ladder of length 15 feet is leaning against a house. The foot of the ladder is 9 feet away from the house and is being pushed towards the house at a speed of 2 feet per second. How fast is the top of the ladder moving upward?

7. (10) A shed with a horizontal square roof and four posts of equal lengths, open on all sides, is to be constructed (see the picture on the right). Its volume must be 12 cubic meters. The cost for the plywood roof is $4 per square meter. The cost for the metal posts is $3 per meter. Find the dimensions that minimize the total cost.



8. (12) Let $g(x) = \dfrac{3x}{x^2+1} + 1$. Then $g'(x) = 3\dfrac{1-x^2}{\left(1+x^2\right)^2}$ and $g''(x) = \dfrac{6x\left(x^2-3\right)}{\left(x^2+1\right)^3}$.

- Find the interval(s) on which g increases and on which it decreases.
- Find the interval(s) on which g is concave up and on which it is concave down.
- Find all critical points of g.
- Find all points of inflection.
- Find all asymptotes.
- Sketch the graph in the coordinate system below, showing critical points and points of inflection and indicating asymptotes with dotted lines.



9. (12) Find the following integrals:

$$\int \left(x^{17} + 3\sin(x)\right) dx$$

$$\int x \cdot e^{\frac{-x^2}{4}} dx$$

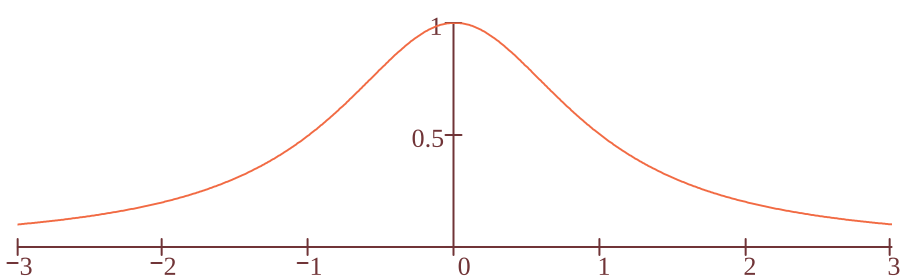
$$\int_{-4}^{2} \left(3x^2 + 1\right) dx$$

10. (5) If $y''(t) = -5$, $y'(0) = 60$, and $y(0) = 200$, find $y(t)$.

11. (6) a)Write down the Riemann sum approximation for $\displaystyle\int_{\frac{-3}{2}}^{\frac{5}{2}} \frac{1}{1+x^2}\, dx$, using four equal subintervals and midpoints as sample points.

Do not compute the value of the Riemann sum or the integral.

b) In the graph below, draw the area which exactly equals the Riemann sum in part a).



12. (4) True or False?

- A function that is concave up for all x cannot have a horizontal asymptote.

- If the function f is continuous on the open interval (a,b), then f must achieve a local minimum in that interval.

- If $f(x) \geq g(x)$ for every x in the interval $[a,b]$, then $f'(x) \geq g'(x)$ on (a,b).

- If $f'(c) = 0$, then f has a local minimum or a local maximum at c.

13. (3) Let $F(x) = \int_0^x g(t)\,dt$ for $0 \leq x \leq 5$, where the graph of g is given below. Identify all local maxima and local minima of F in the open interval $(0,5)$.