Robustness Checks for Example 2 in Genicot-Ray [2002]

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Genicot and Ray[1] study the following example

Example 2. Consider a community of ten individuals with the same functional form for utility as in Example 1 of Genicot and Ray [2002];

$$u(c) = \frac{1}{1-\rho} c^{1-\rho},$$

where ρ is the Arrow-Pratt coefficient of relative risk aversion. We also use the same specific parameters as in Example 1: $\delta = 0.83$, $\rho = 1.6$, $p = 0.4$, $\ell = 2$, and $h = 3$.

We evaluate — for each group size ranging from 1 to 10 — the return to informal insurance. One natural way to do this is to look at the gain over and above autarky, compared to the corresponding per-capita gain that the first-best provides *in the community of all ten*. If $\tilde{v}$ denotes this latter value and $\hat{v}(n)$ is the i-stable value for a group of size n, then the *i*-stable gain may be reported as

$$\frac{\hat{v}(n) - v(1)}{\tilde{v} - v(1)} \times 100$$

in percentage terms. Similarly, if $v^*(n)$ is the stable value for a group of size n, then the stable gain is described as

$$\frac{v^*(n) - v(1)}{\tilde{v} - v(1)} \times 100,$$

again in percentage terms. The results for this example are reported in Table 1.

Much has been written on "social capital" in the past few years. In the insurance context one could measure the return to such capital very much as we have done here. Clearly, recognizing the possibility of coalition deviations dramatically reduces the estimated return on social capital. In Example 2, the highest return a member of a community of 10 could expect is less than half the return we would have evaluated were we not accounting only for coalition formation (38% instead of 85%).

Computations for several parameter values, given below, reveal both robustness and sensitivity, in the following sense. In general, the "grand community" of all individuals will be destabilized by a smaller-sized community, causing a large fraction of the potential benefits from insurance not to be reaped. [Notice that the phrase "potential benefits" already corrects for the single-person deviation constraint, as in Table 1.]

[1]Genicot, G. and D. Ray (2002), "Group Formation in Risk-Sharing Arrangements," forthcoming, *Review of Economic Studies*.

n	Stable?	i-Stable Gain (%)	Stable Gain (%)
1	$\surd$	0	0
2	$\surd$	10	10
3	$\surd$	50	38
4	$\times$	61	$\emptyset$
5	$\times$	69	$\emptyset$
6	$\times$	75	$\emptyset$
7	$\times$	78	$\emptyset$
8	$\times$	81	$\emptyset$
9	$\times$	84	$\emptyset$
10	$\times$	85	$\emptyset$

Table 1. Stable Gains are Limited.

However, equilibrium group sizes and the degree of insurance are very sensitive to the parameters. As Coate and Ravallion [1993] observed in their computations, "[o]ne striking feature of the results ... is how sharply the performance varies. Even quite a successful risk-sharing arrangement may vanish with certain seemingly modest perturbations to parameter values, such a small decline in the participants' aversion to risk". These observations are compounded by an order of magnitude in our model. Even an *increase* in risk (or aversion to it) may destroy previously sucessful insurance arrangements as previously non-viable subgroups now become viable, destroying the viability of the larger community.

In this example, for instance, increasing the need for insurance θ from 0.91 to 1 causes a group of size 3 to become unstable. Several perturbations of θ and p cause the stable gain to fluctuate from 20% (a fifth of the corresponding i-stable gain) to 37% (45% of the corresponding i-stable value). This suggests a great deal of sensitivity in the quantitative magnitudes. However, the results are surprisingly robust in the sense that potential coalition deviations inevitably cause a large fraction of the potential benefits from insurance not to be reaped.

Now for details.

$p \backslash \theta$	.75	1	2	3
.2	1	1	1,2	1,2,3,8,9,10
.4	1,3	1,2	1,2	1,2,3,9,10
.6	1,3	1,2,5	1,2	1,2,7
.8	1,3,5	1,3	1,2,7	1,2,9

Table 2. Stable Sizes

To build Table 2 we take similar parameters as in Example 2: a population of 10 with constant relative risk aversion $\rho = 1.6$, a discount rate of $\delta = 0.83$, and a low income of $\ell = 2$; but we consider a range of different values for the probability of a high income p and different values for the need for insurance θ. Table 2 reports the stable group sizes for the different values of p and θ.

We now turn to a sensitivity analysis in the neighborhood of Example 2. The results reported in Table 1 are based on a value of $p = 0.4$ and $\theta = 0.91$ for which groups of 2 and 3 are stable. In Table 3 we can see that for the same value of $p = 0.4$ but with a higher need for insurance, only groups of size 2 are stable.

$p\backslash\theta$	0.9	0.91	0.9136	1	1.1
.39	1,2,3	1,2,3	1,2,3	1,2	1,2
.4	1,2,3	1,2,3	1,2,3	1,2	1,2
.41	1,2,3	1,2,3	1,2,3	1,2	1,2

Table 3. Local Sensitivity Analysis

The following tables report the i-stable and stable gain for the above values.

n	Stable?	i-Stable Gain (%)	Stable Gain (%)
1	$\surd$	0	0
2	$\surd$	9	9
3	$\surd$	50	41
4	$\times$	62	$\emptyset$
5	$\times$	70	$\emptyset$
6	$\times$	75	$\emptyset$
7	$\times$	79	$\emptyset$
8	$\times$	82	$\emptyset$
9	$\times$	84	$\emptyset$
10	$\times$	86	$\emptyset$

Table 4. example 2bis - $\theta = .9$, $p = 0.41$

n	Stable?	i-Stable Gain (%)	Stable Gain (%)
1	$\surd$	0	0
2	$\surd$	11	11
3	$\surd$	51	38
4	$\times$	62	$\emptyset$
5	$\times$	70	$\emptyset$
6	$\times$	76	$\emptyset$
7	$\times$	79	$\emptyset$
8	$\times$	82	$\emptyset$
9	$\times$	85	$\emptyset$
10	$\times$	87	$\emptyset$

TABLE 5. example 2bis - $\theta = .91$, $p = 0.41$

n	Stable?	i-Stable Gain (%)	Stable Gain (%)
1	$\surd$	0	0
2	$\surd$	11	11
3	$\surd$	51	37
4	$\times$	62	$\emptyset$
5	$\times$	71	$\emptyset$
6	$\times$	76	$\emptyset$
7	$\times$	80	$\emptyset$
8	$\times$	83	$\emptyset$
9	$\times$	85	$\emptyset$
10	$\times$	87	$\emptyset$

TABLE 6. example 2bis - $\theta = .9136$, $p = 0.41$

n	Stable?	i-Stable Gain (%)	Stable Gain (%)
1	$\surd$	0	0
2	$\surd$	22	22
3	$\times$	56	$\emptyset$
4	$\times$	67	$\emptyset$
5	$\times$	75	$\emptyset$
6	$\times$	80	$\emptyset$
7	$\times$	84	$\emptyset$
8	$\times$	87	$\emptyset$
9	$\times$	89	$\emptyset$
10	$\times$	91	$\emptyset$

TABLE 7. example 2bis - $\theta = 1$, $p = 0.41$

n	Stable?	i-Stable Gain (%)	Stable Gain (%)
1	$\surd$	0	0
2	$\surd$	31	31
3	$\times$	60	$\emptyset$
4	$\times$	71	$\emptyset$
5	$\times$	79	$\emptyset$
6	$\times$	84	$\emptyset$
7	$\times$	87	$\emptyset$
8	$\times$	90	$\emptyset$
9	$\times$	92	$\emptyset$
10	$\times$	94	$\emptyset$

TABLE 8. example 2bis - $\theta = 1.1$, $p = 0.41$

n	Stable?	i-Stable Gain (%)	Stable Gain (%)
1	$\surd$	0	0
2	$\surd$	8	8
3	$\surd$	49	41
4	$\times$	61	$\emptyset$
5	$\times$	68	$\emptyset$
6	$\times$	74	$\emptyset$
7	$\times$	77	$\emptyset$
8	$\times$	80	$\emptyset$
9	$\times$	83	$\emptyset$
10	$\times$	85	$\emptyset$

TABLE 9. example 2bis - $\theta = .9$, $p = 0.4$

n	Stable?	i-Stable Gain (%)	Stable Gain (%)
1	$\surd$	0	0
2	$\surd$	10	10
3	$\surd$	45	39
4	$\times$	61	$\emptyset$
5	$\times$	69	$\emptyset$
6	$\times$	74	$\emptyset$
7	$\times$	78	$\emptyset$
8	$\times$	81	$\emptyset$
9	$\times$	83	$\emptyset$
10	$\times$	85	$\emptyset$

TABLE 10. example 2bis - $\theta = .91$, $p = 0.4$

n	Stable?	i-Stable Gain (%)	Stable Gain (%)
1	$\surd$	0	0
2	$\surd$	21	21
3	$\times$	55	$\emptyset$
4	$\times$	61	$\emptyset$
5	$\times$	66	$\emptyset$
6	$\times$	74	$\emptyset$
7	$\times$	79	$\emptyset$
8	$\times$	86	$\emptyset$
9	$\times$	88	$\emptyset$
10	$\times$	90	$\emptyset$

TABLE 11. example 2bis - $\theta = 1$, $p = 0.4$

n	Stable?	i-Stable Gain (%)	Stable Gain (%)
1	$\surd$	0	0
2	$\surd$	30	30
3	$\times$	59	$\emptyset$
4	$\times$	70	$\emptyset$
5	$\times$	78	$\emptyset$
6	$\times$	83	$\emptyset$
7	$\times$	86	$\emptyset$
8	$\times$	89	$\emptyset$
9	$\times$	91	$\emptyset$
10	$\times$	93	$\emptyset$

TABLE 12. example 2bis - $\theta = 1.1$, $p = 0.4$

n	Stable?	i-Stable Gain (%)	Stable Gain (%)
1	$\surd$	0	0
2	$\surd$	7	7
3	$\surd$	48	41
4	$\times$	59	$\emptyset$
5	$\times$	67	$\emptyset$
6	$\times$	72	$\emptyset$
7	$\times$	76	$\emptyset$
8	$\times$	79	$\emptyset$
9	$\times$	81	$\emptyset$
10	$\times$	83	$\emptyset$

TABLE 13. example 2bis - $\theta = .9$, $p = .39$

n	Stable?	i-Stable Gain (%)	Stable Gain (%)
1	$\surd$	0	0
2	$\surd$	8	8
3	$\surd$	48	40
4	$\times$	60	$\emptyset$
5	$\times$	68	$\emptyset$
6	$\times$	73	$\emptyset$
7	$\times$	77	$\emptyset$
8	$\times$	80	$\emptyset$
9	$\times$	82	$\emptyset$
10	$\times$	84	$\emptyset$

TABLE 14. example 2bis - $\theta = .91$, $p = .39$

n	Stable?	i-Stable Gain (%)	Stable Gain (%)
1	$\surd$	0	0
2	$\surd$	9	9
3	$\surd$	49	39
4	$\times$	60	$\emptyset$
5	$\times$	68	$\emptyset$
6	$\times$	73	$\emptyset$
7	$\times$	77	$\emptyset$
8	$\times$	80	$\emptyset$
9	$\times$	82	$\emptyset$
10	$\times$	84	$\emptyset$

TABLE 15. example 2bis - $\theta = .9136$, $p = 3.9$

n	Stable?	i-Stable Gain (%)	Stable Gain (%)
1	$\surd$	0	0
2	$\surd$	20	20
3	$\times$	54	$\emptyset$
4	$\times$	65	$\emptyset$
5	$\times$	73	$\emptyset$
6	$\times$	78	$\emptyset$
7	$\times$	82	$\emptyset$
8	$\times$	85	$\emptyset$
9	$\times$	87	$\emptyset$
10	$\times$	89	$\emptyset$

TABLE 16. example 2bis - $\theta = 1$, $p = 3.9$

n	Stable?	i-Stable Gain (%)	Stable Gain (%)
1	$\surd$	0	0
2	$\surd$	29	29
3	$\times$	58	$\emptyset$
4	$\times$	70	$\emptyset$
5	$\times$	77	$\emptyset$
6	$\times$	82	$\emptyset$
7	$\times$	86	$\emptyset$
8	$\times$	89	$\emptyset$
9	$\times$	91	$\emptyset$
10	$\times$	92	$\emptyset$

TABLE 17. example 2bis - $\theta = 1.1$, $p = 3.9$