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Peso Problems: Their Theoretical and Empirical Implications*

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This paper examines how the theoretical and empirical implications of asset pricing models are affected by the presence of a "peso problem"; a situation where the potential for discrete shifts in the distribution of future shocks to the economy affects the rational expectations held by market participants. The paper examines the ways in which "peso problems" can induce behavior in asset prices that apparently contradicts conventional rational expectations assumptions. This analysis covers the relationship between realized and expected returns, asset prices and fundamentals, and the determination of risk premia.

1. Introduction

One common feature of asset pricing models is that current asset prices incorporate market participants' expectations of future economic variables. When market participants act in a stable economic environment, their rational expectations are based on a subjective probability distribution for shocks hitting the economy that coincides with the distribution generating past realizations of variables. In an unstable environment, by contrast, expectations may be based on a subjective probability distribution that differs from the distribution generating past realizations if market participants rationally anticipate discrete shifts in the distribution of future shocks. The "peso problem" refers to the behavior of asset prices in this situation. In particular, "peso problem" models focus on how the potential for discrete shifts in the distribution of future shocks to the economy can affect the rational expectations held by market participants, and hence the behavior of asset prices.

In this chapter, I shall review how the presence of "peso problems" can affect the predictions of standard asset pricing models. In particular, I shall show how discrete shifts in the distribution of economic determinants can induce behavior in

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asset prices that apparently contradicts conventional rational expectations assumptions. Since these assumptions are widely used in empirical research, "peso problems" can have potentially far-reaching implications for the estimation and evaluation of asset pricing models.

Although the precise origins of the term "peso problem" are unknown, a number of economists attribute its first use to Milton Friedman in his examination of the Mexican peso market during the early 1970's. During the period, Mexican deposit rates remained substantially above U.S. dollar interest rates even though the exchange rate remained fixed at 0.08 dollars per peso. Friedman argued that this interest differential reflected the market's expectation of a devaluation of the peso. Subsequently, in August 1976, these expectations became justified when the peso was allowed to float because it fell in value by 46% to a new rate of 0.05 dollars per peso.

The first written discussion of the "peso problem" appears in Rogoff (1980). He argued that the behavior of Mexican peso futures prices and spot exchange rates from June 1974 to June 1976 was consistent with participants anticipating the devaluation of the peso [see also Frankel (1980)]. Krasker (1980) and Lizondo (1983) provide models that make the reasoning behind this argument clear. Let s_{t+1} be the logarithm of the spot exchange rate (dollars per peso). From April 1954 to August 1976 the spot exchange rate was fixed at 0.08 dollars per peso, $s_t = s^0$. If $s^1 (< s^0)$ is the level of the spot rate after devaluation, the expected spot rate can be written as

$$\mathrm{E}[s_{t+1}|\Omega_t] = \pi_t s^1 + (1 - \pi_t)s^0 \ ,$$

where π_t is the market's assessed probability that the peso will be devalued between period t and $t+1$. While the peso remained fixed at s^0, the difference between the realized spot rate and the rate expected in the market was

$$s^0 - \mathrm{E}[s_{t+1}|\Omega_t] = \pi_t(s^0 - s^1) \ .$$

Thus, so long as market participants assessed there to be a positive probability of devaluation so that $\pi_t > 0$, their forecast errors would be systematically positive.

This example illustrates how the potential for discrete events can affect the forecast errors made by market participants during periods where the events do not materialize. This idea lies at the heart of recent models that allow for the presence of "peso problems". One important difference between these models and the analysis of the Mexican peso market is that they generally do not focus on a single event. Rather, they examine the extent to which repeated but infrequent discrete shifts in the distribution of shocks hitting the economy could induce "peso problems" in the observed behavior of asset prices. This is an important distinction because "peso problem" models designed to explain the behavior of asset prices around a particular event have little predictive content. In the case of the Mexican peso, for example, the model places no restrictions on market expectations unless the probability of devaluation, π_t, and the new value for the exchange rate, s^1, are pinned down.

The problem of how to identify market expectations in the presence of a "peso problem" is tricky. It is always possible that market expectations are being influenced by the possibility of discrete shifts in the distribution of economic determinants that are never observed in the data. In such circumstances, it is impossible to distinguish between rational expectations influenced by a "peso problem" and irrational expectations. Many recent models avoid these "pathological peso problems" by explicitly linking market expectations to discrete shifts estimated in the data. For this purpose, researchers have used variants on the regime switching model originally due to Hamilton (1988, 1989). Regime switching models provide a simple, tractable framework in which to identify the rational expectations of market participants influenced by the possibility of discrete shifts. Importantly, this modelling approach allows us to make a distinction between irrational expectations and the expectations of rational market participants affected by the presence of "peso problems".

In this chapter, I shall use the regime switching framework to discuss how the presence of "peso problems" can affect both the theoretical and empirical implications of asset pricing models. In recent years, "peso problem" models have been developed to examine the behavior of stock prices, interest rates and foreign exchange returns. This chapter makes no attempt to survey the general literature on these topics. Rather, I shall focus on the potential for "peso problem" models to shed light on some of the well-documented puzzles, such as the equity premium and forward premium puzzles.

I begin, in Section 2, by considering how the presence of "peso problems" affect the properties of forecast errors made by rational market participants. Section 3 examines how the presence of "peso problems" can affect the relationships between asset prices and fundamentals. This analysis identifies the conditions under which regime switching in the process for fundamentals will lead to "peso problems". Section 4 considers how "peso problems" can affect the assessment of risk. Here I evaluate several recent models of the equity risk premium that employ regime switching. In Section 5, I consider a number of econometric issues that arise in the modelling of "peso problems". The paper concludes in Section 6 with a discussion of the directions future research on "peso problems" might usefully take.

2. Peso problems and forecast errors

Although "peso problems" can affect the behavior of asset prices through a number of different channels, in the literature researchers have paid most attention to their impact on the errors made by rational market participants when forecasting returns. In this section, I examine both the theoretical origins and empirical implications of these effects. I will begin by considering cases where market participants face uncertainty about the future regime. Here there exists a "pure peso problem" in the sense that there is no uncertainty about the current regime. I then consider the implications of "generalized peso problems". Here the effects of "pure peso problems" and learning combine to alter the properties of forecast errors in cases where market participants are uncertain about both current and future regimes.

2.1. Pure peso problems

2.1.1. Theoretical implications

Let R_{t+1} be the return on an asset between periods t and $t+1$. By definition, we can write this as the sum of the *ex ante* expected return held by market participants given information at $t, \mathrm{E}[R_{t+1}|\Omega_t]$, and the forecast error:

$$R_{t+1} \equiv \mathrm{E}[R_{t+1}|\Omega_t] + e_{t+1} \quad . \tag{1}$$

Under standard rational expectations assumptions, the forecast error, e_t, should have mean zero and be uncorrelated with variables in the markets' information set, Ω_t.

To see how these properties of the forecast errors are affected by the presence of discrete shifts in the returns process, consider the simple case where R_{t+1} can switch between two processes. Throughout this chapter I shall assume that switches in the process are indicated by changes in a discrete-valued variable, $Z_t = \{0, 1\}$. Let $R_{t+1}(z)$ denote realized returns in regime $Z_{t+1} = z$. Our aim, therefore, is to consider the behavior of the forecast errors, $R_{t+1}(z) - \mathrm{E}[R_{t+1}|\Omega_t]$. For this purpose, it is useful to decompose realized returns into the conditionally expected return in regime $z, \mathrm{E}[R_{t+1}(z)|\Omega_t]$, and a residual w_{t+1}:

$$R_{t+1} = \mathrm{E}[R_{t+1}(0)|\Omega_t] + \nabla \mathrm{E}[R_{t+1}|\Omega_t]Z_{t+1} + w_{t+1} \quad , \tag{2}$$

with $\nabla \mathrm{E}[R_{t+1}|\Omega_t] \equiv \mathrm{E}[R_{t+1}(1)|\Omega_t] - \mathrm{E}[R_{t+1}(0)|\Omega_t]$. Notice that it will always be possible to decompose returns in this way irrespective of the process they follow in each regime or the specification of the markets' information set, Ω_t.

In order for (2) to be useful in the analysis of market forecast errors, we have to say something about the properties of the residuals, w_{t+1}. When market participants hold rational expectations, their forecasts, $\mathrm{E}[R_{t+1}(z)|\Omega_t]$, coincide with the mathematical expectation of R_{t+1} conditioned on the market's information set. Taking expectations on both sides of (2) conditioned on Ω_t for $Z_{t+1} = \{0, 1\}$ implies that $\mathrm{E}[w_{t+1}|\Omega_t] = 0$. Thus, the residual, w_{t+1}, inherits the properties of conventional rational expectations forecast errors. Since it represents the error the rational market participants would make when the $t+1$ regime is known, I shall refer to it as the *within-regime* forecast error.

When market participants are unaware of the time $t+1$ regime, their forecast errors will differ from the *within-regime* errors. To see this, we must first identify the market's forecasts by taking expectations on both sides of (2). Using the fact that $\mathrm{E}[w_{t+1}|\Omega_t] = 0$, this gives

$$\mathrm{E}[R_{t+1}|\Omega_t] = \mathrm{E}[R_{t+1}(0)|\Omega_t] + \nabla \mathrm{E}[R_{t+1}|\Omega_t]\mathrm{E}[Z_{t+1}|\Omega_t] \quad . \tag{3}$$

Substituting (2) and (3) into (1) and rearranging, we obtain the following expression for the market's forecast errors, $R_{t+1} - \mathrm{E}[R_{t+1}|\Omega_t]$,

$$e_{t+1} = w_{t+1} + \nabla \mathrm{E}[R_{t+1}|\Omega_t](Z_{t+1} - \mathrm{E}[Z_{t+1}|\Omega_t]) \ . \tag{4}$$

This equation shows how the market's forecast errors, e_{t+1}, are related to the *within-regime* errors, w_{t+1}. Clearly, when the future regime is known, $Z_{t+1} = \mathrm{E}[Z_{t+1}|\Omega_t]$, so the second term vanishes. In this case there is no "peso problem" and the market's forecast errors inherit the conventional rational expectations properties of the *within-regime* errors.[1] When the future regime is unknown, the second term in (4) makes a contribution to the market's forecast errors. It is under these circumstances that the presence of a "peso problem" may affect the properties of the market's forecast errors.

To see this more clearly, suppose that returns are generated from the regime 1 process in period $t+1$. Under these circumstances, the market's *ex post* forecast error in (4) is

$$\begin{aligned} e_{t+1}(1) &= w_{t+1} + \nabla \mathrm{E}[R_{t+1}|\Omega_t](1 - \mathrm{E}[Z_{t+1}|\Omega_t]) \\ &= w_{t+1} + \nabla \mathrm{E}[R_{t+1}|\Omega_t]\Pr(Z_{t+1} = 0|\Omega_t) \ . \end{aligned} \tag{5}$$

As noted above, when market participants have rational expectations, the first term on the right, has mean zero and is uncorrelated with any variables in Ω_t. The second term is equal to the difference between the *within-regime* forecasts, $\nabla \mathrm{E}[R_{t+1}|\Omega_t]$, multiplied by the market's subjective probability that regime 0 occurs next period. A "peso problem" will exist in this case if the market believes that regime 0 is possible so that $\Pr(Z_{t+1} = 0|\Omega_t) > 0$. These beliefs will make the second term in (5) non-zero provided the *within-regime* forecasts differ from one another. If they do, the term may have a non-zero mean and may be correlated with elements in Ω_t. Thus, the presence of a "peso problem" can cause the markets' forecast errors to appear biased and correlated with *ex ante* information when viewed *ex post* even though market participants form their expectations rationally.

The presence of a "peso problem" can have these effects on *ex post* forecast errors more generally. As (4) shows, so long as some uncertainty exists about the future regimes governing returns, the term $\nabla \mathrm{E}[R_{t+1}|\Omega_t](Z_{t+1} - \mathrm{E}[Z_{t+1}|\Omega_t])$ will be present in the realized forecast errors within a regime. As a result, these errors may appear biased and correlated with *ex ante* information when viewed *ex post*.

The extent to which these properties are found in a particular sample of forecast errors depends upon the frequency of regime shifts in the sample. In the extreme case where only regime 1 occurs, the sample properties of the forecast errors will match those of $e_{t+1}(1)$ in (5). Alternatively, when there are a number of regime changes during the sample, the forecast errors will inherit a combination

[1] Fullenkamp and Wizman (1992) coin the term "surety" when referring to a situation where market participants know the process governing realizations of future returns. Here "surety" implies that $Z_{t+1} = \mathrm{E}[Z_{t+1}|\Omega_t]$.

of the properties of $e_{t+1}(1)$ and $e_{t+1}(0)$ [defined analogously with $e_{t+1}(1)$]. As (4) indicates, in this case, the resulting effect on the forecast errors depends on the sample properties of $Z_{t+1} - \mathrm{E}[Z_{t+1}|\Omega_t]$. If the frequency of regime shifts in the sample is representative of the underlying distribution of regime changes upon which rational market participants base their forecasts, in a typical sample $Z_{t+1} - \mathrm{E}[Z_{t+1}|\Omega_t]$ will have a mean close to zero and will be uncorrelated with elements in Ω_t. Equation (4) shows that the sample forecast errors will inherit these properties because, as we noted above, $\mathrm{E}[w_{t+1}|\Omega_t] = 0$. Thus, under these circumstances, the forecast errors will display the conventional rational expectations properties.

From this discussion, it should be clear that the impact of a "peso problem" on the forecast errors made by rational market participants depends upon the frequency of regime shifts in the sample. When the number of shifts is representative of the underlying distribution, the forecast errors will display the conventional rational expectations properties. In other cases where the number of shifts is unrepresentative, the forecast errors may appear biased and correlated with *ex ante* information. Thus, there is a sense in which the presence of a "peso problem" can only impact upon the forecast errors made by rational market participants in "small" samples. Of course the term "small" in this context refers to a sample with an unrepresentative number of regime shifts rather than the number of observations on returns, or even the time span of the data.

2.1.2. Empirical implications

A number of papers have examined whether "peso problems" can account for some of the anomolous behavior of asset returns. To summarize this research, it will prove useful to write returns in terms of spot and forward rates. Define s_t as the logarithm of the spot rate on an asset at time t and f_t^k as the logarithm of the time t forward rate on a contract to buy or sell the asset k periods in the future. Then, the speculative return on a forward contract to sell the asset in the future period is,

$$s_{t+k} - f_t^k = \phi_t + \epsilon_{t+k} \quad , \tag{6}$$

where ϕ_t is the risk premium on this speculative position and ϵ_{t+k} is the market's error in forecasting the spot rate given information available at time t.

The forward premium puzzle: It is natural, given the origins of the term, that the foreign exchange literature has paid a good deal of attention to the potential role of "peso problems". In particular, researchers have considered whether "peso problems" could account for the behavior of foreign exchange returns implied by the following regression of the change in the (log) spot exchange rate, Δs_t, on the forward premium, $f_t^1 - s_t$, due to Fama (1984):

$$\Delta s_{t+1} = b_0 + b(f_t^1 - s_t) + u_{t+1} \quad . \tag{7}$$

Using the fact that $\Delta s_{t+1} \equiv f_t^1 - s_t + \phi_t + \epsilon_{t+1}$, and the standard rational expectations assumption that the covariance between $f_t^1 - s_t$ and the forecast error,

ϵ_{t+1}, is zero, least squares theory implies that in a sample of T observations, the estimate of b is:

$$\hat{b} = 1 + \frac{\text{Cov}_T(\phi_t, f_t^1 - s_t)}{\text{Var}_T(f_t^1 - s_t)} , \tag{8}$$

where $\text{Var}_T(.)$ and $\text{Cov}_T(.)$ denote the sample variance and covariance. Thus, under conventional rational expectations assumptions, an estimate of b different from one implies that the risk premium covaries with the forward premium. Since excess returns can be written as the sum of the risk premium and forecast error, this is equivalent to saying that excess returns can be predicted with the forward premium.

Table 1 shows the results from estimating this regression with dollar exchange rates against the German Mark, British Pound and Japanese Yen over the period

Table 1
This table reports the results of estimating the Fama regression

$$\Delta s_{t+1} = b_0 + b(f_t^1 - s_t) + u_{t+1}$$

where s_t and f_t^1 are the spot and the one-period forward exchange rates, over the period 1975-1989. Column (1) reports OLS estimates of b. Column (2) reports the p-value for $H_0 : \hat{b} = 1$, based on Wald tests that allow for heteroskedasticity in the residuals u_{t+1}. Column (3) reports the bias in the estimate of c implied by $\hat{b}$ under the hypothesis that the risk premium is related to the forward discount by:

$$\phi_t = c_0 + c(f_t^1 - s_t) + v_t.$$

The bias is measured as $c^* - c$ where c^* is the value of implied from the Fama regression based on simulated data from a switching model. The table reports the mean bias with the standard deviation in parenthesis of the empirical distribution based on 1000 simulations. Column (4) reports the mean and standard deviation of the ratio c^*/c.

Currency	(1)	(2) p-value	(3) Monte Carlo Experiments	(4)
	$\hat{b}$	$H_0 : \hat{b} = 1$	Bias	Ratio
Monthly Data				
Pound	-2.266	< 0.001	-0.726	1.222
			(3.438)	(1.053)
Mark	-3.502	0.001	-1.068	1.237
			(3.253)	0.722)
Yen	-2.022	< 0.001	-0.107	1.035
			(0.607)	(0.201)
Quarterly Data				
Pound	-2.347	0.001	-0.724	1.216
			(2.691)	(0.804)
Mark	-3.448	0.004	-0.720	1.162
			(2.735)	(0.615)
Yen	-2.955	< 0.001	-0.124	1.031
			(0.700)	(0.177)

Source: Evans and Lewis (1995b)

1975 to 1989. In common with the findings of other researchers, all the estimates of b are significantly less than zero. Based upon the decomposition of $\hat{b}$ in (8), these negative coefficient estimates imply that the variance of the risk premium is greater than the variance of the forward premium [see Fama (1984)].

There is now quite a large literature trying to reconcile this interpretation of the regression results with the predictions of theoretical asset-pricing models [see, for example, Backus, Foresi and Telmer (1994)]. However, as Lewis (1994) notes in a recent survey, none of the models in the literature have been very successful in generating variability in the risk premia sufficient to explain the regression results. From this perspective therefore, the results in Table 1 present something of a puzzle.

"Peso problems" provide one potential resolution to this puzzle because their presence provides an additional channel through which the forward premium can have predictive power for excess returns within a sample. This can be seen if we rewrite the expression for the OLS estimate of b as

$$\hat{b} = 1 + \frac{\mathrm{Cov}_T(\phi_t, f_t^1 - s_t)}{\mathrm{Var}_T(f_t^1 - s_t)} + \frac{\mathrm{Cov}_T(\epsilon_{t+1}, f_t^1 - s_t)}{\mathrm{Var}_T(f_t^1 - s_t)} \quad , \tag{9}$$

where $\epsilon_{t+1} \equiv s_{t+1} - \mathrm{E}[s_{t+1}|\Omega_t]$. As we have seen, the presence of a "peso problem" can create a small sample correlation between the rational forecast errors, ϵ_{t+1}, and variables in Ω_t, such as the forward premium $f_t^1 - s_t$. Thus, in contrast to Fama's analysis, the third term on the right may actually contribute to the estimate of b in "small" samples where a "peso problem" exists.

Evans and Lewis (1995b) provide some evidence on the size of the third term in (9). Using estimates of a switching model for the spot exchange rates, they ran Monte Carlo experiments to look at the small sample bias in $\hat{b}$ due to "peso problems". In these experiments, the forward rates are driven by both market expectations of future spot rates (which incorporate the effects of potential switches in the spot rate process) and variations in the risk premia according to

$$\phi_t = c_0 + c(f_t^1 - s_t) + v_t \quad , \tag{10}$$

where v_t is an i.i.d. error. In each experiment, a sample of spot and forward rates was generated and used to find the estimate of c implied by the regression in (7), i.e., $c^* = \hat{b} - 1$. An empirical distribution for c^* was built by repeating this procedure.

Columns (3) and (4) of Table 1 reproduce the results of these Monte Carlo experiments. Column (3) reports the mean value of $c^* - c$. This is negative for all three currencies indicating that the Fama coefficient may indeed be biased downwards by the presence of a "peso problem". Column (4) reports the mean and standard deviation of c^*/c. This ratio measures the ratio of lower bounds on the standard deviations of the risk premia and gives an indication of how much "peso problems" may contribute to the apparent variability of the risk premia. For all currencies, the mean value of c^*/c implies that the standard deviation of the measured risk premium exceeds the true risk premium from the model. In the case of the Pound and the Mark, the standard deviations are about 20% higher.

Thus standard inferences may overstate the variability of the risk premia when "peso problems" are not taken into account.

These results illustrate how the presence of a "peso problem" can affect coefficient estimates found in conventional regressions that characterize the short run properties of returns. "Peso problems" may also affect inferences about the long-run properties of asset prices and returns as represented by cointegration relationships estimated in the data.

Cointegration: A good deal of recent empirical research has focused on the long-run properties of asset prices and returns. This interest has been spurred by the observation that many asset prices and returns appear to be well characterized as following processes with permanent shocks. Under these circumstances, many asset pricing models make predictions about the long-run behavior of prices and returns. These predictions can be easily understood by referring back to the expression for returns in (6):

$$s_{t+k} - f_t^k = \phi_t + \epsilon_{t+k} \quad . \tag{6}$$

Standard models with rational expectations imply that both the risk premia, ϕ_t, and forecast errors, ϵ_{t+k}, should follow a covariance stationary process, called "I(0)" in the literature. Since the sum of two stationary variables must be stationary, (6) implies that $s_{t+k} - f_t^k$ must also follow a stationary process. By contrast, observed spot and forward rates have typically been found to contain very persistent shocks, well-approximated as permanent disturbances which cumulate into so called "stochastic trends". These processes are covariance stationary after first differencing, called "I(1)" in the literature.

Clearly, if spot and forward rates are I(1), $s_{t+k} - f_t^k$ will only be I(0) stationary when the permanent shocks to s_{t+k} and f_t^k cancel out. For this to happen two requirements must be met. First, the variables in the vector $X_t \equiv [s_{t+k}, f_t^k]$ must be cointegrated. That is to say, there exists a "cointegrating vector" α such that $\alpha' X_t$ is I(0) stationary. Second, the cointegrating vector must be $\alpha' = [1, -1]$ since premultiplying by this vector, $\alpha' X_t$, gives the excess returns.

Testing for the number of trends: Evans and Lewis (1993) provide an example of how to test the first of these requirements. First they test for the number of trends in a vector of spot rates and a vector of forward rates individually using the methodology developed by Johansen (1988). Next, they test for the number of trends in a vector that combines all the spot and forward rates. If each pair of spot and forward rates share a common trend, the number of trends should not increase when the spot and forward rates are combined in the same vector.

Using data for the US Dollar against the German Mark, British Pound and Japanese Yen currencies over the period 1975 to 1989, Evans and Lewis find that vectors containing spot and forward rates contain one more trend than the vector of spot rates. They then examine whether these results could reflect the presence of a "peso problem". Using the estimates from a switching model for the Dollar/Pound rate, their Monte Carlo study shows that there is a reasonably high probability of observing an additional trend in forward rates when market participants rationally anticipate shifts in the spot rate process. They also show that standard tests would be very unlikely to detect the trends in excess returns due to the "peso problem" associated with these shifts.

Testing for one-to-one cointegration: "Peso problems" may also affect estimates of the cointegrating vector between spot and forward rates. Recall that excess returns will only be stationary when spot and forward rates are cointegrated one-for-one. Thus, in the context of the cointegrating regression,

$$s_{t+k} = a_0 + a_1 f_t^k + v_{t+k} \ , \tag{11}$$

a_1 must be equal to one under the null hypothesis of stationary excess returns. Comparing (11) with the identity, $s_{t+k} - f_t^k \equiv \phi_t + \epsilon_{t+k}$, reveals that we should find $a_1 = 1$ if the sum of the risk premium and forecast errors follow a stationary I(0) process.

Evans and Lewis (1994) examine the relationship in (11) using monthly returns from the U.S. Term Structure for the period June 1964 to December 1988. In this application, s_{t+k} is the rate on a one month T-bill at $t+k$, and f_t^k is the forward rate on a contract at month t for a one month bill at month $t+k$. They show that the null hypothesis of $a_1 = 1$ can be rejected at horizons of $k = 1$, to 10 months.

Could these results be attributable to a "peso problem"? To address this possibility, consider the case where $k = 1$ and let $R_{t+1} = s_{t+1}$ and $f_t^1 = \mathrm{E}[R_{t+1}|\Omega_t] - \phi_t$. Let us also assume that the one period rate switches between two processes that share the same trend:

$$R_{t+1}(z) = \psi_z \tau_{t+1} + e_{t+1}(z), \qquad \tau_{t+1} = \tau_t + \eta_{t+1} \ , \tag{12}$$

for $z = \{0, 1\}$, where τ_t is the common stochastic trend with i.i.d. innovations η_t and $e_{t+1}(z)$ following stationary I(0) processes. Using (12) to find the forecasts of $R_{t+1}(z)$, it is easy to show that

$$\begin{aligned} f_t^1 &= \tau_t[\psi_1 \Pr(Z_{t+1} = 1|\Omega_t) + \psi_0 \Pr(Z_{t+1} = 0|\Omega_t)] + \mathrm{I}(0) \text{ terms} \\ s_{t+1} - f_t^1 &= \tau_t(\psi_1 - \psi_0)(Z_{t+1} - \mathrm{E}[Z_{t+1}|\Omega_t]) + \mathrm{I}(0) \text{ terms.} \end{aligned} \tag{13}$$

In data samples where the frequency of regime shifts differs from the underlying distribution used by market participants in forming their forecasts, $(Z_{t+1} - \mathrm{E}[Z_{t+1}|\Omega_t])$ will be serially correlated. Under these circumstances, (13) shows that the stochastic trend, τ_t, will appear in realized excess returns when $\psi_1 \neq \psi_0$. And, since this same trend drives forward rates, the cointegrating coefficient a_1 in (11) will be different from one.

2.2. *Generalized peso problems*

In the models considered so far, market participants are assumed to know the current regime so that the "small" sample properties of the forecast errors are only affected by uncertainty about *future* regimes. Other models assume that market participants cannot directly observe current or past regimes. These models introduce an element of learning that can be another source of small sample bias and serial correlation into the *ex post* forecast errors.

2.2.1. *Theoretical implications*

To illustrate how learning can contribute to peso effects in forecast errors,, suppose that the only information available to market participants when forecasting future returns are current and past returns so that $\Omega_t = \{R_t, R_{t-1}, \cdots\}$. Under these circumstances, the degree of uncertainty about the current regime is represented by the conditional probability distribution, $\Pr(Z_t|\Omega_t)$. In extreme cases where the observed history of returns is fully revealing about the current regime, $Z_t = z$, there is no uncertainty. Thus, $\Pr(Z_t = z|\Omega_t) = 1$ and the analysis goes through as before. I shall therefore consider cases where the history of returns is not fully revealing so that $1 > \Pr(Z_t|\Omega_t) > 0$ for $Z_t = \{0, 1\}$. Here new observations on returns *within* a regime may allow market participants to learn about the current regime so that $\Pr(Z_t|\Omega_t)$ can vary from period to period.

To see how changes in $\Pr(Z_t|\Omega_t)$ can affect the properties of forecast errors, substitute the identity $\Pr(Z_{t+1} = 0|\Omega_t) \equiv \Pr(Z_{t+1} = 0|Z_t = 1, \Omega_t) - \Pr(Z_{t+1} = 0|Z_t = 1, \Omega_t) - \Pr(Z_{t+1} = 0|\Omega_t))$ into (5) to obtain the following expression for the *ex post* forecast error in regime 1:

$$\begin{aligned} e_{t+1}(1) &= w_{t+1} + \nabla \mathrm{E}[R_{t+1}|\Omega_t]\Pr(Z_{t+1} = 0|Z_t = 1, \Omega_t) \\ &- \nabla \mathrm{E}[R_{t+1}|\Omega_t]\Big(\Pr(Z_{t+1} = 0|Z_t = 1, \Omega_t) - \Pr(Z_{t+1} = 0|\Omega_t)\Big) \ . \end{aligned} \tag{14}$$

The first two terms in this equation are the same as those in (5). The third term shows how learning about the current regime can affect the forecast error. We can rewrite this term as

$$\begin{aligned} &\nabla \mathrm{E}[R_{t+1}|\Omega_t]\Big(\Pr(Z_{t+1} = 0|Z_t = 1, \Omega_t) - \Pr(Z_{t+1} = 0|Z_t = 0, \Omega_t)\Big) \\ &\times \ \Pr(Z_t = 0|\Omega_t) \ . \end{aligned} \tag{15}$$

Notice that this term will be zero if the probability of regime 0 occurring in $t + 1$ is independent of the current regime. In this special case, uncertainty about the current regime, as measured by $\Pr(Z_t = 0|\Omega_t)$, makes no contribution to the forecast errors. In other cases, changes in $\Pr(Z_t = 0|\Omega_t)$ due to learning will contribute to the dynamics of this term. Kaminsky (1993) refers to the combined effect of the second and third terms in (14) as the "generalized peso problem".

If market participants use Bayes Law to update their probability distributions on the current state using current and past returns, we can describe the learning dynamics by

$$\Pr(Z_t = 0|\Omega_t) = \frac{\Pr(Z_t = 0|\Omega_{t-1})\mathcal{L}(R_t|Z_t = 0, \Omega_{t-1})}{\sum_z \Pr(Z_t = z|\Omega_{t-1})\mathcal{L}(R_t|Z_t = z, \Omega_{t-1})} \ , \tag{16}$$

and $$\Pr(Z_t = z|\Omega_{t-1}) = \sum_{Z_{t-1}} \Pr(Z_t = z|Z_{t-1}, \Omega_{t-1}) \Pr(Z_{t-1}|\Omega_{t-1}) \ , \tag{17}$$

where $\mathcal{L}(.|Z_t, \Omega_{t-1})$ denotes the likelihood of observing the return given regime Z_t and past information, Ω_t. The first equation is simply a statement of Bayes' Law showing how observations on current returns are used to update the markets' probability of being in regime 0. The second equation shows how the probability distributions of future and current regimes are linked.

Equations (16) and (17) have two potentially important implications for the evolution of $\Pr(Z_t = 0|\Omega_t)$ and hence the behavior of the forecast errors. First, uncertainty about the current regime will persist while market participants place some likelihood on current returns coming from regime 0, i.e., while $\mathcal{L}(R_t|Z_t = 0, \Omega_{t-1}) > 0$. Second, as the number of consecutive observations from regime 1 become large, $\Pr(Z_t = 0|\Omega_t)$ will approach zero. In other words, if a regime persists long enough, rational market participants will eventually learn which regime they are in.

These features of the learning process suggest that uncertainty about the current regime is unlikely to make a large contribution to the small sample bias and serial correlation of the forecast errors within a single regime if i) current and past returns contain a lot of information about the current regime, and ii) the regime persists for a long time. Both these features depend upon whether market participants view regime changes as being once-and-for-all or not.

Lewis (1989a,b) studies the effects of learning on asset prices. In particular, she considers how the exchange rate would behave during a period where market participants are learning about a *past* change in regime induced by a once-and-for all shift in the process for fundamentals. In the context of equation (14), this situation is equivalent to the case where the switch to regime $z = 1$ is viewed as permanent so that $\Pr(Z_{t+1} = 0|Z_t = 1, \Omega_t) = 0$. Imposing this restriction on (14), we can write the forecast errors following the regime switch as

$$e_{t+1}(1) = w_{t+1} + \nabla \mathrm{E}[R_{t+1}|\Omega_t]\Pr(Z_{t+1} = 0|\Omega_t) \ .$$

Thus, the *ex post* forecast errors will only differ from the *within* regime errors until market participants have learned that the switch in regime has taken place. In such circumstances, forecast errors are affected by a pure learning problem rather than a "generalized peso problem".

2.2.2. Empirical implications

To what extent are the empirical implications of "peso problems" affected by the presence of learning? This issue has recently been addressed in papers by Kaminsky (1993) and Evans and Lewis (1995a).

Evans and Lewis consider the effects of "peso problems" caused by shifts in the inflation process on the long-term relationship between nominal interest rates and realized inflation; the so called long-term Fisher relation. As part of this study, they conduct Monte Carlo experiments on the following cointegrating regression,

$$E[\pi_{t+1}|\Omega_t^m] = d_0 + d_1\pi_{t+1}^m + v_t \ , \tag{18}$$

where $E[\pi_{t+1}|\Omega_t^m]$ is the expected inflation rate and π_{t+1}^m is the realized inflation rate, both generated from a switching model for quarterly inflation. The experiments reveal that the presence of both a "pure" and "generalized peso problem" creates bias in the estimates of d_1 in typical data samples. They also show that the bias is smaller in the "generalized peso" case. Thus, it is quite possible for pure peso and learning effects to have partially offsetting influences on forecast errors.

Kaminsky (1993) provides another perspective on the effects of learning in her study of the dollar/pound exchange rate. She examines the properties of exchange rate forecast errors using a variant of the switching model in Engel and Hamilton (1990) where market participants use both the past history of exchange rates and monetary policy announcements made by the Federal Reserve to make inferences about the current regime. As in (14) and (15), the forecast errors depend upon $\Pr(Z_t|\Omega_t)$. These filtered probabilities are found from the Bayesian updating equations in (16) and (17) using the maximized value of a likelihood function that combines data on the spot exchange rate with a monetary policy indicator.[2]

Kaminsky shows that the forecast errors obtained from the model contain a good deal of small sample bias. She then compares them with forecast errors that are constructed using the "smoothed" probabilities, $\Pr(Z_t|\Omega_T)$, in place of the filtered probabilities. These probabilities can be calculated recursively from

$$\Pr(Z_{t-i}|\Omega_t) = \frac{\mathcal{L}(R_t|Z_{t-i}, \Omega_{t-1})\Pr(Z_{t-i}|\Omega_{t-1})}{\sum_z \mathcal{L}(R_t|Z_{t-1} = z, \Omega_{t-1})\Pr(Z_{t-1} = z|\Omega_{t-1})} \tag{19}$$

starting with $t = T$, $i = 1$, and working back through the sample. Notice that these probabilities incorporate all the information in the sample. Thus, if the subsequent behavior of the exchange rate makes clear what process was being followed at t, this new set of forecast errors will be purged of the effects of learning. Kaminsky shows that there is little difference between the sample properties of the two sets of errors. Again, learning appears to contribute little to the small sample effects of the "peso problem".

2.3. *Summary*

In this section, we have seen how the presence of a "peso problem" can affect the forecast errors made by rational market participants. In"small" data samples where the number of regime shifts are unrepresentative of the underlying distribution used by market participants to forecast, their forecast errors may appear biased and correlated with *ex ante* information when viewed *ex post* by a researcher. In these cases, the size of these peso effects depends upon the difference

[2] Kaminsky refers to this model as an "Imperfect Regime Classification" model because market participants recognize that policy announcements may not provide correct information about the regime. Kaminsky and Lewis (1992) use a similar model to study the impact of foreign exchange intervention.

between the *within-regime* forecasts of future returns, $\nabla \mathrm{E}[R_{t+1}|\Omega_t]$, the dynamics of Z_t, and the degree to which the current regime is known. Examples from the literature show that the presence of "peso problems" can significantly affect the relationship between asset prices and returns estimated from typical data samples. Moreover, these effects appear robust to the presence of learning.

3. Peso problems, asset prices and fundamentals

So far we have seen how the presence of "peso problems" can affect the properties of forecast errors via their impact on the rational market forecasts. Since asset prices also incorporate forecasts of future fundamentals, the analysis above suggests that the presence of "peso problems" will also affect the link between asset prices and their economic fundamentals. In this section, I shall examine these effects.

3.1. Peso problems in present value models

Present value models are among the simplest asset pricing models in which market expectations of future variables affect current asset prices and returns. I shall examine the impact of "peso problems' in the context of a generic present value model:

$$P_t = \theta_0 + \theta(1-\rho)\sum_{i=0}^{\infty}\rho^i \mathrm{E}[X_{t+i}|\Omega_t] \ , \tag{20}$$

where θ_0 is a constant, θ is a coefficient of proportionality, and ρ is the discount factor. Models of this form have been used to examine the behavior of interest rates, stock prices, and exchange rates. For the present, I shall simply refer to P_t and X_t as the asset price and fundamental.

Since P_t and X_t often appear to follow non-stationary I(1) processes in applications, it is useful to consider an alternative form of (20) expressed in terms of stationary I(0) variables. Subtracting θX_t from both sides of the equation and rearranging, we obtain the following expression for the "spread":

$$Y_t \equiv P_t - \theta X_t = \theta_0 + \theta\sum_{i=1}^{\infty}\rho^i \mathrm{E}[\Delta X_{t+i}|\Omega_t] \ . \tag{21}$$

Notice that when X_t follows a non-stationary I(1) process, $\mathrm{E}[\Delta X_{t+i}|\Omega_t]$ must be stationary under conventional rational expectations assumptions. Thus, the spread, Y_t, will follow a stationary I(0) process even when P_t is I(1).

To see how the presence of a "peso problem" affects the link between asset prices and fundamentals, I shall focus on (21) and study how switches in the process for ΔX_t affect the behavior of the spread. As above, I shall confine my attention to the case where ΔX_t switches between two processes governed by the discrete value state variable $Z_t = \{0, 1\}$. Realizations of ΔX_{t+1} are assumed to depend upon the regime during period t determined by the value of $Z_t = z$, and will be written as $\Delta X_{t+1}(z)$.

Since $\mathrm{E}[\Delta X_{t+i}|\Omega_t] = \sum_z \mathrm{E}[\Delta X_{t+i}|\Omega_t, Z_t = z]\Pr(Z_t = z|\Omega_t)$, we can take expectations on both sides of (21) conditioned on the market's information Ω_t [with $Y_t \in \Omega_t$] to obtain

$$Y_t = Y_t(0)\Pr(Z_t = 0|\Omega_t) + Y_t(1)\Pr(Z_t = 1|\Omega_t) \ , \tag{22}$$

where $$Y_t(z) = \theta_0 + \theta\sum_{i=1}^{\infty} \rho^i \mathrm{E}[\Delta X_{t+i}|\Omega_t, Z_t = z] \ . \tag{23}$$

The observed spread is shown in (22) as a probability weighted average of the regime-contingent spreads, $Y_t(z)$. These are defined in (23) as the value of the spread when market participants know the current regime.

To examine the effects of switching, we need to solve for the regime-contingent spreads, $Y_t(z)$. The first step is to iterate (23) one period forward:

$$Y_t(z_t) = \theta_0 + \theta\sum_{i=2}^{\infty} \rho^i \mathrm{E}[\Delta X_{t+i}|\Omega_t, Z_t = z] + \theta\rho\mathrm{E}[\Delta X_{t+1}|\Omega_t, Z_t = z] \ . \tag{24}$$

Next, we note that,

$$\mathrm{E}[\Delta X_{t+i}|\Omega_t, Z_t] = \sum_z \mathrm{E}\big[\mathrm{E}[\Delta X_{t+i}|\Omega_{t+1}, Z_{t+1} = z]|\Omega_t, Z_{t+1} = z\big]\Pr(Z_{t+1} = z|\Omega_t, Z_t).$$

Substituting this expression in the second term on the right hand side of (24) and rearranging, gives

$$\begin{aligned} Y_t(z) = {} & \theta_0(1-\rho) + \rho\sum_{z'} \mathrm{E}[Y_{t+1}(z')|\Omega_t]\Pr(Z_{t+1} = z'|\Omega_t, Z_t = z) \\ & + \theta\rho\mathrm{E}[\Delta X_{t+1}(z)|\Omega_t] \ , \end{aligned} \tag{25}$$

where $$\mathrm{E}[\Delta X_{t+1}(z)|\Omega_t] = \mathrm{E}[\Delta X_{t+1}|\Omega_t, Z_t = z].$$

The next step is to solve (25) for both regimes, $z = \{0, 1\}$. In models where the transition probabilities governing regime switches are either unknown to market participants or depend upon other variables, the probabilities $\Pr(Z_{t+1} = z'|\Omega_t, Z_t = z)$ will be time-varying making (25) a non-linear difference equation. To avoid the complications of solving such an equation, I shall consider the case where Z_t follows an independent Markov process with constant transition probabilities known to market participants. Under these circumstances, we can rewrite (25) as a linear matrix difference equation:

$$\begin{bmatrix} Y_t(1) \\ Y_t(0) \end{bmatrix} = \begin{bmatrix} \theta_0(1-\rho) \\ \theta_0(1-\rho) \end{bmatrix} + \rho\Lambda\begin{bmatrix} \mathrm{E}[Y_{t+1}(1)|\Omega_t] \\ \mathrm{E}[Y_{t+1}(0)|\Omega_t] \end{bmatrix} + \theta\rho\begin{bmatrix} \mathrm{E}[\Delta X_{t+1}(1)|\Omega_t] \\ \mathrm{E}[\Delta X_{t+1}(0)|\Omega_t] \end{bmatrix} \ , \tag{26}$$

where Λ is the matrix of transition probabilities with ij'th element equal to $\Pr(Z_{t+1} = i|Z_t = j, \Omega_t)$. Iterating (26) forward and applying the condition, $\lim_{t\to\infty}\rho^i \mathrm{E}[Y_{t+i}(z)|\Omega_t] = 0$, we obtain

$$Y_t(1) = \theta_0 + \theta \sum_{i=1}^{\infty} \rho^i E[\Delta X_{t+i}(1)|\Omega_t] - (1 - \lambda_1)\Phi_t \ , \tag{27}$$

$$Y_t(0) = \theta_0 + \theta \sum_{i=1}^{\infty} \rho^i \mathrm{E}[\Delta X_{t+i}(0)|\Omega_t] + (1 - \lambda_0)\Phi_t,$$

where λ_z is the probability of remaining in regime $z = \{0, 1\}$ from one period to the next, and

$$\Phi_t \equiv \sum_{i=1}^{\infty} \rho^i E[Y_{t+i}(1) - Y_{t+i}(0)|\Omega_t] \ .$$

Equations (22) and (27) allow us to examine how switches in the process for ΔX_t affect the behavior of the spread under a variety of conditions. For example, consider the case of a "pure peso problem" in which market participants only face uncertainty about the future regime. Here $Y_t = Y_t(z)$ so all the effects of switching can be examined using (27). This equation shows that news about fundamentals can affect the spread through two channels. First, news that leads to revisions in the expected present value of ΔX_{t+i} within the current regime, affects $Y(z)$ through the second term on the right of each equation. Second, new information on the expected size of the jump in dividend prices when a regime switch occurs affects $Y_t(z)$ through Φ_t. This jump term is equal to the present value of expected future changes in the regime-contingent spread induced by switches in regimes. Since $Y_t = Y_t(z)$, in the "pure peso problem" case, Φ_t represents the effects of expected capital gains induced by future regime switching.

In the case of a "generalized peso problem", where market participants face uncertainty about both the current and future regimes, news can affect the spread through a third channel. Recall that under these circumstances the observed spread is linked to the regime-contingent spreads by

$$Y_t = Y_t(0)\Pr(Z_t = 0|\Omega_t) + Y_t(1)\Pr(Z_t = 1|\Omega_t) \ ,$$

with$1 > \Pr(Z_t|\Omega_t) > 0$. Thus news that leads market participants to revise their estimate of the current state will in general lead to a change in the spread even when the regime-contingent spreads remain unchanged.

Equation (27) makes clear that the presence of a "peso problem" affects the relationship between $Y_t(z)$ and the present value of expected future fundamentals growth *within* a regime because market participants take account of future capital gains and losses associated with regime switches. To examine these capital gains, we need to solve for $Y_t(1) - Y_t(0)$. Taking the difference between the two equations in (27), and rearranging, we find that

$$Y_t(1) - Y_t(0) = \theta\rho \sum_{j=1}^{\infty} \varphi^{j-1} E[\Delta X_{t+j}(1) - \Delta X_{t+j}(0)|\Omega_t] \ , \tag{28}$$

where $\varphi \equiv \rho(\lambda_1 + \lambda_0 - 1)$. Thus, the jump in the regime-contingent spread when a switch in regime occurs depends upon the present value of the difference between the *within* regime forecasts of the future ΔX_t's.

Equation (28) has two important implications for the behavior of the spread when there is a change in regime. First, the size of any jump in $Y_t(z)$ depends upon *both* the difference in expected future fundamentals growth across regimes and the dynamics of regime switching. In this two regime example, the value of $\lambda_1 + \lambda_0 - 1$ determines the serial correlation structure of regimes. If $\lambda_1 + \lambda_0 = 1$, regimes are serially independent so the continuation of the current regime is as likely as a switch. In this case, (28) shows that $Y_t(1) - Y_t(0) = \mathrm{E}[\Delta X_{t+1}(0) - \Delta X_{t+1}(1)|\Omega_t]$. Thus, cross-regime differences in future ΔX_t's have no effect on the size of the jump. The reason is that a switch in regime this period has no impact on markets' expectations for future ΔX_t's when regimes are serially independent. In other cases where there is serial dependence in the regimes (i.e. when $\lambda_1 + \lambda_0 \neq 1$), market participants will revise their forecasts of future ΔX_t's when the regime switches so that the cross-regime differences in forecasts far into the future can affect the size of the jump. For example, in the case where $\lambda_1 + \lambda_0 > 1$ so that continuation of the current regime is more likely than a switch, (28) indicates that the spread will jump upwards when there is a switch from regime 0 to 1 if $\mathrm{E}[\Delta X_{t+j}(1)|\Omega_t] > \mathrm{E}[\Delta X_{t+j}(0)|\Omega_t]$ for $j > 0$.

The second implication of (28) is that jumps can occur in $Y_t(z)$ even when the change in regime is not accompanied by a jump in ΔX_{t+1}. For example, suppose that a switch in regime only affects forecasts of ΔX_{t+2}. So long as regimes are not serially independent, a change in regime at t will be accompanied by a jump in the regime-contingent spread. In the case of a "pure peso problem", this jump will be matched by the observed spread. Thus, a regime switch can generate jumps in the spread, even when there is no change in the *current* behavior of fundamentals. In this case, a switch in regime could have the appearance of a financial crisis, or crash.

We can also use (22) and (27) to see how switches in the process for fundamentals can give rise to the appearance of a rational bubble. In the context of the present value model, the spread contains a bubble when Y_t satisfies the difference equation implied by (21), namely,

$$Y_t = \theta_0(1 - \rho) + \rho E[Y_{t+1}|\Omega_t] + \rho E[\Delta X_{t+1}|\Omega_t] \ ,$$

but not the transversality condition, $\lim_{T\to\infty}\mathrm{E}[\rho^T Y_{t+T}|\Omega_t] = 0$. For example, if ΔX_{t+1} is constant, one bubble process for the spread is

$$Y_{t+1} = \text{const.} + \frac{1}{\rho} Y_t + \eta_{t+1}$$

with $\mathrm{E}[\eta_{t+1}|\Omega_t] = 0$. In this case, the spread varies because expectations of future spreads vary and not because there is any fundamentals' news. Bubble models are therefore quite different from present value models with switching in the fundamentals process because in switching models all the variations in Y_t are driven by fundamentals' news.

Flood and Hodrick (1986) noted that this theoretical distinction between peso and bubble models may be impossible to spot empirically. Suppose that during regime one, news arrives about the future fundamental in regime zero. Equations

(22) and (27) indicate that this news would affect the current spread insofar as it alters the expected future capital gain in the event of a regime switch. If this news is uncorrelated with the behavior of fundamentals in regime one, some of the variations in the spread in regime one would appear unrelated to the *observed* fundamentals. In the extreme case where all the observations come from a single regime, there would be no way to distinguish between this manifestation of a "peso problem" and the presence of a bubble.

3.2. Empirical implications

3.2.1. The term structure of interest rates

The first application of a switching model to a fundamentals-based asset pricing model appears in Hamilton (1988). He considers the following model [based on Shiller (1979)] for the yield on ten-year Treasury bonds, R_t^l, and the three month T-bill rate, R_t^1:

$$R_t^l = \theta_0 + \theta(1-\rho)\sum_{i=0}^{l-1}\rho^i \mathrm{E}\ [R_{t+i}^1|\Omega_t] \ , \tag{29}$$

$$R_t^1 = \alpha_o + \alpha_1 Z_t + v_t \ , \tag{30}$$

with $0 < \rho < 1$. Here v_t follows an AR(4) process with regime dependent heteroskedasticity, and $Z_t = \{0, 1\}$ follows an independent first-order Markov process. Market participants are assumed to forecast future short rates only using the past history of short rates [i.e., $\Omega_t = \{R_t^1, R_{t-1}^1, \ldots\}$] so a "generalized peso problem" is present.

The model places a complicated set of rational expectations restrictions on the joint behavior of the long and short rates. Using quarterly U.S. data from 1962:1 to 1978:3, Hamilton estimates the restricted process for the long rate as

$$\begin{aligned} R_t^l = 0.051 + 2.454 Pr(Z_t = 1|\Omega_t) + 1.89\mathrm{E}[v_t|\Omega_t] + 0.009\mathrm{E}[v_{t-1}|\Omega_t] \\ + 0.011\mathrm{E}[v_{t-2}|\Omega_t] + 0.001\mathrm{E}[v_{t-3}|\Omega_t] + \varepsilon_t \ , \end{aligned} \tag{31}$$

with $\quad Pr(Z_t = 1|Z_{t-1} = 1) = 0.997, \quad$ and $\quad Pr(Z_t = 0|Z_{t-1} = 0) = 0.998.$

What do these model estimates imply about the importance of a "peso problem" in the U.S. term structure? Suprisingly, they suggest that "peso problems" were almost completely absent. In the analysis above, we saw that "peso problems" will only affect the spread when market participants take account of the capital gains and losses associated with future changes in regime [i.e., via $(1-\lambda_z)\Phi_t$ in (27)]. Although the estimated coefficient of 2.452% on the $\Pr(Z_t = 1|\Omega_t)$ term in (31) indicates that these capital gains are quite large, market participants largely ignore them because the estimates of $\Pr(Z_t|Z_{t-1})$ indicate that the probability of a regime switch from one period to the next is very close to zero.

Sola and Driffill (1994) come to somewhat different conclusions in their study of the U.S. term structure. Unlike Hamilton, they consider the implications for behavior of the yield spread when there are switches in the process for short rate changes. With this formulation, the variables in the switching model are I(0) stationary even when long and short rates follow I(1) processes. This is an important feature, because as Pagan and Schwert (1990) point out, the validity of Hamilton's procedure for modelling regime switching requires that the variables in the model are I(0).

Although the estimated timing of regime switches in Sola and Driffill's model are very similar to those found in Hamilton (1988), their estimated transition probabilities are a good deal smaller. As a result, their model estimates indicate that the behavior of the U.S. term structure was significantly affected by "peso problems".[3] The contrast between these results suggests that it is perilous to draw conclusions about the importance of peso effects from the estimates of a single switching model.

3.2.2. Stock prices

Switching models have also been used to examine the behavior of stock prices. For example, in Evans (1993), I examine the effects of switches in dividend growth within the context of the dividend ratio model developed by Campbell and Shiller (1989). This model relates the natural log of the dividend price ratio at the beginning of period t, δ_t, to expected future dividend growth:

$$\delta_t = \theta_0 - \sum_{j=1}^{\infty} \rho^j \, \mathrm{E}[\Delta d_{t+j} | \Omega_t] \ , \tag{32}$$

where Δd_{t+1} is the dividend growth rate during year t and ρ is close to but smaller than one. Notice that this equation has the same form as the equation for the spread in (21) with $\Delta d_t = -\Delta X_t$ and $\theta = 1$ so the analysis above can be used to examine the effects of switching in the dividend growth process.

I assume that market participants observe the current regime and dividend growth switches between two processes, with switches determined by $Z_t = \{0, 1\}$ following an independent first order Markov process. As in Campbell and Shiller (1989), the empirical implications of the model are derived within a VAR framework for the joint behavior of log dividend prices and dividend growth. For the case of a first-order system, the VAR takes the form:

$$\begin{bmatrix} \delta_{t+1} \\ \Delta d_{t+1} \end{bmatrix} = \begin{bmatrix} \pi(Z_{t+1})\beta(Z_t) & \pi(Z_{t+1})\alpha(Z_t) \\ \beta(Z_t) & \alpha(Z_t) \end{bmatrix} \begin{bmatrix} \delta_t \\ \Delta d_t \end{bmatrix}$$

$$+ \begin{bmatrix} \gamma(Z_{t+1}) + \pi(Z_{t+1})g(Z_t) \\ g(Z_t) \end{bmatrix} + \begin{bmatrix} \pi(Z_{t+1})v_{t+1} + \eta_{t+1} \\ v_{t+1} \end{bmatrix} , \tag{33}$$

[3] This finding is consistent with the results of Lewis (1991) and Evans and Lewis (1994) for U.S. rates and Kugler (1994) for Eurodollar rates.

where $\alpha(z), \beta(z), g(z), \gamma(z)$ and $\pi(z)$ are coefficients that depend upon the regime and $E[\eta_{t+1}|\delta_t, \Delta d_t] = E[v_{t+1}|\delta_t, \Delta d_t] = 0$. Under rational expectations, the dividend ratio model in (32) imposes a complicated set of restrictions on these coefficients.

Table 2 shows estimates of the model in (33) using annual series for stock prices and dividends for the Standard and Poors Composite Stock price index from 1871 to 1987. The estimates of $\alpha(z)$ and $\beta(z)$ show how the predictability of dividend growth varies across regimes. In particular, the estimates of $\alpha(z)$ indicate that past dividend growth is a useful predictor of future dividend growth over short to medium forecasting horizons in regime 1 but not regime 0. As we saw above, differences in the forecasts of fundamentals across regimes only create "peso problems" when market participants place a significant probability on a regime switching from one period to the next. In this model, the probabilities are approximately 10% when in regime 1 and 1% in regime 0 so "peso problems" do affect the behavior of dividend-prices.

One way to gauge the importance of "peso problems" is to examine the sample behavior of stock returns implied by the model estimates. Campbell and Shiller (1989) show that the log return on stocks between periods t and $t+1$ can be well approximated by

$$r_{t+1} \simeq \kappa + \delta_t - \rho\delta_{t+1} + \Delta d_{t+1} \ , \tag{34}$$

where κ is a constant. Iterating this approximation forward, imposing the terminal condition, $\lim_{i\to\infty}\rho^i\delta_{t+i} = 0$, and taking expectations conditioned on Ω_t, gives,

$$\delta_t = \frac{-\kappa}{1-\rho} - \theta\sum_{j=1}^{\infty}\rho^j\,E[\Delta d_{t+j}|\Omega_t] + \theta\sum_{j=1}^{\infty}\rho^j\,E[r_{t+j}|\Omega_t] \ . \tag{35}$$

Comparing (35) and (32), we see that *ex ante* expected stock returns are constant in the dividend ratio model. Thus, variations in r_{t+1} should not be forecastable with any variables in Ω_t when market participants hold rational expectations and "peso problems" are absent. When they are present, realized returns will appear forecastable in "small" samples for the reasons discussed in Section 2.

The lower panels of Table 2 examine the predictability of returns with the regressions

$$r^m_{t+m} = a_0 + a_1\delta_t + u_{t+m} \ ,$$

and

$$r_{t+1} = b_0 + b_1\sum_{j=0}^{m-1}\delta_{t-j} + w_{t+1} \ ,$$

where $r^m_{t+m} \equiv \sum_{i=1}^{m} r_{t+i}$ is the m-period return. Under the null hypotheses of no predictability, $a_1 = 0$ and $b_1 = 0$.[4] As the upper rows of the panel show, this null

[4] See Hodrick (1992) for a discussion of these regression tests.

Table 2

The upper panel of the table reports the maximum likelihood estimates of the switching VAR model in (33). The parameters $\gamma(z)$ and $\pi(z)$ depend on $\alpha(z), \beta(z)$, and $g(z)$ through the cross-equation restrictions implied by the dividend ratio model in which rational market participants anticipate switches between two regimes. Switches are governed by $Z_t = \{0, 1\}$ which follows an independent first-order Markov Process, with transition probabilities, $\Pr(Z_t = z | Z_{t-1} = z) \equiv \lambda_z$. The model is estimated with S&P annual data of 117 years starting in 1879. The lower panels of the table report the percentiles of the empirical distribution for the t-statistics in the return regressions A and B. The empirical distribution is derived from 1000 replications of Monte Carlo experiments based on the estimated switching model. All the t-statistics correct for the presence of conditional heteroskedasticity. In addition, the statistics in Panel A correct for the presence of an Ma($m-1$) process in the residuals induced by the forecast overlap under the null hypothesis of no predictability in returns.

Maximum Likelihood Estimates

Parameter	Estimates	Std. Error	Parameter	Estimates	Std. Error
$\alpha(1)$	0.575	0.133	$g(1)$	−22.367	20.100
$\alpha(0)$	0.095	0.070	$g(0)$	−89.889	13.881
$\beta(1)$	−0.066	0.584	λ_1	0.898	0.067
$\beta(0)$	−0.307	0.048	λ_0	0.985	0.026

Return Predictability

A : $r^m_{t+m} = a_0 + a_1\delta_t + u_{t+m}$					B : $r_{t+1} = b_0 + b_1 \sum_{j=0}^{m-1} \delta_{t-j} + w_{t+1}$			
	$m=1$	$m=2$	$m=3$	$m=4$		$m=2$	$m=3$	$m=4$
$\hat{a}_1$	0.115	0.285	0.379	0.540	$\hat{b}_1$	0.087	0.058	0.059
t-statistics	2.175	3.073	3.168	3.739	t-statistics	2.717	2.574	2.847
Percentiles					Percentiles			
5	4.560	4.118	3.799	3.397	5	2.909	2.189	1.771
10	5.101	4.588	4.201	3.987	10	3.172	2.419	2.003
25	5.794	5.365	5.054	4.896	35	3.630	2.825	2.382
50	6.627	6.311	6.036	5.994	50	4.180	3.292	2.835
75	7.437	7.224	7.093	7.224	75	4.758	3.768	3.271
90	8.295	8.157	8.228	8.327	90	5.244	4.175	3.713
95	8.725	8.834	8.960	9.076	95	5.555	4.562	3.937

Source: Evans (1993)

can be rejected at standard significance levels when the regressions are estimated with the S&P data. The conventional interpretation of this regression evidence is that market participants' forecasts of future returns vary with the log dividend-price ratio. The lower rows of the panel provide us with an alternative interpretation. Reported here are Monte Carlo distributions for the t-statistics associated with a_1 and b_1 estimated from simulated data based on the maximum likelihood estimates of the switching model in (33). There is only one case where there is a greater than 5% probability of observing a t-statistic less than the asymptotic critical value 1.95. Thus, peso effects appear to have a significant impact on stock returns in this model.

3.3. Summary

In this section, I have examined how the prospect of discrete shifts in the behavior of fundamentals can affect the forecasts of rational market participants, and hence the behavior of asset prices. When market participants anticipate a switch in the fundamentals' process, current asset prices will depend on both the forecasts of fundamentals under the current process, and forecasts of the jump in prices if a switch takes place in the future. In "small" samples, variations in this latter term can induce movements in asset prices that appear unrelated to fundamentals and can complicate inferences about the link between prices and fundamentals in particular applications.

To illustrate how important these effects may be in practice, I considered models of the term structure and stock prices that incorporate switching in fundamentals. The findings from these models exemplify two important points. First, the presence of switching in fundamentals need not imply that "peso problems" significantly affect the behavior of asset prices. Second, it can be perilous to draw conclusions about the importance of "peso problems" from the estimates of a single switching model.

4. Risk aversion and peso problems

So far we have seen how the presence of "peso problems" can affect the behavior of asset prices and returns through their effect on market participants' expectations. In particular, we have seen how the prospect of a shift in regime can affect the link between asset prices and fundamentals and the properties of rational forecast errors in "small" samples. In this section, I shall consider how the prospect of regime shifts affects the market's assessment of risk.

I will begin by examining the impact of "peso problems" in a fairly general theoretical setting. This provides us with the framework to consider recent research on the behavior of asset prices in general equilibrium models with regime switching. In the second half of this section, I will examine how regime switching may provide a potential explanation for the equity premium and forward premium puzzles.

4.1. Peso problems in dynamic asset pricing models

In modern dynamic asset pricing theory, the asset prices are constrained by the behavior of a pricing kernel: a stochastic process governing prices of state-contingent claims. Let γ_{t+1} be a random variable that prices one-period state-contingent claims. If the economy admits no pure arbitrage opportunities, it can be shown that the one-period returns on all traded assets, i, must satisfy

$$\mathrm{E}[\gamma_{t+1} R^i_{t+1} | \Omega_t] = 1 \ , \tag{36}$$

where R^i_{t+1} is the real gross return on asset i between t and $t+1$ [see Duffie (1992)]. I shall refer to γ_{t+1} as the pricing kernel. In economies where there is a complete set of markets for state-contingent claims, there is a unique random variable γ_t satisfying (36). Under other circumstances, this no arbitrage condition still holds but for a range of γ_t's. In economies with a representative agent, γ_{t+1} is the intertemporal marginal rate of substitution so that (36) also represents a first-order condition. For the present, I shall keep the specification of γ_{t+1} general so that the analysis of "peso problems" can be applied to a wide class of asset pricing models.

Since (36) applies to all traded assets, the pricing kernel will be related to the return on a risk-free asset, R^0_{t+1}, by $\mathrm{E}[\gamma_{t+1}|\Omega_t] = 1/R^0_{t+1}$. Combining this expression with (36), we obtain an equation for the risk premium on asset i:

$$\mathrm{E}\ [R^i_{t+1}/R^0_{t+1}|\Omega_t] = 1 - \mathrm{Cov}(\gamma_{t+1}, R^i_{t+1}|\Omega_t)\ . \tag{37}$$

It is clear from (37) that the presence of a "peso problem" will only affect the risk premium insofar as it influences the conditional covariance term. To examine this influence, consider the simple case where the vector $X'_{t+1} \equiv [R^i_{t+1}, \gamma_{t+1}]$ switches between two regimes. As in Section 2, we can write the realized values of X_{t+1} as

$$X_{t+1} = \mathrm{E}[X_{t+1}(0)|\Omega_t] + \nabla\mathrm{E}[X_{t+1}|\Omega_t]Z_{t+1} + W_{t+1}\ , \tag{38}$$

where $\nabla\mathrm{E}[z_{t+1}|\Omega_t] \equiv \mathrm{E}[z_{t+1}(1)|\Omega_t] - \mathrm{E}[z_{t+1}(0)|\Omega_t]$ and $W'_{t+1} \equiv [w^R_{t+1}, w^\gamma_{t+1}]$ with $\mathrm{E}[W_{t+1}|\Omega_t] = 0$. From (38), it is easy to show that

$$\mathrm{Cov}(\gamma_{t+1}, R^i_{t+1}|\Omega_t) = \mathrm{cov}(w^R_{t+1}, w^\gamma_{t+1}|\Omega_t)$$
$$+\nabla\mathrm{E}[R^i_{t+1}|\Omega_t]\nabla\mathrm{E}[\gamma_{t+1}|\Omega_t]\mathrm{Var}\ (Z_{t+1}|\Omega_t)\ . \tag{39}$$

This decomposition of the conditional covariance allows us to see clearly how the presence of a "peso problem" can affect the risk premium. In the cases where the future regime is known [i.e., $Z_{t+1} \in \Omega_t$], there is no "peso problem" and the risk premium only depends on the conditional covariance between the *within-regime* forecast errors, $\mathrm{cov}(w^R_{t+1}, w^\gamma_{t+1}|\Omega)$. Here the variations in the risk premium originate from conditional heteroskedasticity in a regime [i.e., changes in $\mathrm{cov}(w^R_{t+1}, w^\gamma_{t+1}|\Omega)$ for a given value of Z_{t+1}] and/or conditional heteroskedasticity induced by a change in Z_{t+1}. By contrast, when a "peso problem" is present [i.e., $Z_{t+1} \notin \Omega_t$], the risk premium includes the conditional covariance between $\mathrm{E}[R_{t+1}(z)|\Omega_t]$ and $\mathrm{E}[\gamma_{t+1}(z)|\Omega_t]$. This term accounts for the forecast uncertainty market participants face across regimes.

It is clear from (39) that the importance of a "peso problem" depends on several factors. In particular, the second term in (39) will make no contribution to the risk premium in cases where the *within-regime* forecasts of the pricing kernel are the same so that $\nabla\mathrm{E}[\gamma_{t+1}|\Omega_t] = 0$. Thus, it is quite possible for a "peso problem" to generate small sample bias and serial correlation in $R_{t+1} - \mathrm{E}[R^i_{t+1}|\Omega_t]$ because $\nabla\mathrm{E}[R^i_{t+1}|\Omega_t] \neq 0$, and yet have no effect on the risk premium. While this may appear to be a special case and therefore of limited interest, it turns out to be a feature of some models in the literature.

"Peso problems" will contribute to the risk premium in varying degrees depending upon the amount of information market participants have about the future regime. This is easily seen by writing the conditional variance of Z_{t+1} in (39) as

$$\mathrm{Var}(Z_{t+1}|\Omega_t) = \mathrm{E}[\mathrm{Var}(Z_{t+1}|\Omega_t, Z_t)|\Omega_t] + \mathrm{Var}(\mathrm{E}[Z_{t+1}|\Omega_t, Z_t]|\Omega_t) \ . \tag{40}$$

When market participants observe the current regime, the second term in (40) vanishes. The behavior of $\mathrm{Var}(Z_{t+1}|\Omega_t)$ will then depend entirely on the dynamics governing regime changes. For example, when there is no serial dependence in Z_t, $\mathrm{Var}(Z_{t+1}|\Omega_t, Z_t)$ will be a constant. In this case, the presence of a "peso problem" introduces a constant into the risk premium. Otherwise, $\mathrm{Var}(Z_{t+1}|\Omega_t, Z_t)$ will vary with Z_t so that the "peso problems" will introduce another source of variability in the risk premium when there is a change in regime. In cases where market participants do not observe the current regime, the presence of a "peso problem" can contribute to variations in the risk premium *within* a regime. Here the probabilities $\Pr(Z_t = z|\Omega_t)$ will change as market participants learn about the current regime and this will lead to variations in both the terms on the right of (40).

4.1.1. *Peso problems and the equity premium puzzle*

A number of papers have recently used switching models in an effort to relate the observed behavior of the equity returns to general equilibrium asset pricing models. In particular, Cecchetti, Lam and Mark (1990, 1993) and Kandel and Stambaugh (1990) have used estimates of switching processes for consumption and dividends to examine the behavior of stock returns in variants of Lucas' model [Lucas (1978)]. These papers nicely illustrate the conditions under which "peso problems" can contribute to the behavior of the returns.

In all the papers, the presence of a representative agent with isoelastic utility makes $\gamma_{t+1} = \beta(C_{t+1}/C_t)^{-\eta}$ where C_t is equilibrium consumption, η is the coefficient of relative risk aversion, and $0 < \beta < 1$. One important difference between the papers is their specification for the switching process governing consumption and dividends. These specifications are summarized in the table below:

Model	Dividend and Consumption growth	Paper
I	$\Delta d_{t+1} = \mu_0, +\mu_1 Z_1 + \varepsilon_{t+1}$	Cecchetti, Lam and Mark (1990)
	$\Delta c_{t+1} = \mu_0, +\mu_1 Z_t + \varepsilon_{t+1}$	
II	$\Delta d_{t+1} = I_\mu(Z_t)$	Kandel and Stambaugh (1990)
	$\Delta c_{t+1} = I_\mu(Z_t)$	
III	$\Delta d_{t+1} = \mu_{0,d} + \mu_{1,d} Z_{t+1} + \varepsilon_{d,t+1}$	Cecchetti, Lam and Mark (1993)
	$\Delta c_{t+1} = \mu_{0,c} + \mu_{1,c} Z_{t+1} + \varepsilon_{c,t+1}$	

In Models I and III, Z_t is assumed to follow an independent first-order Markov process that switches between two regimes $z = \{0, 1\}$. The errors, ε_{t+1}, are assumed to be independent and identically distributed normal variates with zero mean. The presence of these errors creates uncertainty about growth within each regime. By contrast, in model II, all the variations in growth originate from changes in Z_t via the indicator function $I_\mu(.)$ that takes a different value according to the regime. Here Z_t follows an independent first-order Markov process between four regimes.

Although these models are similar in many respects, they have quite different implications for the role played by "peso problems" in determining the behavior of equity returns. In Model I, equilibrium dividends and consumption are identically equal. Moreover, growth between period t and $t+1$ depends upon the current regime Z_t. Since market participants are assumed to observe the current regime in all the models, this implies that there is no uncertainty about the distribution of growth over the next period.

To understand the implications this timing assumption has for the role of "peso problems", consider the equilibrium expressions for the pricing kernel and stock returns derived from model I:

$$\gamma_{t+1} = \beta \exp(-\eta\mu_0 - \eta\mu_1 Z_t - \eta\varepsilon_{t+1}) \tag{41}$$

$$R^s_{t+1} = \left[\exp\left(\delta(Z_t) - \delta(Z_{t+1})\right) + \exp\left(\delta(Z_t)\right)\right] \exp(\mu_0 + \mu_1 Z_t + \varepsilon_{t+1})$$

where $\delta(z)$ is the equilibrium log dividend price ratio in regime z. The important thing to note in (41) is that Z_{t+1} only affects realized stock returns. This means that there is no difference between the *within-regime* forecasts of the pricing kernel, i.e., $\nabla \mathrm{E}[\gamma_{t+1}|\Omega_t] = 0$. As a result, uncertainty about the future regime makes no contribution to the equity risk premium because the coefficient on $\mathrm{Var}(Z_{t+1}|\Omega_t)$ is zero in the expression for $\mathrm{Cov}(\gamma_{t+1}, R^i_{t+1}|\Omega_t)$ shown in (39).

While "peso problems" have no effect on the equity premium in this model, they do affect the small sample properties of equity returns, R^s_{t+1}. As the second equation in (41) shows, realized returns depend upon Z_{t+1} through the log dividend-price ratio in $t+1, \delta(Z_{t+1})$. Provided the ratio varies across regime [i.e., $\delta(1) \neq \delta(0)$], the *within-regime* forecast of future returns will differ from one another so that $\nabla \mathrm{E}[R^s_{t+1}|\Omega_t] \neq 0$. As we saw in Section 2, "peso problems" will affect the small sample properties of the rational forecast errors under these circumstances.

Model II has very similar implications. Although Kandel and Stambaugh's model implies a somewhat different expression for the equilibrium log dividend price ratio, the pricing kernel in their model depends upon the current regime as in (41). Consequently, "peso problems" have no effect on the equity premium or expected returns, $\mathrm{E}[R^s_{t+1}|\Omega_t]$. As in Model I, the dividend price ratio does vary across regimes creating a dependence between realized returns and the future regime. This, in turn, is the source of a "peso problem" in the rational forecast errors which is reflected in realized returns.

Model III allows uncertainty about the future regime to affect the pricing kernel. This can be clearly seen from the equilibrium expression for the pricing kernel and stock returns:

$$\gamma_{t+1} = \beta \exp(-\eta\mu_{0,c} - \eta\mu_{1,c}Z_{t+1} - \eta\varepsilon_{c,t+1}) \tag{42}$$
$$R^s_{t+1} = \left[\exp\left(\delta(Z_t) - \delta(Z_{t+1})\right) + \exp\left(\delta(Z_t)\right)\right] \exp(\mu_{0,d} + \mu_{1,d}Z_{t+1} + \varepsilon_{d,t+1})$$

The most important difference between (42) and (41) is that the pricing kernel now depends upon the future regime, Z_{t+1} rather than the current regime. This means that there is now the potential for "peso problems" to affect the size of $\text{Cov}(\gamma_{t+1}, R^i_{t+1}|\Omega_t)$ through the second term in (39), and hence the behavior of the equity premium.

To examine the strength of this peso effect, it is useful to reconsider equation (39), shown below:

$$\text{Cov}(\gamma_{t+1}, R^i_{t+1}|\Omega_t) = cov(w^R_{t+1}, w^\gamma_{t+1}|\Omega_t) + \nabla\text{E}[R^i_{t+1}|\Omega_t]\nabla\text{E}[\gamma_{t+1}|\Omega_t]Var\ (Z_{t+1}|\Omega_t)\ .$$

As the last term in the equation shows, uncertainty about the future regime will only affect $\text{Cov}(\gamma_{t+1}, R^i_{t+1}|\Omega_t)$ when both $\nabla\text{E}[\gamma_{t+1}|\Omega_t]$ and $\nabla\text{E}[R^i_{t+1}|\Omega_t]$ are non-zero. From (42) we see that the size of $\nabla\text{E}[\gamma_{t+1}|\Omega_t]$ depends upon the degree of risk aversion via the term $-\eta\mu_{1,c}$ and the size of $\nabla\text{E}[R^i_{t+1}|\Omega_t]$ depends upon the cross-regime differences in the equilibrium log dividend pricing ratio, $\delta(1) - \delta(0)$. Cecchetti, Lam and Mark's estimates imply that $\delta(1) - \delta(0)$ is close to zero because there is very little serial dependence in regimes, [the estimated value of $\lambda_1 + \lambda_0 - 1$ is only 0.06]. As a result, "peso problems" have little impact on the equity risk premium in this model.

There are two lessons to be drawn from the analysis of these models. The first is that the presence of switching need not lead to peso effects in risk premia even though market participants are aware that small sample problems will exist in the errors they make in forecasting future returns. As models I and II illustrate, peso effects on the risk premia can be ruled out by the (implicit) choice of specification for the equilibrium pricing kernel. The second lesson is more subtle. Even if the specification for the pricing kernel means that peso effects can potentially affect risk premia, the importance of these effects depends upon the dynamics of regime changes. Thus, the presence of switching in fundamentals need not imply that "peso problems" contribute significantly to the behavior of returns.

So far I have only examined the implications of these switching models for the behavior of the *conditional* equity premium, $\text{E}[R^s_{t+1}/R^0_{t+1}|\Omega_t]$. Abel (1993) considers their implications for the *unconditional* premium, $\text{E}[R^s_{t+1}/R^0_{t+1}]$. Taking unconditional expectations on both sides of (37), and applying the law of iterated expectations, we can write the unconditional premium as

$$\begin{aligned}\text{E}[R^s_{t+1}/R^0_{t+1}] &= 1 - \text{E}[\text{Cov}(\gamma_{t+1}, R^s_{t+1}|\Omega_t)] \\ &= 1 - \text{Cov}(\gamma_{t+1}, R^s_{t+1}) + \text{Cov}(\text{E}[\gamma_{t+1}|\Omega_t], \text{E}[R^s_{t+1}|\Omega_t])\end{aligned} \tag{43}$$

where Cov (.) denotes the unconditional covariance. Abel points out that if the conditionally expected growth rates of consumption and dividends are positively correlated, the last term on the right hand side of (43) will be negative in models with conditional lognormality and constant relative risk aversion. Thus, in these cases, the unconditional risk premium will be lower in the presence of Markov switching than would emerge from a model using the unconditional distribution of shocks. Abel confirms this prediction for the Markov switching specifications in Models I, II, and III.

What implications do these findings have for the potential effects of "peso problems" on the unconditional equity premium? Equation (43) shows that switching in fundamentals will affect the size of the unconditional risk premium through the covariance between $\mathrm{E}[\gamma_{t+1}|\Omega_t]$ and $\mathrm{E}[R^s_{t+1}|\Omega_t]$. "Peso problems" will therefore only affect the unconditional equity premium to the extent they alter this covariance. This observation suggests that "peso problems" will be of little help in resolving the equity premium puzzle in models where $\mathrm{Cov}(\mathrm{E}[\gamma_{t+1}|\Omega_t], \mathrm{E}[R^s_{t+1}|\Omega_t]) < 0$. However, as we shall see, "peso problems" can have significant effects on the unconditional moments of returns estimated in "small" samples. It is therefore possible that the sample estimates of $\mathrm{E}[R^s_{t+1}/R^0_{t+1}]$ and $\mathrm{Cov}(\gamma_{t+1}, R^s_{t+1})$ used to characterize the equity premium puzzle are quite different from the unconditional population moments.

4.1.2. Peso problems and the forward premium puzzle

In Section 2, we saw how the presence of switching in the spot exchange rate process could generate "peso problems" in exchange rate forecast errors. We also saw how estimates of peso effects could explain some, but not all of the predictability of foreign exchange returns in the context of Fama's regression. In view of these findings, it is worthwhile investigating whether "peso problems" could contribute to the predictability of returns via the foreign exchange risk premia. Hansen and Jagannathan (1991) provide a suitable framework for this purpose.

To begin, write the nominal return on asset i as $R^i_{t+1} \equiv L^i_{t+1}/V^i_t$ where V^i_t is the dollar value of the asset at t and L^i_{t+1} is the cash flow one period later. The no arbitrage condition in (36) can now be written as $V^i_t = \mathrm{E}[\gamma_{t+1}L^i_{t+1}|\Omega_t]$ where γ_{t+1} is the nominal pricing kernel denominated in dollars. Note that γ_{t+1} will be equal to the nominal intertemporal marginal rate of substitution in representative agent models. Next, let $L^i_{t+1} = F_t - S_{t+1}$ where F_t is the one period forward price and S_{t+1} is the future spot price of foreign currency. Since this cash flow can be generated by selling domestic currency to buy the forward contract, it involves no (net) payments at time t. Thus, the no arbitrage condition in (36) implies that $\mathrm{E}[\gamma_{t+1}(F_t - S_{t+1})|\Omega_t] = 0$. Applying the law of iterated expectations, we can rewrite this restriction as

$$\mathrm{Cov}_T(\gamma_{t+1}, F_t - S_{t+1}) = -\mathrm{E}_T[\gamma_{t+1}]\mathrm{E}_T[F_t - S_{t+1}] \tag{44}$$

where $\mathrm{E}_T[.]$ and $\mathrm{Cov}_T(.)$ represent the mean and covariance based on a sample of T observations. Using the Cauchy-Schwarz inequality, (44) implies the following bound on the coefficient of variation for the nominal pricing kernel:

$$\frac{\sqrt{\mathrm{Var}_T(\gamma_{t+1})}}{\mathrm{E}_t[\gamma_{t+1}]} \geq \frac{|\mathrm{E}_T[F_t - S_{t+1}]|}{\sqrt{\mathrm{Var}_T(F_t - S_{t+1})}}\;. \tag{45}$$

The Hansen-Jagannathan bound in (45) applies not only to investments in foreign exchange but also to investments in equities or bonds, or in portfolios that combine all these assets, so long as the associated cash flow at data t is zero. Bekaert and Hodrick (1992) estimate the bounds using equity and foreign exchange returns in the U.S., Japan, U.K. and Germany. For the three exchange rates, they estimate the bound to be as large as 0.48 with a standard error of 0.08. By contrast, the bound for U.S. equity is estimated to be 0.12 with a standard error of 0.10. These estimates appear to be very high when compared against the behavior of the pricing kernel implied by standard asset pricing models with moderate degrees of risk aversion. For example, Bekaert (1994) calculates the left hand side of (45) from an extended version of the Lucas (1982) model to be approximately 0.01 assuming the coefficient of relative risk aversion is equal to 2. From this perspective, the behavior of foreign exchange appears to be even more of a challenge for asset pricing theory than the behavior of equity returns.

To see how the presence of a "peso problem" might help explain these results, consider an economy where equilibrium foreign exchange returns and the nominal pricing kernel switch between two processes. In particular, let $X'_{t+1} \equiv [F_t - S_{t+1}, \gamma_{t+1}]$ so that the joint switching process for the two variables can be represented by (38). Further, let us assume that $\gamma_{t+1}(0)$ is constant. Now suppose that the researcher calculates the variance bound from a sample of foreign exchange returns that only contains observations from regime zero. Under these circumstances, the no arbitrage condition in (36) implies that

$$\mathrm{E}[F_t - S_{t+1}|\Omega_t] = \frac{-\mathrm{Cov}(\gamma_{t+1}, F_t - S_{t+1}|\Omega_t)}{\mathrm{E}[\gamma_{t+1}|\Omega_t]} \quad ,$$

where $\mathrm{Cov}(\gamma_{t+1}, F_t - S_{t+1}|\Omega_t) = \nabla\mathrm{E}[F_t - S_{t+1}|\Omega_t]\nabla\mathrm{E}[\gamma_{t+1}|\Omega_t]\mathrm{Var}(Z_{t+1}|\Omega_t)$. The absolute value of the mean excess return from such a sample is therefore

$$\left|\mathrm{E}_T[F_t - S_{t+1}]\right| = \left|\mathrm{E}_T\left[\frac{\nabla\mathrm{E}[F_t - S_{t+1}|\Omega_t]\nabla\mathrm{E}[\gamma_{t+1}|\Omega_t]\mathrm{Var}\ (Z_{t+1}|\Omega_t)}{\mathrm{E}[\gamma_{t+1}(0)|\Omega_t] + \nabla\mathrm{E}[\gamma_{t+1}|\Omega_t]\mathrm{E}[Z_{t+1}|\Omega_t]}\right]\right| . \quad (46)$$

Thus, the absolute value of the mean excess return will be greater than zero whenever the term in the numerator is non-zero. We saw above that this term determines whether a "peso problem" is present is the risk premium. When a "peso problem" is present, (46) indicates that the sample estimate of the lower bound on the right hand side of (45) is greater than zero.

Now suppose that a researcher compared the predictions of a particular general equilibrium asset pricing model against this bound. If the model ignored regime switching, and the data used to calibrate the model was from regime zero, the implied value of $\sqrt{\mathrm{Var}_T(\gamma_{t+1})}/\mathrm{E}_T[\gamma_{t+1}]$ will be close to zero. This value could easily violate the lower bound in (45) based on the sample behavior of returns.

This example illustrates the potential effects of "peso problems" on variance bound calculations. The violation of the variance bounds in the example occurs because the sample distribution of $F_t - S_{t+1}$ and γ_{t+1} is unrepresentative of the underlying distribution used by market participants in their assessment of risk. In this particular case, the sample distribution of the pricing kernel implied that there was no foreign exchange risk premium because $\mathrm{Cov}(\gamma_{t+1}(0), F_t - S_{t+1}) = 0$. In reality however, market participants accounted for the risk associated with the switch to regime 1 through $\nabla \mathrm{E}[F_t - S_{t+1}|\Omega_t] \nabla \mathrm{E}[\gamma_{t+1}|\Omega_t] \mathrm{Var}\,(Z_{t+1}|\Omega_t)$. Of course, these effects should disappear in large samples as the sample distribution of data approaches the underlying distribution.

4.1.3. Summary

The discussion above shows that "peso problems" can potentially affect the behavior of returns through their implications for the market's assessment of risk. I have identified the conditions under which uncertainty about the process driving future fundamentals can lead to a peso effect in the risk premium. Importantly, these conditions differ from those needed to generate "peso problems" in forecast errors and may not be met by every switching model. I have also shown how variance bounds can be affected in "small" samples when "peso problems" affect the risk premia. One question for future research is whether standard general equilibrium models extended to include peso effects in the risk premia are capable of meeting the bound requirements implied by the observed behavior of equity and foreign exchange returns.

5. Econometric issues

The central point to emerge from the analysis above is that the presence of a "peso problem" can complicate inferences about the behavior of asset prices and returns in "small" samples. Once this point has been recognized, the researcher faces two related problems.

The first concerns the size of the available data sample. As we have seen, size in this context means much more than the number of data periods. Theoretically, the size of a sample depends on the difference between the sample distribution of the data and the underlying distribution used by market participants. A data sample is "small" when there is a significant difference between the two. In conventional rational expectations models without regime switching, the span of the data set is often used as a reliable indicator of size. While there are no hard and fast rules, researchers have routinely used asymptotic inferences in data sets as short as 15 years. Unfortunately, the simulation results in the literature indicate that data spans of over 100 years can be considered "small" when regimes switch infrequently. This suggests that there is no way to judge whether a data set is "small" without a model characterizing regime switches in the sample.

The second problem concerns the modelling of regime switching. Following the pioneering work of Hamilton (1988, 1989), a plethora of switching specifications have been used to characterize regime switching in various applications. As we saw above, the choice of switching specifications can have far-reaching consequences for the potential role of peso effects. It is therefore important that the switching model be appropriately specified if we want to accurately gauge the importance of "peso problems". Unfortunately, this requirement forces the researcher to face some thorny econometric issues.

In this section, I will try to provide some practical guidance towards addressing these problems. I will not discuss the techniques used to estimate particular switching models since they are well covered in Hamilton (1994).

5.1. Small samples

At the outset, it should be clear that there is no way to definitively tell whether a data sample is "small" in a finite sample. It is always possible that market participants are influenced by the possibility of a switch to a regime that never occurred during the sample period. In this case, we can never hope to uncover the underlying distribution used by market participants in decision-making however well we manage to characterize the distribution of regime switches that took place in the sample. Pathological small sample problems of this type could only be detected in an infinite sample.

Putting these pathological cases aside, how might a researcher proceed? One approach is to assume that the sample is well characterized by a single regime and then look for evidence against this null hypothesis. Although the details of this approach will vary according to the application, the general idea is that the presence of regime switching will manifest itself as parameter instability in the reduced form equations of the model. For example, for the dividend ratio model described in Section 3, regime switching generates parameter instability in a standard VAR for δ_t and Δd_t:

$$\begin{bmatrix} \delta_{t+1} \\ \Delta d_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \delta_t \\ \Delta d_t \end{bmatrix} + \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} v_{1,t+1} \\ v_{2,t+1} \end{bmatrix} . \tag{47}$$

In this case, the proposed procedure would be to estimate (47) and then test for instability in the estimated coefficients A_{ii} and μ_i. The tests developed by Hansen (1991) could be used for this purpose.

Of course, evidence of parameter instability need not imply that the samples contain more than one regime. It may reflect other forms of misspecification instead. Nevertheless, finding evidence of parameter instability should lead to the consideration of regime switching.

5.2. Alternative switching models

Once the researcher finds some evidence of parameter instability and decides to investigate the possibility of regime switching, the natural question arises of how to model the switching process. Since economic theory rarely provides any specific guidance on this issue, the common approach has been to select a model on econometric grounds. In particular, researchers have typically first estimated an *ad hoc* switching specification and then evaluated how well it characterizes the data sample with a series of specification tests. As switching models are highly nonlinear, inferences from these tests are usually based on asymptotic distribution theory. Unfortunately, as Hansen (1992) points out, the regularity conditions used in standard asymptotic theory are often violated in situations where we want to conduct specification tests on switching models. In particular, tests for the number of regimes require non-standard distribution theory.

To address this problem, Lam (1990) and Cecchetti, Lam and Mark (1990) use Monte Carlo simulations in which they repeatedly estimate their proposed switching model on data generated under the null hypothesis of a single regime, i.e., no switching. The results from these simulations are then used to derive the empirical distribution of the test statistics under the null hypothesis. Although this procedure appears reasonably straight forward, it may not be easy to implement in practice for two reasons. First, the switching model has to be repeatedly estimated in order to build the empirical distribution. This can require a significant amount of computation. Second, since the data used to estimate these models is generated under the null hypothesis of no switching, the likelihood function for the switching model is likely to be very ill-behaved. As a result, nonlinear optimization techniques may have a very hard time finding the global maximum.

Hansen (1992) has advocated an alternative to this Monte Carlo simulation approach. He uses the theory of empirical processes to derive a bound on the asymptotic distribution of a standardized likelihood ratio statistic that is applicable even when conventional regularity conditions are violated. Unfortunately, calculating this bound also requires an enormous amount of computation in all but the simplest models.

Where does this leave the researcher? At present, there does not appear to be an easy way to conduct correct asymptotic inferences about the number of regimes to include in a model. In simple models it may be feasible to use either of the methods described above, but in others the CPU requirements appear well beyond the reach of most researchers. Perhaps the best approach in these latter cases is to consider the implications of alternative models with a different number of regimes. Recall from Sections 3 and 4 that the presence of regime switching need not lead to peso effects in asset pricing models. In particular, we examined switching models that did not generate peso effects because the estimated transition probabilities implied that the regimes were serially independent. Thus, there is little *a priori* reason to think that spurious peso effects will be present in a model with "too many" regimes. We may be able to side-step the question of how many regimes exist by showing that similar peso effects are present in models that use switching processes with different numbers of regimes.

Aside from choosing the number of regimes, the researcher also has to specify the process for regime switching. Following Hamilton (1988, 1989), most models in the literature have assumed that the process governing the regime, Z_t, follows an independent first-order Markov process. As we saw in Section 3, this assumption simplifies the calculations needed to quantify the effects of switching in dynamic asset pricing models. However, a number of authors have argued that this assumption may be unduly restrictive in certain applications. As an alternative, Diebold, Lee and Weinbach (1992) suggest that the transition probabilities be modelled as logistic functions of a vector of variables x_t. In the case of a two regime model, the transition probabilities are given by

$$Pr(Z_{t+1} = z | Z_t = z, x_t) = \frac{\exp(x_t' \beta_z)}{1 + \exp(x_t' \beta_z)} , \tag{48}$$

for $z = \{0,1\}$. When x_t includes a constant, the constant probability model is nested within this specification. Papers using this more flexible switching specification include Engel and Hakkio (1994) and Filardo (1994).

If our objective is to provide a parsimonious yet flexible switching representation for a time series process, allowing for endogenous transition probabilities is certainly attractive. But if the estimated switching model is to be used to represent the dynamics of fundamentals in an asset pricing model, the presence of endogenous transition probabilities greatly complicates the model. In this situation, it may be more attractive to think about alternative specifications for the switching process maintaining the assumption of constant probabilities.

5.3. Summary

Researchers interested in examining the empirical importance of "peso problems" face a number of difficulties. Since the theoretical impact of "peso problems" are confined to "small" samples, the question of whether a particular sample is "small enough" is an important one. Unfortunately, it is very hard to judge whether a sample is "small" without the explicit use of switching models. Furthermore, modelling regime switching presents a number of challenges. Since conventional asymptotic inference cannot be used to differentiate between models with different numbers of regimes, in practice it will often be impossible to provide sound statistical evidence supporting a particular switching specification. Thus, the best practical way forward may be to make sure that the significance of estimated peso effects using a particular switching specification are robust to alternative specifications.

6. Conclusion

In this chapter, I have examined the channels through which the presence of "peso problems" may affect the behavior of asset prices. Although the peso effects described above will only be present in "small" samples, this theoretical constraint does not appear to limit the potential for "peso problems" to affect the observed behavior of asset prices in many applications using typical data sets. Thus, the question of whether "peso problems" contribute to the well-known asset pricing puzzles in the literature is largely an empirical one. If there is strong econometric evidence to support the presence of discrete shifts in the distribution of the data, "peso problems" can *potentially* affect asset prices. Going beyond this to make a strong case for the significance of peso effects in a particular application is challenging.

Nevertheless, there are a number of directions that future research on "peso problems" may profitably take. Although most research to date has focused on the implications of "peso problems" for the behavior of rational forecast errors, "peso problems" can also affect the link between fundamentals and asset prices and the assessment of risk. To examine these effects, we need to consider the behavior of asset prices in a general equilibrium setting allowing for both risk aversion and switching in the fundamental processes. With such models, we will be able to consider *all* the potential implications of "peso problems" for the behavior of a single asset price. These models will also allow us to consider the implications of "peso problems" across asset markets. Insofar as "peso problems" have a common source, like shifts in government policy, it seems likely that cross-market information will be very useful in estimating the significance of peso effects.

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