Econometrics II Final Exam

December 13, 2004

INSTRUCTIONS: The exam must be completed before Jan 12’th. 2005. All the questions should be attempted. You may consult any books or notes but you must not communicate in any way with other members of the class regarding the exam until after Jan. 12’th. Any questions should be referred to me. Question 4 requires estimation. It is very important that you fully describe how the estimation was carried out in your answer (i.e., in words and math). Your Gauss code should be submitted as an appendix. A small portion of your grade will be determined by the clarity with which you present your results.

1. Consider the following simultaneous equation model:

\[ y_t = \alpha x_t + \delta z_t + e_t \]  
\[ x_t = \beta y_t + v_t \]

where \( z_t \sim i.i.d.N(\mu_z, \sigma_z^2) \), \( e_t \sim i.i.d.N(0, \sigma_e^2) \) and \( v_t \sim i.i.d.N(0, \sigma_v^2) \). The parameters \( \{\alpha, \beta, \delta\} \) all differ from zero and \( \alpha\beta < 1 \).

(a) Compute the reduced form equations for \( x_t \) and \( y_t \). What does the reduced form tell us about the unconditional joint distribution of \( y_t \) and \( x_t \)?

(b) Compute the probability limit of the OLS estimate of \( a \) from the regression

\[ y_t = ax_t + u_t \]

(c) Use the results from parts (a) and (b) to write an expression for \( u_t \) in terms of \( z_t, v_t \) and \( e_t \) and the model parameters \( \{\alpha, \beta, \delta, \sigma_e^2, \sigma_z^2, \sigma_v^2, \mu_z\} \).

(d) Is \( x_t, u_t \) a martingale difference sequence? Explain your answer.

(e) Now suppose we estimate \( b \) from the regression

\[ x_t = by_t + \eta_t \]

using \( z_t \) to instrument for \( y_t \). Derive the asymptotic distribution of the IV estimate of \( b \).

2. Let us now amend the model from question 1 to the following:

\[ y_t = \alpha x_t + e_t \]  
\[ x_t = \beta y_t + v_t \]

where \( e_t \sim i.i.d.N(0, \sigma_e^2) \) and \( v_t \sim i.i.d.N(0, \omega_t^2) \) where \( \omega_t^2 = \exp(\omega_0 + \omega z_t) \). Notice that \( z_t \) is now absent from equation (3). Instead it appears in the conditional variance specification for \( v_t \).

(a) Compute the elements of the covariance matrix for \( \{y_t, x_t\} \) conditional on the value of \( z_t \).
(b) Using your answer to part (a), construct a set of moment restrictions that can be used to estimate all the model parameters \( \{ \alpha, \beta, \omega, \omega_0 \text{ and } \sigma^2 \} \) by GMM. Explain your choice of instruments.

(c) Now assume, in addition, that \( z_t \sim i.i.d.(\mu_z, \sigma^2_z) \). Derive the set of moment conditions that implicitly define the maximum likelihood estimates of the model parameters.

(d) Examine the moment conditions derived in part (c). Explain what would happen if we attempted to estimate the model using these conditions if \( \sigma^2_z = 0 \).

3. Let \( z_t \) follow a random walk
\[
z_t = z_{t-1} + \eta_t
\]
with \( \eta_t \sim i.i.d. N(0, \sigma^2_\eta) \). Suppose that we do not observe \( z_t \) directly, but instead receive a noisy signal of the innovations \( \tilde{\eta}_t \), where
\[
\tilde{\eta}_t = \eta_t + \varpi_t
\]
with \( \varpi_t \sim i.i.d. N(0, \sigma^2_\varpi) \).

(a) Write the \( z_t \) and \( \tilde{\eta}_t \) processes in state space form.

(b) Use the state space form and the kalman filtering equations to derive the steady state kalman gain in terms of the model parameters.

(c) Let \( z_{t}^{err} \) denote the error in estimating the current level of \( z_t \) based on the history of observations on \( \tilde{\eta}_t \):
\[
z_{t}^{err} = z_t - E \left[ z_t | \{ \tilde{\eta}_{t-i} \}_{i \geq 0} \right]
\]
Prove that \( z_{t}^{err} \) converges to the following process
\[
z_{t}^{err} = z_{t-1}^{err} + \nu_t,
\]
where \( E[\nu_t] = 0 \) and \( Var[\nu_t] > 0 \) for any \( \sigma^2_\varpi > 0 \).

4. Consider the following model:
\[
\begin{align*}
    y_t &= \alpha_0 + \alpha \sigma^2_t + \varepsilon_t \quad \varepsilon_t \sim i.i.d. N(0, \sigma^2_t) \\
    \sigma^2_t &= \gamma_0 + \gamma \varepsilon_{t-1}
\end{align*}
\]
Data on \( y_t \) is provided in the file final_04.asc on my web page.

(a) Compute the maximum likelihood estimates of the model.

(b) Compute a wald test for the null hypothesis that \( \gamma = 0 \).

(c) A researcher claims that it is unnecessary to examine the significance of the maximum likelihood estimates of \( \gamma \) with a wald test. Instead all we have to do is regress \( y_t \) on a constant and its squared lagged value:
\[
y_t = \delta_0 + \delta y_{t-1}^2 + \xi_t.
\]

We can then test the significance of \( \delta \) with a t-test. Use the maximum likelihood estimates obtained from part (a) to compute the finite power of this test with a monte carlo simulation.

Your simulation should take the data sample as the relevant sample size. Be sure to describe the design of your simulation and the calculation used to compute the power of the test.

(d) Explain the result of your simulation.