The Behavior of Consumption in the RBC Open Economy Model

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Abstract

The RBC open economy model has been extensively used in the literature. An incomplete list of recent influential papers that use this model include Schmitt-Grohe and Uribe (2003), Neumeyer and Perri (2004), Aguiar and Gopinath (2007), and Boz, Daude, and Durdu (2011). In this note we demonstrate a theorem stating that in this model, for the calibration usually adopted in this literature, consumption is almost entirely and solely determined by the long-run level of TFP: After an impulse to TFP, consumption jumps to a level determined by the long-run level of TFP and stays there. This result has important implications for the interpretation of the results in this literature.

We make our point using exactly the same model as Aguiar and Gopinath (2007) (henceforth AG). This also makes clear that the limit result in Cao and L’Huillier (2014) can be extended to an economy with capital and labor supply.

We use the linearization in AG (note). The only difference is that

\[ b_{t+1} = B_{t+1} - \bar{B} \]

instead of the one used in the original paper in order to accommodate the case where \( \bar{B} = 0 \).

The proof is fairly involved. To make it readable, we simply consider the case with \( \gamma = 1 \), with corresponds to labor supply being exogenously given. One can restate the arguments for the case \( \gamma \neq 1 \) but \( \sigma = 1 \), which corresponds to endogenous labor supply, but additively separable from consumption (log-log preferences).

If \( \gamma = 1 \), \( n_t = 0 \). Under this assumption, the linearization of the production technology (AG 22) becomes

\[ \hat{y}_t = z_t + (1 - \alpha) k_t + \alpha g_t. \]

Plugging this in the linearization of the budget constraint (AG 20) and substituting \( x_t \) from (AG 21) implies:

\[ 0 = Y (z_t + (1 - \alpha) k_t + \alpha g_t) + GQ \bar{B} (g_t - \psi \bar{Q} b_{t+1}) + GQ b_{t+1} - b_t - K (Gk_{t+1} - (1 - \delta) k_t + \bar{C} g_t) - \bar{C} c_t. \]

Let \( \mathbf{X}_t = \begin{bmatrix} z_t & g_t & b_t & k_t \end{bmatrix}' \) denote the state space, and for further use we will also use \( \mathbf{X}_t^0 = \begin{bmatrix} b_t & k_t \end{bmatrix}' \) and \( \mathbf{X}_t^1 = \begin{bmatrix} z_t & g_t \end{bmatrix}' \). We use the general notation that for an economic variable \( y \),

\[ y_t = D_y \mathbf{X}_t = D_y^0 \mathbf{X}_t^0 + D_y^1 \mathbf{X}_t^1 = D_{yz} z_t + D_{yg} g_t + D_{yk} k_t + D_{yb} b_t. \]
We conjecture that:

\[ c_t = D_c X_t \]
\[ = D_c^0 X_t^0 + D_c^1 X_t^1 \]
\[ = D_{cz} z_t + D_{cg} g_t + D_{ck} k_t + D_{cb} b_t \]

**Proposition:** We will show the following limiting result similar to the one in Cao and L’Huillier (2014):

\[
\begin{align*}
\lim_{GQ \to 1} & \lim_{\psi \to 0} D_{ck} = 0 \\
\lim_{GQ \to 1} & \lim_{\psi \to 0} D_{cb} = 0 \\
\lim_{GQ \to 1} & \lim_{\psi \to 0} D_{cz} = 0
\end{align*}
\]

and

\[
\lim_{GQ \to 1} \lim_{\psi \to 0} D_{cg} = \frac{1}{1 - \rho_g}.
\]

In order to do so, we will solve for the loglinear dynamics of capital as well.

We conjecture that:

\[ k_{t+1} = D_k X_t \]
\[ = D_k^0 X_t^0 + D_k^1 X_t^1 \]
\[ = D_{kz} z_t + D_{kg} g_t + D_{kk} k_t + D_{kb} b_t \]

or

\[
\begin{bmatrix} c_t \\ k_{t+1} \end{bmatrix} = \begin{bmatrix} D_{cb} & D_{ck} \\ D_{kb} & D_{kk} \end{bmatrix} \begin{bmatrix} b_t \\ k_t \end{bmatrix} + \begin{bmatrix} D_{cz} & D_{cg} \\ D_{kz} & D_{kg} \end{bmatrix} \begin{bmatrix} z_t \\ g_t \end{bmatrix}.
\]

Substituting the conjecture for \( c_t \) and \( k_{t+1} \) into the budget constraint, we obtain:
\[ b_{t+1} GQ (1 - \psi QB) = b_t - Y z_t - (\alpha Y - KG - GQB) g_t \\
- (Y (1 - \alpha) - K (1 - \delta)) k_t \\
+ \left[ \begin{array}{c} C \\ KG \end{array} \right] \left[ \begin{array}{c} c_t \\ k_{t+1} \end{array} \right] \\
= \left[ 1 - (Y (1 - \alpha) - K (1 - \delta)) \right] X_0^t \\
+ \left[ - Y - (\alpha Y - KG - GQB) \right] X_1^t \\
+ CD_c^0 X_0^t + KG D_k^0 X_0^t \\
+ CD_c^1 X_1^t + KG D_k^1 X_1^t. \]

We regroup the coefficients on \( X_0^t \) and \( X_1^t \) to obtain:

\[ D_b^0 = \frac{1}{GQ (1 - \psi QB)} \left( \left[ 1 - (Y (1 - \alpha) - K (1 - \delta)) \right] + CD_c^0 + KG D_k^0 \right) \]

(1)

and

\[ D_b^1 = \frac{1}{GQ (1 - \psi QB)} \left( \left[ - Y - (\alpha Y - KG - GQB) \right] + CD_c^1 + KG D_k^1 \right). \]

More explicitly:

\[ D_{bb} = \frac{1}{GQ (1 - \psi QB)} \left( 1 + KG D_{kb} + CD_{cb} \right) \]

\[ D_{bk} = \frac{1}{GQ (1 - \psi QB)} \left( KG D_{kk} + CD_{ck} - (Y (1 - \alpha) - K (1 - \delta)) \right) \]

\[ D_{bz} = \frac{1}{GQ (1 - \psi QB)} \left( KG D_{kz} + CD_{cz} - Y \right) \]

\[ D_{bg} = \frac{1}{GQ (1 - \psi QB)} \left( KG D_{kg} + CD_{cg} - (\alpha Y - KG - GQB) \right) \]

From the loglinearizing equation of the Euler equation with respect to \( k \)
(AG 12), we have

\[
0 = \sigma E_t [c_{t+1} - c_t] \\
+ \beta G^{-\alpha} \phi E_t [g_{t+1}] \\
+ \beta G^{-\alpha} (1 - \alpha) E_t [Y (z_{t+1} + (1 - \alpha) k_{t+1} + \alpha g_{t+1})] \\
+ \beta G^{-\alpha} \phi E_t [k_{t+2}] \\
- \left( \beta G^{-\alpha} \left( (1 - \alpha) \frac{Y}{K} + \phi G^2 \right) + \phi G \right) k_{t+1} \\
- \left( \sigma + \phi G \right) g_t + \phi G k_t
\]

We will use the fact that

\[
E_t [g_{t+1}] = \rho g_t \\
E_t [z_{t+1}] = \rho z_t
\]

to simplify \( E_t [g_{t+1}] \) and \( E_t [z_{t+1}] \). We also use

\[
E_t [k_{t+2}] = E_t [D_k X_{t+1}] \\
= E_t [D_{k,h} D_b X_t + D_{k,b} D_k X_t] + D_{k,z} \rho_z z_t + D_{k,g} \rho_g g_t \\
= (D_{k,h} D_b + D_{k,b} D_k) X_t + D_{k,z} \rho_z z_t + D_{k,g} \rho_g g_t
\]

and

\[
E_t [c_{t+1}] = E_t [D_c X_{t+1}] \\
= (D_{c,b} D_b + D_{c,k} D_k) X_t + D_{c,z} \rho_z z_t + D_{c,g} \rho_g g_t
\]

and

\[
E_t [c_t] = D_c X_t.
\]
So (2) simplifies to

\[ 0 = \sigma \left( (D_{c,b}D_b + D_{c,k}D_k) X_t + D_{c,z} \rho_z z_t + D_{c,g} \rho_g g_t - D_c X_t \right) \]
\[ + \beta \bar{G}^{-\sigma} \phi \rho_g g_t \]
\[ + \beta \bar{G}^{-\sigma} (1 - \alpha) \frac{\bar{Y}}{K} (\rho_z z_t + \alpha \rho_g g_t) \]
\[ + \beta \bar{G}^{-\sigma} (1 - \alpha) \frac{\bar{Y}}{K} (1 - \alpha) D_k X_t \]
\[ + \beta \bar{G}^{-\sigma} \phi \left( (D_{k,b}D_b + D_{k,k}D_k) X_t + D_{k,z} \rho_z z_t + D_{k,g} \rho_g g_t \right) \]
\[ - \left( \beta \bar{G}^{-\sigma} \left( (1 - \alpha) \frac{\bar{Y}}{K} + \phi \bar{G}^2 \right) + \phi \bar{G} \right) D_k X_t \]
\[ - (\sigma + \phi \bar{G}) g_t + \phi \bar{G} k_t. \]

Now, extracting the components related to \( X_t^0 \) from this equation, we have

\[ 0 = \sigma \left( (D_{c,b}D_b^0 + D_{c,k}D_k^0) X_t^0 - D_c^0 X_t^0 \right) \]
\[ + \beta \bar{G}^{-\sigma} (1 - \alpha) \frac{\bar{Y}}{K} (1 - \alpha) D_k^0 X_t^0 \]
\[ + \beta \bar{G}^{-\sigma} \phi \left( (D_{k,b}D_b^0 + D_{k,k}D_k^0) X_t^0 \right) \]
\[ - \left( \beta \bar{G}^{-\sigma} \left( (1 - \alpha) \frac{\bar{Y}}{K} + \phi \bar{G}^2 \right) + \phi \bar{G} \right) D_k^0 X_t^0 \]
\[ + \phi \bar{G} k_t \]

for all \( X_t^0 \). This implies

\[ 0 = \sigma \left( (D_{c,b}D_b^0 + D_{c,k}D_k^0) - D_c^0 \right) \]
\[ + \beta \bar{G}^{-\sigma} (1 - \alpha) \bar{Y} (1 - \alpha) D_k^0 \]
\[ + \beta \bar{G}^{-\sigma} \phi \left( D_{k,b}D_b^0 + D_{k,k}D_k^0 \right) \]
\[ - \left( \beta \bar{G}^{-\sigma} \left( (1 - \alpha) \frac{\bar{Y}}{K} + \phi \bar{G}^2 \right) + \phi \bar{G} \right) D_k^0 \]
\[ + \phi \bar{G} \begin{bmatrix} 0 & 1 \end{bmatrix} \]
or

\[
0 = \sigma ((D_{c,b}D_{0}^b + D_{c,k}D_{0}^b) - D_c^0) \\
+ \beta \overline{G}^{-\sigma} (1 - \alpha) \overline{Y} (1 - \alpha) D_k^0 \\
+ \beta \overline{G}^{2-\sigma} \phi (D_{k,b}D_{0}^b + D_{k,k}D_{0}^b) \\
- \left( \beta \overline{G}^{-\sigma} \left( (1 - \alpha) \frac{\overline{Y}}{K} + \phi \overline{G}^2 \right) + \phi \overline{G} \right) D_k^0 \\
+ \phi \overline{D}_k
\]

(3)

where \( \overline{D}_k = \overline{G} \left[ \begin{array}{c} 0 \\ 1 \end{array} \right] \). This equation helps determines \( D_k^0 \), i.e., \( D_{k,b} \) and \( D_{k,k} \) as functions of \( D_{c,b} \) and \( D_{c,k} \). In particular when \( D_{c,b} \) and \( D_{c,k} \) are close to zero, we have the Taylor expansion:

\[
D_{k,b} = \alpha_1 D_{c,b} + \beta_1 D_{c,k} + o(D_{c,b}) + o(D_{c,k}) \\
D_{k,k} = D_{k,k}^* + \alpha_2 D_{c,b} + \beta_2 D_{c,k} + o(D_{c,b}) + o(D_{c,k}),
\]

(4)

where \( D_{k,k}^* \) is the solution of

\[
0 = \left( -\beta \overline{G}^{-\sigma} \left( (1 - \alpha) \frac{\overline{Y}}{K} + \phi \overline{G}^2 \right) + \phi \overline{G} \right) D_{k,k}^* \\
+ \beta \overline{G}^{2-\sigma} \phi D_{k,k}^* D_{k,k}^* + \phi \overline{G},
\]

i.e. equation 3 for \( D_{k,k}^* \) when \( D_c^0 = 0 \).

Armed with the solution (4), we now use the first order condition for \( b' \) (equation AG 17):

\[
0 = \sigma \mathbf{E}_t [c_{t+1} - c_t] + \sigma g_t - \psi \overline{Q} b_{t+1}
\]

(5)

and extract the coefficients on \( X_t^0 \) to obtain

\[
0 = \sigma ((D_{c,b}D_{0}^b + D_{c,k}D_{0}^b) - D_c^0) - \psi \overline{Q} D_b^0.
\]

From (1), we have

\[
D_b^0 = \frac{1}{GQ (1 - \psi \overline{Q} B)} (\overline{D}_b + \overline{C} D_c^0 + \overline{K} \overline{G} D_k^0),
\]

where

\[
\overline{D}_b = \left[ 1 - (\overline{Y} (1 - \alpha) - \overline{K} (1 - \delta)) \right].
\]
Therefore
\[ 0 = \left( D_{c,b} - \frac{\psi}{\sigma} Q \right) \frac{1}{GQ (1 - \psi QB)} \left( \overline{D}_b + \overline{C} D_c^0 + \overline{K} G D_k^0 \right) + D_{c,k} D_k^0 - D_c^0. \]

We separate the equations for \( D_{c,b} \) and \( D_{c,k} \) to obtain

\[ 0 = \left( D_{c,b} - \frac{\psi}{\sigma} Q \right) \frac{1}{GQ (1 - \psi QB)} \left( \overline{D}_{b,b} + \overline{C} D_{c,b} \right) - D_{c,b} \]
\[ + \left\{ \left( D_{c,b} - \frac{\psi}{\sigma} Q \right) \frac{1}{GQ (1 - \psi QB)} \overline{K} G + D_{c,k} \right\} D_{k,b} \tag{6} \]

and

\[ 0 = \left( D_{c,b} - \frac{\psi}{\sigma} Q \right) \frac{1}{GQ (1 - \psi QB)} \left( \overline{D}_{b,k} + \overline{C} D_{c,k} \right) - D_{c,k} \]
\[ + \left\{ \left( D_{c,b} - \frac{\psi}{\sigma} Q \right) \frac{1}{GQ (1 - \psi QB)} \overline{K} G + D_{c,k} \right\} D_{k,k}. \tag{7} \]

We first solve for \( D_{c,k} \) from the second equation (7):

\[ D_{c,k} = \frac{\left( D_{c,b} - \frac{\psi}{\sigma} Q \right) \left( \frac{1}{GQ (1 - \psi QB)} \overline{D}_{b,k} + \frac{1}{GQ (1 - \psi QB)} \overline{K} G D_{k,k} \right)}{1 - \left( D_{c,b} - \frac{\psi}{\sigma} Q \right) \frac{1}{GQ (1 - \psi QB)} \frac{1}{GQ (1 - \psi QB)} \overline{C}}. \]

As \( \psi \) goes to zero, this equation simplifies to

\[ D_{c,k} = \frac{D_{c,b} \left( \frac{1}{GQ} \overline{D}_{b,k} + \frac{1}{GQ} \overline{K} G D_{k,k} \right)}{1 - D_{c,b} \frac{1}{GQ} \frac{1}{GQ} \overline{C}}. \tag{8} \]

Moreover equation (6) simplifies to

\[ 0 = D_{c,b} \frac{1}{GQ} \left( \overline{D}_{b,b} + \overline{C} D_{c,b} \right) - D_{c,b} \]
\[ + \left\{ D_{c,b} \frac{1}{GQ} \overline{K} G + D_{c,k} \right\} D_{k,b}. \tag{9} \]

Plugging (8) into (9) and grouping by \( D_{c,b} \) (also by definition \( \overline{D}_{b,b} = 1 \)), we
have

\[ 0 = \frac{1}{\bar{G}Q} \left(1 + \bar{C}D_{c,b}\right) - 1 \]
\[ + \left\{ \frac{1}{\bar{G}Q}K\bar{G} + \frac{1}{\bar{G}Q}D_{b,k} + \frac{1}{\bar{G}Q}K\bar{G}D_{k,k} \right\} D_{k,b}. \]

At this point, we can use the solution for \( D_{k,b} \) and \( D_{k,k} \) from (4), and simplify to obtain

\[ \bar{G}Q - 1 = \alpha_c D_{c,b} + o(D_{c,b}), \]

where the constant \( \alpha_c > 0 \) also depends on the constant \( \alpha_1, \beta_1, \alpha_2, \beta_2 \) from (4).

Therefore

\[ \lim_{\bar{G}Q \to 1} \lim_{\psi \to 0} D_{c,b} = 0. \]

Then (8) implies that

\[ \lim_{\bar{G}Q \to 1} \lim_{\psi \to 0} D_{c,k} = 0. \]

To determine the coefficients \( D_{c,g} \) and \( D_{c,z} \), we extract the coefficient on \( X_1^t \) from equation (5): extract the coefficients on \( X_0^t \) to obtain

\[ \sigma (\rho_g - 1) D_{c,g} + \sigma - \psi Q D_{b,g} = 0 \]

and

\[ \sigma (\rho_z - 1) D_{c,z} - \psi Q D_{b,z} = 0 \]

so

\[ \lim_{\psi \to 0} D_{c,g} = \frac{1}{1 - \rho_g} \]

and

\[ \lim_{\psi \to 0} D_{c,z} = 0. \]

References


