Optimal Money and Debt Management: 
Friedman and Barro Revisited

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Abstract

How do the fundamental insights of Milton Friedman[16] and Robert Barro[6] fare when we recognize the fact that government bonds provide liquidity services? To answer this question, we extend a standard cash and credit good model by assuming that bonds may be used as collateral in the purchase of some kinds of consumption goods. In the extended model, the government cannot use its liabilities to finance deficits without affecting the amount of liquidity available to support consumption. The insights of Friedman and Barro survive largely intact if the government has an additional debt instrument that is Ricardian – in the sense that its outstanding stock has no direct effect on consumption decisions. In particular, an extended Friedman Rule is optimal when prices are flexible, and when prices are sticky, the Ramsey planner will choose to smooth short run tax distortions at the expense of longer run consumption (when adjusting to, say, an increase in government spending). Absent a Ricardian debt instrument, the planner’s options are much more limited. It would not appear that governments currently have available to them such a debt instrument.

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1 Introduction

Milton Friedman[16] and Robert Barro[6] made seminal contributions to our understanding of the optimal management of money and public debt. Friedman held that the cost of producing fiat money is negligible, and therefore real money balances should be driven to satiation levels. This could be accomplished using the celebrated Friedman Rule: drive the interest rate that measures the opportunity cost of holding money to zero. In a largely forgotten part of his essay, Friedman noted that bonds also provide liquidity. However, Friedman had private debt in mind; so he missed a potentially important link between monetary and fiscal policy.

Barro held that short run tax distortions could be alleviated at the expense of some long run consumption. In response to, say, a temporary increase in government spending, new liabilities should be issued to smooth the path of the tax hikes that would ultimately be needed; long run consumption would of course have to fall a little to service the higher level of debt. In stochastic versions of Barro’s world, debt and taxes follow unit root processes as they move to efficiently absorb fiscal shocks.\footnote{Barro’s original contribution did not include uncertainty and focused on real debt. The unit root implication of his tax smoothing result is highlighted in discussions of stochastic general equilibrium models with price rigidities. See for example Albanesi[3].} However, Barro did not contemplate the possibility that government bonds might provide liquidity to the private sector, and that the paths of both money and debt may affect the path of equilibrium allocations. So he too missed a potentially important link between monetary and fiscal policy.

In this paper, we ask how the insights of Friedman and Barro fare when we assume that money and government bonds provide differing degrees of liquidity to the private sector. While the Friedman Rule is optimal in many models, it is not optimal in all
models. So, we begin with a commonly used cash and credit good model in which the Friedman Rule is optimal under flexible prices; adding price rigidities, public liabilities should be used to smooth tax distortions, ala Barro. Then, we expand the model in a natural way to allow for the liquidity of government bonds. We do this by assuming that bonds may be used as collateral in the purchase of some kinds of consumption goods.

In what follows, we will refer to a government debt instrument as being "Ricardian" if its outstanding (real) stock has no direct effect on consumption decisions. Money is clearly not a Ricardian debt instrument since it provides liquidity that is necessary to purchase cash goods. In the standard cash and credit good model, government bonds are Ricardian in this sense. But when we extend the model to let government bonds provide liquidity, then public debt is no longer Ricardian. Absent a Ricardian debt instrument, the government cannot choose to finance a short run deficit (ala Barro) independently of its decision of how much liquidity to provide for consumption. This basic fact will play a prominent role in our results.

If the government has a Ricardian debt instrument, then the Ramsey problem for our expanded model is similar to that of the standard cash and credit good model. An Extended Friedman Rule is optimal when prices are flexible; that is, optimality requires satiation in both money and government bonds, and this can be achieved by

\[2\text{See Woodford.[28] and DeFiore and Teles.[14]}\]

\[3\text{We are of course not the first to study the liquidity services of bonds. In addition to Friedman, contributions to the literature include: Patinkin[24], Bansal and Coleman[8], Holmstrom and Tirole[18], Calvo and Vegh[10], Hu and Kam[19], and Linnemann and Schabert[23]. Woodford[29], Aiyagari and McGratten[2] and Angeletos et al[4] study the role of bonds as collateral in real models with heterogenous agents.}\]

\[4\text{Note however that the standard model does not obey the "Ricardian Principle" unless the government is able to finance its spending with lump sum taxes.}\]

driving the interest rates that measure the opportunity costs of holding money and bonds to zero. Absent a Ricardian debt instrument, the Extended Friedman Rule is optimal in the steady state, but the cash and collateral constraints may be binding in the transition to it.

When we introduce staggered price setting (thereby entering an environment considered by Barro), the need for a Ricardian debt instrument becomes even more apparent. Without such an instrument, the government would not want to smooth short run tax distortions by running deficits; issuing new liabilities would distort consumption patterns. In this case, the Ramsey solution is stationary; the unit root characterization of optimal taxes and debt is lost.

In this paper, we characterize the Ricardian debt instrument that is needed as a nominal government asset that can be used to finance a temporary deficit. We model these assets as government loans to the private sector. We will discuss the possible interpretations of this in the conclusion.

A brief review of the literature will put our work in context. Chari, Christiano, and Kehoe[11][12] – using a cash and credit goods framework – extend Friedman’s discussion to a stochastic general equilibrium model with nominal government bonds and a distortionary tax on labor. In their model, the Friedman Rule characterizes monetary policy in the Ramsey solution. Their basic insight is that unanticipated inflation provides a non-distortionary tax on nominal assets, and these tax revenues can be used to limit fluctuations in the distortionary wage tax. Optimal inflation is very volatile in their model, but that does not matter since they assume prices are flexible. More recently, Benigno and Woodford[7] and Schmitt-Grohe and Uribe[26][27] add staggered price setting to the mix. The inefficient price dispersion created by stag-

\footnote{In a calibrated exercise, the authors show that the labor tax is quite stable, while the standard deviation of annual inflation is 20 percent.}
gered price setting leads to a fundamental inflation tradeoff for the Ramsey planner: the price dispersion can be eliminated by holding the aggregate price level constant, but this conflicts with the use of unanticipated inflation as a fiscal shock absorber and with implementation of the Friedman Rule. In numerical exercises, the tradeoff is generally resolved in favor of price stability, and the Friedman Rule is lost.

Finally, Correia, Nicolini, and Teles [13] reach a striking conclusion: with a sufficiently rich menu of taxes, nominal rigidities are irrelevant to the optimal conduct of monetary policy. They consider a stochastic economy with cash goods and credit goods, monopolistically competitive firms, and nominal debt. Optimal tax rates are set on labor income, dividends, and consumption. The authors show that the Ramsey solution for an economy with nominal rigidities is identical to that for an economy with flexible prices; the Friedman rule is optimal despite the nominal rigidities.

We reviewed this literature extensively in Canzoneri, Cumby and Diba[9]. The model we used there was essentially the Correia-Nicolini-Teles model, minus the consumption tax; here, we call that model the Standard Model. We extend the Standard Model in a natural way to allow for the liquidity of government bonds; that is, we add a third good – a bond good – for which consumers hold bonds as collateral. We call the extended model the Liquid Bonds (or LB) Model. The LB Model comes in two forms: in the Benchmark LB Model, the government is able to buy private

\[ \text{Equation} \]

Eliminating this tax breaks up Correia et al’s basic result, and restores the fundamental inflation tradeoff described above. Our reasons for eliminating this tax were discussed at length in our handbook chapter.

Unfortunately, neither the Standard Model nor the LB Model offers a deep theory of liquidity demand. Such theories do exist. For example, Holmstrom and Tirole[18] show private provision of liquidity will be inefficiently low and that public debt can fill the gap. And, other theories of money abound. Here, we use the financial frictions that have generally been used in discussions of optimal money and debt management.
sector bonds (which have no liquidity value); that is, it has available the Ricardian
debt instrument discussed above. In the Constrained LB Model, the government is
not able to lend to the private sector.

Before we proceed to the analysis, we should perhaps discuss our assertion that
government bonds provide liquidity services. The basic premise should not be con-
troversial. U.S. Treasuries facilitate transactions in a number of ways: they serve
as collateral in many financial markets, banks hold them to manage the liquidity of
their portfolios, individuals hold them in money market accounts that offer checking
services, and importers and exporters use them as transactions balances (since so
much trade is invoiced in dollars). The empirical literature finds a liquidity premium
on government debt, and moreover that the premium depends upon the quantity of
debt.

Empirical contributions to the this literature include: Friedman and Kuttner[15],
who study the imperfect substitutability of commercial paper and U.S. Treasuries;
Greenwood and Vayanos[17], who find that the supply of long-term relative to short-
term bonds is positively related to – and predicts – the term spread. Krishnamurthy
and Vissing-Jorgensen[20], who find that the spread between liquid treasury securities
and less liquid AAA debt moves systematically with the quantity of government
debt; and Pflueger and Viceira[25], who document a liquidity premium in the spread
between inflation-indexed and nominal government bonds.

The paper proceeds as follows: In section 2, we present both versions of the LB
Model, and show that it encompasses the Standard Model. In the LB Model the
government has two seigniorage taxes at its disposal: the usual one on cash holdings,
and a new one on government bond holdings. We also discuss how the various
taxes distort household decisions. In section 3, we compare the Ramsey solutions to
these models. And in section 4, we briefly summarize our conclusions and discuss
the availability of Ricardian debt instruments. In an appendix, we provide a full derivation of the Ramsey solution to the LB Model in the case of flexible prices.

2 The Liquid Bonds (LB) Model

The basic structure of our LB Model is easily explained. There are three consumption goods: a cash good, $c_m$, a bond good, $c_b$, and a credit good, $c_c$. Households face a cash in advance constraint for the cash good and a collateral constraint for the bond good; government bonds serve as collateral. Households pay for their credit and bond goods, receive their wages and dividends, and pay their taxes at the beginning of the period that follows. A competitive bundler produces a perishable final product, $y$, which can be sold as a cash good, a bond good, or a credit good. Government purchases, $g$, are assumed to be credit goods. Consolidating the treasury and the central bank, the government has four taxes at its disposal: a labor tax, $\tau_w$, a profits tax, $\tau_Y$, and two seigniorage taxes, one on money and the other on bonds. All of the taxes are distortionary except for the profits tax. In the Benchmark LB Model, the government is able to lend to the private sector; that is, it can hold bonds issued by the private sector. These private sector bonds cannot be used as collateral in purchasing the bond good. In the Constrained LB Model, the government is not allowed to hold private sector bonds, even though (as we shall see) it would be welfare improving for it to do so.

The LB Model encompasses the Standard Model, which is obtained by dropping the bond good (and therefore the seigniorage tax on bonds). Government bonds do not provide liquidity in the Standard Model, and there is not an optimum quantity of debt in the steady state. In our numerical exercises, the steady state value of the debt is pinned down by initial conditions; to facilitate comparisons, we set the debt
to GDP ratio equal to the optimal debt to GDP ratio in the LB Model. Apart from such issues, the Standard Model is identical to the model we analyzed in some detail in Canzoneri, Cumby and Diba[9]. In the next subsection, we present the LB Model, and describe the way in which it reduces to the Standard Model.

2.1 Structure of the Liquid Bonds (LB) Model

In each period $t$, one of a finite number of events, $s_t$, occurs. $s^t$ denotes the history of events, $(s_0, s_1, ..., s_t)$, up until period $t$. The initial realization, $s_0$, is given. The probability of the occurrence of state $s^t$ is $\rho(s^t)$.

A continuum of monopolistically competitive firms produce intermediate goods using a common technology, which is linear in labor and has an aggregate productivity shock, $z(s^t)$. Competitive retailers buy the intermediate goods and bundle them into the final good, using a CES aggregator with elasticity $\sigma$. The final goods are then sold to households (as cash goods, bond goods, or credit goods), and to the government (as credit goods). The feasibility constraint is then

$$y(s^t) = c_m(s^t) + c_b(s^t) + c_c(s^t) + g(s^t)$$

(1)

Government purchases are assumed to follow an exogenous AR(1) processes in our numerical exercises. There is no $c_b(s^t)$ in the Standard Model.

Labor markets are competitive; there is no wage rigidity. However, intermediate goods producers engage in Calvo price setting. In every period, each producer gets to reset its price with probability $1 - \alpha$; otherwise, its price remains unchanged from the previous period. There is no price indexation to lagged inflation, or to steady state inflation.
The utility of the representative household is

\[ U = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \rho(s^t) u \left[ c_m \left( s^t \right), c_b \left( s^t \right), c_c \left( s^t \right), n_t \left( s^t \right) \right] \tag{2} \]

\[ = E \sum_{t=0}^{\infty} \beta^t u \left[ c_{m,t}, c_{b,t}, c_{c,t}, n_t \right] \]

where \( n_t \) is hours worked. As these equations suggest, we will suppress the notation for the state, \( s^t \), when we think that it will cause no confusion.

In period \( t \) (or more precisely, state \( s^t \)), households may purchase state contingent securities, \( x(s^{t+1}) \), that pay one dollar in state \( s^{t+1} \) and cost \( Q(s^{t+1}|s^t) \). Households can construct a riskless bond, \( B^c \), by purchasing a portfolio of claims that costs one dollar and pays a gross rate of return \( I^c(s^t) \) in every future state \( s^{t+1} \):

\[ 1 = I^c(s^t) \sum_{s^{t+1}|s^t} Q(s^{t+1}|s^t) \tag{3} \]

In our LB Model, there are actually two risk free bonds, \( B^c \) and \( B \). \( B^c \) is issued by the private sector and cannot be used as collateral to purchase bond goods. We will refer to \( I^c \) as the CCAPM rate (for reasons that will become evident in the next subsection). Since households are identical in our models, there will be no private sector demand for \( B^c \) in equilibrium. However, the government can hold private sector bonds in the Benchmark LB Model; so, the net demand for these bonds will be \( B^c_g(s^t) \), the government’s holding of \( B^c \). In the Constrained LB Model, \( B^c_g = 0 \) is a constraint on government behavior. \( B \) is the bond issued by the government, and it pays a gross rate of return \( I \). This is the liquid bond that can be used as collateral in the purchase of bond goods. In the Standard Model, government bonds do not provide liquidity, and \( B \) and \( B^c \) are perfect substitutes and \( I = I^c \).

Each period is divided into two exchanges. In the financial exchange, public and private agents do all of their transacting except for the actual buying and selling.
of the final product; purchases of the final good occur in the goods exchange that follows, subject to the cash and collateral constraint: $M_t \geq P_t c_{m,t}$ and $B_t \geq P_t c_{b,t}$, where $P_t$ is the price of the final product. Once again, there is of course no bond good or collateral constraint in the Standard Model. Households come into the financial exchange in period $t$ with nominal wealth $A(s^t)$. Their budget constraint in the financial exchange is

$$M(s^t) + B(s^t) - B^c(s^t) + \sum_{s^{t+1}|s^t} Q(s^{t+1}|s^t)x(s^{t+1}) \leq A(s^t)$$

(4)

where $x(s^{t+1})$ is the number of dollar claims purchased for state $s^{t+1}$. The evolution of wealth is governed by

$$A(s^{t+1}) = I(s^t)B(s^t) - \Gamma(s^t)B^c(s^t) + x(s^{t+1}) + [M(s^t) - P(s^t)c_m(s^t)]$$

$$- P(s^t)[c_b(s^t) + c_c(s^t)] + [1 - \tau_w(s^t)]W(s^t)n(s^t) + [1 - \tau_Y(s^t)]\Upsilon(s^t)$$

(5)

where $\Upsilon$ is profits.

First, we discuss the pricing of state contingent claims, $Q(s^{t+1}|s^t)$, and conditions required for the cash and collateral constraints to be binding. This will be useful when we derive the Ramsey solution. Then, we turn to the first order conditions that illustrate the tax distortions.
2.2 Pricing of State Contingent Claims

Households maximize

$$V[A(s^t)] = \max \{ u \left[ c_m(s^t), c_b(s^t), c_c(s^t), n_t(s^t) \right]$$

$$+ \lambda^m(s^t) [M(s^t) - P(s^t) c_m(s^t)]$$

$$+ \lambda^b(s^t) [B(s^t) - P(s^t) c_b(s^t)]$$

$$+ \lambda^A(s^t) [A(s^t) - M(s^t) - B(s^t) + B^c(s^t) - \sum_{s^{t+1} | s^t} Q(s^{t+1} | s^t) z(s^{t+1})]$$

$$+ \beta \sum_{s^{t+1} | s^t} \rho(s^{t+1} | s^t) V[A(s^{t+1})] \}$$

First order conditions from this maximization give the pricing equation for state contingent claims

$$Q(s^{t+1} | s^t) = \beta \rho(s^{t+1} | s^t) \left[ \frac{u_m(s^{t+1})}{u_m(s^t)} \frac{P(s^t)}{P(s^{t+1})} \right]$$

The first order conditions also imply that $$I_t \leq I^c_t$$; this reflects the fact that government bonds pay a non-pecuniary return (or yield transactions services). Moreover, the cash in advance constraint is binding ($$\lambda^m(s^t) > 0$$) if $$I^c_t > 1$$ and the collateral constraint is binding ($$\lambda^b(s^t) > 0$$) if $$I^c_t > I_t$$.

2.3 Tax Distortions and Euler Equations

The household’s first order conditions imply

$$u_{m,t} = I_t^c u_{c,t}$$

If the household gives up one dollar’s worth of the cash good, it can spend $$I_t^c$$ dollars on the credit good, because credit goods avoid the cash in advance constraint. Since the final good can be sold as either a cash good or a credit good, the marginal rate of transformation is one for one. So if $$I_t^c > 1$$, the household buys too few cash goods.
$I^c_t - 1$ is the standard seigniorage tax on cash goods, and it is distortionary. This tax is also available to the government in the Standard Model.

Similarly, the first order conditions imply

$$u_{b,t} = (1 + I^c_t - I_t) u_{c,t}$$

If the household gives up one dollar’s worth of the bond good, it can spend $1 + I^c_t - I_t$ dollars on the credit good, because credit goods avoid the collateral constraint. So if $I^c_t - I_t > 0$, the household buys too few bond goods. $I^c_t - I_t$ is a new seigniorage tax on bond goods, and again this tax is distortionary. This tax is not available in the Standard Model.

The bond good - cash good margin can also be distorted by the seigniorage taxes. From (8) and (9),

$$u_{b,t} = \left( \frac{1 + I^c_t - I_t}{I_t^c} \right) u_{m,t}$$

The wage tax, $\tau_{w,t}$, is also distortionary. First order conditions imply

$$-u_{n,t} = (1 - \tau_{w,t}) \left( \frac{W_t}{P_t} \right) u_{c,t}$$

$$-u_{n,t} = \left( \frac{1 - \tau_{w,t}}{I_t^c} \right) \left( \frac{W_t}{P_t} \right) u_{m,t}$$

$$-u_{n,t} = \left( \frac{1 - \tau_{w,t}}{1 + (I_t^c - I_t)} \right) \left( \frac{W_t}{P_t} \right) u_{b,t}$$

Note that $\tau_{w,t}$, $I^c_t - 1$ and $I^c_t - I_t$ distort the consumption - leisure margins; they make the household want to work less than is efficient.

Now, we can derive the Euler Equations for the three goods. Recalling that the gross nominal CCAPM rate is defined by

$$1 = I^c(s^t) \sum_{s^{t+1}|s^t} Q(s^{t+1}|s^t)$$
and using the pricing equation for contingent claims, (7), we arrive at the standard Euler equation

$$1 = I_t c^e E_t \left[ \frac{u_{m, t+1}}{u_{m, t}} \frac{P_t}{P_{t+1}} \right]$$

(15)

Then, (8) implies the Euler equation for credit goods

$$1 = \beta E_t \left[ \frac{I_{t+1} c}{u_{c, t+1}} \frac{P_t}{P_{t+1}} \right]$$

(16)

Note that intertemporal smoothing of the credit good depends on next period’s interest rate; that is, $I_{t+1} c$ appears in this Euler equation instead of $I_t c$; The reason is that the credit good is not paid for until period $t + 1$. Finally, (9) implies the Euler Equation for bond goods

$$1 = \beta E_t \left[ I_{t+1} c \left( \frac{1 + I_t c - I_t}{1 + I_{t+1} c - I_{t+1}} \right) \left( \frac{u_{b, t+1}}{u_{b, t}} \frac{P_t}{P_{t+1}} \right) \right]$$

(17)

The difference in the timing of these seigniorage taxes will play a role in the next section.

2.4 The Government’s Present Value Budget Constraint

So, where does the second seigniorage tax – the one on bonds – show up in the government’s fiscal accounting? In the LB Model, the consolidated government flow budget constraint can be written as

$$M_t + B_t - B_{g,t}^c + S_t = M_{t-1} + I_{t-1} B_{t-1} - I_{t-1} c B_{g,t-1}^c$$

(18)

where $S_t$ is the primary surplus, and where it will be recalled that $B_{g,t}^c$ is the government’s stock of private sector bonds. Let $L_t \equiv M_{t-1} + I_{t-1} B_{t-1} - I_{t-1} c B_{g,t-1}^c$ be total government liabilities at the beginning of period $t$. Then, iterating the flow budget constraint forward, and applying the household transversality condition, yields the
present value budget constraint.

\[ \frac{L_t}{P_t} = E_t \sum_{j=0}^{\infty} \alpha_t^{c_{t+j}} \left[ S_{t+j} + \left( I_{t+j}^c - 1 \right) \left( M_{t+j} - I_{t+j}^c \right) \right] \quad (19) \]

where \( \alpha_t^{c_{t+j}} \) is the stochastic discount factor. The next to last term in the equation is the standard seigniorage tax. The last term is the seigniorage tax on bonds; it reflects the fact that the government borrows at a discount, due to the non-pecuniary return on its bonds. Of course, this last tax is not available in the Standard Model.

3 Optimal Money and Debt Management

We can solve the Ramsey problem analytically for the Benchmark LB Model with flexible prices, and we will begin with that case. Then, we will discuss the complications that arise in the Constrained LB Model, where the government does not have access to a Ricardian debt instrument. And finally, we will relate our results to those found (by others) in the Standard Model.

With staggered price setting, we can no longer solve the Ramsey problem analytically. The reason is that, as noted by Schmitt-Grohe and Uribe\cite{26} and others, price rigidity adds a new state variable, one cannot reduce the sequence of flow implementability constraints to a single present value constraint. However, we can illustrate the optimal solution using numerical methods. Here, the results are quite different for the Benchmark LB Model and the Constrained LB Model.

In what follows, we will work with the utility function

\[ u(c_{m,t}, c_{b,t}, c_{c,t}, n_t) = \phi_m \log(c_{m,t}) + \phi_b \log(c_{b,t}) + \phi_c \log c_{c,t} - \frac{1}{2} \eta_t^2, \quad (20) \]

although our results readily extend to more general preferences. In particular, as explained in the Appendix, our derivation of the Ramsey solution for the Benchmark
LB Model with flexible prices extends to preferences that are homothetic over the three consumption goods and weakly separable across employment and consumption. In our numerical exercises, we will also let \( \phi_m = \phi_b = \phi_c \); this symmetry will make some of the impulse response functions easier to interpret.

One final note before we begin the derivations: \( \Lambda(s^t) \equiv m(s^t) + b(s^t) - b_g(s^t) \) represents net real government liabilities. We will let the Ramsey planner set the initial values of \( m(s^0) \), \( b(s^0) \), and \( b_g(s^0) \) subject to a given initial value of \( \Lambda(s^0) \) \((\geq 0)\). This allows the planner to choose arbitrarily large values of initial gross liabilities, \( m(s^0) + b(s^0) \), in the Benchmark LB Model (the only model for which \( b_g(s^t) \) is available). The choice of initial gross liabilities will be important in what follows.

### 3.1 The Case of Flexible Prices

We begin with the Benchmark LB Model. We present the full derivation of the Ramsey Solution in an appendix. In the main text, we present a sketch of the derivation, give intuition for how the solution works, and illustrate the intuition with impulse response functions from a numerical example.\(^{10}\)

\(^{9}\)There are important questions about how we should treat the choice of the initial conditions (or the price level, given some initial nominal liabilities). However, these issues are well known, and we have nothing new to contribute.

\(^{10}\)For our numerical examples, the Calvo parameter, \( \alpha \), is set at 0.75 for the case of staggered price setting. Government spending follows an AR(1) process, with the auto-regressive parameter set at 0.9. \( \sigma \), the CES aggregator for the final good is set equal to 7. The steady state debt to GDP ratio in the Standard Model is set equal to the optimal steady state debt to GDP ratio in the Benchmark LB Model.
3.1.1 The Benchmark LB Model with Flexible Prices

We follow the usual procedure in solving for the Ramsey problem: we derive flow implementability conditions by using household first order conditions to eliminate relative prices and tax rates in the household’s flow budget constraints; then, we choose the quantities of consumption and labor that optimize household utility subject to the relevant constraints. However, the Ramsey planner will also choose \( m(s^t) \) and \( b(s^t) \), because we have inequality constraints on the consumption of cash and bond goods. The Ramsey planner will also choose its holdings of private sector debt, \( b^c_g(s^t) \). As we shall see, the flow implementability conditions reduce to a single present value condition if this Ricardian debt instrument is available to the planner.

The first step is to derive the flow implementability conditions, and incorporate the complimentary slackness conditions into them. As shown in the appendix, the household’s flow budget constraint can be written as

\[
A(s^t) = \sum_{s^{t+1} | s^t} [Q(s^{t+1} | s^t)A(s^{t+1})] + M(s^t) \left[ 1 - \frac{1}{I^c(s^t)} \right] + B(s^t) \left[ 1 - \frac{I(s^t)}{I^c(s^t)} \right] + \frac{P(s^t)}{I^c(s^t)} \left[ c_m(s^t) + c_b(s^t) + c_c(s^t) \right] - \left[ 1 - \frac{\tau_w(s^t)}{I^c(s^t)} \right] W(s^t)n(s^t)
\]  

(21)

The cash in advance constraint is binding for \( I^c(s^t) > 1 \), and we have \( I^c(s^t) = 1 \) if this constraint is not binding. So for all \( I^c(s^t) \geq 1 \), we have

\[
M(s^t) \left[ 1 - \frac{1}{I^c(s^t)} \right] = P(s^t)c_m(s^t) \left[ 1 - \frac{1}{I^c(s^t)} \right]
\]  

(22)

or, rearranging terms,

\[
M(s^t) \left[ 1 - \frac{1}{I^c(s^t)} \right] + \frac{P(s^t)c_m(s^t)}{I^c(s^t)} = P(s^t)c_m(s^t)
\]  

(23)

Similarly, the collateral constraint is binding if \( I^c(s^t) > I(s^t) \), and we have \( I^c(s^t) = I(s^t) \) if this constraint is not binding. So, for all values of \( I(s^t) \) satisfying \( 1 \leq I(s^t) \)
\[ \leq I^c(s^t), \text{ we have} \]
\[
B(s^t) \left[ 1 - \frac{I(s^t)}{I^c(s^t)} \right] = P(s^t)c_b(s^t) \left[ 1 - \frac{I(s^t)}{I^c(s^t)} \right] = P(s^t)c_b(s^t) \left[ \frac{I^c(s^t) - I(s^t)}{I^c(s^t)} \right] \quad (24)
\]
or, rearranging terms,
\[
B(s^t) \left[ 1 - \frac{I(s^t)}{I^c(s^t)} \right] + \frac{P(s^t)c_b(s^t)}{I^c(s^t)} = P(s^t)c_b(s^t) \left[ \frac{1 + I^c(s^t) - I(s^t)}{I^c(s^t)} \right] \quad (25)
\]
Substituting the complimentary slackness conditions – (23) and (25) – into (21), and using household first order conditions to eliminate relative prices and \( Q(s^{t+1}|s^t) \), we arrive at the flow implementability conditions
\[
\frac{\phi_mA(s^t)}{P(s^t)c_m(s^t)} = \beta \sum_{s^{t+1}|s^t} \rho(s^{t+1}|s^t) \left[ \frac{\phi_m A(s^{t+1})}{P(s^{t+1})c_m(s^{t+1})} \right] + 1 - [n(s^t)]^2 \quad (26)
\]
The Ramsey planner maximizes household utility subject to (26), the feasibility constraint
\[
c_m(s^t) + c_b(s^t) + c_c(s^t) + g(s^t) = z(s^t)n(s^t) , \quad (27)
\]
and the inequality constraints for cash and bond goods. We have put the complementary slackness conditions into the implementability conditions, they are valid whether or not the constraints are binding. So, all we have left are the inequality constraints themselves.

As shown in the appendix, two of the first order conditions for this maximization problem are
\[
\lambda(s^{t+1}) - \lambda(s^t) = \gamma^m(s^{t+1}) \frac{c_m(s^{t+1})}{\phi_m} \quad (28)
\]
and
\[
\gamma^m(s^t) = \gamma^b(s^t) \quad (\equiv \gamma(s^t)) \quad (29)
\]
where \( \gamma^m(s^t) \) and \( \gamma^b(s^t) \) are the Lagrange multipliers for the inequality constraints on the cash and bond goods, and \( \lambda(s^t) \) is the Lagrange multiplier on the implementability conditions (26).
The cash and collateral constraints are both binding, or they are both slack. If they are binding \((\gamma(s^t) > 0)\), then (28) says that the Ramsey planner issues more liabilities, easing the constraints but increasing the tax burden and \(\lambda(s^{t+1})\). If the inequality constraints are never binding \((\gamma(s^t) = 0)\), then \(\lambda(s^t) = \lambda\), a constant over time and across states. In this case, we can iterate the flow implementability conditions to obtain a single present value condition. If however the cash and collateral constraints are binding, then \(\lambda(s^t)\) increases over time, and the flow constraints do not collapse into a present value constraint.

So, are the cash and collateral constraints binding in the Benchmark LB Model, or are they not? We have made no mention of the Ramsey planner’s use of its Ricardian debt instrument, \(b^\gamma_g(s^t)\). It turns out that the derivative with respect to this instrument implies

\[
\gamma^m(s^t) = 0
\]

and therefore \(\lambda(s^t) = \lambda\). We will give a fuller explanation of the role played by the Ricardian debt instrument later in the section.

As shown in the appendix, the Ramsey planner’s first order conditions for consumption imply

\[
\frac{\phi_m}{c_m(s^t)} = \frac{\phi_b}{c_b(s^t)} = \frac{\phi_c}{c_c(s^t)}
\]

when \(\gamma(s^t) = 0\). And since the implementation of the optimal allocation is subject to household first order conditions (8) and (9), we must have

\[
I^c(s^t) = I(s^t) = 1
\]

So, the Ramsey planner follows an Extended Friedman Rule: the cost of holding money is eliminated \((I^c(s^t) - 1 = 0)\), and the cost of holding government bonds is eliminated \((I^c(s^t) - I(s^t) = 0)\). We have satiation in both money and government
debt. And of course, \( I^c(s^t) = 1 \) implies that prices are expected to deflate at the real rate of interest.

The wage tax is the third distortionary tax. As shown in the appendix, the optimal tax rate is positive.

\[
\tau_w(s^t) = 1 - \left( \frac{1}{1 + 2\lambda(s^t)} \right) \left( \frac{\sigma}{\sigma - 1} \right) > 0
\]  

(33)

And since \( \lambda(s^t) = \lambda \), the optimal wage tax rate is constant. The wage tax distortion is perfectly smoothed over time and across states. Why is the wage tax rate positive while the the two seigniorage taxes are set to zero? As seen in section 2.3, any of these taxes would distort the labor - leisure margin. However, using either of the seigniorage taxes would also distort the relative consumption margins. The wage tax does not distort the consumption margins; so it is efficient to use it to tax work.\(^{11}\)

But if the wage and seigniorage tax rates are held constant, how does the Ramsey planner accommodate shocks that have fiscal consequences? Figure 1 illustrates the Ramsey planner’s response to an auto-corellated increase in government spending.\(^{12}\) The planner engineers a surprise (shock dependent) inflation; this is a non-distortionary tax that lowers the real value of government liabilities, or equivalently households’ real assets. Households respond by decreasing consumption and increasing their work effort to rebuild their wealth. The three goods are perfect substitutes in production, and they enter utility in an identical way. So, their consumption paths are identical, declining by equal amounts and then returning to their steady state values at equal rates. A rising path of consumption requires a real interest rate that is above its steady state value. The nominal interest rate enters the three Euler equations – (15), (16) and (17) – in different ways, and the Extended Friedman Rule

\(^{11}\)Chari et al[11] relate this result to Atkinson and Stiglitz[5].

\(^{12}\)We use numerical methods for illustration only. We provide an analytical derivation of the Ramsey solution in the Appendix.
\((I_c(s^t) = I(s^t) = 1)\) nullifies these differences. The required movements in the real interest rate are accomplished by expected inflation.

Since all of the tax rates are unchanged, the persistent increase in government purchases results in a deficit. The Ramsey planner finances the deficit by issuing new liabilities, and this is of course what allows the households to rebuild their wealth.

But what if the initial inflation tax on money and bonds makes the cash and collateral constraints binding? This would distort consumption patterns and raise interest rates, none of which happens. Here is where \(b^e_g(s^t)\) comes in. Recall that we have let the Ramsey planner set the initial values of its liabilities, \(m(s^0)\) and \(b(s^0)\), subject to a given (possibly zero) initial value of net liabilities, \(\Lambda(s^0) = m(s^0) + b(s^0) - b^e_g(s^0)\). By lending to the private sector, the planner can build up \(m(s^0)\) and \(b(s^0)\) to a level where the cash and collateral constraint will be never binding. This debt instrument, unlike government bonds, is Ricardian; its supply has no direct effect on household consumption decisions.

### 3.1.2 The Constrained LB Model with Flexible Prices

In the Constrained LB Model, the government is not allowed to hold private sector debt; that is, \(b^e_g(s^t) \equiv 0\). The government does not have a Ricardian debt instrument. The steady state solution and all of the first order conditions for this problem are the same as in the Benchmark case except for the condition with respect to \(b^e_g(s^t)\), which made \(\lambda_t(s^t)\) constant and kept the cash and collateral constraints from binding. If initial liabilities are small, then the cash and collateral constraints will bind for large shocks. In this case, (28) implies that the government will issue more liabilities going into states with binding inequality constraints (recall that a rising shadow price \(\lambda\) corresponds to higher public-sector liabilities). We will assume that initial liabilities are large enough for the inequality constraints not to
bind. In this case the Ramsey solution coincides with the solution when we let the
government buy private bonds.

3.1.3 The Standard Model with Flexible Prices

In the Standard Model, there is no bond good and no collateral constraint. As
Chari, Christiano and Kehoe[11] have shown, the original Friedman Rule is optimal in
the Standard Model with flexible prices.13 There is no need for government lending,
b^*_d(s^t), here. Public debt, b(s^t), is a Ricardian instrument. It can be issued to finance
temporary deficits. And if, say, the response to an increase in government spending
requires an initial inflation tax that would make the cash constraint binding, then
the Ramsey planner can use open market operations to change the composition of
total liabilities and avoid the problem.

Surprise inflation is quite volatile in Chari, Christiano and Kehoe’s numerical
simulations. In their benchmark calibration, inflation has a standard deviation of
20 percent per annum. This price volatility can also be seen in Figure 1 for the
Benchmark LB Model.

3.2 The Case of Staggered Price Setting

With flexible prices, we saw that the Friedman Rule was optimal and the ex-
pected rate of deflation was equal to the real rate of interest; moreover, the Ramsey
Planner made substantial use of unanticipated inflation to absorb the fiscal conse-
quences of macroeconomic shocks. This is costless when prices are flexible. But, as
is well known, staggered price setting can create a dispersion of intermediate goods’
prices that distorts household consumption decisions.14 And if (as we assume) prices

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13 Canzoneri, Cumby and Diba[9] provide a fuller discussion the earlier literature.
14 Woodford[30] is the classic reference.
are not indexed to inflation, there can also be a dispersion of prices in the steady state. This relative price distortion can be eliminated if the Ramsey Planner fixes the price level; in this case, price setters will not want to change their prices even when given the opportunity.

So, staggered price setting creates a fundamental inflation tradeoff for the Ramsey Planner. Fixing the price level to eliminate the price dispersion clashes with the implementation of the Friedman Rule, and with the use of unanticipated inflation as a fiscal shock absorber. The question becomes: how important is the distortion created by the price dispersion relative to the distortions created by the seigniorage taxes and the wage tax? Benigno and Wooldridge[7] and Schmitt-Grohe and Uribe[26], using calibrated models similar to our Standard Model, found that the inflation tradeoff was basically resolved in favor of price stability.

As noted above, we cannot solve the Ramsey problem analytically with staggered price setting. So, we have to resort to numerical methods. We will begin with a discussion of optimal taxation and inflation in the various models’ steady states. Then, we will show how the Ramsey planner responds to an unexpected increase in government spending. We will begin with the Standard Model, since the planner’s response reflects the conventional wisdom that grew out of the work of Barro. Then, we will proceed to the Benchmark LB Model, which retains the basic insights of Barro. Finally, we will discuss the Constrained LB Model, in which the government does not have available a Ricardian debt instrument and the insights of Barro are lost.

More precisely, we use the Get Ramsey program developed by Levin and Lopez-Salido[21] and Levin, Otrnski, Williams, and Williams[22].
3.2.1 Optimal Tax Rates and Inflation in the Steady State

Table 1 reports optimal steady state tax rates and inflation for our numerical calibrations of the various models. With flexible prices, the seigniorage taxes are set to zero and the wage and profit taxes service the public debt; deflation is equal to the real rate of interest. With staggered price setting, prices are virtually fixed, and the seigniorage taxes are positive. Calvo trumps Friedman: the inflation tradeoff is resolved decidedly in favor of price stability.

Table 1 shows that $I^e > I > 1$. Why is this? If $I$ were set equal to 1, (10) implies that the bond good - cash good margin would not be distorted, but (9) implies that the bond good - credit good margin would be distorted. Similarly setting $I^e = I$ would leave the bond good - credit good margin undistorted, but the bond good - cash good margin would be distorted. Optimal policy spreads the consumption distortions by setting $I^e > I > 1$. Moreover, the cash seigniorage tax is larger than the bond seigniorage tax. This must be the case, since $I > 1$ implies that $I^e - 1 > I^c - I$.\footnote{With the symmetry in our utility function, $I$ is set exactly half way between zero and $I^e$.}

3.2.2 Optimal Responses in the Standard Model

Figure 2 shows the Ramsey planner’s response to an auto-correlated increase in government spending, and it reflects the conventional wisdom. As before, households respond to the increased tax burden by decreasing consumption and working more. Here however, there is virtually no unanticipated inflation; it is just too costly in terms of the price dispersion it creates. The spending increase has to be financed another way.

Raising the seigniorage tax, $I$, or the wage tax, $\tau_w$, creates a distortion, and the Ramsey planner minimizes these distortions by issuing liabilities to finance part of
the increase in government spending. This leaves a larger public debt to be rolled over in the new steady state, and the wage tax is raised permanently to finance that. In this way, the Ramsey planner trades off short run tax smoothing against lower long run consumption. This tradeoff, and the unit root it engenders, captures the fundamental insight of Barro.

The consumption paths of the cash good and the credit good are very similar after the first few periods, and they resemble the consumption paths for the flexible price case. As before, the optimal allocation requires that consumption falls initially and then rises over time, requiring that the real interest rate be above its steady state value. Now, however, the nominal interest rate needs to change because inflation is too costly to move. And as a result, the paths of cash and credit good consumption are similar, but not identical.

The Ramsey planner takes some artful steps in the first few periods to smooth initial distortions. The need for these steps arises because of the difference in the timing of the interest rates in Euler equations (15) and (16). From (16), a drop in period 1 credit good consumption requires an increase in $I^c_2$, the nominal interest rate in period 2. But this will distort the relative consumption of cash and credit goods in period 2; that is, cash good consumption will be too low. To mitigate the cost of this, the planner makes cash good consumption fall by less than credit good consumption in period 1, which requires a decrease in $I^c_1$.

In order to achieve the optimal paths of consumption, the Ramsey planner has to alter the composition of government liabilities so that household money balances are consistent with cash good consumption. Since public debt is Ricardian, open market sales of bonds allow the cash in advance constraint to be satisfied with no further consequences.

The details of the artful steps taken by the Ramsey planner in the first few periods
depend on the special timing in the cash and credit good apparatus. They may not be robust to changes in the way of modeling liquidity. The important result here is that optimal policy involves short run tax smoothing and permanent effects on debt, consumption, and the wage tax rate.

3.2.3 Optimal Responses in the Benchmark LB Model

Figure 3 shows the Ramsey planner’s response to a persistent increase in government spending, and it is similar to that in Figure 2. In particular, Barro’s fundamental insight survives: short run tax distortions are smoothed at the expense of longer run consumption. There is, however, a twist in the Ramsey planner’s implementation of the policy due to the fact that government bonds are now used as collateral; they are no longer Ricardian. Government assets, $b^c$, are Ricardian, and they play an important role in what follows; we return to this issue at the end of the section.

As before, the increase in government spending crowds out consumption and increases work effort in the short run, and once again the three consumption goods’ paths are very similar after the first few periods. Inflation (not pictured) is too costly to move. So, the real interest rate increases, associated with the rising consumption profiles, require changes in the seigniorage taxes, $I^c - 1$ and $I^c - I$. Consequently, the consumption paths are not identical.

As with the standard model, the Ramsey planner takes some complicated steps in the first few periods. The increase in $I^c_2$ again implies that period 2 cash good consumption will fall too much relative to credit good consumption, and this loss is mitigated by a decrease in $I^c_1$. But the divergence in period 1 cash and credit good consumption has a utility cost. The Ramsey planner must choose bond good consumption in a way that mitigates that utility cost while achieving a drop in over-
all period 1 consumption large enough to accommodate the increase in government purchases. This is achieved by balancing the smaller drop in cash good consumption with a larger drop in bond good consumption; that is, the decline in credit good consumption is between these two.

The difference in bond and credit good consumption must be reflected in the interest rate spread, $I^c - I$. From (17), the interest rate entering the Euler equation for the bond good is $I^c \frac{\text{spread}_t}{\text{spread}_{t+1}}$, which differs from that in the Euler equation for the credit good by the spread terms. If the spread were to remain constant at its steady state value, the response of bond good consumption would be identical to that of credit good consumption. But since bond good consumption falls by more than credit good consumption and is therefore expected to rise by more, the real interest rate for bond goods must be higher and the spread must be expected to fall. And since the spread returns to its steady state value, a declining spread implies an initial increase in the spread.

Now, we return to the role played by $b_g$. The Ramsey planner cannot simply alter the composition of government liabilities to achieve the optimal allocation: satisfying the cash in advance constraint requires selling government bonds. But in the LB Model, this would loosen the collateral constraint, and too many bond goods would be consumed. The government can finance its short run deficit in the normal way (by issuing more total liabilities), but then the government sells $b_g^c$ for $m$ and $b$. In this way, the cash and collateral constraints can be made consistent with optimal allocation of cash and bond good consumption.

### 3.2.4 Optimal Responses in the Constrained LB Model

Figure 4 shows the Ramsey planner’s response to a sustained increase in government spending, and it clearly violates the conventional wisdom. The Ramsey
solution is stationary; the government does not smooth short run tax distortions at the expense of longer run consumption.

The reason is that both money and government bonds provide liquidity in the LB Model, and the path of consumption is tied to their supplies. Without a Ricardian debt instrument (like \( b \) in the Standard Model or \( b^*_g \) in the Benchmark LB Model), the planner cannot finance temporary surpluses or deficits without affecting the supply of liquidity to households. Stated differently, without the Ricardian instrument, fiscal policy and liquidity provision cannot be determined independently.

Consumption is again crowded out by the increase in government purchases. The decline in cash and bond good consumption requires a corresponding decline in both \( m \) and \( b \). (This means of course that the government has to run a surplus.) Comparing Figures 3 and 4, the Ramsey planner has to raise the wage tax more by a full order of magnitude. And this means that consumption has to fall by about twice as much and the additional work effort is curtailed.

The Ramsey planner's steps in the first few periods are even more artful than in the previous cases. Private sector holdings of nominal government liabilities are inherited from the previous period and real liabilities will initially change only as a result of inflation. Since inflation is small, the sum of the change in \( m \) and \( b \) – and therefore the sum of the changes in cash and bond good consumptions – is close to zero. But since interest rates must change to reflect the optimal consumption paths, the ratio of consumption of the cash and bond goods must change. If the sum of the changes is nearly zero and the ratio changes, one must rise and one must fall. From above, we know that it is cash good consumption that will rise because \( I^*_1 \) can be used to mitigate the distortion resulting from an increase in \( I^*_2 \). In order to mitigate the utility cost of differing consumption of the 3 goods, bond good consumption will fall less than credit good consumption. (If it were to fall by more, cash good consumption
would need to rise by even more.) In addition, to mitigate the effect of this distortion, inflation while negligible is much greater here than in the previous case.

Because the decline of credit good consumption exceeds the decline of bond good consumption, expected growth of credit good consumption exceeds expected growth of bond good consumption. Therefore the interest rate in the credit good Euler equation exceeds that in the bond good Euler equation. The spread is therefore expected to grow and must therefore decline initially.

3.2.5 The Benefit of having a Government Lending Instrument

Figure 5 compares the Ramsey planner’s responses to an increase in government spending in three models: the Benchmark LB Model with flexible prices, the Benchmark LB Model with staggered price setting, and the Constrained LB Model with staggered price setting. The figure illustrates the potential benefit of the government’s having a Ricardian debt instrument like $b_g^c$.

Staggered price setting adds a nominal distortion to the model. But when $b_g^c$ is available, the consumption paths are quite similar to those with flexible prices. When $b_g^c$ is not available, consumption of all three goods falls much further. Of course, long run consumption is a little lower when $b_g^c$ is used.

In contrast, the response of government liabilities to a spending shock is much smaller when the Ramsey planner does not have access to a Ricardian debt instrument. This stands in sharp contrast to the much greater response seen for consumption and the wage tax rate. The reason is, once again, that tax policy and liquidity provision cannot be determined independently without a Ricardian debt instrument. The liquidity value of government liabilities introduces a motive for the Ramsey planner to smooth debt as well as the labor tax rate. Alternatively, the Ramsey planner needs to trade off smoothing the wage tax rate and smoothing the bond seigniorage
tax rates. That trade-off is resolved in this model by a much greater wage tax rate volatility.

4 Conclusion

In this paper, we asked how the fundamental insights of Friedman and Barro fare when we recognize the fact that bonds provide liquidity services. To investigate this question, we augmented a standard cash and credit good model by allowing government bonds to be used as collateral in purchasing certain types of consumption goods. In this environment, the government cannot use its bonds to finance a short run deficit (ala Barro) independently of its decision of how much liquidity to provide for consumption. We found that the insights of Friedman and Barro survive largely intact if the government has at its disposal an additional debt instrument that is “Ricardian” – in the sense that it does not directly affect economic activity. Absent a Ricardian debt instrument, the Friedman Rule may not be optimal in the transition to a steady state, even with flexible prices. And with sticky prices, the Ramsey planner would no longer choose to smooth short run tax distortions at the expense of some long run consumption (ala Barro).

We model the Ricardian debt instrument as government holdings of private sector bonds, which by our assumptions did not provide liquidity services. But, it would not seem that governments currently utilize a Ricardian debt instrument. They certainly do not use private sector debt in the way our Ramsey planner would. They do of course have non-liquid assets such as parks, oil rights, grazing rights, spectrum rights, and state owned enterprises. However these assets would be hard to use to finance short run deficits without creating disruptions in the real economy.

Finally, future research might usefully focus on relaxing some of the sharp distinc-
tions that we have made in our modeling of money and bond liquidity. For example, some kinds of private debt may well compete with government bonds in the provision of liquidity services. Such debt would not qualify as a Ricardian debt instrument for use in the manner described here. A more elaborate modeling of the use of various financial assets would seem to be warranted. In a similar vein, long term government debt may not provide the liquidity services envisioned in our model. If this is the case, then long term government bonds may qualify as the Ricardian debt instrument that is needed here. Modeling a term structure of debt may help differentiate the financial assets that do and do not provide liquidity. More generally yet, the substitutability between money and bonds in the provision of liquidity could be explored further. This could be done by say increasing the substitutability of the cash and bond goods in utility. As the cash and bond goods became perfect substitutes, money would presumably be crowded out. Or alternatively, holding the elasticity of substitution constant, one could explore the implications of the central bank’s paying interest on reserves. This could lead to a discussion of the optimal approach to a winding down of the "unconventional policies" pursued by central banks in recent years.

5 Appendix: Ramsey Solution to the LB Model with Flexible Prices

We can solve the Ramsey problem analytically for the LB Model with flexible prices. In the Benchmark LB Model, the government is able to lend to the private sector; that is, $b_g^c$ is a policy instrument. In the Constrained LB Model $b_g^c$ is not available.
We follow the usual procedure in solving for the Ramsey solution: we derive flow implementability conditions by using household first order conditions to eliminate relative prices and tax rates from the household’s flow budget constraints; then, we choose the quantities of consumption and labor that optimize household utility subject to the relevant constraints. However, the Ramsey Planner will also choose $m(s^t)$ and $b(s^t)$, because we have inequality constraints on the consumption of cash and bond goods. We have to insert complementary slackness conditions into the implementability conditions to allow for the possibility that these constraints are not binding in the optimal solution. Finally, the Ramsey planner will also choose its holdings of private sector debt, $b_y(s^t)$. As we shall see, the flow implementability conditions reduce to a single present value condition if this Ricardian debt instrument is available to the planner.

One final note before we begin the derivations: there are important questions about how we should treat the choice of the initial government liabilities (or the price level, given some initial nominal liabilities). However, these issues are well known, and we have nothing new to contribute. In the benchmark model, we will let the Ramsey planner set the initial values $m(s^0)$, $b(s^0)$, and $b_y(s^0)$, subject to a given (possibly zero) value of net liabilities, $m(s^0) + b(s^0) - b_y(s^0)$. We will return to the question of where the planner would set these initial values later in the appendix.

We begin by deriving the flow implementability constraint. Assuming non-satiation in some state, the household budget constraint, (4), holds with equality; using (5) to eliminate $x(s^{t+1})$, we get

$$A(s^t) = M(s^t) + B(s^t) + B_c(s^t) + \sum_{s^{t+1} | s^t} Q(s^{t+1} | s^t) \{ A(s^{t+1})$$

$$- M(s^t) - I(s^t) B(s^t) - I_c(s^t) B_c(s^t)$$

$$+ P(s^t) [ c_m(s^t) + c_b(s^t) + c_c(s^t) ] - [ 1 - \tau_w(s^t) ] W(s^t) n(s^t) \}$$

$$A(s^t) = M(s^t) + B(s^t) + B_c(s^t) + \sum_{s^{t+1} | s^t} Q(s^{t+1} | s^t) \{ A(s^{t+1})$$

$$- M(s^t) - I(s^t) B(s^t) - I_c(s^t) B_c(s^t)$$

$$+ P(s^t) [ c_m(s^t) + c_b(s^t) + c_c(s^t) ] - [ 1 - \tau_w(s^t) ] W(s^t) n(s^t) \}$$

(34)

31
We have set after tax profits to zero because profits are pure monopoly rents in our model, and we know that the Ramsey planner will tax them away. Using \( \sum_{s^{t+1}|s^t} Q(s^{t+1}|s^t) = \frac{1}{I^c(s^t)} \), the budget constraint becomes

\[
A(s^t) = \sum_{s^{t+1}|s^t} [Q(s^{t+1}|s^t) A(s^{t+1})] + M(s^t) \left[ 1 - \frac{1}{I^c(s^t)} \right] + B(s^t) \left[ 1 - \frac{I(s^t)}{I^c(s^t)} \right] + P(s^t) \left[ c_m(s^t) + c_b(s^t) + c_c(s^t) \right] - \left[ \frac{1 - \tau_w(s^t)}{I^c(s^t)} \right] W(s^t) n(s^t)
\]  

(35)

Next, we incorporate the complementary slackness conditions for the cash and collateral constraints. The cash in advance constraint is binding for \( I^c(s^t) > 1 \), and we have \( I^c(s^t) = 1 \) if this constraint is not binding. So for all \( I^c(s^t) \geq 1 \), we have

\[
M(s^t) \left[ 1 - \frac{1}{I^c(s^t)} \right] = P(s^t) c_m(s^t) \left[ 1 - \frac{1}{I^c(s^t)} \right]
\]

or

\[
M(s^t) \left[ 1 - \frac{1}{I^c(s^t)} \right] + \frac{P(s^t) c_m(s^t)}{I^c(s^t)} = P(s^t) c_m(s^t)
\]

(37)

Similarly, the collateral constraint is binding if \( I^c(s^t) > I(s^t) \), and we have \( I^c(s^t) = I(s^t) \) if this constraint is not binding. So, for all values of \( I(s^t) \) satisfying \( 1 \leq I(s^t) \leq I^c(s^t) \), we have

\[
B(s^t) \left[ 1 - \frac{I(s^t)}{I^c(s^t)} \right] = P(s^t) c_b(s^t) \left[ 1 - \frac{I(s^t)}{I^c(s^t)} \right] = P(s^t) c_b(s^t) \left[ \frac{I^c(s^t) - I(s^t)}{I^c(s^t)} \right]
\]

or

\[
B(s^t) \left[ 1 - \frac{I(s^t)}{I^c(s^t)} \right] + \frac{P(s^t) c_b(s^t)}{I^c(s^t)} = P(s^t) c_b(s^t) \left[ \frac{1 + I^c(s^t) - I(s^t)}{I^c(s^t)} \right]
\]

(39)

Substituting (37) and (39) into (35), the budget constraint becomes

\[
A(s^t) = \sum_{s^{t+1}|s^t} [Q(s^{t+1}|s^t) A(s^{t+1})] + P(s^t) \left\{ c_m(s^t) + \frac{c_b(s^t) + c_c(s^t)}{I^c(s^t)} \right\} + \left[ \frac{I^c(s^t) - I(s^t)}{I^c(s^t)} \right] \left[ \frac{1 - \tau_w(s^t)}{I^c(s^t)} \right] W(s^t) n(s^t)
\]

(40)
Next, we use the household’s first order conditions (8), (9), and (12) to eliminate relative prices and tax rates,

\[ A(s^t) = \sum_{s^{t+1}|s^t} [Q(s^{t+1}|s^t)A(s^{t+1})] + P(s^t) \left\{ c_m(s^t) + \frac{u_b(s^t)c_b(s^t)}{u_m(s^t)} \right\} \]

And using (7), we get

\[ \frac{A(s^t)u_m(s^t)}{P(s^t)} = \beta \sum_{s^{t+1}|s^t} \rho(s^{t+1}|s^t) \left[ \frac{A(s^{t+1})u_m(s^{t+1})}{P(s^{t+1})} \right] + u_m(s^t)c_m(s^t) \]

Equation (42) is a series of implementibility constraints, one for each period and state, \( s^t \). With flexible prices, in the Standard Model the flow constraints can be reduced to a single present value constraint beginning in \( s^0 \). See Canzoneri, Cumby and Diba ([9]) This is also true in the LB Model if \( b^c_g \) is available. However, if \( b^c_g \) is not available, the cash and collateral constraints can be binding and we have to work with the flow implementibility constraints, even with flexible prices.

We should also note that there are implementability constraints implied by \( 1 \leq I(s^t) \leq I^c(s^t) \) (see equations (9) and (10)),

\[ u_c(s^t) \leq u_b(s^t) \leq u_m(s^t) \]

We will tentatively ignore these constraints and leave them to be verified later.

To lighten our exposition, we will work with the utility function

\[ \phi_m \log(c_{m,t}) + \phi_b \log(c_{b,t}) + \phi_c \log c_{c,t} - \frac{1}{2} \theta_t^2, \]

although our results readily extend to preferences that are homothetic over the three consumption goods and weakly separable across employment and consumption, as in
With this specification of preferences, the implementability constraints (42) simplify to

$$
\frac{\phi_m A(s^t)}{P(s^t)c_m(s^t)} = \beta \sum_{s^{t+1}|s^t} \rho(s^{t+1}|s^t) \left[ \frac{\phi_m A(s^{t+1})}{P(s^{t+1})c_m(s^{t+1})} \right] + 1 - \left[n(s^t)\right]^2
$$

(45)

The Ramsey planner maximizes household utility subject to (45), the feasibility constraint

$$
c_m(s^t) + c_b(s^t) + c_c(s^t) + g(s^t) = z(s^t)n(s^t),
$$

(46)

and the inequality constraints for cash and bond goods. We have already put the complementary slackness conditions into the implementability constraints; they are valid whether or not these constraints are binding. So, all we have left are the inequality constraints themselves,

$$
c_m(s^t) \leq m(s^t)
$$

(47)

and

$$
c_b(s^t) \leq b(s^t)
$$

(48)

We can now solve the planner’s problem. Let \( \omega(s^t) \) denote the left hand side of the implementability constraint,

$$
\omega(s^t) \equiv \frac{\phi_m A(s^t)}{P(s^t)c_m(s^t)} = \frac{\phi_m [m(s^t) + b(s^t) - b^*_c(s^t)]}{c_m(s^t)}
$$

(49)

Let \( \lambda(s^t) \) denote the Lagrange multipliers for the implementability constraints, and incorporate these constraints in the Lagrange function as

$$
\beta^t \rho(s^t) \lambda(s^t) \left\{ \beta \sum_{s^{t+1}|s^t} \rho(s^{t+1}|s^t) \omega(s^{t+1}) + 1 - \left[n(s^t)\right]^2 - \omega(s^t) \right\}
$$

(50)

17 We don’t present these derivations because they mimic very closely the derivations in Chari et. al.[12]
Let \( \mu(s^t) \) denote the multipliers for the feasibility condition, (46), and incorporate these constraints in the Lagrange function as

\[
\beta^t \rho(s^t) \mu(s^t) \left[ z(s^t) n(s^t) - c_m(s^t) - c_b(s^t) - c_c(s^t) - g(s^t) \right]
\]

Finally, let \( \gamma^m(s^t) \) and \( \gamma^b(s^t) \) be the multipliers for the inequality constraints, and incorporate them in the Lagrange function as

\[
\beta^t \rho(s^t) \left\{ \gamma^m(s^t) \left[ \frac{\omega(s^t)c_m(s^t)}{\phi_m} - b(s^t) - b^*_g(s^t) - c_m(s^t) \right] \right. \\
\left. + \gamma^b(s^t) [b(s^t) - c_b(s^t)] \right\}
\]

We begin with the first order conditions for consumption and work effort:

The FOC for \( c_c(s^t) \) gives

\[
\frac{\phi_c}{c_c(s^t)} = \mu(s^t)
\]

The FOC for \( c_b(s^t) \) gives

\[
\frac{\phi_b}{c_b(s^t)} = \mu(s^t) + \gamma^b(s^t)
\]

The FOC for \( c_m(s^t) \) gives

\[
\frac{\phi_m}{c_m(s^t)} = \mu(s^t) + \gamma^m(s^t) \left[ 1 - \frac{\omega(s^t)}{\phi_m} \right]
\]

or

\[
\frac{\phi_m}{c_m(s^t)} = \mu(s^t) + \gamma^m(s^t) \left[ c_m(s^t) - A(s^t) \right]
\]

And, the FOC for \( n(s^t) \) gives

\[
[1 + 2\lambda(s^t)] n(s^t) = \mu(s^t) z(s^t)
\]

Next, we have the first order conditions for the financial variables:

The FOC for \( \omega(s^{t+1}) \) gives

\[
\beta^t \rho(s^t) \lambda(s^t) \beta \rho(s^{t+1} | s^t) - \beta^{t+1} \rho(s^{t+1}) \lambda(s^{t+1}) + \beta^{t+1} \rho(s^{t+1}) \gamma^m(s^{t+1}) \frac{c_m(s^{t+1})}{\phi_m} = 0
\]
We have $\rho(s^t)\rho(s^{t+1}|s^t) = \rho(s^{t+1})$, or else $\rho(s^{t+1}|s^t) = 0$, because $s^{t+1}$ is just the history $s^t$ followed by the realization $s_{t+1}$ of the random variables at date $t + 1$. So, this FOC reduces to

$$\lambda(s^{t+1}) - \lambda(s^t) = \gamma^m(s^{t+1}) \frac{c_m(s^{t+1})}{\phi_m}$$

(59)

This equation says that the shadow price of the implementability constraint is increasing over time if the inequality constraint for cash goods is binding.

The FOC for $b(s^t)$ gives

$$\gamma^m(s^t) = \gamma^b(s^t) \equiv \gamma(s^t)$$

(60)

We also have the non-negativity conditions with the complementary slackness conditions

$$\gamma^m(s^t) [m(s^t) - c_m(s^t)] = \gamma^b(s^t) [b(s^t) - c_b(s^t)] = 0$$

(61)

In the Benchmark LB Model, the government is allowed to buy household debt; that is, it has another policy instrument, $b_g(s^t)$. In this case, the Ramsey solution simplifies quite dramatically:

The FOC for $b_g(s^t)$ gives

$$\gamma^m(s^t) = 0$$

Then, (59) implies that $\lambda(s^0) = \lambda(s^t) \equiv \lambda$ for all states and dates; the series of implementability constraints can be iterated forward to obtain a single present value implementability constraint.

More importantly, the inequality constraints on cash and bond holdings will never bind if the government can buy private debt. And this is why the Lagrange multipliers, $\lambda(s^t)$, are constant. The Ramsey planner does not have to use $b(s^t)$ to finance fiscal imbalances, which could be distortionary. The Constrained LB Model does not share these features.
When $\gamma(s^t) = 0$, the FOC’s for consumption imply

$$\frac{\phi_m}{c_m(s^t)} = \frac{\phi_b}{c_b(s^t)} = \frac{\phi_c}{c_c(s^t)} = \mu(s^t)$$  \hspace{1cm} (62)

Since the optimal allocation satisfies (62), and its implementation is subject to (8) and (9), we have

$$I^c(s^t) = I(s^t) = 1$$

The extended Friedman Rule is optimal. This also confirms that (43) is satisfied.

Using (62) and (57), the Ramsey allocation satisfies

$$(1 + 2\lambda)n(s^t)c_m(s^t) = \phi_m z(s^t)$$ \hspace{1cm} (63)

Since the implementation of optimal policy is subject to (12), with the real wage equal to $(\sigma - 1)z(s^t)/\sigma$, (63) and $I^c(s^t) = 1$ imply

$$(1 + 2\lambda)[1 - \tau_w(s^t)] = \frac{\sigma}{\sigma - 1}$$

and a constant optimal tax rate

$$\tau_w(s^t) = 1 - \left(\frac{1}{1 + 2\lambda}\right) \left(\frac{\sigma}{\sigma - 1}\right)$$  \hspace{1cm} (64)

To calculate optimal allocations, we need to pin down the value of $\lambda$. Using (46), (62) and (63), we get

$$n_t = \frac{1}{2z_t} \left[g_t + \sqrt{(g_t)^2 + \left(\frac{z_t}{1 + 2\lambda}\right)^2}\right]$$ \hspace{1cm} (65)

Iterating (45), we get

$$\frac{\phi_m a_0}{c_m 0} = E_0 \sum_{t=0}^{+\infty} \beta^t \{1 - [n_t]^2\}$$
which, together with (55) and (57), implies

\[
\frac{a_0(1 + 2\lambda)n_0}{z_0} = E_0 \sum_{t=0}^{+\infty} \beta^t \{1 - [n_t]^2\}
\]

Substituting (65) and taking expectations will pin down \(\lambda\).

References


Table 1: Steady State Tax Rates and Inflation  (annual rates)

<table>
<thead>
<tr>
<th>Flexible Prices</th>
<th>$R^c$</th>
<th>$R^c - R$</th>
<th>wage tax</th>
<th>inflation</th>
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<td>0.005</td>
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</tbody>
</table>

* Assumes initial government liabilities are large enough for inequality constraints not to bind.
** Assumes initial government liabilities are limited, and the inequality constraints are binding.
Figure 1: Benchmark LB Model with flexible prices, government spending shock

- Cash good consumption
- Bond good consumption
- Credit good consumption
- Work
- Inflation
- Deficit
Figure 2: Standard Model, staggered price setting
Figure 3: Benchmark LG, staggered price setting
Figure 4: Constrained LB Model, staggered price setting

- Cash good
- Bond good
- Credit good
- Work
- Inflation
- I
- Ic - I
- Wage tax rate
- Government liabilities
Figure 5: Comparison of responses in the LB models

Note: solid line = Benchmark LB Model (flexible prices);
dashed line = Benchmark LB Model (staggered price setting);
dotted line = Constrained LB Model (staggered price setting).