An Equilibrium Model of Structural Unemployment

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1. Introduction

There has been a striking increase in unemployment in most European countries over the past two decades. At the same time, the share of unemployment that is long-term (e.g., of one year's duration or more) has also gone up substantially. Unemployment has not trended upwards in the United States over the same period, although the importance of long-term unemployment has also increased, albeit to minuscule shares of the total by European standards.[1]

The fact that European unemployment rates have been persistently high for more than a decade suggests that there has been a change in equilibrium unemployment rates in these countries, i.e., it does not appear that this situation can be explained in terms of cyclical unemployment. This apparent change in equilibrium unemployment rates as well as the growing importance of long-term unemployment is what motivates the model developed in this paper. Our model considers equilibrium unemployment as the sum of two related components – search unemployment and wait unemployment. We identify search unemployment with looking for a new job, e.g., job search by new entrants to the labor force, by individuals who have decided to move from one occupation, industry, or location to another, etc. We think of wait unemployment in terms of not actively searching, i.e., waiting in a sector in the hope that one's old job will come back. The choice between searching and waiting is central to our model. The incentive

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[1] Data on unemployment rates and on long-term unemployment in Europe and the United States are available in the annual Employment Outlook volumes of the OECD. A summary and discussion of these developments is given in Bertola and Ichino [1995].

to search is simply that it makes sense to look where jobs are more plentiful. The incentive to wait that is built into our model is that workers can accumulate sector-specific human capital, and leaving one's sector to search means giving up the possibility of earning a return on those skills.

While some economists would classify search unemployment as frictional and wait unemployment as structural, we believe both are influenced by changes in the underlying structure of the economy. Our model investigates the extent to which structural changes can increase equilibrium unemployment of either variety. It also looks at factors that can change the relative shares of wait and search unemployment. To the extent that structural changes of the sort we model have contributed to the rise in equilibrium unemployment in Europe, constructing an equilibrium model that allows for both search and wait unemployment is useful. That is what we do in this paper.

Specifically, we develop a model of an economy that consists of a large number of sectors that are subject to persistent, idiosyncratic shocks. It is the reaction of individuals to the changes in the structure of the economy that are induced by these shocks that creates both search and wait unemployment. At any time, individuals in this economy are either attached to a particular sector or searching. Those who are attached to a sector are either employed or unemployed (i.e., waiting for a job in their old line of work), and those who are employed are either unskilled (not endowed with sector-specific human capital) or skilled (endowed with sector-specific human capital). We then specify the law of motion for the economy, both within sectors and between sectors. That is, we specify how the measures of workers of each possible type (namely, unskilled employed, skilled employed, and resters[2]) change from period to period within each sector. This law of motion depends in part on the choices that individuals make. In particular, we examine the decision faced by workers endowed with sector-specific human capital who have lost their jobs: shall they stay in their sector waiting to be rehired or shall they search elsewhere and give up their sector-specific skills?

We look for a stationary equilibrium in this economy, by which we mean the following. In any period, the state of a sector is defined by (i) the realized value of its idiosyncratic shock and (ii) the measures of the three worker types in the sector. In stationary equilibrium, the density of states across sectors must be constant through time, even though the state of each sector will not be constant through time. The Markovian structure of our model ensures the existence of a unique stationary equilibrium. We want to use this stationary equilibrium to address the following questions. To what extent can an equilibrium model of

[2] We use the term resters to denote those who remain in a sector waiting to be rehired. Obviously, waiters would seem the more appropriate term, but it has other connotations.

structural unemployment explain the observed increase in European unemployment? Can our model generate series for duration, incidence and mobility that are similar to what we observe in reality? What features of our model determine the relative importance of search versus wait unemployment? How do search and wait unemployment vary over the life cycle?[3]

There is a rich literature, both theoretical and empirical, related to our model. On the theoretical side, our starting point is Lucas and Prescott [1974] in which unemployment results from workers moving from sectors that have fallen on hard times to sectors in which conditions are currently better. There is no unemployment within sectors in their model: once migration decisions have been made, all workers within a sector are employed at the market-clearing wage. Unemployment is generated solely by the movement of job-seekers across sectors and is thus purely search unemployment.

We augment the Lucas and Prescott model by (i) allowing for unemployment within sectors and (ii) allowing individuals to accumulate sector-specific human capital. With respect to within-sector unemployment, we assume that matches break up with a probability that depends solely on conditions within the sector (e.g., whether demand is high or low). Job accessions within a sector also depend on conditions there. With respect to sector-specific human capital, we imagine that a worker is either unskilled (has not been endowed with sector-specific human capital) or skilled in his or her sector. The acquisition of sector-specific skill is stochastic; in particular, we assume that an unskilled worker, in any period of employment, learns the sector-specific skill with some constant probability. So long as this individual continues to work, we assume that he or she does not forget what was learned. (An alternative interpretation is that, conditional on employment, a skilled worker has a sufficient base to be able to adapt to changing skill requirements within the sector.) Once a skilled worker's match breaks up, however, matters change. If the worker decides to exit in search of a job in another sector, then skills in his or her old sector are forfeited. Alternatively, if the worker decides to stay in the sector, then he or she suffers a constant risk per period of unemployment of forgetting, i.e., of losing his or her sector-specific capital.

The key idea in the above specification is that some individuals may choose to stay behind in their sectors, even though they have lost their jobs, in order to retain the possibility of benefiting from their accumulated sector-specific human capital should their sectors recover. Thus, both search unemployment (due to the movers) and wait unemployment (due to the stayers) arise in our model.

[3] We do not explicitly index individuals by age. However, we model lifetimes as geometric random variables, so we can track individuals to see how search and wait unemployment vary with age.

The notion that workers might choose not to leave a sector, even though current conditions there are bleak is, of course, not new. A model in the spirit of Lucas and Prescott [1974] that captures this idea is presented in Jovanovic [1987].[4] A worker's productivity in a particular job (sectors and jobs are identical in his model) is the sum of two independent Markovian shocks, a match-specific component and an aggregate component. The worker cannot distinguish perfectly between the two components, and when productivity falls below a threshold value on the current job, he or she may choose to "rest" rather than spending the costs associated with search for a new job. The reason is that with the passage of time, the worker's information improves in the sense of being better able to distinguish between bad times specific to the current job (in which case search is called for) versus bad times all over (in which case the better option is to continue to rest until conditions improve). Jovanovic's model thus generates both search unemployment and wait unemployment (i.e., "resting"). The mechanism behind wait unemployment is, however, quite different from ours, namely, a combination of signal extraction (it is better not to search until one is reasonably sure that conditions aren't bad all over) and intertemporal substitution (if conditions are in fact bad all over, then it is better to consume leisure now while waiting for good times to return).

Likewise, the idea that human capital might be accumulated stochastically while employed and decumulated stochastically while unemployed as a central component of a model of unemployment has precedents. For example, the model of Ljungqvist and Sargent [1997] is one in which workers stochastically move up a human capital ladder while employed and down the same ladder when unemployed. The unemployed draw (at a rate determined by their search intensities) from an exogenous distribution over wages per unit of human capital; thus an individual's income when employed is the product of his or her wage per unit of human capital and his or her position on the ladder. The Ljungqvist and Sargent explanation of long-term European unemployment is essentially that unemployment benefits are linked to previous employment incomes. When a worker becomes unemployed, if he or she falls far enough down the ladder, it will become virtually impossible to draw a high enough wage to enable the worker to better the value associated with continued unemployment. Again, however, the role of human capital accumulation and decumulation in Ljungqvist and Sargent [1997]

[4] There is also a substantial literature on wait unemployment in strictly static models. In these models, workers wait for a higher wage in the "good" sector rather than accept a lower wage in the "bad" sector. Burda [1988] is an example of this sort of model in a dual labor market context. This type of model has also been used to analyze the effects of a minimum wage. See, for example, Mincer [1976].

is quite different from its role in our model. Specifically, human capital in their model is general, while in our model it is sector-specific. This means that there is no wait unemployment in the Ljungqvist and Sargent model (necessarily, since there are no sectors in which to wait). There is, however, a connection between our models in the sense that long-term unemployment arises because workers who previously enjoyed high incomes associated with human capital do not find it worthwhile to search actively for new jobs.[5]

In the next section, we set out our model. Numerical methods are required to solve for the stationary equilibrium. Before we do this, however, we present a simplified, analytical version of our model. This is given in Section 3. The key simplification is that we suppress any element of choice in the law of motion for the economy. This means that equilibrium can be defined purely in terms of steady-state conditions. By manipulating these conditions and differentiating with respect to the key parameters of our model, we can get some intuition about which factors are most important in determining the relative importance of wait unemployment.

In Section 4, we describe the algorithm that we use to solve for the stationary equilibrium. The value (expected lifetime utility) of search is a key element of this stationary equilibrium. The essence of our algorithm is an iterative solution for this value. In Section 5, we present our results. We set the parameters for our baseline case so that the equilibrium unemployment rate is close to 12 percent. Wait unemployment represents about 60 percent of unemployment and the duration of wait unemployment is significantly greater than that of search unemployment. To further give some sense of how the model works, we present the distributions of employment and resters across good sectors and across bad sectors along with the equilibrium wage distributions. We compute two alternative solutions to the model in an effort to assess the effects of changes in turbulence on the equilibrium. First, we increase the magnitude of the shocks; i.e., we make the good shock "better" and the bad shock "worse." This leads to an increase in unemployment, and the proportion of unemployment accounted for by resters decreases. In our second alternative, we hold the magnitude of the shocks at the level of the baseline case, but we increase their persistence. An increase in persistence reduces the unemployment rate. This reduction occurs largely through a decrease in search unemployment. Section 6 contains our conclusions.

[5] Other relevant theoretical papers include Pissarides [1992] and Rogerson [1996]. Relevant empirical papers include Lilien [1982] and Abraham and Katz [1986].

2. Model

2.1. Basic Structure

We consider a discrete-time, infinite-horizon economy with a continuum of workers. The measure of workers is exogenous and normalized to one. Workers are finitely lived, discount the future at rate β, and are replaced upon death. There is a large number of sectors in the economy, indexed by s. These can be interpreted as industries, regions, or firms. Conditions in a sector are either "good" or "bad": a shock $z \in \{G, B\}$ is realized in each sector in each period. These shocks follow a first-order Markov process and can be interpreted either as demand shocks or as productivity shocks.

In any time period, each individual is either attached to a sector or searching. Those who are attached to a sector are characterized by a level of sector-specific human capital and an employment status. An individual within a sector is indexed by $b \in \{e_0, e_1, r\}$, where e_0 denotes employed with no sector-specific human capital, e_1 denotes employed with sector-specific human capital, and r denotes resting, i.e., attached to the sector having no job but having sector-specific human capital. In each period, a sector is thus described by four numbers: its value of z and a vector $\mu = (\mu_0, \mu_1, \mu_r)$, where μ_b is the measure of workers in category b attached to that sector. Searchers are not attached to a sector and hence have neither jobs nor sector-specific human capital.

We can now describe the basic within-period timing in the model. A sector enters a period with vector μ and realizes a shock, z. The realization of z triggers a series of exogenous changes in μ (death, hiring/firing, human capital appreciation/depreciation, etc.); i.e., the vector of measures of worker types in the sector is updated according to $\widehat{\mu} = f(\mu, z)$. Next, wages and output within the sector are determined by $(z, \widehat{\mu})$. Finally, the $\widehat{\mu}_r$ workers in the sector with specific human capital who are unemployed decide whether to stay in the sector or to leave to join the pool of unemployed searchers. This final decision step updates $\widehat{\mu}$ to μ'.[6] The next realization of the shock, z', at the start of the next period, then starts the process again.

The details of the exogenous changes that occur within each sector at the beginning of each period are as follows. Some workers die, some workers (from the group that entered the period in category e_0) acquire sector-specific human capital, some workers (from the group that entered in category r) lose their sector-specific human capital and leave to search elsewhere, some unskilled workers (who entered the period employed) lose their jobs and become searchers, some skilled

[6] Note that $\widehat{\mu}_0 = \mu_0'$ and $\widehat{\mu}_1 = \mu_1'$; i.e., the decision step only affects the measure of resters in the sector.

workers (who entered the period employed) lose their jobs and become resters, and some workers (from the group that entered the period in category r) find a job.[7] We use the following notation:

ρ is the probability that z switches value (i.e., from G to B or vice versa)
δ is the probability of death
α_e is the probability (for a worker of type e_0) of acquiring human capital
α_r is the probability (for a worker of type r) of losing human capital
$\gamma(z)$ is the probability of losing one's job
$\pi_r(z, \mu)$ is the probability (for a worker of type r) of finding a job.

Note that while we assume ρ, δ, α_e, α_r to be given constants, we allow the separation rate, $\gamma(z)$, to depend on the currently realized value of the shock. Similarly, we assume that the measure of new jobs in a sector in the current period depends on the currently realized value of the shock. We denote this measure of new hires by $H(z)$. The probability that a rester will be rehired then depends on $H(z)$, on how these new hires are allocated between resters and searchers, and on how many other resters are available for rehire. We specify a precise form for $\pi_r(z, \mu)$ in Section 5. Searchers are hired into the sector to the extent that new hires exceed the measure of rehired resters. Letting λ be the index for searchers, the number of searchers hired into the sector is $H_\lambda(z, \mu) = H(z) - (1-\delta)\pi_r(z, \mu)\mu_r$. Finally, the workers who die are replaced by new entrants into the pool of searchers.

The vector of measures of worker types in the sector after these changes is denoted by $\widehat{\mu} = f(z, \mu)$. (Note that $\widehat{\mu}$ is a deterministic function of μ and z.) Thus

$$
\begin{aligned}
\widehat{\mu}_0 &= (1-\delta)(1-\alpha_e)(1-\gamma(z))\mu_0 + H_\lambda(z, \mu) \\
\widehat{\mu}_1 &= (1-\delta)(1-\gamma(z))\mu_1 + (1-\delta)\alpha_e(1-\gamma(z))\mu_0 + (1-\delta)(1-\alpha_r)\pi_r(z, \mu)\mu_r \\
\widehat{\mu}_r &= (1-\delta)\gamma(z)\mu_1 + (1-\delta)(1-\alpha_r)(1-\pi_r(z, \mu))\mu_r.
\end{aligned}
$$

Next, wages are determined, output is produced, and workers are paid. The wage a worker receives depends on whether he or she is endowed with sector-specific human capital and on labor market conditions in the sector at the time the wage is determined. That is, we assume wages of the form $w_0(z, \widehat{\mu})$ and $w_1(z, \widehat{\mu})$ for unskilled and skilled workers, respectively. Output depends on z,

[7] For convenience, we disallow the possibility that an individual can be affected by more than one shock at a time. In particular, we assume that workers of type e_0 who lose their jobs cannot at the same time acquire sector-specific human capital and that resters who lose their sector-specific human capital do not simultaneously find a job.

μ_0, and μ_1. We specify particular functional forms for wages and output in the numerical exercises that we carry out in Section 5 below.

Finally, resters decide whether to stay in their sector (r) or to enter the pool of searchers (λ). This decision (described in detail below) depends on conditions in the worker's own sector and on conditions elsewhere in the economy. In stationary equilibrium, this latter is completely summarized by the value of search, which we denote by v_λ. The decision made by each worker takes into account the effect of the decisions being made simultaneously by all other workers. The vector, $\widehat{\mu}$, together with these decisions, which depend on $\widehat{\mu}$, z, and v_λ, generates the final vector of worker measures within the sector. That is, the vector of worker measures carried into the next period is given by $\mu' = G(z, \widehat{\mu}, v_\lambda) = G(z, f(z,\mu), v_\lambda) \equiv g(z, \mu, v_\lambda)$, where the law of motion $g(\cdot)$ is as described above.

The above describes the characteristics of a single sector, which is completely described by the shock that has been realized and by the composition of its labor force: in a particular time period, the state of sector s is described by $(z(s), \mu(s))$. In stationary equilibrium, the distribution of states in the economy is fixed through time, although the identity of the sectors that are associated with particular states will vary. If we let S denote a collection of possible states and $1_{\{\cdot\}}$ the indicator function, stationary equilibrium requires

$$\sum_s 1_{\{((z'(s),\mu'(s))\in S\}} = \sum_s 1_{\{((z(s),\mu(s))\in S\}}$$

for all possible S.

2.2. Value Functions and Resters' Decisions

To model the decisions of the resters, we need the value functions associated with searching (v_λ), with employment as an unskilled worker ($v_0(z,\mu)$), with employment as a skilled worker ($v_1(z,\mu)$), and with resting ($v_r(z,\mu)$). These are:

$$\begin{aligned}
v_0(z,\mu) &= w_0(z,\widehat{\mu}) + \beta(1-\delta)E\{\gamma(z')v_\lambda + [1-\gamma(z')][(1-\alpha_e)v_0(z',\mu') \\
&\quad +\alpha_e v_1(z',\mu')]\} \\
v_1(z,\mu) &= w_1(z,\widehat{\mu}) + \beta(1-\delta)E\{\gamma(z')v_r(z',\mu') + [1-\gamma(z')]v_1(z',\mu')\} \\
v_r(z,\mu) &= \max[v_\lambda, \beta(1-\delta)E\{[1-\pi_r(z',\mu')][\alpha_r v_\lambda + (1-\alpha_r)v_r(z',\mu')] \\
&\quad +\pi_r(z',\mu')v_1(z',\mu')\}] \\
v_\lambda &= \beta(1-\delta)\{(1-\pi_\lambda)v_\lambda + \pi_\lambda \sum_s v_0(z(s),\mu(s))\frac{H_\lambda(z(s),\mu(s))}{\sum_\sigma H_\lambda(z(\sigma),\mu(\sigma))}\}.
\end{aligned}$$

Note that these value functions are written at the point within each period before resters' decisions are made but after the exogenous changes.[8] Thus, the expectations are being taken with respect to the joint density of (z', μ') conditional on (z, μ).

The expression for v_λ requires some explanation. Consider an individual who ends a period as a searcher. With probability $(1-\delta)$ he or she survives to enter the next period. In this case, a job is found with probability π_λ. This probability is computed as follows. The total measure of new jobs for searchers in a period equals $\sum_s H_\lambda(z(s), \mu(s))$, while the total measure of searchers is $1 - \sum_s [\mu_0(s) + \mu_1(s) + \mu_r(s)]$. Each searcher has an equal chance at any job that arises, so

$$\pi_\lambda = \min[1, \frac{\sum_s H_\lambda(z(s), \mu(s))}{1 - \sum_s [\mu_0(s) + \mu_1(s) + \mu_r(s)]}].$$

The expected value of taking this job as an unskilled worker (as of the start of the next period) is $\sum_s v_0(z(s), \mu(s)) \frac{H_\lambda(z(s),\mu(s))}{\sum_\sigma H_\lambda(z(\sigma),\mu(\sigma))}$. With probability $1 - \pi_\lambda$, the individual fails to find a job, in which case the value v_λ is retained. Note the implicit assumption that $v_0(z, \mu) \geq v_\lambda$ for all possible (z, μ).

Resters' decisions are based on a comparison $v_r(z, \mu)$ and v_λ. There are three possibilities to consider. If $v_r(z, \widehat{\mu}_0, \widehat{\mu}_1, 0) = v_\lambda$, it is no more valuable to be the last rester in the sector than to search, so all resters leave. If $v_r(z, \widehat{\mu}_0, \widehat{\mu}_1, \widehat{\mu}_r) > v_\lambda$, then all resters stay. Finally, if $v_r(z, \widehat{\mu}_0, \widehat{\mu}_1, x) = v_\lambda$ for some $x \in (0, \widehat{\mu}_r)$, then some resters stay and some leave.

3. Analytical Solution for a Simplified Version of the Model

To get a general sense for the model, we make a number of simplifying assumptions that allow us to solve for the steady state analytically. With these simplifications, we cannot answer many of the questions that are addressed by the full model. Nonetheless this simplified version provides some useful intuition.

The simplifying assumptions are:

A1. $\delta = 0$ - Individuals live forever.

Rather than allowing for birth and death, we simply recycle individuals through the various states that they can occupy.

A2. $\gamma(G) = 0$ - The layoff rate in sectors with a good shock is zero.

We let $\gamma(B) = \gamma$, a constant, denote the layoff rate in sectors with a bad

[8] We could write these values as functions of $\widehat{\mu}$, e.g., $v_r(z, \widehat{\mu})$, but since $\widehat{\mu} = f(z, \mu)$ we chose the simpler notation.

shock.

A3. $\pi_r(B, \mu) = 0$ - Resters are not rehired into a sector when $z = B$.

A4. $\pi_\lambda(B, \mu) = 0$ - Searchers are not hired into a sector when $z = B$.

A5. $\pi_r(G, \mu) = \pi_r$ - The rate at which resters are rehired into sectors in which times are good is a constant, independent of the sector's workforce.

A6. $\pi_\lambda(G, \mu) = \pi_\lambda$ - The rate at which searchers are hired into sectors with a good shock is a constant, independent of the sector's workforce.

A7. Resters stay in the sector in which they were laid off unless their skill depreciates, in which case they join the pool of searchers.

Assumption A7 eliminates any aspect of individual choice in the model; hence, equilibrium can be expressed solely as a set of steady-state conditions. The full set of assumptions imply that we need not keep track of individual sectors, only of what fractions of employment (unskilled and skilled) and resters are in sectors in which $z = B$ and what fractions are in sectors in which $z = G$. This means that we can define our steady-state equilibrium in terms of the states in which individuals find themselves, as opposed to the more complicated problem of tracking sectors.

With these assumptions, the following aggregate flow equations must be satisfied in steady-state equilibrium:

$$(\rho + \alpha_e + \gamma)E_{0B} = \rho E_{0G}$$
$$(\rho + \alpha_e)E_{0G} = \rho E_{0B} + \pi_\lambda S$$
$$(\rho + \gamma)E_{1B} = \alpha_e E_{0B} + \rho E_{1G}$$
$$\rho E_{1G} = \alpha_e E_{0G} + \pi_r R_G + \rho E_{1B}$$
$$(\rho + \alpha_r)R_B = \gamma E_{1B} + \rho R_G$$
$$(\rho + \alpha_r + \pi_r)R_G = \rho R_B$$
$$\pi_\lambda S = \gamma E_{0B} + \alpha_r(R_B + R_G)$$
$$E_{0B} + E_{0G} + E_{1B} + E_{1G} + R_B + R_G + S = 1.$$

Here E_{0B} is the measure of unskilled who are employed in sectors with $z = B$, E_{0G} is the analogous measure for sectors with $z = G$, E_{1B} is the measure of skilled workers who are employed in sectors with $z = B$, E_{1G} is the analogous measure for sectors with $z = G$, R_B is the measure of resters in sectors with $z = B$, R_G is the measure of resters in sectors with $z = G$, and S is the measure of searchers in the economy. The interpretation of these conditions is straightforward. For example, the first condition states that the flow of workers out of the category "unskilled, employed in a sector with $z = B$" must equal the corresponding flow of workers into that category. Workers exit this category when their sector switches from B to G (this occurs at rate ρ), when they acquire sector-specific human capital (this occurs at rate α_e), and when they lose their jobs (this occurs at rate γ).

The corresponding inflow comes from employed unskilled workers whose sector switches from G to B.

This system of equations can be solved for the various measures, but the expressions are complicated and of little intuitive value. A more interesting approach is to look at the ratio of resters ($R_B + R_G$) to total unemployed ($R_B + R_G + S$); i.e., wait unemployment as a fraction of total unemployment:

$$F = \frac{(R_B + R_G)}{(R_B + R_G + S)} = \frac{\alpha_e \pi_\lambda (2\rho + \alpha_e + \gamma)}{\alpha_e \pi_\lambda (2\rho + \alpha_e + \gamma) + \alpha_r [\rho\gamma + \alpha_e(2\rho + \alpha_e + \gamma)]}.$$

This expression is relatively simple and allows for the following straightforward comparative statics calculations:

$$\frac{\partial F}{\partial \rho} = \frac{-\alpha_e \alpha_r \pi_\lambda \gamma (\alpha_e + \gamma)}{(den)^2} < 0$$
$$\frac{\partial F}{\partial \alpha_e} = \frac{\alpha_r \pi_\lambda \rho \gamma (2\rho + 2\alpha_e + \gamma)}{(den)^2} > 0$$
$$\frac{\partial F}{\partial \alpha_r} = \frac{-\alpha_e \pi_\lambda (2\rho + \alpha_e + \gamma)[\rho\gamma + \alpha_e(2\rho + \alpha_e + \gamma)]}{(den)^2} < 0$$
$$\frac{\partial F}{\partial \pi_\lambda} = \frac{\alpha_e \alpha_r (2\rho + \alpha_e + \gamma)[\rho\gamma + \alpha_e(2\rho + \alpha_e + \gamma)]}{(den)^2} > 0$$
$$\frac{\partial F}{\partial \pi_r} = 0$$
$$\frac{\partial F}{\partial \gamma} = \frac{-\alpha_e \alpha_r \pi_\lambda \rho (2\rho + \alpha_e)}{(den)^2} < 0,$$

where $(den)^2 = \{\alpha_e \pi_\lambda (2\rho + \alpha_e + \gamma) + \alpha_r [\rho\gamma + \alpha_e(2\rho + \alpha_e + \gamma)]\}^2$.

The first comparative statics result indicates that as ρ increases, wait unemployment decreases as a fraction of total unemployment. An increase in ρ is equivalent to a decrease in the persistence of shocks, i.e., it becomes more likely that a shock will be reversed in the next period. Thus, as ρ increases, the expected duration of wait unemployment decreases. The second result is quite straightforward. As unskilled workers become more likely to acquire sector-specific skill, the importance of wait unemployment increases. This follows from the fact that an increase in skill acquisition increases the fraction of laid-off workers who have sector-specific skills and so can possibly wait in the sector. The third result is also straightforward. As α_r increases, more of those who wait lose their sector-specific skills and become searchers so that the fraction of unemployment made up of resters falls. (In the full model, there would be an additional indirect effect since it would become less advantageous to wait and more skilled unemployed would find it optimal to search elsewhere in the economy.) The next result states that as the likelihood of getting a job when searching increases, the fraction of unemployment composed of resters rises.

An increase in π_r, the rate at which resters are hired in good sectors, has no influence on the relative share of wait unemployment. This is an artifact of our analytical model. In the full model, resters' decisions are influenced by this parameter and we expect that it would have an effect, although the direction is unclear. The direct effect is to reduce the duration of wait unemployment in good sectors, but the indirect effect is to make wait unemployment more attractive relative to searching. Here the lack of effect arises because the reduction of wait unemployment in the good sectors is balanced by an equivalent reduction in search unemployment. The final result states that an increase in the rate at which workers are laid off, γ, decreases the share of wait unemployment. This follows from our assumption that layoffs only occur in bad sectors. Since it takes time to accumulate sector-specific human capital, high-skilled workers are relatively underrepresented in bad sectors. Thus an increase in γ contributes more to search unemployment than to wait unemployment. Again, in the full model this result may change since the rise in γ will influence the decisions made by resters.

4. Numerical Solution

The stationary equilibrium for the full model must be computed numerically. In equilibrium, the density of (z, μ) across sectors must be constant through time, even though the state of each sector will vary from period to period. For any potential stationary distribution of (z, μ) there will be a corresponding collection of value functions for individuals in the economy, all of which depend on the value of search, v_λ. These value functions, together with the exogenous processes (the Markov process for z, the layoff risks, etc.), in turn determine the evolution of (z, μ) within and between sectors. We thus have the following fixed point problem. Given a value of search, v_λ, find the corresponding stationary distribution for (z, μ). Then, given that stationary distribution, compute the corresponding value of search, say v'_λ. In equilibrium, we seek a value of search that returns itself in this process.

The details of our solution algorithm are as follows:

Step 1: Choose a starting value, v^0_λ.

Step 2: Given v^0_λ, compute $v_0(z, \mu; v^0_\lambda)$, $v_1(z, \mu; v^0_\lambda)$, and $v_r(z, \mu; v^0_\lambda)$. Use these values to compute the law of motion for μ; i.e., $\mu' = g(z, \mu; v^0_\lambda)$.

This step is relatively complicated. We begin by specifying a grid for (z, μ). Next, we specify starting points for the value functions, i.e., $v^0_0(z, \mu; v^0_\lambda)$, $v^0_1(z, \mu; v^0_\lambda)$, and $v^0_r(z, \mu; v^0_\lambda)$. This is relatively simple to do by assuming, for example, that no resters ever leave the sector. Given the starting values, we then "shock" each grid point and compute optimal rester choices. There are three possibilities at each grid point:

(i) All resters leave; i.e., $v^0_r(z, \mu; v^0_\lambda) = v^0_\lambda$.

In this case $\mu' = (\widehat{\mu}_0, \widehat{\mu}_1, 0) = (f_0(z,\mu), f_1(z,\mu), 0)$.

(ii) All resters stay; i.e., $v_r^0(z,\mu;v_\lambda^0) > v_\lambda^0$.

In this case $\mu' = (\widehat{\mu}_0, \widehat{\mu}_1, \widehat{\mu}_r) = (f_0(z,\mu), f_1(z,\mu), f_r(z,\mu))$.

(iii) Some, but not all, resters leave.

In this case, $\mu' = (\widehat{\mu}_0, \widehat{\mu}_1, x)$, where x is such that $v_r^0(z,\mu;v_\lambda^0) = v_\lambda^0$.

Given resters' decisions, we have our law of motion $g(z,\mu)$. Given $v_0^0(\cdot)$, $v_1^0(\cdot)$, and $v_r^0(\cdot)$, we then recompute the value functions to get $v_0^1(\cdot)$, $v_1^1(\cdot)$, and $v_r^1(\cdot)$. If the updated values are sufficiently close to the starting point, we proceed to the next step. Otherwise, we recompute optimal rester choices and iterate until the value functions converge.

Step 3: Simulate the economy, using the value functions computed in step 2, over a large number of sectors and for enough time periods in order to reach the stationary distribution for (z,μ).

Step 4: Compute v_λ^1.

Sample a large number of agents from the pool of searchers. Simulate the economy again, using the stationary distribution computed in step 3 and the value functions computed in step 2. Track the realized utilities for these agents over a large number of periods, and use these realized series to estimate v_λ^1.

Step 5: If $v_\lambda^1 \approx v_\lambda^0$, stop. Otherwise, return to step 2, using an updated value of search.

5. Results

In this section, we present our numerical results. In our baseline case, each period is one month long and our simulations are based on 1000 sectors, 500 of which are "good" in any period and 500 of which are "bad." We have chosen the death risk, δ, so that an individual's expected working life is 40 years; i.e., $\delta = \frac{1}{12\times 40} \approx 0.002$. Our persistence parameter ρ is set so that the expected duration of either a good or bad shock is 10 years; i.e., $\rho = \frac{1}{12\times 10} \approx 0.008$. We assume that it takes on average two years to acquire sector-specific skill and 5 years to forget it, so $\alpha_e \approx 0.042$ and $\alpha_r = \frac{\alpha_e}{2.5}$. The layoff risks are assumed to be $\gamma(G) = .0185$ and $\gamma(B) = .075$. These translate into expected durations of employment of 4.5 years in a good sector and 1.1 years in a bad sector. We use a discount rate of $\beta = 0.97$.[9]

We fix the hiring rate in such a way, given our assumed layoff rates, as to generate an overall unemployment rate that is close to the European Union aver-

[9] Since this is a monthly discount rate, it is, of course, quite low. We have chosen such a low value to help speed convergence.

age. Specifically, we assume hires of about 0.044 per good sector per period and of about 0.002 per bad sector per period. Thus, the overall hiring rate is about 2.2 percent per period. Half of all new hires in a sector are allocated to resters and half to searchers. If the measure of resters in the sector is insufficient, the unfilled positions are allocated to searchers.

We assume that output in a sector is determined by the function:

$$y = z\left(h_0\mu_0 + h_1\mu_1\right)^{1-\theta}$$

where h_0 is the productivity level of unskilled workers and h_1 is the productivity level of skilled workers. Wages equal marginal product for each worker type, i.e.,

$$w_0 = zh_0\left(1-\theta\right)\left(h_0\mu_0 + h_1\mu_1\right)^{-\theta}$$

$$w_1 = zh_1\left(1-\theta\right)\left(h_0\mu_0 + h_1\mu_1\right)^{-\theta}.$$

We assume in our baseline case that $h_0 = 1$, $h_1 = 1.5$, and $\theta = 0.1$. Thus, in a particular sector, the wage for workers with sector-specific skill is 1.5 times the wage of an unskilled worker. We set $z = 0.75$ when conditions in a sector are bad and $z = 1.25$ when conditions in a sector are good. Wages thus vary across sectors depending on whether the shock is good or bad and on the level and composition of employment. The latter depends on the sector's history. For example, a sector that is currently bad, but was previously good for a long while will have higher employment than a sector that is bad and has been bad for a long while. The latter will have higher wages since it has fewer workers.

Of primary interest are our results for unemployment. In the stationary equilibrium (after 150 periods), we find an overall unemployment rate of about 11.94 percent (searchers plus resters divided by the labor force) with 58 per cent of unemployment made up of resters. The resters also represent the relatively long term unemployed. The expected duration of unemployment for searchers is 3.56 months, while the expected duration of unemployment for resters is 7.06 months in good sectors and 16.60 months in bad sectors.

To illustrate the characteristics of our equilibrium more clearly, we have graphed the distributions of employment and unemployment across sectors. Figure 1 gives the distribution of employment across good sectors, and Figure 2 illustrates the distribution of employment across bad sectors. The units in the figures should be interpreted as thousandths of the labor force, e.g., 2 is equivalent to .002 times the labor force. Note that good sectors are generally much larger than bad sectors. This reflects the high degree of persistence in the economy. Nonetheless, there are some very small good sectors and a few large bad sectors. The small good sectors are sectors that were bad for a long time and recently became good, while the large good sectors are sectors that have been good for a long time. Similarly, small bad sectors are sectors that have been bad for a long time, while the large bad sectors were good for a long time but recently suffered a bad shock.

Figures 3 and 4 give the distributions of resters in good and bad sectors. These graphs are quite different. Among good sectors, there is a wider distribution of resters. The distribution is bimodal. The sectors with very few resters (near 0) are sectors that have recently become good. The sectors with relatively large measures of resters are those sectors that been good for many periods. (This reflects high employment in these sectors.) Among bad sectors, the distribution of resters is quite narrow. There is a small mass between 0.08 and 0.09, but all other sectors have fewer than 0.02 resters. Between 0 and 0.02, the distributions in the two types of sectors are quite different. There appear to be more bad sectors at the high end of this range.

The remaining figures present wage distributions. Figures 5 and 6 present the wage distributions over sectors for unskilled workers in good and bad sectors, respectively. Since the wage for workers with sector-specific skill is 1.5 times the wage of the unskilled, there is no need to show this distribution. First, note that the wages in bad sectors are uniformly below the wages in good sectors. Further, the wage distribution for good sectors is skewed to the right while that for bad sectors is skewed to the left. Good sectors with relatively low wages are those that have been good for a relatively long time. As employment grows in the sector, the marginal product falls, leading to relatively low wages in these sectors. Bad sectors with relatively high wages are those that have been bad for a long time and hence have lower employment.

Figures 7 and 8 present wage distributions for unskilled workers and skilled workers across individuals. Each of these includes workers in both good and bad sectors. The shapes reflect the shapes of the distributions in figures 5 and 6, but are adjusted by employment weights. Thus, the highest wage bad sectors, where employment is relatively low, appear less dramatically, while the low wage good sectors are more prominent.

We have performed two alternative computations of our model in order to assess the effect of changing turbulence on the results. In the first alternative, we increase the conditional variance of z holding persistence of the shocks (ρ) constant. By the conditional variance, we mean the variance of z in any period conditional on its value in the previous period. We do this by changing the magnitude of the good and bad shocks. That is, we assume that the z for the bad shock falls to 0.65, while the z for the good shock rises to 1.35. Increasing the conditional variance captures the idea of increasing turbulence in the economy. The effect is to raise the overall unemployment rate to 12.56 percent with the fraction of resters falling to 57 percent. Thus, there is a greater increase in unemployment among searchers. The duration of all types of unemployment increases. For searchers, it becomes 3.9 months, for resters in good sectors it is 7.2 months, and for resters in bad sectors it is 18 months.

Our second alternative is to change the persistence of the shocks without changing their magnitudes. This also changes the conditional variance of z, but, given our specification of the shock process, it does not change the unconditional variance of z, which was changed in the first alternative. Specifically, we decrease ρ, so that the expected duration of a good or bad shock is about 12 years rather than 10 years, i.e., $\rho = \frac{1}{12 \times 12} \approx 0.007$. This decreases the conditional variance of z and can be interpreted as a decrease in turbulence. In the new stationary equilibrium, the unemployment rate is 9.91 percent and the fraction of resters is .7013. That is, the unemployment rate falls compared to the baseline case, but while both the measures of searchers and resters fall, the reduction in unemployment is primarily among the searchers. The duration of unemployment falls for all unemployed. For searchers, it becomes 2.16 months, for resters in good sectors it is 6.92 months, and for resters in bad sectors, it is 14.51 months.

In sum, when we increase "turbulence" by increasing the magnitude of the shocks, we find that unemployment increases. Consistent with this, when we decrease "turbulence" by making ρ smaller (having shocks persist longer), unemployment falls for both searchers and resters, although the effect for searchers is greater.

6. Conclusions

In this paper, we constructed a model in which structural changes in an economy affect the equilibrium rates of search and wait unemployment. The model captures the idea that high-skilled workers who are laid off in a sector that is temporarily on bad times may rationally choose to wait for conditions to improve in order to avoid giving up the premium associated with their skills. We began by presenting a simplified, analytical version of our model in which we suppressed any element of choice in the law of motion for the economy so that equilibrium could be defined purely in terms of steady-state conditions. In this simplified model, we were able to examine how various parameters affect the relative importance of wait unemployment. For example, we found that an increase in the persistence of shocks increases the expected duration of wait unemployment.

We then presented a numerical solution to the full model. The parameters for our baseline case were set to generate an equilibrium unemployment rate close to 12 percent. In our results, wait unemployment represents about 60 percent of unemployment and the duration of wait unemployment was significantly greater than that of search unemployment. We also computed two alternative solutions to the model to assess the effects of turbulence on the equilibrium. When we increased the magnitude of the shocks, we found an increase in unemployment, and the proportion of unemployment accounted for by resters decreased. When we increased the persistence of shocks, the unemployment rate fell, largely via a decrease in search unemployment.

The results that we have presented are preliminary. A full calibration of the model would require matching the distributions of wages and unemployment durations generated in our numerical solutions with the corresponding distributions in the data. Nonetheless, we feel that our results cast some light on the relationship between structural changes and equilibrium unemployment.

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Figure 1: Distribution of employed workers in good sectors

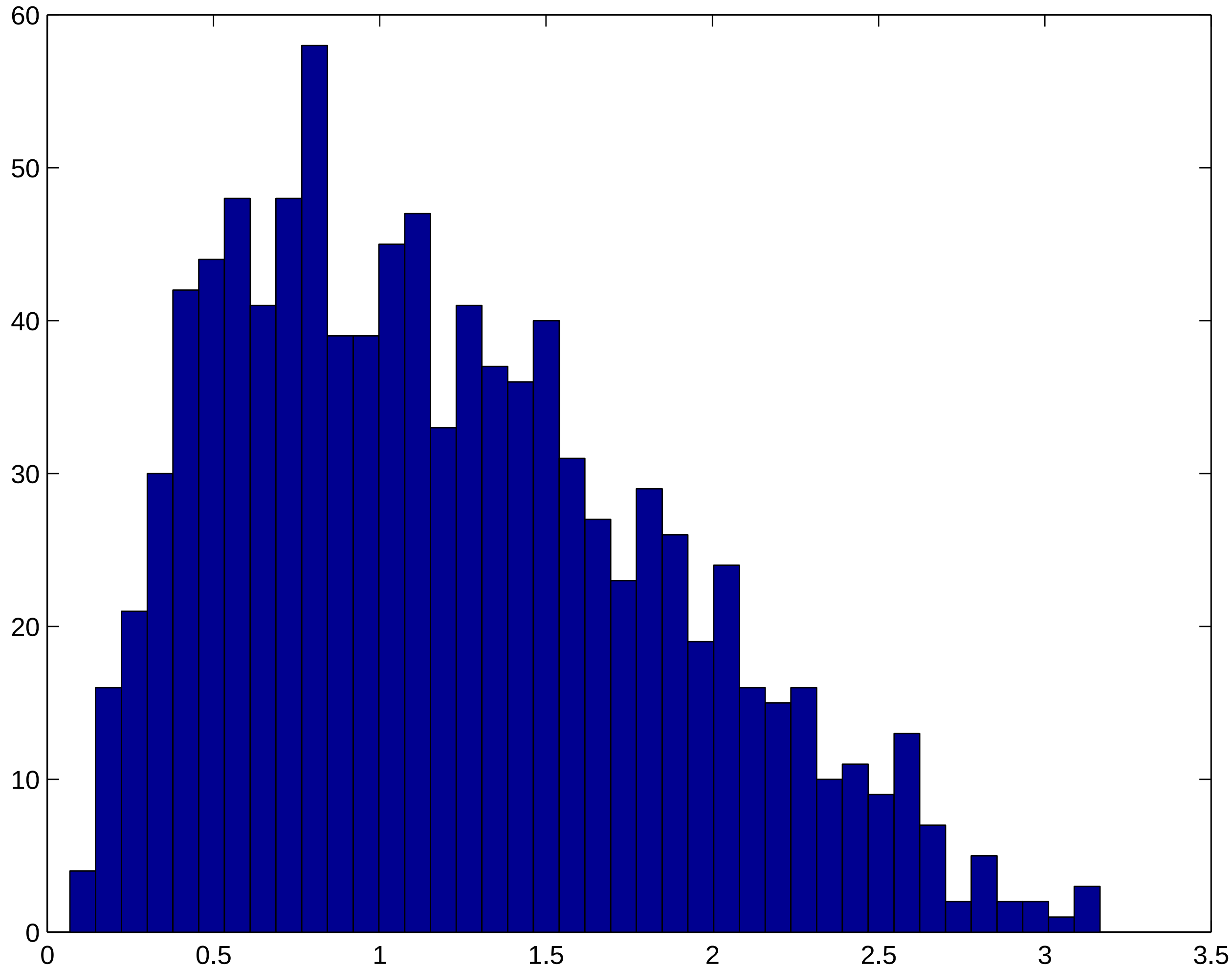

Figure 2: Distribution of employed workers in bad sectors

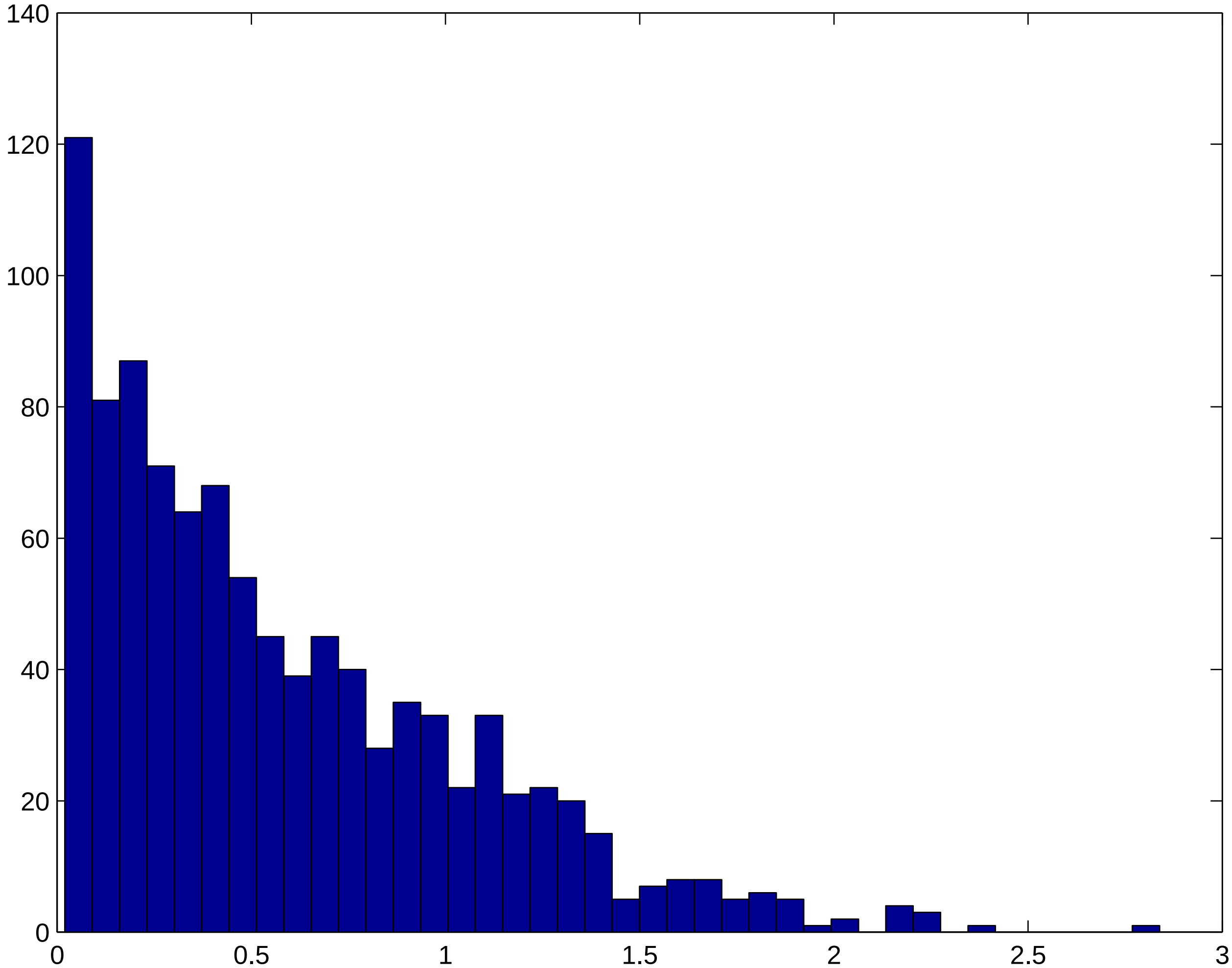

Figure 3: Distribution of resters in good sectors

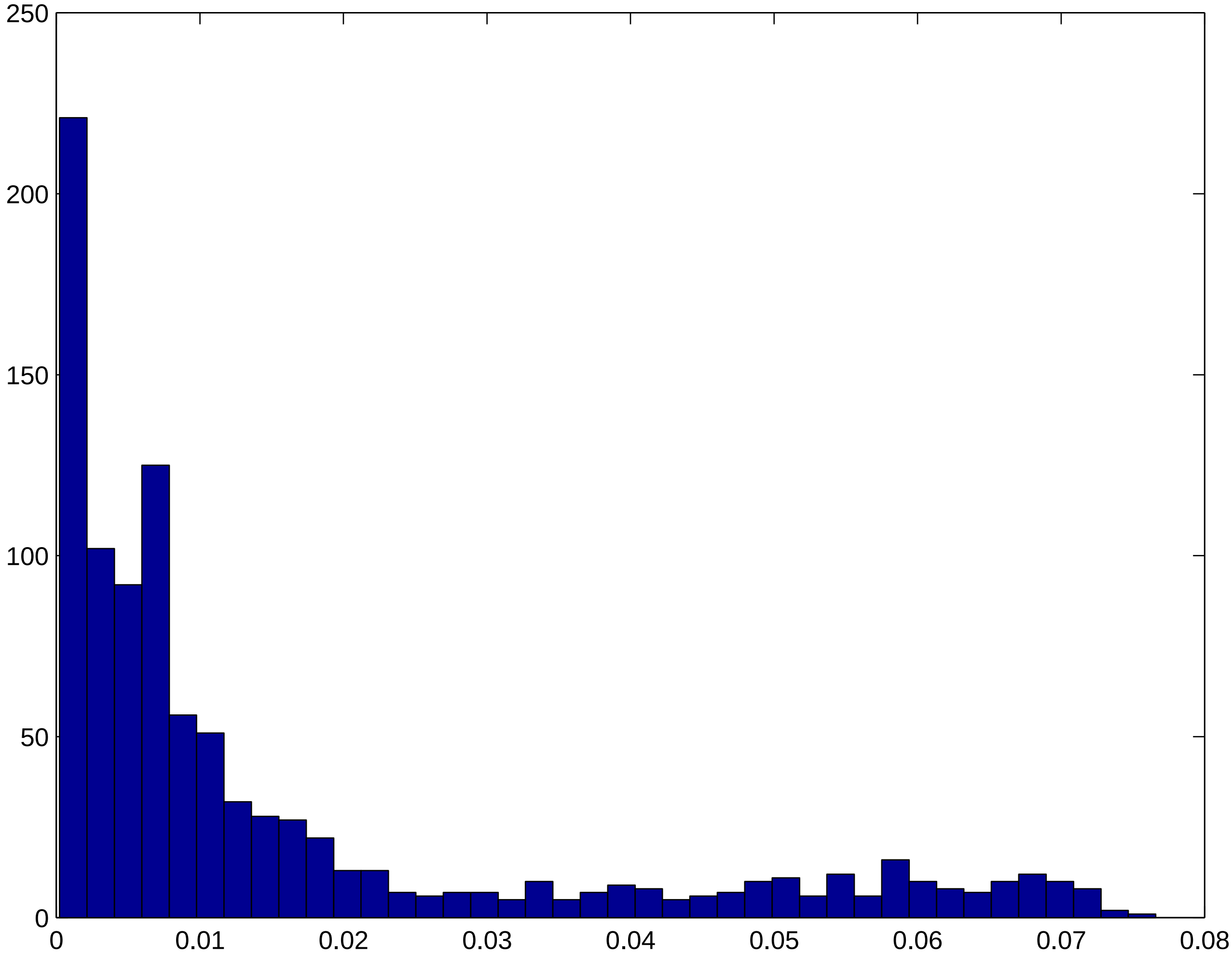

Figure 4: Distribution of resters in bad sectors