A Note on Peters and Severinov, "Competition Among Sellers Who Offer Auctions Instead of Prices"[1]

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Abstract: We consider a market in which sellers compete for buyers by advertising reserve prices for second-price auctions. Applying the limit equilibrium concept developed in [1], we show that the competitive matching equilibrium is characterized by a reserve price of zero. This corrects a result in [1].

In [1], Peters and Severinov (PS) consider a market with many buyers and many sellers of a homogeneous good. Sellers each hold one unit of the good and compete by advertising auctions; specifically, each seller posts a reserve price for a second-price auction for her good. Each buyer, after observing all posted reserve prices, chooses a seller and then competes in the seller's auction with any other buyers who have also chosen that seller. The main contribution of PS is to develop a limit equilibrium concept that can be applied to markets like these when there are infinitely many buyers and sellers. Their equilibrium concept is the standard one in directed search models in which sellers compete by posting auctions.

PS consider two cases. In the first, they assume that each buyer learns his valuation for the good only after selecting a seller. In the second, buyers learn their valuations before choosing which seller to visit. The contribution of our note is to point out an error in PS's characterization of the "competitive matching equilibrium" for the first case. The error in PS is their claim (p. 156) that "Despite the fact that sellers compete in price in this problem, the reserve price does not fall to zero in equilibrium." We now show that this claim is incorrect.

Lemma 2 (p.154) of PS shows that the competitive matching equilibrium for their first case is characterized by the (r^*, k^*) that solves

$$\max\ \Pi(r,k) \text{ subject to } V(r,k) = \beta,$$

where r is the reserve price, k is the Poisson arrival rate of buyers, and β is the market level of buyer utility. The seller and buyer payoffs are $\Pi(\cdot)$ and $V(\cdot)$, respectively, with

$$\begin{aligned} \Pi(r,k) &= k\int_r^1 v(x)e^{-k(1-F(x))}f(x)dx \\ V(r,k) &= \int_r^1 (1-F(x))e^{-k(1-F(x))}dx, \end{aligned}$$

where

$$v(x) = x - \frac{1-F(x)}{f(x)}$$

is the "virtual valuation function." As PS note (p.154), "it is straightforward to show that the solution to this maximization problem is unique."

We now show that $r^* = 0$ solves the above problem. The buyer arrival rate, k^*, is then determined by $V(0, k^*) = \beta$. The Lagrangean for the constrained maximization problem posed in Lemma 2 of PS is

$$L(r, k, \lambda) = \Pi(r, k) + \lambda\,(V(r, k) - \beta)$$

with first-order conditions

$$\begin{aligned} \frac{\partial L(\cdot)}{\partial r} &= \Pi_r(r^*, k^*) + \lambda^* V_r(r^*, k^*) = 0 \\ \frac{\partial L(\cdot)}{\partial k} &= \Pi_k(r^*, k^*) + \lambda^* V_k(r^*, k^*) = 0 \\ \frac{\partial L(\cdot)}{\partial \lambda} &= V(r^*, k^*) - \beta = 0. \end{aligned}$$

To show that these conditions hold when $r^* = 0$, note first that

$$\Pi_r(0, k^*) + \lambda^* V_r(0, k^*) = 0$$

implies

$$\lambda^* = k^*.$$

This follows from

$$\Pi_r(0, k^*) = k^* e^{-k^*} \quad \text{and} \quad V_r(0, k^*) = -e^{-k^*}.$$

We thus need to verify that

$$\Pi_k(0, k^*) + k^* V_k(0, k^*) = 0, \tag{1}$$

where k^* is the solution to $V(0, k^*) = \beta$. Since β, as a parameter of the problem, can take on any positive value, so too can k^*. We thus need to verify (1) for any positive value of k^*. This is done by direct computation.

Note first that

$$V_k(0, k^*) = -\int_0^1 (1 - F(x))^2\, e^{-k^*(1-F(x))} dx.$$

We then have

$$\begin{aligned}
\Pi_k(0,k^*) &= \int_0^1 \left(x - \frac{1-F(x)}{f(x)}\right) e^{-k^*(1-F(x))} f(x)dx \\
&\quad -k^* \int_0^1 \left(x - \frac{1-F(x)}{f(x)}\right)(1-F(x))e^{-k^*(1-F(x))} f(x)dx \\
&= \int_0^1 \left(x - \frac{1-F(x)}{f(x)}\right) e^{-k^*(1-F(x))} f(x)dx \\
&\quad -k^* \int_0^1 x(1-F(x))e^{-k^*(1-F(x))} f(x)dx - k^* V_k(0,k^*).
\end{aligned}$$

To prove (1) we thus need to show

$$\int_0^1 \left(x - \frac{1-F(x)}{f(x)}\right) e^{-k^*(1-F(x))} f(x)dx = k^* \int_0^1 x(1-F(x))e^{-k^*(1-F(x))} f(x)dx. \tag{2}$$

This final equality is verified by integrating the right-hand side of (2) by parts with $u = x(1-F(x))$ and $dv = k^* f(x)e^{-k^*(1-F(x))}dx$. This concludes the proof that the equilibrium reserve price equals zero.

Figure 1 (a corrected version of Figure 1 on p. 155 in PS), computed for the case in which buyer valuations are draws from a standard uniform distribution and $\beta = 0.2$, shows that the tangency between the seller and buyer indifference curves holds at $r^* = 0$.

References

[1] M. Peters, S. Severinov, Competition Among Sellers Who Offer Auctions Instead of Prices, J. Econ. Theory 75 (1997), 141-179.

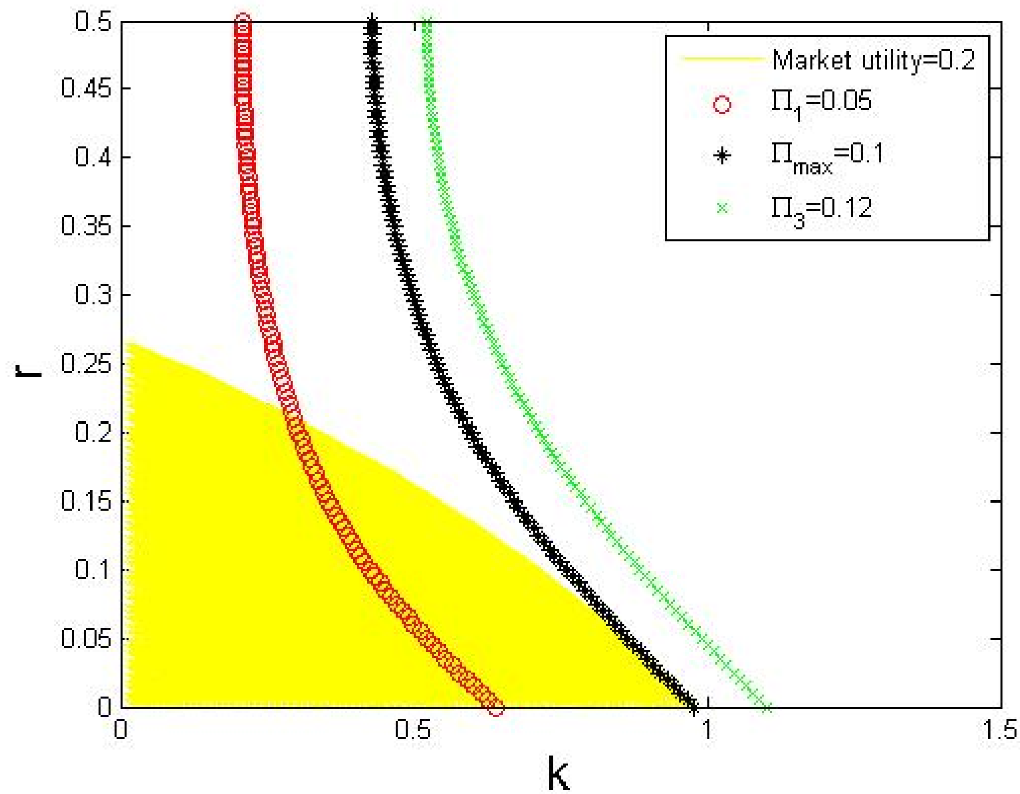
Market utility=0.2
Π_1=0.05
Π_{max}=0.1
Π_3=0.12
r
k
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
1
1.5

Figure 1