Prescribing Medications: Complicating Factors I and II

Teacher Materials

Mathematics Contained in the Lesson

This lesson covers linear systems of equations in 2 and 3 variables. Method of solution is not prescribed in the materials. The equations could be solved by methods of substitution or addition, or by matrix methods using a graphing calculator. It can be used at the level of Intermediate Algebra; it could also be used at the level of Pre-calculus.

Set-up

Students should work together in threes or fours. The simulation in **Prescribing Medications: Complicating Factors I** requires three students, but a fourth person can keep the group records. Students should also work together in groups of 3 or 4 on the homework following this first section if at all possible.

Materials required for the simulations are only paper and pencil; of course, if you are truly inspired, you or your students could make "checkbooks" with blank, check-sized slips of paper on which students can write the amounts they are giving away and a register in which they can record their debits and credits.

Foursomes can split into pairs of pairs when they work on the last section. Two students working together can check their progress periodically with the second pair, or get insights into the problem by talking with the others.

Organization

This lesson might follow the three lessons, **Reading this could help you sleep: Caffeine in your body**, **Get the lead out**, and **So much coffee, so little time**. In those lessons, students become familiar with the idea of an equilibrium value of a system; that is the key idea in the situation in these problems. Familiarity with that kind of process would make these problems easier for students. However, this lesson does not depend on having done those lessons and the idea of equilibrium is sufficiently developed in this material so that students who have not had prior experience will be able to develop an understanding of the idea here.

Students should read the Reading Assignment before coming to class. In class, distribute pages 3-5 of the student materials. The game simulating chemical interconversion is explained on page 3 of the student materials; however, it will be efficient if you introduce the rules of the game. It is important to note that the percent of money A gives to C is a percent of the **original amount of money**, not the amount of money A has after giving some to B. Thus, A gives 5% of $52 to B, and 40% of $52 dollars to C. A similar comment must be noted for B.

This project was supported, in part, by the National Science Foundation. Opinions expressed are those of the authors and not necessarily those of the Foundation

Once students have heard the rules, ask them to explain the analogy between the game and chemical interconversion as described in their Reading Assignment. Have them think about the connections for a few minutes, then hold a full class discussion on the connections. These points should be made in the discussion:

* The percent of money A gives B corresponds to the percent of chemical A that is converted to chemical B, and vice versa.
*The constant amounts of money that C gives A and B correspond to constant amounts of the two chemicals that a person may take each day.
*The percent of money A and B each give C correspond to percents of the chemicals A and B that are eliminated by the kidneys each day.

If students don't come up with these ideas themselves, help them to see the analogies.

Following completion of **Prescribing Medications: Complicating Factors I**, students have homework problems in which they make a mathematical model of interconversion of two chemicals in the body. The homework opens with another simulation with "money" which is best done with 3 (or 4) students, so students should be encouraged to arrange to do the homework in groups if at all possible.

Discuss the homework when students return to class. At this point students will have had several opportunities to solve a pair of equations in two variables. Pause here to review methods of solving equations and the meaning of those methods. Give the students additional problems in which to try these methods. Then distribute **Prescribing Medications: Complicating Factors II**. The readings at the end of this section should be done at home.

Answers to Problems and Teaching Suggestions for Prescribing Medications: Complicating Factors I

1.

Checkbook for person A			
Description	Debit	Deposit	Total
beginning of day 1			$52.00
gave 5% to B	– $2.60		$49.40
gave 40% to C	– $20.80		$28.60
received from B		+$16.40	$45.00
received from C		+$7.00	$52.00
beginning of day 2			$52.00
gave 5% to B	-$2.60		$49.40
gave 40% to C	-$20.80		$28.60
received from B		+$12.40	$41.00
received from C		+$7.00	$48.00
beginning of day 3			$48.00
gave 5% to B	-$2.40		$45.60
gave 40% to C	-$19.20		$26.40
received from B		+$9.60	$36.00
received from C		+$7.00	$43.00
beginning of day 4			$43.00

Checkbook for person B			
Description	Debit	Deposit	Total
beginning of day 1			\$82.00
gave 20% to A	– \$16.40		\$65.60
gave 10% to C	– \$8.20		\$57.40
received from A		+\$2.60	\$60.00
received from C		+\$2.00	\$62.00
beginning of day 2			\$62.00
gave 20% to A	-\$12.40		\$49.60
gave 10% to C	-\$6.20		\$43.40
received from A		+\$2.60	\$46.00
received from C		+\$2.00	\$48.00
beginning of day 3			\$48.00
gave 20% to A	-\$9.60		\$38.40
gave 10% to C	-\$4.80		\$33.60
received from A		+\$2.40	\$36.00
received from C		+\$2.00	\$38.00
beginning of day 4			\$38.00

Checkbook for person C			
Description	Debit	Deposit	Total
beginning of day 1			\$20.00
gave \$7 to A	–\$7		\$13.00
gave \$2 to C	–\$2		\$11.00
received from A		+\$20.80	\$31.80
received from B		+\$8.20	\$40.00
beginning of day 2			\$40.00
gave \$7 to A	–\$7		\$33.00
gave \$2 to C	–\$2		\$31.00
received from A		+\$20.80	\$51.80
received from B		+\$6.20	\$58.00
beginning of day 3			\$58.00
gave \$7 to A	–\$7		\$51.00
gave \$2 to B	–\$2		\$49.00
received from A		+\$19.20	\$68.20
received from B		+\$4.80	\$73.00
beginning of day 4			\$73.00

Totals at beginning of day				
	Day 1	Day 2	Day 3	Day 4
A	\$52	\$52	\$48	\$43
B	\$82	\$62	\$48	\$38
C	\$20	\$40	\$58	\$73

2. **a.** $a - 0.05a - 0.4a = 0.55a$

b. $0.2b + 7$

c. $a_{\text{next}} = 0.55a + 0.2b + 7$

d. $b_{\text{next}} = b - 0.2b - 0.1b + 0.05a + 2 = 0.05a + 0.7b + 2$

f. Solve $a = 0.55a + 0.2b + 7$ and $b = 0.05a + 0.7b + 2$ getting $a = 20,\ b = 10$.

g. $a(5) = 38.25$; $b(5) = 30.75$
$a(10) = 24.743$; $b(10) = 14.821$
$a(20) = 20.271$; $b(20) = 10.271$

Although part g is labeled optional, we would strongly encourage you to assign it. It is helpful for students' understanding of the physical/chemical process described and will help them to recognize the power of the elementary mathematics they are using.

One way to do the computation is on a TI-82 or TI-83 graphing calculator, using the SEQuence mode. Set the minimum n value to 1. Set u(1)=52, v(1)=82, $u(n) = 0.55u(n-1) + 0.2v(n-1) + 7,\ v(n) = 0.7v(n-1) + 0.05u(n-1) + 2.$ *Then use TABLE to read off the values of u and v when* $n = 5, 10, 20.$ *(Set Table minimum at 5 and use increments of 5.) Encourage students to investigate higher values of n if they do not do so on their own.*

Another way to do this problem is to work it on a spreadsheet, where the computer screen is displayed so the whole class can see. Students might find it valuable to understand how spreadsheets work and to see their connection to this problem.

h. Where $a(1) = 20\, and\, b(1) = 30$,
$a(5) = 24.063$; $b(5) = 15.313$
$a(10) = 21.17$; $b(10) = 11.209$
$a(20) = 20.068$; $b(20) = 10.068$

As before, the system tends toward its equilibrium value.

3.

Totals at beginning of day			
	Day 1	Day 2	Day 3
A	$20	$20	$20
B	$10	$10	$10
C	$124	$124	$124

Because we begin with the values of a and b for which $a_{\text{next}} = a$ and $b_{\text{next}} = b$, we expect each next day's amount to be the same amount as at the beginning of the day.

4. **a.** $a(2) = 65$ $b(2) = 73$ $c(2) = 72$

b. $a(3) = 68.90$ $b(3) = 45.20$ $c(3) = 95.90$

c. $a_{\text{next}} = 0.6a + 0.3b + 8$ and $b_{\text{next}} = 0.2a + 0.4b + 3$

e. Solve $a = 0.6a + 0.3b + 8$ and $b = 0.2a + 0.4b + 3$, getting $a = \frac{95}{3} = 31.67,\ b = \frac{140}{9} = 15.56$.

f. You are finding the values of a and b for which the next day's values are equal to the values at the beginning of this day.

g. Where $a(1) = 10$, $b(1) = 170$, $c(1) = 30$;
$a(5) = 56.198$, $b(5) = 29.524$, $c(5) = 124.28$;
$a(10) = 38.152$, $b(10) = 19.113$, $c(10) = 152.73$;
$a(20) = 32.109$, $b(20) = 15.798$, $c(20) = 162.09$.

5. Step 1 gives $x - .1x - .4x = 0.5x$; Step 2 gives $0.3y + 30$; and Step 3 gives the final answer $x_{\text{next}} = 0.5x + 0.3y + 30$.

6. $y_{\text{next}} = y - .15y - .3y + .4x + 50 = 0.55y + 0.4x + 50$

7. Solve $x = 0.5x + 0.3y + 30$ and $y = 0.55y + 0.4x + 50$ getting $x = 271.4$ and $y = 352.4$.

8. Students should compute these values with a calculator or spreadsheet program. Values of $x(10),\ y(10), x(20),\ y(20), x(30),\ y(30)$ will vary because students will use various initial values, but in all cases the values should approach $x = 271.4$ and $y = 352.4$.

Reflection point: Students have had several opportunities now to solve a pair of equations in two variables. Pause here to review methods and meaning. In **Prescribing Medications: Complicating Factors II**, students will solve systems in 3 variables.

Answers to Problems for **Prescribing Medications: Complicating Factors II**

1. a. (i) $x_{\text{next}} = x - 0.015x - 0.038x + 0.002y + 6$
$y_{\text{next}} = y + 0.038x - 0.002y$
or
$x_{\text{next}} = 0.947x + 0.002y + 6$
$y_{\text{next}} = 0.998y + 0.038x$

(ii) $x = 0.947x + 0.002y + 6$ and $y = 0.998y + 0.038x$
with solution $x = 400,\ y = 7600$.

Comment: At 900 μg of lead in the bloodstream, a child is considered to be in severe danger and in need of treatment.

b. $x_{\text{next}} = 0.947x + 0.002y + 6 + 0.02z - 0.01x = 0.937x + 0.002y + 0.02z + 6$
$y_{\text{next}} = 0.998y + 0.038x$
$z_{\text{next}} = z + 0.01x - 0.02z = 0.98z + 0.01x$
To find the equilibrium values, solve the equations:
$x = 0.937x + 0.002y + 0.02z + 6$
$y = 0.998y + 0.038x$
$z = 0.98z + 0.01x$
with solution $x = 400\ \mu\text{g},\ y = 7600\ \mu\text{g},\ z = 200\ \mu\text{g}$.

2. Beginning with equations

$$x_{\text{next}} = x - 0.4x - 0.3x + 0.01y + 1 \text{ and } y_{\text{next}} = y + 0.3x - 0.01y,$$

solve $0.7x - 0.01y = 1$ and $0.3x - 0.01y = 0$.
Solution: $x = 2.5$ mg of Vitamin A in the plasma and $y = 75$ mg in the liver.