Money Investigation 3
Teacher Materials

Mathematics Contained in the Lesson

The preliminary homework reviews plotting of points and naming of coordinates. The lesson develops students' skill in writing linear functions algebraically, relating slope to context, and understanding the significance of the y-intercept. The in-class lesson focuses on concrete illustration of slope; the homework problems include focus on slope as a rate of change. In the homework to follow the lesson in class (and subsequent discussion), students learn to tell the slope and y-intercept of a line from its equation.

Set-up

Ask each student to bring to class: 10 pennies
15 nickels
7 dimes
4 quarters

This should be enough for each group even if one member of a four person group forgets to bring coins. Students could make "paper" coins, but real coins would be better.

Students should have graph paper and a straightedge (the edge of a book would suffice). Students may use graphing calculators, but they are not essential for the lesson.

Organization

This activity should take one class period, with one homework assignment handed out the class before this activity is done, and a second homework to be handed out at the end of the class in which the activity is done.

Students should complete Homework 1, before you do **Money Investigation 3**, in class. If some students have questions about the homework, have other students present their solutions

Have students work together on the Student Classroom Materials in groups of 4 during class. As you observe their work, encourage them to do the concrete steps of the procedures; that is, to actually build the piles of coins described in the problems.

When all students have completed the problems (or 10 minutes before the end of the class, whichever occurs first) initiate a whole-class discussion to synthesize what they have learned.

Assign Homework 2, and <u>discuss what is learned in it during the next class meeting</u>. This homework gives students more practice in linear functions, extends the analysis of slope as a rate of change, and helps students learn to determine the slope and y-intercept of a line from its equation.

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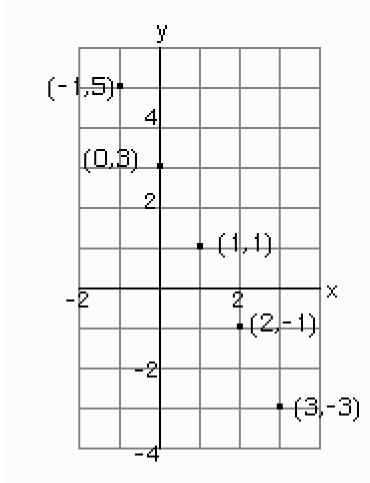
Answers to Problems and Teaching Suggestions for Homework 1

1. a.



b. Students should draw a straight line through the points on the graph.

c. 3; or $(0, 3)$.

d. 1.5, or $(1.5, 0)$.

e. Students are to choose any pair of points on their line and compute $\frac{\Delta y}{\Delta x} = -2$.

Take care that students understand that the order of subtraction of the coordinates must be the same for x and y. Encourage students to compare the results of the computation with *__different__* *pairs of points so that students note that the slope of a line is constant. When discussing this problem in class, it would be valuable to show graphs of $y = x^2$ and $y = x^3$, for example, to demonstrate that varying slope is what differentiates lines from other curves by pointing out the steep places, the less steep, the almost flat, possibly letting them compute $\frac{\Delta y}{\Delta x}$ for different pairs of points on these curves to see that the slope between points is not constant.*

2. a. The y-intercept is 0 or $(0, 0)$. That tells us that when he hasn't traveled any miles yet, it would require 0 gallons to fill the tank; thus we know that he started with a full tank of gas.

b. The slope of the line is $\frac{1}{20}$. This tells us that the car is using $\frac{1}{20}$ of a gallon of gas for each mile traveled; that is, the car gets 20 miles per gallon of gas.

Answers to Problems and Teaching Suggestions for Money Investigation 3

Encourage students to do the concrete step of the procedures, that is, to actually build the piles of coins described in the problems, as they work together.

1. a. Students should give some value for x and some value for y such as, "when $x = 2$ then $y = 10$." From this, they will hopefully write a statement similar to, "There are 5 times as many pennies as nickels."

b. Students should make a table of values for x and y that satisfy $y = 5x$; they should also plot the corresponding points on a graph and draw the line that passes through them (encourage the use of a straightedge).

c. $y = 5x$. Slope is 5; physical significance: when 1 more nickel is added to the first pile, 5 more pennies must be added to the second pile. Thus, increasing x by 1 causes a corresponding increase in y of 5.

It is important that you write the verbal description and the equation on the board when discussing this problem, so that students understand the relationship between them and that equations are just "short-hand" sentences. If students write an incorrect equation when working this problem, such as $5y = x$, have them substitute their values for x and y into their equation to see they are wrong.

d. Students should make a table such as

x	1	2	3	$\cdots$
y	15	20	25	$\cdots$

and plot the ordered pairs in their table on the same coordinate plane they used in part c, then drawing the line through them.

e. $y = 5x + 10$. As in part c: the slope is 5 and its physical significance is that when 1 more nickel is added to the first pile, 5 more pennies must be added to the second pile. Thus, increasing x by 1 causes a corresponding increase in y of 5. The vertical intercept is at $y = 10$, meaning that the first pile started with a dime in it.

f.

x	3	4	5	$\cdots$
y	0	5	10	$\cdots$

Students should graph these points on the same coordinate plane as before. Again, the slope is 5 and its physical significance is that when 1 more nickel is added to the first pile, 5 more pennies must be added to the second pile. The smallest number of nickels you can use in this problem is 3. You could use this to mention the concept of domain. The y-intercept of the line is $y = -15$; its "physical significance" is a little bit metaphysical: it suggests that if we <u>could</u> have 0 nickels, we would have to <u>remove the value of 15 pennies</u> from the quarter in the second pile.

If students remark that the three lines are parallel, ask why. Otherwise, wait until they have done part 2d. Then return to the three lines in #1 as a second example of a set of parallel lines.

2. a.

q	0	2	4	6	$\cdots$
d	0	5	10	15	$\cdots$

Students should plot the points in their table on a graph.

b. Students may write a relationship between q and d in words. Do not discourage that, but also get them to write an algebraic relationship, $5q = 2d$ or $d = \frac{5}{2}q$ or any equivalent equation. What "makes the relationship linear" is that each increase of 2 in q always produces the same increase in d of 5. The slope of the line is 2.5. Students might say that the physical significance of the slope is that 1 quarter has the same value as 2.5 dimes. This problem may be harder for the students. The verbal description of the slope should clarify matters.

c.

q	0	2	4	6	$\cdots$
d	2	7	12	17	$\cdots$

The equation is $d = \frac{5}{2}q + 2$ or some equivalent form.

The slope is 2.5, with the same physical significance as in part b.

d. The vertical intercept is 2. It signifies that if you have no quarters in the first pile you will need 2 dimes in the second pile.

e. Students should graph the lines $d = \frac{5q}{2}$ and $d = \frac{5q}{2} + 2$ on the same coordinate plane and explain why they are parallel. For example, "They are parallel because a change in q produces the same change in d on both of them."

At this point, refer back to the three lines of problem 1.

3. a.

d	0	1	2	$\cdots$	20
n	40	38	36	$\cdots$	0

with graph.

b. $n = 40 - 2d$. The slope is -2. Physical significance: for each dime you remove, you need two less nickels in the other pile of coins. Largest possible d is 20. The vertical intercept is 40. It means that if you remove 0 dimes, you need a matching pile of 40 nickels.

4. This question is intended to cause students to reflect on the work they have done and what they learned from it. You could use students' work here for the focus of whole-class discussion of slopes and y-intercepts of linear functions, or you could hold such a discussion before you ask them to do the writing.

Answers to Problems and Teaching Suggestions for Homework 2 to follow **Money Investigation 3**

1. a.

$x =$ games	$y =$ total income
0	250,000
1	350,000
5	750,000
8	1,050,000

b. Students should plot the points from the table in part a.

c. The y-intercept of the line joining the points gives the amount of money Magic Lightning makes this season if he never plays at all; that is, the signing bonus.

d. The slope of the line joining the points tells the rate at which his income increases in dollars per game.

e. $y = 100\,000x + 250\,000$, so he receives $10,250,000 for 100 games.

2. a. Line A: The y-intercept is 50 which tells us that Train A is 50 miles north of Union Station at noon, and the slope of 50 tells us that Train A traveled at 50 miles/hour northward.
Line B: The y-intercept is 0 which tells us that Train B is at Union Station at noon, and the slope is 70 which tells us that Train B traveled at 70 miles/hour northward. (Students will need to estimate the slope for this line. Anything near 70 miles/hour is acceptable.)

b. Train B is traveling faster; we can tell because it has the greater slope. We can see it on the graph since Line B is steeper.

c. The slope of line C is -50 and the y-intercept is 50.

d. Lines A and C have the same y-intercept; they were both 50 miles north of Union Station at noon. They have slopes with the same absolute value but with opposite signs; the trains are both moving at 50 mph, but Train A is moving north and Train C is moving south.

e. Line A is given by $y = 50x + 50$, line B is given by $y = 70x$, and line C is given by $y = -50x + 50$. We hope that students will see that the equation can be written in a form $y = mx + b$, or $y = b + mx$, from which they can read off the y-intercept as the constant term and the slope as the coefficient of x.

Reflection points: Give students time to clarify their ideas about y-intercept and slope of a linear function. Emphasize the concept of rate of change in the homework problems. In problem 1, the slope is the rate of change of an income with respect to number of games played. In problem 2, the slope is the rate of change of distance with respect to time.

Help students develop the idea that a linear function can always be written in the form $y = mx + b$, where m is the slope of the line and b is the y-intercept.