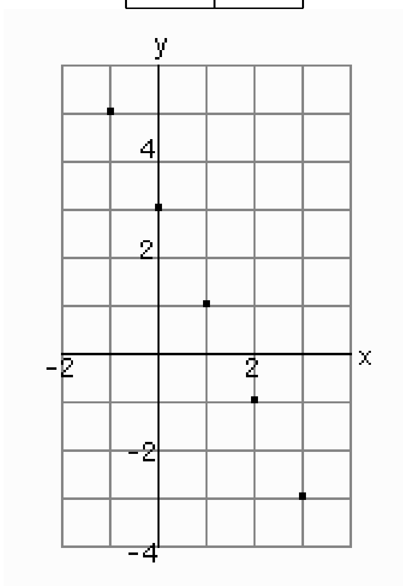
Money Investigation 3

Student Materials: Homework 1

In this set of activities, you will learn about lines and linear equations. By formalizing your understanding of the relationship between coins of different values and other everyday relationships, you will develop concepts about linear functions. With this understanding, you will be equipped to investigate many real problems because much of the real world seems to be linear, at least in the short run!

1. The set of points given in the following table are plotted on the graph

x	y
-1	5
0	3
1	1
2	-1
3	-3



a. Label each point on this graph with its coordinates, which can be found in the table.

b. Using a straightedge, draw the line that passes through these points.

c. The point where a line intersects the y-axis is called the ***y*-intercept**. What is the y-intercept of the line you drew?

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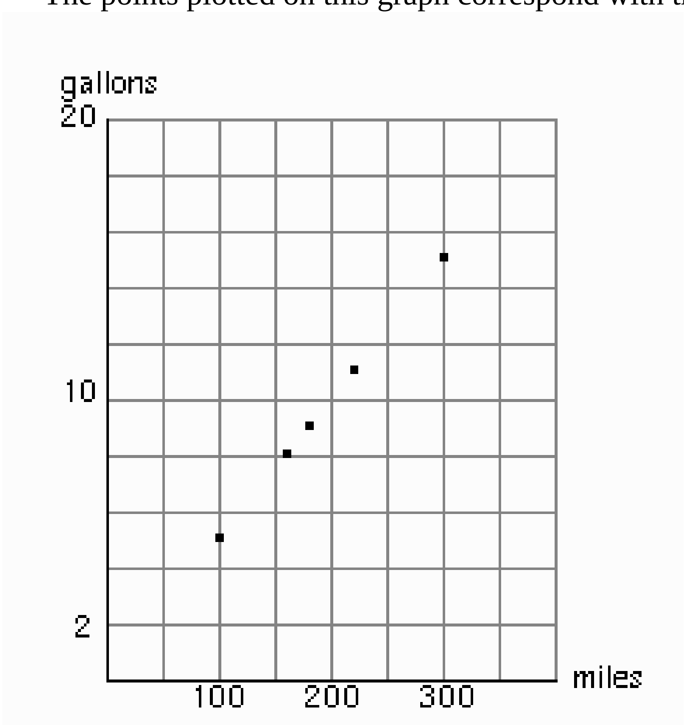
d. The point where a line intersects the x-axis is called the $\boldsymbol{x}$**-intercept.** What is the x-intercept of the line you drew?

e. The slope of a line is an important property. Notice this line goes downhill as it goes to the right; such lines have negative slope. More specifically, this line goes down 2 units in the y direction for each 1 unit it goes to the right. This line has a slope of -2. Choose any two points on the line that are more than one unit apart horizontally. Find the difference between their y-coordinates; divide that by the difference between their x-coordinates. Note that since subtraction isn't commutative, once you choose which point's y-coordinate is to be subtracted from the other, you should subtract the x-coordinates in the same order. Compare your answer with others in your class who may have chosen different points on the line.

2. When Sr. Jiminez drove his car from Mexico City to Washington, DC, he kept a record of his mileage between fill-ups and the amount of gas it took to fill his car's tank at each fill-up. Here is part of his record. The tank's capacity was 20 gallons, and he started the trip with a full tank of gas.

miles traveled	number of gallons to fill tank
100	5
160	8
300	15
220	11
180	9

The points plotted on this graph correspond with the data in the table.



a. Use a straightedge to draw a line connecting the points. What is the y-intercept of the line? What does that tell us? (That is, how does it relate to the story?)

b. Find the slope of the line. How is that related to the story?

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Student Classroom Materials

You will need a supply of pennies, nickels, dimes, and quarters to do the following.

1. In this problem, you will be making two piles of coins having the same monetary value. Throughout this problem, x will represent the number of nickels in the first pile and y will represent the number of pennies in the second pile. In some cases, each pile will have other coins besides pennies and nickels.

a. Make a small pile of nickels. This is the first pile of coins. Make a second pile of pennies that has the same monetary value as the first pile. Record the values of x and y. Write a sentence that describes the relationship between the two values.

b. Repeat part a. using different size piles. Make a table of values of x and y. Also plot the pairs of values (x, y) on a graph. Use a straightedge to draw the line containing these pairs of points.

c. Write the equation for a line that relates the number of pennies y in the second pile to the number of nickels x in the first pile. What is the slope of this line and what is its physical significance? By how much must y change when x is increased by one; that is, when one more nickel is added to the first pile?

d. Put a dime in the first pile, then add some number of nickels. Remember that x represents the number of **nickels** in the first pile. For example, if the first pile has the dime and 3 nickels, then $x = 3$. Put enough pennies in the second pile so that it has the same monetary value as the first pile. Let y represent the number of pennies in the second pile. Record the values for x and y. Make several different first piles, containing the one dime and some number of nickels. In each case, make a second pile of only pennies that has the same value as the first pile. Record x and y in each case. From this table, plot points on the same coordinate plane you used for part c's graph. Use a straightedge to draw the line containing this new set of points.

e. Write the equation for a line that relates x and y where x is the number of nickels in a pile that contains one dime and the rest nickels, and y is the number of pennies in a second pile that has only pennies and the same monetary value as the first pile. What is the slope of the line and what is its physical significance? Where does the line intersect the vertical axis? This is called the **vertical intercept.** (When the vertical axis is for y-values, it is called the y-intercept.) What is the physical significance of the vertical intercept in this problem.

f. Repeat parts d and e, except that the first pile starts with a dime and the second pile starts with a quarter. You then add x nickels to the first pile and enough pennies y to the second pile so that the two piles have the same value. In working this problem, think carefully about the physical significance of the y-intercept. What is the smallest number of nickels that will work?

2. In this problem, you will have two piles of coins. Let q represent the number of quarters in the first pile and let d represent the number of dimes in the second pile. Each pile may have coins other than quarters or dimes, according to instructions for that part.

a. Put an even number of quarters in the first pile and some number of dimes in the second pile so that both piles have the same monetary value. There are only quarters in the first pile and only dimes in the second. Record your values for q and d. Repeat this using different numbers of quarters in the first pile, remembering that you must use an even number of quarters. Make a table of the values of q and d with q in the first column. Plot points on a graph using the values from your table; put q on the horizontal axis and d on the vertical axis.

b. Write a relationship between q and d from part a. What makes this relationship linear? What is the slope of this line? What is the physical significance of the slope in this problem?

c. Repeat parts a and b, but this time put 4 nickels in the first pile with the q quarters.

d. What is the vertical intercept of the linear relation found in part c? What is the physical significance of this vertical intercept?

e. Graph the two lines found in this problem on the same coordinate plane. Why are these lines parallel?

3. In this problem, we have a first pile of coins that consists of exactly 20 dimes. Let d represent the number of dimes that are **removed** from this pile of coins. Let n represent the number of nickels that must be placed in a second pile of coins so that it has the same monetary value as the first pile, once the d dimes are removed.

a. Actually make the first pile of coins, then remove some number of dimes. Record d. Make a second pile of nickels that has the same monetary value as the first pile. Record n. Repeat this using different values for d. Record the values for d and n in a table. Use these values to plot points on a graph.

b. Write a relationship between d and n. What is the slope of this linear equation and what is its physical significance? What is the largest possible value for d? What is the vertical intercept of this line and what is its physical significance?

4. Write a paragraph about your understanding of slope and vertical intercept, often called the y-intercept. Use this note for future reference.

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Student Materials: Homework 2

1. When Magic Lightning signed on to play basketball for the Big City Tornadoes, the newest team in the NBA, he received a $250,000 signing bonus. His contract said that he would also receive $100,000 for each game in which he played.

a. Complete this table to show some of the possible amounts Mr. Lightning might make this season, depending on how many games he plays in.

$x =$ games	$y =$ total income
0	
1	
5	
8	

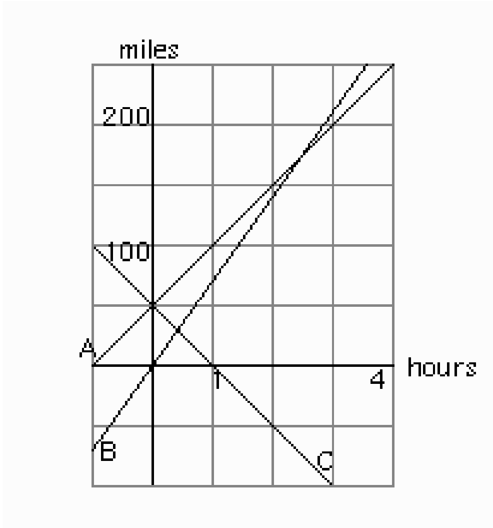
b. Plot points on graph paper to correspond with the information in the table. On the y-axis, choose units that are easy to work with.

c. What information can we get from the y-intercept of the line that joins these points?

d. What information does the slope of the line convey?

e. Write an equation relating x and y so that the equation tells how much money, y, Mr. Lightning makes if he plays x games this season. Use this line to compute how much Mr. Lightning makes if the season is 100 games long. When you have an equation, check to make sure all the ordered pairs in your table will satisfy it.

2. On this graph, line A shows the distance train A is from Union Station at time t (in hours after noon). Similarly, lines B and C show the distance trains B and C are from Union Station at time t.



a. Find the slope and the y-intercept of lines A and B. What do the slopes tell you? What do the y-intercepts tell you?

b. Which train is traveling faster, A or B? How do you know?

c. Find the slope and the y-intercept of line C.

d. Compare the graph for line C with the graph for line A. How are they alike? How are they different? Interpret what this means in the situation.

e. Write an equation for each of the lines A, B, and C. Studying the equations you have written, can you see <u>in the equation</u> how to tell what the slope of each line is? Can you see how to tell from the equation what the y-intercept is? If so, explain.