Money Investigation 1
Teacher Materials

Mathematics Contained in the Lesson

The mathematics in this activity includes:
- understanding and using the four basic binary operations in algebraic expressions;
- solving linear equations in one variable;
- solving a formula for an indicated variable.

Set-up

Ask each student to bring to class at least:

10 nickels,
5 dimes
8 quarters, and

This is enough so that if one member of a group of four students forgets, the group will still have enough to work with. Alternatively, you could make paper "coins", although we strongly recommend using **real** coins. To use paper coins, have each student write the appropriate amount on pieces of paper. It would help if all paper "pennies" were one size, all paper "nickels" a second size, and so on.

Organization

Students should do the homework on pages 1 and 2 of **Money Investigation 1** before they do the lesson together in class. For the in-class lessons, students should work in groups of 4, insofar as possible**.**

The main idea is to give students an opportunity to model with familiar concrete objects, coins, in order to develop their conceptual understanding of the four basic binary operations in algebraic expressions, which in turn will help them develop a conceptual understanding of the meaning of linear equations in one variable and how to solve them. It is important to begin by giving students an opportunity to realize they already know how to do mathematics and how to construct their own formulas. The introductory questions in the following suggestions for presentation are intended to provide that opportunity. During discussion, you may want to share this idea with them.

Begin by posing the following problem:

Suppose you have 10 nickels , a bunch of quarters, and the total value of these coins is z cents. Find the number of quarters you have.

Give students time to think about the question. If there are students who are able to answer it, ask them to explain it to the class; then continue. If no one is ready to answer it, leave it where it can be seen and continue; do not discuss this question.

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Tell the class: **To make this kind of question easier to understand, we will model it, and similar questions, with coins. That will help you understand the situation better by allowing you to actually see it. To get started, answer this first, easy, question:**

How many nickels are there in a pile of nickels that is worth 80 cents? Actually make a pile of nickels like that.

Do this in your 4-groups so that you can share coins. One pile of nickels per group is quite enough. This question should be easy for everyone, so when it has been answered, move right on. **Answer: 16.**

Next, **Suppose the only money you have is several rolls of nickels and you wish to purchase some item in a store. Describe how you would determine the number of nickels you need to make this purchase.**

Answer: Divide the cost of the item, in cents, by 5; if it doesn't 'come out even', round up. Be sure everyone is thoroughly comfortable with the language and the thinking before you go on. If a student suggests an alternative answer, such as, "multiply the cost in dollars by 20", it is an excellent opportunity to note that there is usually more than one approach to questions in mathematics.

Now tell the students to: **Make a pile of coins that includes 5 quarters and a bunch of nickels, and which has a value of $2.00. How many nickels are in the pile?** Assure that groups are working effectively together. **Answer:** They need 15 nickels. In essence, they have computed

$$n = \frac{200 - 125}{5}$$

In the discussion, ask students to explain their approaches; encourage differences, even slight ones.

After this introduction, you can see if there are any questions on the homework. Then distribute the Student Classroom Materials, pages 3, 4, and 5 of **Money Investigation 1** and let students work.

Answers to Homework, pages 1 and 2, of **Money Investigation 1**

1. a. 155 cents

b. For example, students might write: "Add the total amount of money you have in nickels, dimes and half dollars, which is $1.05, then add 25 times the number of quarters to that.

c.

x	w
0	105
1	130
2	155
3	180

d. $w = 105 + 25x$

e. $w = 105 + 25 \times 13 = 430$

2. a.

$x =$ number of quarters	$y =$ number of nickles
0	0
1	5
2	10
3	15

b. $5y = 25x$ or $y = 5x.$

c. When $y = 60,$ then $x = 12$ since 60 nickels is equivalent to 12 quarters.

Answers to pages 3 and 4, of **Money Investigation 1**, along with **teaching suggestions**

1. 80 cents

2. $1p + 5n + 10d + 25q,$ up to order, is the goal. *Do not let students skip this problem; students who are unused to group work or to hands-on work may have a tendency to do so. This problem will help students to establish a better understanding of why the variable in these problems should represent the number of each kind of coin instead of its value and thus facilitate their understanding of later problems.*

3. a. 17 quarters

b. "I multiplied 12 by 5 and then subtracted the answer from 475. I got 425. I divided that by 25 to get the number of quarters."

c. "I add up the money. 12 nickels is 60 cents. 17 quarters is \$4.25. Add them together and get the right amount of money: 485 cents," for example.

d. $\frac{t-60}{25} = q,$ using part b. You might also accept $60 + 25q = t$ with the comment that you solve for q after you are given t.

e. "The amount of money in quarters has to be a multiple of 25. If we subtract away the money in nickels, 60 cents, we'd have 380-60 = 320, which isn't a multiple of 25."

Reflection point: If students answer part d in two or more forms, take advantage of the opportunity to have students discuss them and why they are equivalent. This will be helpful to them both in (1) gaining confidence in their own thinking rather than believing that they have to write what the teacher is thinking and (2) recognizing how to manipulate expressions in equations in order to solve them.

4. a. For example:

$x =$ number of dimes	$y =$ number of nickels
0	5
1	7
2	9
3	11
4	13

Students who have not had experience making tables may need guidance here.

b. $25 + 10x =$ amount of money if first pile; $5y =$ amount of money in second pile; $25 + 10x = 5y$

c. "The change in y is twice as much as the change in x." Or, "Each time x goes up one, y goes up 2."
d. When $y = 12$, you would have 60 cents in the nickel pile. There is already a quarter in the other pile. That leaves 35 cents to be made up in dimes. We can't do that!
e. If $x = 4$, then $y = 13$. There would be 13 nickels in one pile, and 1 quarter and 4 dimes in the other.
f. $y = 2x + 5$. "Because x is multiplied by 2, any change in x is doubled."

Reflection Point: The purpose of parts c and f are to begin helping students know the purpose of slope of a line, when they get to that.

5. a. In the pile with the quarters, there is twenty cents besides the quarters; in the pile with the dimes, there is another 15 cents, perhaps three nickels.
b. x could be 1, 3, 5, 7,...
c. $5 + 25x = 10y$. Physically, this would mean removing the extra 15 cents from the pile with the dimes; and removing 15 cents from the other pile so that the two piles still have equal amounts of money. After division by 10, you get $0.5 + 2.5x = y$.

Reflection Point: You might discuss the meaning of the 2.5 in the equation in part c. When you add 2 quarters, you add 5 dimes, $2\frac{1}{2}$ times as many. Part b is intended to develop preliminary thinking about domain of functions in context.

6. *If students have had difficulty with the assignment of variables to the* <u>*number*</u> *of coins as opposed to the value of the coin, they will likely go astray on this problem. For groups where this happens, suggest reviewing problem 2. Then ask, "In problem 6, what does the 75 tell us? What does the 15 tell us? What about the 45?" Then let students return to the question in 6a.*
a. x is the number of coins worth 15 "cents"; y is the number of coins worth 1 "cent". One pile has some 15-"cent"-pieces and another 75 "cents"; the other pile has some 1-"cent"-pieces and another 45 "cents".
b. Since $y = 30 + 15x$, y could be 30, 45, 60, 75,...
c. Physically, subtracting 45 from each side means removing 45 cents from the constant extra money on each side.

For homework, assign problems that students did not work in class, along with the homework assignment on page 6 of **Money Investigation 1**. After this activity, you can refer to this approach for students that are having difficulty with linear expressions; that is, you can have them learn to put their problems in a context. When they apply a binary operation incorrectly, you can ask them if what they did makes sense in the context of a money problem.

Answers to problems and teaching suggestions for Homework, page 5, to **Money Investigation I**

1. a. $25a$ gives the amount of money in cents that I have in quarters; $50b$ gives the amount of money in cents that I have in half dollars; $25a + 50b$ gives the amount of money in cents that I have in my pocket altogether in quarters and half dollars. 1000 tells what amount that is; $10.

b. $25a + 50 \times 9 = 1000;\ a = 22.$ "If I have 9 half dollars, then I have 22 quarters" or $a = \frac{1000-450}{25}$.

c. For example:

a	b
0	20
10	15
20	10
30	5
40	0

d. $a + 2b = 40$. "Yes." Explanations for this answer will vary; try to encourage the essence of good mathematical thinking rather than searching for a rigorously correct explanation. Ideas like, "If two numbers are equal, then dividing them both by 25 has to get you the same answer," is of course only part of the reason, but it is important. The other issue is that, "To divide the number $25a + 50b$ by 25, you have to divide both parts of it by 25." Another explanation might be that they are computing the number of "quarters" in each term when they divide by 25 and the total number of "quarters" must be the same on each side of the equation.

Reflection point: Help students recognize the uses here of (1) the multiplication property of equality, (2) a multiplicative inverse, (3) and the distributive property. It is not important to stress the language at this stage, but it is helpful for students to see that what they are doing is part of a larger abstract idea.

2. a. So she could determine the weight of the mercury apart from the weight of the tube by subtracting it from the total 108 grams.

b. $30 + 3v = 108 - 42$

c. $v = 12$

3. a. $x = 5$ **b.** $x = 9$ **c.** $x = 3$ **d.** $x = 6$

4. In discussing these answers, look for the good in students' thinking and try to reinforce it. As above, it can be useful to help students recognize that the mathematical ideas they express informally is so important that it has been stated in careful rigorous language for formal occasions.

Following this assignment, you could assign homework problems on (1) evaluating linear expressions, (2) solving linear equations with one variable and (3) solving a formula for a given variable.