# Money Investigation 1

Student Material: Homework

## Introduction

In this unit, you will learn to understand the meaning of statements like $3x + 2y = 5$ and $15 + 4a = 3b$. You will also learn how to use such equations to answers questions. The "applications" you will work with in this unit are connected to your own experiences in order to help you learn the mathematics. The hands-on nature of this lesson is intended to help you develop a feel for the meaning of the mathematical language. With time, you can develop enough facility with the language of mathematics to be able to work applications that help people solve some of the real problems in our world.

## Example

Suppose I have 7 dimes, 3 quarters, and $x$ nickels. The total value of all these coins is $w$ cents. If you knew $x$, how could you find $w$? First try it with $x = 8$. That is, suppose in one instance you have 8 nickels. How much money do you have?

You probably said, "I have 70 cents in dimes, 75 cents in quarters and 40 cents in nickels; so that is 70 + 75 + 40 = 185 cents altogether."

Once you have solved this problem for a specific number of nickels, 8 nickels in this case, you could write a verbal description of the solution to the original problem. For example, you could write: "Find the monetary value of the dimes by multiplying the number of dimes by 10: $(7 \times 10)$; find the monetary value of the quarters by multiplying the number of quarters by $25$: $(3 \times 25)$; and find the monetary value of the nickels by multiplying the number of nickels by 5: $(x \times 5 = 5x)$. Then add the three results to get the total." Once you have the verbal description, you can use it to write a formula for $w$, such as,

$$7 \times 10 + 3 \times 25 + 5x = w \quad \text{or} \quad 70 + 75 + 5x = w \quad \text{or} \quad 145 + 5x = w.$$

In this example, how much money would there be if $x = 3$?

Mathematicians (and just about everyone else) often store information in tables; tables organize the information and make it easier to see patterns. We can make a table that shows how much money we would have in our collection of 7 dimes, 3 quarters, and $x$ nickels for various values of $x$. When $x = 0$, we have 145 cents (7 dimes and 3 quarters). When $x = 1$, we have 150 cents (7 dimes, 3 quarters, 1 nickel). Put the information into a table like Table 1:



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| Table 1 | |
|---|---|
| $x =$ number of nickels | $w =$ Total amount of money in cents |
| 0 | 145 |
| 1 | 150 |
| 2 | . |
| 3 | . |
| . | . |
| . | . |

We can think of the values in the table in terms of the money or in terms of the equation we wrote, $145 + 5x = w$. Which way do you find it easiest to think about this situation? Try thinking about it the other way.

**1.** Now suppose you have 7 nickels, 2 dimes, 1 half dollar, and $x$ quarters. The total value of all these coins is $w$ cents.
**a.** How much money is that if $x = 2$?
**b.** Write a verbal description of how to find $w$ when you know $x$.
**c.** Make a table similar to table 1, putting in values for $w$ when $x$ is 0, 1, 2, 3.
**d.** Write a formula for $w$ in terms of $x$ by following your verbal description.
**e.**. If you put $x = 13$ in the table, what would go in the $w$ column?

**2.** Tables can often be used to investigate a situation. Suppose a friend makes two piles of coins with the same monetary value. One pile has only nickels and the other pile has only quarters. If she only puts one quarter in the second 'pile', she will need 5 nickels in the first pile. Two quarters is 50 cents, so she will need 10 nickels to equal that.

**a.** Make a table of the number of quarters in the first pile versus the number of nickels in the second pile.

**b.** Let $x$ represent the number of quarters in the first pile and let $y$ represent the number of nickels in the second pile. Develop a relationship between $x$ and $y$. Noticing the pattern in your table might help.

**c.** If you put $y = 60$ in the table, what would go in the $x$-column?

**Money Investigation 1**
Student Classroom Materials

**1.** Make a pile of 4 nickels and 6 dimes. You can represent the total monetary value of this pile of coins with the expression

$$5n + 10d$$

where $n$ represents the number of nickels (so $n = 4$) and $d$ represents the number of dimes (here $d = 6$). Find the value of your pile of coins by substituting 4 for $n$ and 6 for $d$ in this expression and check that it gives the correct monetary value, in cents

**2.** Each person in your group will secretly make a pile of coins. Let $p$ be the number of pennies in your pile; $n$, the number of nickels; $d$, the number of dimes; and $q$, the number of quarters. Each person will then tell the rest of your group his/her values for $p, n, d,$ and $q$. Everyone should then determine the monetary value of each individual's pile of coins. Write, symbolically, the expression you used to find the value of the money. Compare your expression with the expressions written by the others in your group.

**3.** Suppose you have 12 nickels and a bunch of quarters. The total value of these coins is $t$ cents.

**a.** Find the number of quarters you have if $t = 485$ cents.

**b.** Write a verbal description detailing how you determined the number of quarters in part a.

**c.** Find a way to check your work. Describe how you do the checking.

**d.** Use your verbal description to find a formula that gives you the number of quarters you have in terms of $t$. Check that your formula gives you the correct answer to part a.

**e.** Why can't $t = 380$?

**4.** Make a pile of coins that includes one quarter and several dimes. Make a second pile of only nickels which has the same monetary value as the first pile. Repeat this process, but with a different number of dimes in the first pile and the corresponding number of nickels in the second pile.

**a.** Make a table of the number of dimes in the first pile versus the number of nickels in the second pile. (Note: write the <u>number of</u> dimes and nickels, not their value.)

**b.** Let $x$ represent the number of dimes in the first pile and let $y$ represent the number of nickels in the second pile. Develop an expression for the total value in cents of the first pile. Develop a second expression for the total value in cents of the second pile. Set the expressions equal to each other, since the piles have equal value. **$\boldsymbol{x}$ is number of dimes; $\boldsymbol{y}$ is number of nickels.**

**c.** How does the value for $y$ change each time you change the value for $x$? Your table in part a may help.

**d.** Use your equation to find $x$ if $y$ is 12. Then use coins to model this situation. Do you get the same answer both ways? Use coins to help you understand how to "solve" the equation.

**e.** Use your equation to find $y$ if $x$ is $4$. Then use coins to model this situation. Again, use the coins to help you understand how to solve the equation.

**f.** Put your equation in a form that gives $y$ in terms of $x$. (Thus, write it as "$y =$ ....") Clearly, if you add one dime to the first pile, you must add two nickels to the second pile so that the piles have the same value. How does the simplified equation show why you need to add two nickels to the second pile each time you add one dime to the first pile?

**5.** Let $x$ represent the number of quarters in a pile of coins and $y$ represent the number of dimes in a second pile of coins. The two piles have the same monetary value, which is indicated by the formula

$$20 + 25x = 15 + 10y.$$

**a.** Explain what else is in each pile of coins.

**b.** What are the possible number of quarters in the first pile? If you have difficulty with this question, make several first piles and then determine for which ones you can make the second pile have the same monetary value.

**c.** Simplify this equation by subtracting 15 from both sides. Describe what that would mean physically in terms of the piles. Then solve the equation for $y$ by dividing by 10.

**6.** Someone made up a problem like the one in problem 5 about piles of coins from a fictional country (with money different from ours). The piles could be described by the equation

$$75 + 15x = 45 + y$$

**a.** What do $x$ and $y$ represent? What is in each pile of coins?

**b.** What are the possible values of $y$ for this coin problem? Explain why.

**c.** Simplify this equation by subtracting 45 from both sides. Describe what that would mean physically in terms of the piles.

**Money Investigation 1**
Student Materials: Homework

**1.** In the equation $25a + 50b = 1000$, $a$ represents the number of quarters in my pocket and $b$ represents the number of half dollars in my pocket.

**a.** Explain what is represented by each of the terms in the equation; that is, by $25a$, $50b$, $25a + 50b$, and $1000$.

**b.** Find the value of $a$ when $b = 9$. Then explain what the answer means in terms of the situation.

**c.** Make a table of any five ordered pairs of values for $a$ and $b$ that satisfy the equation.

**d.** Divide each term of the equation by 25. Test the five pairs of values of $a$ and $b$ in your table. Do they satisfy this new equation, too? Why?

**2.** A chemist had 30 grams of mercury in a tube. She also had three identical vials which contained equal amounts of mercury. She poured all three of them into the tube with the first 30 grams. Then she realized the three vials were different from those she had used on other occasions and she didn't know how much mercury was in each of these vials. So she weighed the tube containing all her mercury. It weighed 108 grams. She had another tube just like it but empty, and she found it weighed 42 grams.

**a.** Why did she need to know the weight of the other tube in order to determine the weight of the mercury in each of the three vials?

**b.** Write an equation for the weight of mercury in her tube where the variable $v$ represents the weight of mercury in one of the small vials.

**c.** Solve your equation for $v$.

**3.** Solve these equations for $x$.

**a.** $2x + 4 = 14$
**b.** $2x - 4 = 14$
**c.** $2x + 4 = 3x + 1$
**d.** $3x + 8 = 6x - 10$

**4.** Explain your process for solving each of the equations in problem 3.