

Light's Speed in Water

Student Materials: Reading Assignment

Introduction

In the 19th century, the French physicist Foucault developed a method for measuring the speed of light in air. Using Foucault's method as a foundation, the 20th century physicist Albert Michaelson was able to measure the speed of light in a vacuum. Because of their work, we know that light travels in a vacuum at a rate of about 30 cm/nanosecond. A nanosecond is one billionth of a second.

1. Use this information to compute the speed of light in miles per second. If you already knew the speed of light in miles per second, does what you knew agree with the result of this computation?

The speed light travels depends on the **optical density** of the medium. Light is slower in water than it is in air. It was Foucault who first proved this statement, but you have probably seen evidence of it yourself. When you place part of a straight object such as a pencil or a ruler into water, the object appears to bend at the surface of the water. This illusion is caused by the difference in the rate at which the light travels through water and through air. The reason for this illusion is that light does not necessarily travel in a straight line. Instead, **light travels the path between two points that takes the least amount of time to traverse**; this fact is called **Fermat's principle**.

For example, when taking a trip in a car, you don't necessarily go by the shortest route. Instead, you may go somewhat out of your way to reach an interstate highway. Even though this route may be longer, you can drive it in less time since your speed on the interstate is much greater than your speed on other roads. Since light travels at a faster speed in air than in water, just like you on the interstate, light travels a route that gets it to the air quickly, then travels through a longer distance in air, but at a faster rate, getting to your eye more quickly than if it traveled a straight line. The bent line (in black) in figure 1 is a representation of the path that light might travel from a fish in a tank to your eye. To see the fish, you look in the direction of the point x at the surface of the water. By looking in that direction, the fish appears to you to be somewhere on the straight line (in gray), such as where the "ghost" fish is, in figure 1. This is why objects are often not where they appear to be when they are under water.



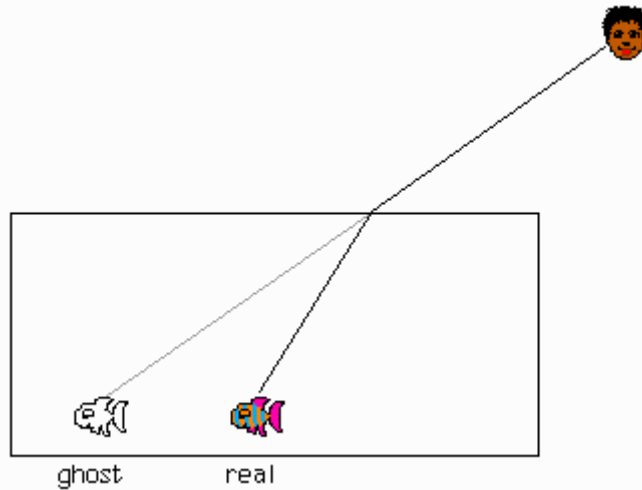


figure 1

To find the speed of light through air and in a vacuum, Foucault and Michaelson required a place for the light to travel a long distance and very precise measurement of time. To do this requires specialized equipment. But because of Fermat's principle, you will be able to find a good approximation for the speed of light in water using just a graphing calculator or computer, some rather simple equipment, and a bit of good mathematics.

Fermat's principle states that light travels the quickest path between two points. Words like "quickest" suggest optimization; least time, most speed, optimal path. Optimization means getting the best possible outcome in a situation. For light, that means finding the path that reaches the destination in the least amount of time. Strategies for optimization often make use of fairly advanced mathematical concepts. However, the key to optimization is getting a function that is a reasonable model of the situation you want to optimize. Building such a function frequently only requires important ideas from quite elementary mathematics. Integration of elementary mathematical concepts to solve seemingly complex problems can be very powerful.

Light's Speed in Water

Student Materials: Class Materials

The Path Light Travels

Your instructor has set up a container of water with a mark A on the bottom end of a container and another mark B on a pole. Light traveling from mark A to mark B will travel through both water and air. When you look at the mark from an observation point, you can "see" that the path from mark A to mark B actually bends at the point where the path emerges from the water. Your instructor has illustrated a path light might take with a string. Figure 2 illustrates this path.

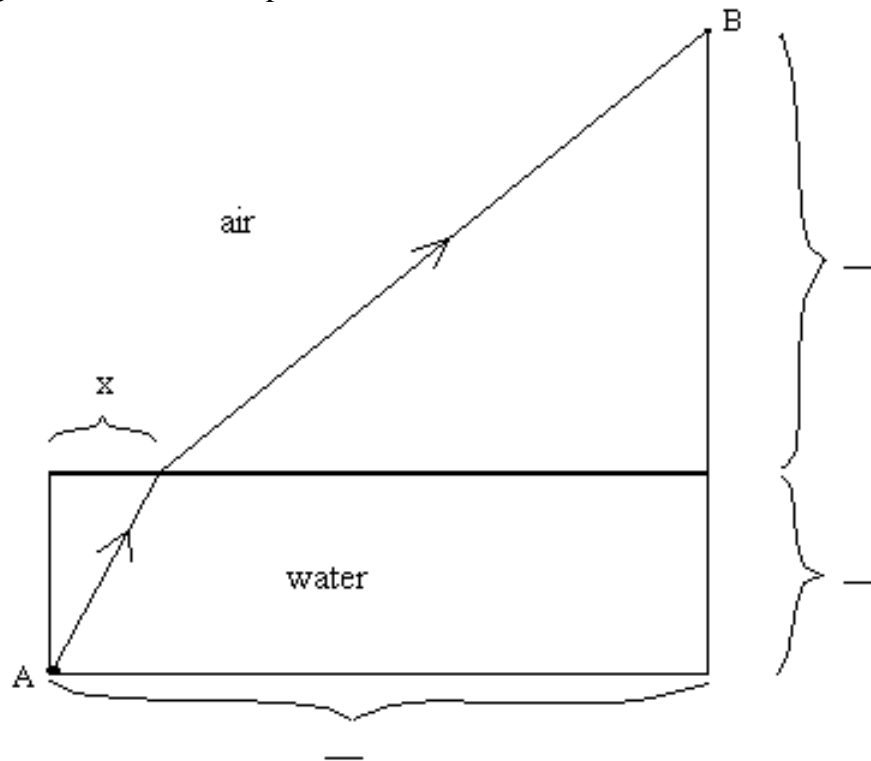


figure 2

2. Make a careful sketch, similar to figure 2. Include a possible path light might travel as illustrated in the set-up. On your diagram, write all known distances. The horizontal distance from the end of the container above mark A to the point where the path of light emerges from the water is labeled as x in figure 2, since we do not know the actual position. Light travels 30 cm/nanosecond in a vacuum; we can use that speed as a good approximation of its speed in air. The speed of light in water is less than 30 cm/nanosecond.
 - a. **Your class has made a guess at the speed of light in water.** Your class has also made a possible path that light might take from mark A to mark B , represented by a particular value for x in figure 2. Find the length of time (number of nanoseconds) it would take for light to travel that path.

- b.** Your group has been given an alternate path that light might take; that is, you have been given another possible value of x . Find the length of time (number of nanoseconds) it would take for light to travel this new path.
- c.** Which of those two paths is the best; that is, takes light the least amount of time to travel?
- d.** Write a function $T(x)$ for the time it would take light to travel from mark A to mark B for varying values of x representing the possible paths that light might travel. Use 30 cm/nanosecond as the speed light travels in the part of the path that is in air and use the speed of light in water that your class has agreed on to represent the speed light travels in the part of the path that is in water. Note that you do not yet know what the correct speed is for light in water.
- e.** We know light travels the quickest possible path. Use this fact, a graphing calculator or computer, and your function $T(x)$ to estimate the path light will take if your class's guess at the speed of light in water is correct.
- f.** Does light take the path you and your class chose as the "possible path" it might travel? If not, consider why not and comment.

Finding the Speed Light Travels in Water

While your class is answering questions 2f and 3, groups can take turns observing and measuring the actual value of x , the distance of light's exit point from the water as it travels from the mark on the bottom of the container to an eye in the specified position of your set-up. Lay a ruler across the top of the container with zero at the string that is taped across the top above the mark. One person should hold a pencil point at the surface of the water at the point where your group computed the light would exit. A second person observes from the specified point. If the observer sees the mark at the position where the pencil point is held, then your computed value of x is the "actual x -value." If not, the observer should tell the person holding the pencil to move the pencil forward or backward until it is at the correct position. A third person should read off the correct value of x from the ruler and write it down. After all groups have done this, you should discuss your results and agree on what you all believe to be the actual value of x . You will use this number in problem 4.

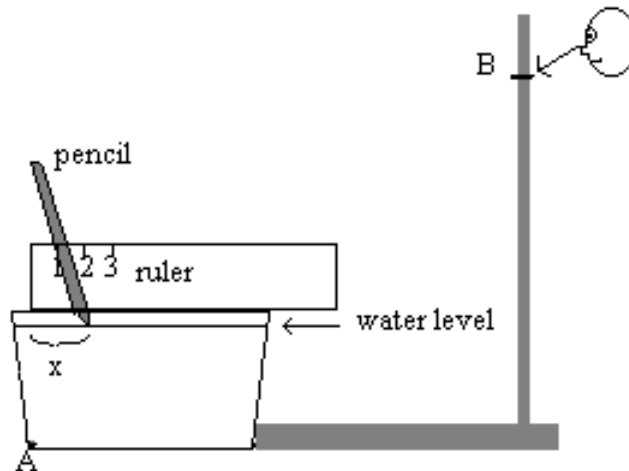


figure 3

3. Your group has been assigned a guess for v , the speed of light in water. Develop a function for $T(x)$, the time it takes light to travel a path from mark A in the container to mark B , if it exits the water at the point x in figure 2. Graph $T(x)$ on your graphing calculator or computer, using your group's assigned value for v . Find the value of x that minimizes your version of $T(x)$. When your group completes your part of the investigation, put your results in a table similar to table 1. Also put your result on a table on the board. Fill in other groups' results in your table. (Your group may be asked to repeat this problem using other values for v , also.)

Guess for speed of light, v	Value of x to minimize $T(x)$

table 1

4. Compare the actual x -value that was measured in your class to the values in table 1. Make an estimate of the correct speed of light in water. After your group has discussed this, share your thinking with other groups. You may be assigned another possible value for v (the speed of light in water) for which you must compute the best x -value so that your class can make a better estimate of the actual speed of light in water. Put your new results in your version of table 1, also.

5. Write a summary of what you did in this lesson, what you learned from it, how you used mathematics to approximate the speed of light in water, and what knowledge you had to have to do the work. Include in the summary your group's best estimate for the speed of light in water based on the work done by your class.

6. (Optional) Using the same set-up as before, fill the container to a different depth than before. Use the result your class got for the speed of light in water to predict where you would see the object; that is predict the value of x where the path of light emerges from the water. Then observe as before to see where it actually emerges. Comment on your result and explain it.

A Light Under the Water

7. Your class has settled on one particular value for v , the speed of light in water. Pick your own point P_1 out of the water (see figure 3) and compute the path that light takes from mark A under the water to point P_1 above the water; that is, compute the horizontal distance, x_1 , from the far end of the container to the point where the light exits the water. Use the same depth for the water that you used in class for problems 2 through 5. Repeat this for two more points P_2 and P_3 . After you have calculated x_2 and x_3 , try to choose other points, P_4 , P_5 , etc., to result in the largest values you can get for x_i . What is the largest value you have found for x_i ?

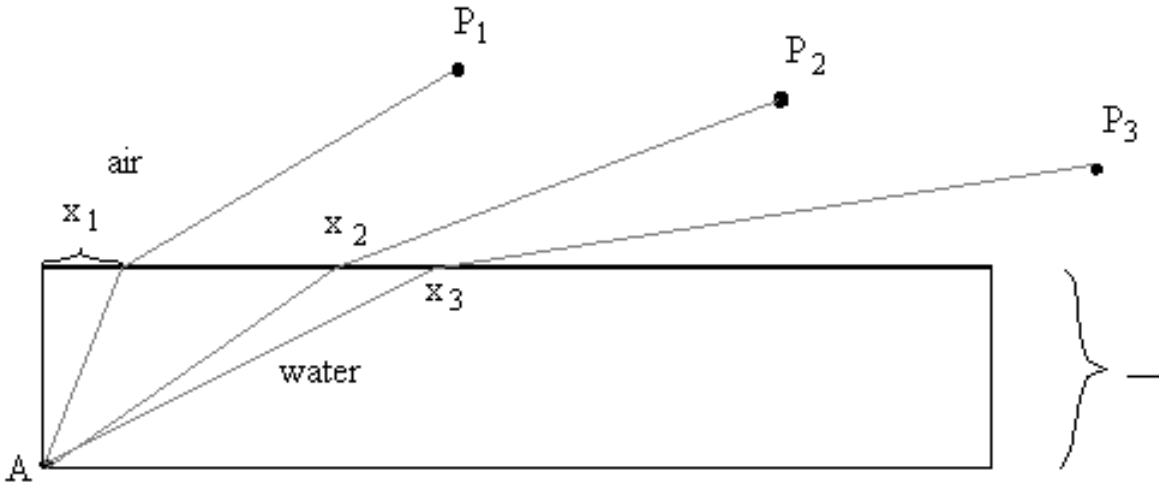


figure 4

8. Suppose a flashlight is underwater where mark A used to be in the container you used in class. The flashlight is pointed in the direction indicated by the line in figure 4. Draw where that beam of light will continue, using what you learned from problem 7. (Use the same depth for the water that you used in problems 2-5.)

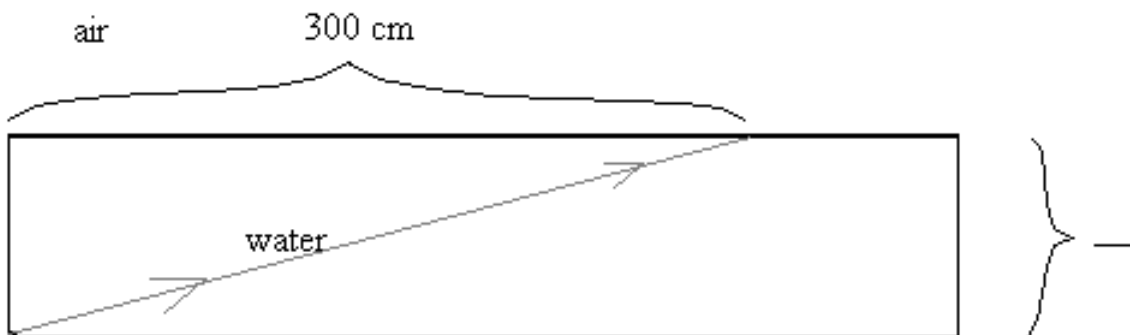


figure 5