Genetic Factors: Effects on Medication
Teacher Materials

Mathematics Contained in the Lesson

The main purpose of the lesson is the introduction of the function

$$y = \frac{k}{x}$$

and the interpretation of its graph using context.

The lesson reinforces proportions, use of proper notation to write sets of the form $a < x < b$, and solution of equations involving percents.

You might briefly discuss the fact that the graph of $y = \frac{k}{x}$ is a hyperbola and demonstrate its rotated form $x^2 - y^2 = c$.

The activity can also be used to introduce or review the meaning of vertical and horizontal asymptotes.

Pronunciation Note: Hydralazine is pronounced hI-'dra-la-"zEn and acetylation is pronounced a-"se-til-'A-shun. (Uppercase denoting long vowel; lowercase, short vowel. Single ' denotes strong accent; " denotes secondary accent.)

Organization

Give students pages 1 and 2, the Reading Assignment, before the lesson is to be done in class. In class, open with discussion to clarify important points from the reading (see suggestions below).

Then distribute page 3 of the student material for students to work together in groups of 3 or 4.

Each student should have a graphing calculator. Alternatively, each group of 3 or 4 students could work at one computer.

Answers to Problems and Teaching Suggestions

1. Fast acetylator: 5 to 10 mg. Students could write, for example, $5 \leq f \leq 10$ or $f \in [5, 10]$. Slow acetylator: 15 to 20 mg; $15 \leq s \leq 20$.
2. 70 mg
3. 0.175 or 17.5%

Reflection point: Students should complete the problems in the Reading Assignment before this lesson is done in class. In opening discussion, ascertain that students understand the following from the reading:

- people metabolize some medicines at different rates,

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This project was supported, in part, by the National Science Foundation. Opinions expressed are those of the authors and not necessarily those of the Foundation

- what fast and slow acetylators are, and
- why it is important to get the right dose of Hydralazine.

So that students are confident about the elementary mathematics used in this lesson, have them reflect on the three problems they worked in the Reading Assignment. In every case, they are working with the relationship

$$xy = k \quad \text{or} \quad \text{fraction} \times \text{dose} = \text{effective amount},$$

solving for a different variable each time.

In the context of this unit, in $y = \frac{k}{x}$, y represents the dose of a medicine, x represents the fraction of the medicine that is absorbed into the body, and k represents the fixed amount of the drug that should be absorbed by the body for the medicine to work well.

The equation "fraction $\times$ dose $=$ effective amount" is very similar to the equation

$$rt = d \quad \text{or} \quad \text{rate} \times \text{time} = \text{distance}$$

with which the students should be familiar. You might find that a brief discussion of the "rate" equation will help clarify the "dose" equation and its meaning.

Problem 1 will allow a quick review of the use of percents and provide an opportunity to present notation for an interval, such as $5 \leq f \leq 10$. In problem 1, students needed to find a given percent of a given dose, so they multiplied "fraction" times "dose" to get the "effective amount."

In problem 2, they needed to find what dose to give when they knew how much is effective and what percent of the dose would be absorbed, which can be found as

$$\text{dose} = \frac{\text{effective amount}}{\text{fraction}}.$$

In problem 3, they needed to know the percent given both a dose and an effective amount, which is found from

$$\text{fraction} = \frac{\text{effective amount}}{\text{dose}}.$$

After review, hand out "Prescribing hydralazine safely", page 3 of the student material, for students to work together in groups of 3 or 4.

4. $xy = 10.5$
Students should recognize that this is simply the relationship they worked with in problems 1 to 3 with the percent and the dose varying while the effective amount remains constant.

5. $y = \frac{10.5}{x}$. A realistic window for the graph might be $0 \leq x \leq 1$, since x represents the fraction of the dose that is absorbed in the body. For y, students might make different decisions; $0 \leq y \leq 200$, for example, shows values for absorption of the medicine at fractions as low as 5%.

6. $26.25 \leq y \leq 35$ mg. You might prescribe, for example, three 10 mg pills.

7. $52.5 \leq y \leq 105$ mg. You might prescribe three 25 mg pills.

8. The width of the range for slow acetylators is only 8.75 mg while for fast acetylators it is 52.5 mg. The reason for this difference can be seen on the graph. The graph is very steep over the interval $0.1 \leq x \leq$ 0.2; the steep slope results in a large change in y over the interval. It is much closer to horizontal over the interval $0.3 \leq x \leq$ 0.4; this much smaller slope means correspondingly a much smaller change in y over the interval even though the x-interval is the same length.

Reflection point: In whole-class discussion, have students summarize the properties of their graph of $y = \frac{10.5}{x}$. They might note that:

- for $x > 0$, y is always positive;
- as x increases y decreases;
- as x becomes large, y approaches 0; that is, $y = 0$ is a horizontal asymptote;
- for small positive x, the graph is very steep (with a negative slope); that is, $x = 0$ is a vertical asymptote.

Have students change their window to $-1 \leq x \leq 1$ and $-200 \leq y \leq 200$ and extend the discussion to the properties of the entire graph. (On a graphing calculator, you can avoid difficulties near the vertical asymptote by putting the calculator in DOT Mode instead of CONNECTED Mode. Students should be advised about such matters and taught to make the graph look the way it should instead of assuming that the function actually does what the calculator may show.) Discussion of slope after students work problem 8 is particularly valuable for students who will go on to study calculus.

You might wish to have students compare graphs of equations of the form $y = \frac{k}{x}$ with graphs of equations of the form $x^2 - y^2 = c$ to prepare them for further study of hyperbolas.

Acknowledgment: We would like to thank Professor Philip Almes of Wayland Baptist University for his helpful comments on the use of these materials.