Trivial Pursuits and Algebraic Models
Teacher Material

The mathematics in this activity includes setting up and **solving problems using linear equations** in one variable. These problems are similar to standard problems for this course, except that students should actually make the physical objects described.

Materials needed:
1) 4 or 5 sheets of 8.5" by 11" paper for each group. Used paper is fine.
2) Each student should have a book (any book will do).
3) 12-inch ruler for each pair of students. We have successfully photocopied rulers.
4) A pair of scissors for each pair of students
5) Lengths of string. For half of the groups, the string should be about 2 yards long. For the other half, lengths of 1 yard should be sufficient.

Organization

Students need to work in pairs and pairs of pairs. Organize groups of four and have students choose two pairs within the foursome. One pair will be "Group A" and the other will be "Group B". In the event that your class is not a multiple of 4, one remaining student can be added to one of the foursomes so that one "pair" becomes three; a second remaining student should be added to a second foursome and a third remaining student can be added to a third foursome.

Have the students begin work on problems 1-6. These are to be done in the groups of 4, or 5 in some cases. For problems 4 and 5, the groups break into pairs (or triads in some cases). For problems 4 and 5, groups of three can play "round robin" games; for example, in problem 4, person 1 gives a page total to person 2, person 2 gives a page total to person 3 and person 3 gives a page total to person 1. The same process will work for problem 5.

Let the students know that you will be interrupting them for a discussion after everyone has finished problem 2, although they should continue to problems 3, 4 and 5 until you interrupt them.

These problems are similar to standard problems for this course, except that students should make the objects described or actually look at page numbers in a book. Make sure each group actually makes the rectangles suggested and tries the games instead of only trying to solve the problems algebraically. This will give them a better sense of the connection between the problem and the mathematics. It will also help make clear why certain algebraic calculations are valid.

After all groups have finished problem 2, have the class discuss the first two problems. **As you observe the work, if you see students working with an appropriate equation, encourage them to share their thinking with the class.** However, if no

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group uses an algebraic approach, have several groups share their processes for solving the problems. Then together, convert these processes into equation form so that students see that algebra is a language for the mathematical thinking they already know how to do. (See Reflection Point after answer to problem 2.)

When a group has completed problem 6 (this will, in most classes, be at the beginning of the second day of the lesson), they should ask for their materials for "Group A Problems" and "Group B Problems." Give the Group A pair in each 4-group a length of string about 2 yards long and 1 or 2 sheets of paper; give the group B pair a length of string about 1 yard long and several sheets of paper. Students doing Group A will solve problems about determining the lengths of three pieces of string to satisfy given conditions; then they will generalize their process to write instructions for their group B counterparts to follow in doing a similar string problem. Students doing Group B will solve problems about determining the length and width of a rectangle to satisfy given conditions; then they will generalize their process to write instructions for their Group A counterparts to follow in doing a similar rectangle problem. **Make it clear that no pair should work on problem 4 until they have received the other pair's instructions for working that problem.**

In the final instructions that they write (part 5 of the Group A/Group B problems), you might suggest that they include an explanation of their instructions. The explanations would most easily be done by writing an equation to describe the situation and explaining its solution.

Answers to Problems and Teaching Suggestions

1. Students might accomplish this by folding the paper into three equal parts and cutting off one section. The lengths should be $3\frac{2}{3}$" and $7\frac{1}{3}$".

2. Students might accomplish this by folding the paper so that two inches are not covered by the fold and then cutting along the fold. The lengths should be 4.5" and 6.5".

Reflection point: Once all groups have worked the first two problems, bring the class together for a discussion. Ask students if they can suggest formulas for these two problems. If students cannot suggest formulas, they probably do not have an adequate understanding of either variables or equations. In this case, help students analyze (with tables, diagrams, or other strategies) the processes <u>they</u> used to solve the problems and use them to develop "formulas" by assigning useful variable names and focusing on general aspects of the reasoning.

One approach to problem 1 is to let the short length be x, giving the equation $x + 2x = 11$. Some students may get this by letting the long length be y so that $x + y = 11$ with $2x = y$. The more methods they see, the better.

One approach to problem 2 is to let the short length be x; thus, the other length would be $x + 2$. This gives the equation $x + (x + 2) = 11$. Some students may get this by letting the long length be y so that $x + y = 11$ with $x + 2 = y$. Another approach is to let the long piece be x, the short piece being $x - 2$, and then solve $2x - 2 = 11$.

Use this discussion to be sure students make connections between the physical situations and the mathematics and to reflect on the connections between the problems.

Once these problems have been discussed, students can continue working on problems 3, 4, 5 and 6. If time is short, all students do not have to finish all of these in class. You can assign students to finish them for homework and to develop relationships that help them answer the questions.

3. $x + x + (x + 1) = 11$. Two pieces each $3\frac{1}{3}$ inches long; the third piece $4\frac{1}{3}$inches.

4. The solution to this problem is a pair of whole numbers, x and $x + 1$ where $x + (x + 1) = K$, the sum received.

5. With x being the number chosen, the number received is $K = \frac{3x+21}{\mathbf{3}}$. The solution is $x = K - 7$.

6. To answer problem 6, students must write an equation to represent each of the previous problems, whether or not that was their approach to solving the problem. Equations will, of course, differ according to their definition of variables. We have given one or more possible equations in the answers to problems 3 through 5 and in the "Reflection Point" following 1 and 2.

Group A problems

The purpose of the Group A/Group B problems is to require students to a) generalize their reasoning to deal with an unknown parameter and b) write their generalized reasoning in clear language. This gets to the heart of algebraic thinking and is worth the time it takes.

1. Where $x =$ length of shortest piece, $x + (x + 1) + (x + 3) = 16$. The three pieces are 4", 5" and 7" long.

2. Where $x =$ length of shortest piece, $x + (x + 1) + (x + 3) = 23$. The three pieces are $\frac{19}{3}$, $\frac{22}{3}$, and $\frac{28}{3}$ inches long.

3. Instructions from Group A to Group B for cutting the string might be something like this: "Subtract 4 from the length of string. Divide that answer by 3. This is the length of the shortest piece. The medium piece is one inch longer and the longest piece is two inches longer than the medium piece." Or they might give an answer like

"The length of the shortest piece $= \dfrac{L-4}{3}$,

the length of the medium piece $= \dfrac{L-1}{3}$, and

the length of the longest piece $= \dfrac{L+5}{3}$

where $L =$ the total length of the 3 pieces of string."

When Group A has prepared their instructions and given them to Group B, they should watch while Group B implements their instructions, but they should not comment or modify their instructions while Group B works.

4. Group A should follow Group B's instructions. If they do not work, they should tell Group B; they may also offer helpful suggestions for improving the instructions if they wish. If the instructions are correct, they will be solving an equation such as $2w + 2(2w) = 28$. Their rectangle would have dimensions $\frac{14}{3}$ inches by $\frac{28}{3}$ inches.

5. Students should rewrite their instructions for problem 3 to make them as clear as possible. Suggest that they might wish to include an equation, explain the relationship between their equation and their instructions, and why their equation works. For example, for cutting the string, the first cut in this process will provide a length x which is the solution to $x + (x+1) + [(x+1)+2] = l$, where l is the length of the string. Others may write $(y-1) + y + (y+2) = l$, where y is the length of the medium piece of string. Encourage students to solve their equation for the length of the piece of string, $x = (l-4)/3$ for the short piece or $y = \frac{l-1}{3}$ for the medium piece. Have students cut the pieces according to their formula to see that it works.

Group B problems

1. Where w is the width of the rectangle, $2w + 2(2w) = 12$. The width of the rectangle should be 2 inches and its length 4 inches.

2. Where w is the width of the rectangle, $2w + 2(2w) = 21$. The width of the rectangle should be $3\frac{1}{2}$ inches and its length 7 inches.

3. Instructions from Group B to Group A might be: "The width is one-sixth of the perimeter and the length is one-third of the perimeter." Or it could be

$$\text{"Width} = \frac{P}{6} \text{ and length} = \frac{P}{3}$$

where $P =$ the perimeter of the rectangle."

When Group B has prepared their instructions and given them to Group A, they should watch while Group A implements their instructions, but they should not comment or modify their instructions while Group A works. It does not matter which group works first.

4. Group B should follow Group A's instructions. If they do not work, they should tell Group A; they may also offer helpful suggestions for improving the instructions if they wish. If the instructions are correct, they will be solving an equation such as $x + (x+1) + (x+3) = 31$. Their segments of string should have lengths 9 inches, 10 inches, and 12 inches.

5. Students should rewrite their instructions for problem 3 to make them as clear as possible. Suggest that they might wish to include an equation, explain the relationship between their equation and their instructions, and explain why their equation works.

Have different groups present solution strategies.

Following these activities, assign problems from the textbook for which students must set up and solve an equation. Encourage students to make models of the situation when needed. In some cases, you will be able to assign problems that are an exact parallel to problems in this activity along with some new problems. Encourage students to try hands-on approaches to help them with any problem that initially presents difficulty.