Computing Speed

Teacher Materials

Mathematics Contained in the Lesson

This lesson should take one day of class time. The purpose of this lesson is to develop an understanding of quadratic functions. We use the linear relation between distance, constant speed and time and the quadratic relation between the vertical distance of a falling object and time. From these, students will develop two new quadratic functions. The graph of one of these provides a picture of the physical phenomenon they have viewed.

The lesson includes:

linear functions,
quadratic functions,
practice in algebraic manipulation,
composition of functions,
solving a 'literal equation' for one of its variables.

Set-up

You could use a ramp set up on a table with the end of the ramp not too far from the edge of the table. (Hot Wheels® make an inexpensive track and car combination). The switch and trapdoor in the imagined experiment in problem 2 are not needed! The calculations will be easiest if the end of the ramp is 4 feet above the floor, if you are measuring in feet. You will need a toy car or a ball that can be rolled down the ramp; there should be no obstructions on the ramp, on the table below the ramp, or on the floor in front of the table, so that the car or ball is unimpeded until it hits the floor. Try your set-up out before class to be sure that you have enough space, or a slow enough initial speed, so that the toy will stop without hitting a wall or some other obstruction. You will need measuring tapes or measuring sticks; you can use either metric or British units. If you run the car down the ramp one time when you get to class so that students see about where the car hits the floor, then you could put some powder on that area of the floor so that it will be easier to see where the car hits the floor when they do the experiment.

If you don't have a track or car, you can just push a ball, coin, or paper clip of a desk.

Organization

For effective use of class time, students must read the Reading Assignment and try problem 1 before coming to class. At the beginning of class, have a trial run of the toy car or ball down the ramp and off the edge of the table to the floor. Point out the distance represented by F (the vertical distance from the edge of the table to the floor) and the horizontal distance H (the horizontal distance from the point on the floor directly below the point on the table where the car left it to the point where it first hits the floor). This

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will help avoid confusion about what the variables represent. Ask students what the variables r and t in the two formulas represent.

Give students time to compare their results on problem 1, and then have one student present it to the class. If necessary, have students who were unable to sketch the path observe the experiment from the side until they see clearly what the situation is. Have others help them transfer their understanding to their diagram if necessary.

Working in groups of 3 or 4, the students can then discuss problem 2. Problems 1 and 2 will serve to help students clarify the information given in the reading and make it their own.

Have two or three students roll the car (or ball) down the ramp and measure the vertical and horizontal distances it travels after it leaves the table until it reaches the floor while the rest of the class observes. Then all students, in their groups, can proceed with problems 3 and 4.

As you walk around the groups while they work on problems 3 and 4, give each group a small object, such as a paperclip or a coin, for their work on problem 5. Problem 5 does not extend the thinking; it is just an opportunity for the students to do the experiment themselves. Each group would be expected to get different results.

You might close by having students summarize what mathematics they learned from the lesson.

Answers to Problems and Teaching Suggestions

1. Since $F = 8 = 16t^2$, we get $t = \sqrt{\frac{1}{2}}$. Thus, $H = 50\sqrt{\frac{1}{2}} = 25\sqrt{2}$, or about 35 feet.

2. They should hit the ground at the same time. The vertical distance each object falls and the speed at which it falls is independent of its horizontal movement. *The purpose of this problem is to focus on this fact. Subsequent problems do not compare two falling objects.*

3. As in problem 1, use $t = \sqrt{\frac{F}{16}} = \frac{\sqrt{F}}{4}$. Thus, $H = rt = \frac{r\sqrt{F}}{4}$. Consequently, $r = \frac{H}{t} = \frac{4H}{\sqrt{F}}$. The results will depend on the measurements of H, the horizontal distance the toy moved from the point where it went off the table, and F, the height of the bottom of the track above the floor. Students will almost certainly use their measured values for this work; they need not take the generalized approach shown here. *Caution students to use feet (not inches) if using $F = 16t^2$, and meters (not centimeters) if using $F = 4.9t^2$.*

4. a. $A = h - 16t^2$, where h is the measured height (in feet) of the bottom of your track above the floor. (Students should write it using the value for the height, h, not a variable.) Or $A = h - 4.9t^2$ if h and A are in meters.

b. Using A $= h - 16t^2 = h - 16(\frac{H}{r})^2 = h - \frac{16}{r^2}H^2$. Replace r with the estimate of its value calculated in part a. (Recall, students have a constant for h, your measured value.) Thus, the graph will be a parabola with its vertex at (0, h), opening downward. The path of the toy is illustrated by the section of the parabola for H values from 0 to the horizontal distance the toy actually traveled; or for A values from h to 0. *Note that here we use H as a variable to represent the range of horizontal positions; students may need assistance in seeing it that way since earlier they measured a value for H at the time of impact. Students may also need assistance in realizing that to graph the function on their calculators, they replace H with x.*

Reflection point. When all students have successfully completed problem 4b, stop the class for a short whole-class discussion to make two points: (1) Be sure all students understand how A was converted from a function of t to a function of H; the ideas of solving $H(t)$ for t and then substituting in $A(t)$ to get A as a function of H are the critical ideas. (2) Explain the term composition of functions to students and point out that their result is $A(H)$, a composition of $A(t)$ and a function $t(H)$ they got from $H = rt$.

c. The y-intercept is the height of the table top, the distance the toy is above the floor just as it leaves the horizontal piece of track.

d. The x-intercept is the horizontal distance the toy has traveled through the air when it hits the floor; the value of H when A = 0.

Reflection point: When all students have completed parts c and d, pause to reflect on the meaning of the graph. It is a reasonable representation of the path students saw since y is the height above the floor and x is the horizontal distance from the table. It would be valuable to have students graph the parametric representation of this parabola on their calculators, where the calculator will accomplish the composition of functions for us. In parametric mode, enter $H(t) = rt$ in X_1 (using the estimated value of r) and $A(t) = h - 16t^2$ in Y_1 (using the measured value of h). Be sure students have a small enough step-value for t to make a nice graph. They should notice that now they can get from their calculator graph the time the toy was in a given position as well as seeing where it was.

e. Students should note that:
(i) because their function $y = h - kx^2$ (where h and k are positive constants) depends only on the square of x, the sign of x doesn't matter, making the graph symmetrical about the y-axis;
(ii) because the bigger the absolute value of x gets, the smaller y becomes, the function increases as x approaches 0 from the left and decreases afterward;
(iii) since the toy was never higher than at the place where $H = 0$, this is where the graph has its maximum point.

Reflection point: In problem 4, students have seen (1) a quadratic function, $A(t)$, built from another quadratic function using subtraction, (2) a quadratic function, $A(H)$,

built from another quadratic function using composition. Be sure they notice these operations!
In part e, they should discuss why their function has the shape it does, relating their algebraic understanding to the geometric picture of the function.

5. Students should have measured a horizontal distance H and a vertical distance F. Then they can find its initial speed r in the same way that was done in problem 3. Their result should satisfy $r = \frac{4H}{\sqrt{F}}$. *If students do this in the classroom, they should probably flick the object off the horizontal surface. Some groups might prefer to do this problem outside with a ball and write up their results. Caution them to throw the ball* *<u>horizontally</u>* *if they want meaningful results; otherwise, the formulas here will not model their situation.*