Computing Speed

Student Materials: Reading Assignment

You probably know that the relationship between the speed an object is traveling, the length of time the object is traveling that speed, and the distance the object travels during that period of time is given by the expression

$$\text{distance} = \text{rate} \times \text{time}.$$

This formula can be used in several ways. For example, if you know you have traveled at about 50 miles per hour for about 3 hours, you can estimate that you have driven 150 miles. If you need to travel 20 miles and estimate you can drive an average speed of 40 miles per hour, you can use the formula in the form

$$\text{time} = \frac{\text{distance}}{\text{rate}}$$

to estimate it will take you about $t = 20/40 = 0.5$ hours or 30 minutes to make your trip.

Suppose an object is pushed or thrown in a horizontal direction. Then, neglecting the force of friction, the only other force acting on the object is gravity which acts only in the vertical direction. Thus, the formula relating the horizontal distance the object travels, the time it takes to travel that distance and the speed the object was traveling is still this same formula, distance $=$ rate $\times$ time. Letting $H =$ horizontal distance traveled, $r =$ rate or speed of the object, and $t =$ time the object travels, we can summarize the formula with

$$H = rt.$$

We see that horizontal distance traveled is a **linear function** of time.

It is up to you to determine if friction can reasonably be neglected in any case you are studying. If you roll a marble across a smooth floor, friction does not appear to be much of a factor, at least for the first few seconds. If you roll a marble across a carpeted floor, friction quickly slows the marble down.

The formula relating distance, rate, and time can sometimes be used to estimate one of the quantities when it would be difficult to compute it directly. For example, you could roll a marble across the floor and measure how far it travels in 3 seconds. You could then use that distance and time to estimate the average velocity of the marble.

Let's now discuss the distance traveled by an object falling under the force of gravity. It was discovered by Newton that the distance an object falls, neglecting friction, is a **quadratic function** of time. If we let $F =$ vertical distance an object **falls** in t seconds and we assume the object starts at rest at time $t = 0$, then

$$F = 16t^2 \text{ feet or } F = 4.9t^2 \text{ meters.}$$

This project was supported, in part, by the National Science Foundation. Opinions expressed are those of the authors and not necessarily those of the Foundation

If your object travels for only a brief time and has a fairly high mass, friction will have only a minimal effect. On the other hand, if your object has a relatively low mass, such as a feather or balloon, or if it is falling for a fairly long period of time, such as an object dropped from an airplane, then friction is fairly significant and you would not use the above formula.

The formula $F = 16t^2$ could be used to estimate the time it would take for an object to reach the ground if it was dropped from different heights, such as from eye-level or from a building.

An important physical result is: If you throw one ball horizontally and simultaneously drop another ball from the same height, both balls will hit the ground at the same time. Said another way, the vertical distance each ball falls is independent of how fast the ball is traveling horizontally. Because of this, if we know the ball is not moving vertically at time $t = 0$, and if we know the speed of the ball in the horizontal direction (r), then we can use the formulas

$$H = rt \text{ feet and } F = 16t^2 \text{ feet}$$

(or $F = 4.9t^2$ if using meters) to find the position of the ball at any time.

1. Suppose a ball is thrown horizontally with a speed of 50 feet per second from a height of 8 feet. Make a rough sketch of the situation, showing the ball's path. On the sketch, mark the horizontal distance the ball moves, H. Also mark the vertical distance it moves, F. Find how long it will take for the ball to hit the ground and use that time to estimate how far it should land from the feet of the person pitching it. *If you have trouble making your initial sketch of the ball's path, get someone else to throw one horizontally, or push a ball off a table. Stand or sit where you can watch the ball's path from the side.*

Note: In the formulas for vertical distance, the coefficients for t^2 are approximate and are related to the acceleration due to gravity. Since acceleration is measured in distance/time/time, the distance units determine this coefficient.

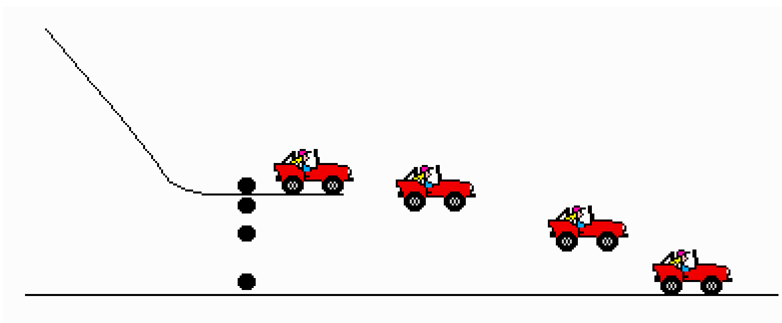
Computing Speed
Student Classroom Material

One of the uses of mathematics is in computing quantities that would be difficult to measure directly. The purpose of this activity is to see how you can use the formulas

$$H = rt \text{ feet and } F = 16t^2 \text{ feet}$$

(or $F = 4.9t^2$ meters) to find the speed a toy is traveling at some point in time. If you had the proper equipment, you could actually measure this speed. But with a little mathematics and nothing more complex than a ruler, you will be able to estimate the speed of this object. Using similar techniques, you can estimate how fast you can throw a ball or shoot a spitball (but don't experiment in class; at least not yet).

2. A toy car runs down a ramp, across a horizontal part, and then goes off the end. As it goes off the end, it trips a switch that releases a ball that falls vertically through a hole in the track behind the car. Does the car or the ball hit the ground first? Explain your answer.



3. You have observed an object as your instructor pushed it off of a table and it fell to the floor, like the car in the above picture (without the trapdoor and the switch). Someone in your class has measured 1) where the toy hit the floor and 2) the height of the table top above the floor. Use their measurements, and the formulas for H and F above, to estimate the speed, r, of the object as it left the edge of the table.

4. In the formula $F = 16t^2$, F gives the distance the object has fallen from the top of the table at time t.

a. Let A represent the vertical distance above the floor where the car is located at time t. Use the information the formula gives about F to write a function A in terms of t.

b. Use the substitution $t = H/r$ in the formula you developed in part a to get a formula for the height of the toy above the floor in terms of the horizontal distance. In this new formula, replace the variable r with the estimated speed you computed in problem 3. Graph that function on your calculator to get a picture of the path the toy took as it traveled from the table to the floor.

c. What is the significance of the y-intercept of the graph in the context of this situation?

d. What is the significance of the x-intercept of the graph in the context of this situation?

e. Although your function does not have real-world meaning for $H < 0$, extend the graph to negative values of H (x on your calculator). Discuss the shape of the graph and the significance of the maximum point on the graph for this situation.

5. (Optional) Throw a ball horizontally or horizontally propel a small object, such as a coin or paperclip by flicking it off a tabletop. Measure appropriate distances from the place where the object starts to where it hits the ground and estimate the initial speed of the object. Be sure to include your measurements in your write-up.