Stacking Cups

Teacher Materials

Mathematics Contained in the Lesson

In this lesson, students learn to develop the equation for a line. The emphasis is on the significance of the slope and the y-intercept.

Set-up

Each group of 3 or 4 students will need 5 styrofoam cups. Cups with a lip are preferred, but any ordinary cups will do provided that all 5 are alike. Each group will also need something with which to measure length. We have used photocopies of a ruler.

Organization

Students can begin work on problem 1 with little or no introduction. Encourage students to think through the questions together. If necessary, ask questions to help groups of students clarify their thinking if they get stuck anywhere.

Following problem 1, hold a whole-class discussion summarizing:

- how to write an equation for a linear function,
- how to recognize a linear function from its equation,
- the geometric meaning of the slope and y-intercept of a line as well as its contextual meaning.

It would be valuable for students to work the homework for this lesson together in their groups.

Answers to Problems and Teaching Suggestions

1. a. Students should measure the vertical height of one cup, then measure the heights of a stack of 2 cups, a stack of 3 cups, 4 cups, and 5 cups. <u>Extreme</u> care in attaining a high degree of accuracy is unnecessary. Encourage groups to record their results in a table. After students measure their stacks, they are to graph the points (x, y), where x is the number of cups in the stack and y is the height. As you observe, if you see that their 5 points do not seem to fall in a line, suggest that they (a) think about the process and whether their numbers make sense, (b) check their graphing and (c) check their measurements.

b. A linear function, $y = mx + b$, satisfying the data should be written here. The value for m will be the height of the lip of one cup and b will be the height of the base of the cup, without the lip.

Be alert. Some students, in an effort to be accurate, may not quite have the same change in height each time a new cup is added. For example, they might have that 1 cup is 8.1 cm, 2 cups is 9.4 cm (1.3 cm more), and 3 cups is 10.8 (1.4 cm more). They may then miss the point that each time a new cup is added, the height increases by the height of the

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lip of a cup. If this happens, ask questions to focus the students on the process and the context.

c. Students are to give the slope of their line. Its 'significance' in this situation is that it tells the height that one more cup adds to a stack of cups; if the cups have a lip, it is probably the vertical height of the lip.

d. The answer to this question will be the y-intercept of their linear function. Its physical significance for a stack of cups with lips may be that it is the height of one cup minus the lip. Otherwise, it can be expressed as the height of one cup minus the amount that does not fit down inside another cup. (Or the base or some such response.)

e. To do this, students will have to figure out that they need to know (a) how high the space under the table is and (b) how high a stack of 100 cups is. For (a), they must measure. For (b), they must substitute 100 for x in their equation from part b.

f. Here students are solving the inequality $mx + b \leq T$ for x where $mx + b$ is their linear function from part b and T is the height under the table measured in part e.
Students may not recognize the mathematics they have accomplished here. When everyone has completed the lesson, be sure they see that they have solved a linear inequality in answering this question.

g. Assuming students have been working in inches, then using $mx + b$ from part b, students must solve their equation $36 = mx + b$ for x and multiply the result by 10. They should not expect an exact answer, but should see how a good estimate can be made. In many situations, the estimate is all that is important. For example, if you are having a party, you only need an estimate for the number of cups to determine if you need more.

Reflection point: When all groups have completed the problem, pause to consider in whole-class discussion what they have learned about linear functions including:
- •useful forms for writing a linear function,
- •how to recognize a linear function from its equation,
- •generalizations about meaning of the slope of a line; use of the term "rate of change",
- •generalizations about meaning of a y-intercept,
- •how to find out when the function has some particular value,
- •how to find out when the function has more than or less than some value,
- •how to find out what the value of the function is at some particular value of either variable.

Answers to Homework Problems and Teaching Suggestions

1. This problem is a repeat of the idea from "Stacking Cups" parts a and b. Its purpose is to allow students to do again what they learned there, this time with the earlier experience as a guide.

2. *Note: If your student population is made up of many students from other countries, you may wish to re-write questions 2 and 3 using the currency from a country familiar to them. Or you might have them write such questions after they have worked these.*

a. 160 guilders for $100
320 guilders for $200
480 guilders for $300

b. Yes, it is linear because every increase of 1 in the value of d causes an equal increase of 1.6 in g. *(Students may express the proportion differently, of course.)*

c. $g = 1.6d$. The slope is 1.6, the number of guilders equal in value to 1 dollar.

d. $d = 0.625g$ The slope is 0.625, which tells him that 1 guilder is worth $0.625, i.e., $62\frac{1}{2}$ cents. For 500 guilders, he got $d = 0.625 \times 500 = 312.5;$ thus, $312.50

3. a. For $100, he could get $g = 1.6 \times 75 = 120$ guilders. For $101, he could get $g = 1.6 \times 76 = 121.6$. For $102, he could get $g = 1.6 \times 77 = 123.2$

b. 1.6 more guilders

c. $g = 1.6(d - 25) = 1.6d - 40.$

d. The slope is 1.6; for every increase of 1 in d, there is an increase of 1.6 in g.

e. $d > 25$

f. Solve $2500 = 1.6(d - 25)$ to get $d =$ $1587.50

g. $d = 0.625g + 25$

h. Students should graph $g = 1.6d$ and $g = 1.6(d - 25) = 1.6d - 40$ on the same coordinate plane. It is probably best to use graph paper to be sure that they can see that these lines are <u>parallel</u>. In answering the question of <u>why</u>, students' writing should in some way convey the idea of equal rates of change.

4. a.

g	m
1	25
2	50
3	75
4	100
5	125

b. $m = 25g$. The slope is 25; its physical significance is that it gives the number of miles that can be driven on 1 gallon (miles/gallon). $0 \le g \le 15$.

c. $g = \frac{m}{25}$. The slope is $\frac{1}{25}$, the number of gallons the car needs to go 1 mile (gallons/mile). $0 \le m \le 375$.

d. $G = 15 - \frac{m}{25}$. The slope is -$\frac{1}{25}$; the change in the number of gallons of gas in the car for each mile it travels (gallons/mile). $0 \le m \le 375$.