So Much Coffee, So Little Time: More on Caffeine in the Body

Student Materials: Reading Assignment

Introduction

Filtration of the blood by the kidneys is a major process in removing chemicals from our bodies. Usually, the kidneys filter a fixed fraction of the chemical from the blood each time period. A second method for removal of chemicals from our body is metabolism by enzymes from the liver; often, this method results in a nearly constant amount of the chemical in the blood being broken down each time period. There are other mechanisms that eliminate chemicals from the body, such as through the hair and nails, and through the respiratory system.

In this lesson, we study caffeine and cadmium, two chemicals that are eliminated primarily by filtration by the kidneys; that is, in which an approximately **fixed percent** is eliminated each time period. In your earlier study of the elimination of chemicals such as caffeine and cadmium (and lead) from the body, you have discovered that a function that reasonably models the amount of the chemical left in the body is an exponential of the form

$$f(t) = A\,r^t$$

Here, t represents the amount of time, A represents the amount of the chemical that was in our body at the start, r represents the **fraction of the chemical that was not eliminated** during the time period, and $f(t)$ represents the amount of the chemical left in our plasma after t time units. For caffeine, we measured t in hours while for cadmium and lead, t was measured in days. For cadmium, we could easily have used years for t, since the elimination was so slow.

In each of our examples, we assumed that no more of the chemical was being added to the body. However, many of us will still continue to consume caffeine. Furthermore, no matter how careful we are, more cadmium and lead will get into our bodies. Thus, we want to model situations in which there is a continuing entry of the chemical into our bodies.

Caffeine

To model effectively, one usually tries to model a simplified situation; thus, we started by modeling a situation where a person drank only one cup of coffee. Once the simple situation has been effectively modeled, one can try to model a slightly more realistic, but still contrived, situation. That is what we are going to do for our study of caffeine. Suppose someone drinks a small cup of coffee every hour all day long, starting at 6 am. Let us say that there is 100 mg of caffeine per cup. We want to know how much caffeine is in this person's body immediately after drinking the first cup, second cup, third cup, and so on. Recall that 13% of the caffeine in our bodies is eliminated each hour. (It

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is unrealistic to assume that a person will drink one cup of coffee every hour. But the approach developed for this model applies to medicines that are taken periodically, say once every 6 hours or once every day.)

1. Find the amount of caffeine in this person's body, immediately after drinking the first, second, third, and fourth cups of coffee.

Let's think about how you might have answered problem 1. You first ingest some caffeine, so you began with 100. Then, because 13% of the caffeine is eliminated each hour, you multiplied by 0.87 to get the amount of caffeine present in the body an hour later, 87 mg. You then added another cup of coffee (100 mg of caffeine) to the new amount giving a total of 187 mg in the body in one hour (after the second cup of coffee). Next, you multiplied the result by 0.87 and then add another 100 mg to get the amount of caffeine in the body two hours later (just after the third cup of coffee). You continued multiplying by 0.87 and adding 100 mg.

We want a function for $f(t)$, the amount of caffeine in the body after t hours. To find such a function, let's think about how we are computing the amount of caffeine in the body after t hours. As we did for our elimination model which resulted in an exponential, we repeatedly multiply by the same number. This suggests that there may be an exponential function for this model, too. But our model must also take into account the addition of a constant amount of caffeine to the system each hour. The exponential function that describes the amount of caffeine in your body after t hours **could** have the form

$$f(t) = Ab^t + C$$

where in this case t is the time in hours since 6am, when we had our first cup of coffee. Since we multiplied by 0.87 each hour to find the amount of the previous hour's caffeine left in the body, we know an appropriate base for the exponential is 0.87, so our function might be

$$f(t) = A(0.87)^t + C.$$

In fact, the function that models the amount of caffeine in the body is of this form. Unlike before, though, A is **not** the amount of caffeine we started with and C is not the amount of caffeine that is consumed each hour. We now need to determine reasonable choices for A and C. We will do this by fitting this function to the known data.

What is $f(t)$ in terms of A and C when $t = 0$ and when $t = 1$? From your knowledge of the situation and your answers to problem 1, $f(0) = 100$ and $f(1) = 187$. This information can be used to find A and C.

Substitution of 0 for t and 100 for $f(0)$ in the equation $f(t) = A(0.87)^t + C$ gives $100 = A + C$ (recall that $0.87^0 = 1$). Substitution of 1 for t and 187 for $f(1)$ into the same equation gives $187 = 0.87A + C$.

2. Solve the equations $100 = A + C$ and $187 = 0.87A + C$ for A and C. Use this function to compute $f(2)$ and $f(3)$, and compare those results to your answers for problem 1. Then graph the function $f(t) = A(.87)^t + C$ and describe what eventually happens to the amount of caffeine in this person's body. In particular, discuss whether a person who actually drank a cup of coffee every hour for a long period would die of an overdose of caffeine (5000 mg).

Smokin'
Student Materials

Introduction

As was discussed before, cadmium is an extremely dangerous chemical. It tends to be in the soil and in plants we eat, at very low levels. One of the properties of cadmium is that it is eliminated from our bodies at a very slow rate. In fact, only about 4% of the cadmium is eliminated from our bodies each year.

Cadmium is most toxic when inhaled. Initial symptoms include chest pains and nausea. Symptoms can progress to emphysema and fatal pulmonary edema. One source of inhalation of cadmium is through cigarettes. Each cigarette contains about 0.001 to 0.002 mg of cadmium. Smokers absorb between 10% and 40% of this cadmium. Suppose a person smokes one pack of 20 cigarettes every day, with each cigarette containing 0.0015 mg of cadmium. Assume this person absorbs 25% of the cadmium in the cigarettes. This means this person would absorb an additional

$$0.0015 \times 20 \times 365 \times 0.25 \approx 2.7\,\mathrm{mg}$$

of cadmium each year from smoking.

3. Suppose this person starts smoking when 20 years old. Let $f(t)$ represent the amount of cadmium in this person's body after t years, from smoking alone. Since the body is eliminating cadmium at the rate of 4% per year but some constant amount of cadmium is being continually added, we expect $f(t)$ to be an exponential of the form

$$f(t) = A(0.96)^t + C.$$

At the time of starting smoking (when $t = 0$) we will assume $f(0) = 0$. After one year, when $t = 1$, then $f(1) \approx 2.7$.

a. Substitute $t = 0$ and $f(0) = 0$ into the formula for the amount of cadmium to get one equation. Substitute $t = 1$ and $f(1) = 2.7$ to get another equation. Solve these equations for A and C to get the function that approximates the amount of cadmium in this person's body which results from smoking.

b. Suppose this person smokes one pack a day for 30 years, that is, until reaching the age of 50. How much additional cadmium is in this person's body due to smoking?

c. What is the horizontal asymptote of the function $f(t)$ as t gets large without bound?

4. Suppose a person smokes one pack of cigarettes each day for 30 years. Each year, between 1.4% and 7% of the cadmium is removed from the body.

a. Assume the best case scenario, that 7% of the cadmium is removed from the body each year. Also assume the best case that only 0.001 mg of cadmium is in each cigarette and that this person only absorbs 10% of this. Find the function for $f(t)$, the amount of cadmium in this person's body as a result of smoking, after t years. Then find $f(30)$.

b. Assume the worst case scenario, that only 1.4% of the cadmium is removed from the body each year. Also assume the worst case that 0.002 mg of cadmium is in each cigarette and that this person absorbs 40% of this. Find the function for $f(t)$, the amount of cadmium in this person's body as a result of smoking, after t years. Then find $f(30)$.

5. It is estimated that a smoker has twice the exposure to cadmium as a nonsmoker. Does this estimate seem reasonable when the answers to problems 3 and 4 are compared to the result that the average 50-year-old adult has about 30 mg of cadmium in her or his body? Discuss what other information you would need to know if you had to make an accurate assessment of the relationship between smokers' and nonsmokers' exposure to cadmium.

Information in this activity was obtained from **Goodman and Gilman's The Pharmacological Basis of Therapeutics**: A.G. Gilman, L. S. Goodman, T.W. Rall, and F. Murad, Macmillan Pub. Co., NY, 1985.