Reading This Could Help You Sleep: Caffeine in Your Body
Student Materials: Reading Assignment

Introduction
In this lesson, we explore the dynamics of caffeine in the body through the use of exponential functions.

Various foods and drinks popular around the world contain caffeine. Caffeine is an alkaloid compound that comes from plants, including coffee, tea, kola nuts, mate, cacao and guarana. Many people drink caffeine drinks because they like the taste of them, others for the physical effect of the caffeine. Most people are aware of differences in the way they feel as a result of drinking caffeine, which stimulates the central nervous system, the heart muscles, and the respiratory system. The way individuals interpret the effects of caffeine as a stimulant varies widely. For many, the effect is pleasant and energizing, a "wake up" or a "pick-me-up", and it can delay fatigue. For others, the effects are unpleasant. Laboratory tests indicate that 1 to 3 cups of coffee can produce an increased capacity for sustained intellectual effort and decrease reaction time, but may adversely affect tasks involving delicate muscular coordination and accurate timing [1].

The effect of coffee is quite different from the effect of alcohol, for example, with regard to increase in mental capacity; such an increase is not seen in persons intoxicated with alcohol. Caffeine is classified as a stimulant, whereas alcohol is a depressant. These two general classes of drugs have very different effects on the human body.

Quantity of caffeine per drink
The amount of caffeine in different drinks varies, and some also contain other alkaloids that act as stimulants or relaxants. Thus, it is difficult to relate the amount of caffeine in a drink to the physical effect it may have on your body.

We list an average, approximate amount of caffeine in some drinks. The caffeine levels in commercial sodas tend to be consistent. The caffeine levels in coffee and tea vary widely according both to the plant and to processing, but these numbers give some idea of the caffeine level.

In an 8oz cup of COFFEE:
- Drip: 165 mg
- Brewed: 130 mg
- Instant: 95 mg
- Decaffeinated: 4 mg

In an 8oz cup of TEA:
- Brewed: 45 mg
- Instant: 35 mg
- Green tea: 30 mg

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In a 12oz can of SODA:

<table>
<thead>
<tr>
<th>Beverage</th>
<th>Caffeine (mg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coca-Cola</td>
<td>45.6</td>
</tr>
<tr>
<td>Diet coke</td>
<td>45.6</td>
</tr>
<tr>
<td>Surge</td>
<td>51</td>
</tr>
<tr>
<td>Dr. Pepper</td>
<td>39.6</td>
</tr>
<tr>
<td>Pepsi</td>
<td>37.2</td>
</tr>
<tr>
<td>Diet Pepsi</td>
<td>35.4</td>
</tr>
<tr>
<td>Mountain Dew</td>
<td>55</td>
</tr>
</tbody>
</table>

In a 1.5oz CHOCOLATE BAR:

<table>
<thead>
<tr>
<th>Chocolate Type</th>
<th>Caffeine (mg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hershey's Special dark chocolate</td>
<td>31</td>
</tr>
<tr>
<td>Hershey Bar (milk chocolate)</td>
<td>10</td>
</tr>
</tbody>
</table>

**Physical responses to caffeine**

The effects of caffeine can only be felt when the caffeine is present in sufficient amounts. For most people, from 32 to 200 mg of caffeine acts as a minor stimulant; these amounts have been shown to speed up reactions in simple routinized tasks in laboratory experiments. As noted above, the minor stimulant effect is experienced by some people as pleasant and by others as unpleasant. Steadiness of the hand has been shown to be worse after 200 mg of caffeine. More than 300 mg is enough to produce temporary insomnia. 480 mg has been known to cause panic attacks in panic disorder patients. Amounts of 5 to 10 g (5000-10,000 mg) of caffeine cause death.

**Elimination of caffeine from the body**

The two primary ways that chemicals are eliminated from the body are through filtration by the kidneys and metabolism by enzymes from the liver. Our bodies eliminate caffeine primarily by the functioning of the kidneys. The kidneys tend to filter out a constant proportion of a chemical, that proportion depending on the particular chemical and individual. In the "average person", about 13% of the caffeine in the body is eliminated each hour.

1. A person starts the day by drinking 3 cups of coffee containing 130 mg of caffeine each.

   a. How much caffeine will there be in this person's body 1 hour later? 2 hours later? 3 hours later?

   b. Reflect on how you computed the answers to part a to help you develop a relatively simple expression that gives the amount of caffeine in this person's body after 24 hours, assuming no additional caffeine is consumed. What is this amount? (You do not need to keep computing the amount of caffeine in the body for one hour after another until you reach 24 hours. You can compute this amount directly.)

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Classroom materials

To answer the following questions, use the information in the reading assignment.

2. Suppose someone quickly consumes 3 cups of coffee, each containing 130 mg of caffeine. Assume that the caffeine is absorbed into the body immediately.

a. Write an equation in the form \( y = f(t) \) for the amount of caffeine in the body \( t \) hours after drinking the coffee containing 390 mg of caffeine.

b. Use a graphing calculator to graph the function you developed in part a.

c. What is the long term behavior (end behavior) of the function you have graphed?

d. Mark your graph at a point where the caffeine is reduced to an amount that should no longer have any stimulant effect on the body.

e. How much caffeine will be in this person's body in 30 minutes (when \( t = \frac{1}{2} \))? 

f. How long will it take until the amount of caffeine in this person's body is cut in half; that is, from 390 to 195?

The time it takes for the amount of a substance present to be reduced by half is called the half-life of the substance. This term is applied in many situations, including the elimination of drugs from the body and the decay of radioactive materials.

g. To understand half-life, first determine how much of the original 390 mg of caffeine is in the body after two half-lives; then after three half-lives. Then determine the value of \( t \) (time) for each of those answers. Note that half-life is a measure of time.
### Homework

3. Make a table similar to the one below to keep a record of your own caffeine consumption over a 24-hour period beginning when you wake in the morning. Estimate the amounts of caffeine in the drinks or chocolate you consume using the information from the reading assignment or information in product labeling. You could get more information on caffeine content on the internet. (Try http://www.austinlinks.com/General/caffeine.html) If you do not consume caffeine, use data from a friend. Use your table to help sketch a graph of the amount of caffeine in your body (or your friend's) for this 24-hour period.

Assume consumption and absorption of caffeine is immediate even though you may actually take some period of time consuming the drink and it may take a short period of time for the caffeine to be absorbed into your body.

<table>
<thead>
<tr>
<th>Time of caffeine consumption.</th>
<th>8:00am</th>
<th>9:30am</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount of caffeine in your body before consuming this drink.</td>
<td>0</td>
<td>105.5mg</td>
<td>...</td>
</tr>
<tr>
<td>Amount of caffeine in this drink.</td>
<td>130mg</td>
<td>40mg</td>
<td>...</td>
</tr>
<tr>
<td>Amount of caffeine in your body after consuming this drink.</td>
<td>130mg</td>
<td>145.5mg</td>
<td>...</td>
</tr>
<tr>
<td>Function that describes amount of caffeine in your body from now until next caffeine consumption.</td>
<td>( f(t) = 130(0.87)^t ) for ( 0 \leq t &lt; 1.5 ), ( f(t) = 145.5(0.87)^{t-1.5} ) for ( 1.5 \leq t \leq ? ).</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use your graph to help write a conditional function that gives the amount of caffeine in your body at any time, \( t \), during the day. To write one function using a single variable for time, you need to note that \( t \) measures the time after some initial starting point. Note that this is different from the use of \( t \) in the last line of the table.

Do you notice any patterns between your caffeine level, when you have a caffeine drink, and how you feel? Comment on this.