Alcohol in Your Body
Teacher Materials

Mathematics Contained in the Lesson

The functions that model the process of the elimination of alcohol from the body serve as an introduction to a study of **rational functions** at an intermediate algebra level. The lesson focuses on graphs of the functions with an emphasis on interpretation of the **horizontal and vertical asymptotes** in the context of elimination of alcohol from the body. Other mathematics involved is algebraic manipulation of the rational functions, solution of equations with rational expressions, realistic domain of a function, inverse functions, and equilibrium state of a dynamic process.

Set-up

Each student should have a graphing calculator <u>or</u> each group should have a computer.

Organization

Time : 1 to 2 class periods. For effective use of class time, students must read the Reading Assignment, pages 1, 2, and 3 of student material, <u>before class</u> and try problems 1 through 3. Begin class with a discussion of the reading material and the problems. Be sure students understand what a and p represent. Make sure, in problem 3, they understand what a horizontal asymptote is, what it says about p, and how it can be recognized from the function.

In class, after the discussion, groups of 3 or 4 students can begin to work immediately on **Alcohol and Your Body:** Classroom materials, part 1. As students work, it is important to help them focus on the mathematics they are learning. End this section with a <u>general</u> discussion of rational functions and horizontal asymptotes, using the functions in the lesson as <u>examples</u>:

- Make sure students understand the relationship between the function and the horizontal asymptote.
- They should understand how to recognize the asymptote from the expression.
- Most importantly, as problem 6 indicates, they should understand how knowledge of a horizontal asymptote can help them solve a problem, that is, for large values of the independent variable, they can "assume" the dependent variable is equal to the asymptote.

The third section, **Alcohol and Your Body:** Classroom materials, part 2, discusses vertical asymptotes. This section could be covered several days later if you wish. If you use this section, we recommend you begin your discussion of vertical asymptotes with this unit. Begin this section with a review of alcohol elimination and help students associate meanings with the variables a, y, and d.

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We have found that students have a great deal of difficulty with problem 7, solving the equation for a; please be alert for this.

After all students have finished problem 10 (although some may have gone on to problems 11 and 12), have a general discussion about equilibrium and vertical asymptotes:

- Make sure students know how the vertical asymptote relates to the rational expression.
- Most importantly, make sure they understand that when the independent variable is near the asymptote, small changes in the independent variable result in large changes in the dependent variable. This is emphasized in problems 9, 10, and 11.

Problems 11 and 12 are extensions and could be assigned as (optional) homework or omitted. Their purpose is to emphasize how careful they must be when analyzing a function near a vertical asymptote. In applications, when the independent variable is near a vertical asymptote, small inaccuracies in measuring the independent variable or in developing the particular function can result in large inaccuracies in the dependent variable.

For students who will study calculus later, it is valuable to have an informal discussion of slopes near asymptotes in terms of the graphs in both sections of this lesson.

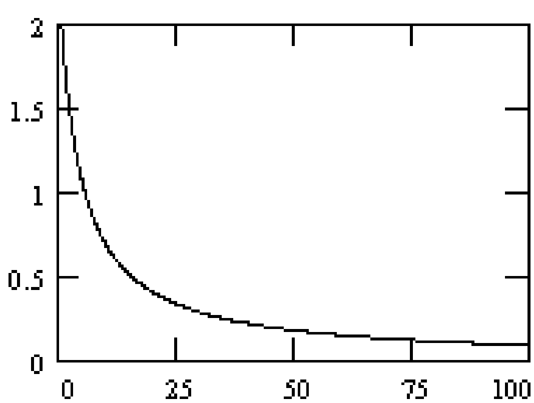
Answers to Problems and Teaching Suggestions for the Reading Assignment

This section deals with the study of horizontal asymptotes.

1. **a.** $p = \frac{10}{4.2+14} \approx 0.549$
b. $p = \frac{10}{4.2+28} \approx 0.311$
c. $p = \frac{10}{4.2+42} \approx 0.216$

2. Solve $1 = \frac{10}{4.2+a}$. $a = 5.8$

3. The graph of $p = \frac{10}{4.2+a}$ is given below



$p < 0.1$ when $a > 95.8$ grams.

Students should have attempted the previous three problems before class. Discuss the reading material with them at the beginning of class as well as these three problems. Make sure they understand how to explore functions such as these with their graphing calculator. Also have some discussion of horizontal asymptotes and their meaning in the context of problem 3. Make sure you discuss what you consider important about functions with $y = 0$ as a horizontal asymptote. This could include finding the asymptote algebraically.

Answers to Problems and Teaching Suggestions for Classroom materials, part 1.

4. $y = \frac{10 \times 14}{4.2+14} \approx 7.69$; $y = \frac{10 \times 28}{4.2+28} \approx 8.69$; $y = \frac{10 \times 42}{4.2+42} \approx 9.09$
Comparison of problem 1 with problem 4 reveals that as the amount of alcohol, a, increases, the <u>amount</u> being eliminated increases (problem 4), but the <u>proportion</u> of the alcohol present that is being eliminated decreases (problem 1). In other words, the function p is decreasing and the function y is increasing. Observe students' work. If some groups do not seem to see the significantly different way the two functions act as a increases and what this means physically, initiate a whole class discussion to be sure that they do.

5. Horizontal asymptote at $y = 10$. This implies that as the amount of alcohol in the body increases, the amount eliminated approaches a ceiling of 10 grams per hour.

6. For $a \in [34, 160]$, the function value is "near" its asymptote, $y = 10$. We can use 10 as an approximation of the number of grams eliminated per hour. Since $160 - 34 = 126$ grams need to be removed from the body, $126/10$ = 12.6 or 13 hours estimates the time needed. 12.6 is an <u>underestimate</u> since 10 is an overestimate for the number of grams eliminated per hour, which can be seen from observing the graph of $y = \frac{10a}{4.2+a}$.

Reflection point: horizontal asymptotes. When all groups have completed the first section, "Process of Elimination", have a short whole-class discussion for students to reflect on (1) what a horizontal asymptote is, (2) what the horizontal asymptote to $y = \frac{10a}{4.2+a}$ means in the alcohol context, (3) how they might have recognized from the equation that the end behavior of this function would be an approach to $y = 10$; that is, how we might easily recognize what the horizontal asymptote of a rational function is. Include as much algebra as you deem necessary for your class. At this point you might want to go into more depth on horizontal asymptotes, what they mean, and how to find them, including homework from their text.

Answers to Problems and Teaching Suggestions for Classroom materials, part 2.

This section deals with vertical asymptotes. This section could be done at the beginning of the next class period or you could continue the discussion of horizontal asymptotes, then come back to this section when you begin the study of vertical asymptotes.

7. $a = \frac{4.2d}{10-d}$. This equation gives the amount of alcohol in the body of a person who drinks d grams of alcohol every hour over a relatively long period. It is the amount of alcohol in the body when it reaches its equilibrium state for this size drink, where continued ingestion at this rate maintains a body level of a grams of alcohol.

Many students will have difficulty solving this equation for a. You might warm them up by having them solve this equation for a particular value for d, say $d = 7$. In this case, you would have them solve

$$7 = \frac{10a}{4.2 + a}$$

for a. This might serve as a good reminder for students not to forget old techniques.

8. $\frac{4.2 \times 7}{10-7} = 9.8$. A 150 pound person may not have any physical response to this rate of alcohol intake even over a long period since the amount in the body remains below 10g.

9. When $d = 9$, the equilibrium value is $a = \frac{4.2 \times 9}{10-9} = 37.8$; This person will be intoxicated (BAC over 0.08) if this drinking pattern continues over a long enough period and so should not drive once near equilibrium. When $d = 9.5$, $a = \frac{4.2 \times 9.5}{10-9.5} = 79.8$; very intoxicated. When $d = 9.9$, $a = \frac{4.2 \times 9.9}{10-9.9} = 415.8$; there is risk of death at 210 grams, well before nearing equilibrium.

10. Vertical asymptote at $d = 10$. As d approaches 10 from the left, a gets large without bound. Very small increases in d produce very large increases in the equilibrium amount a.

Reflection point: vertical asymptotes. When all groups have completed at least problem 10, have a short whole-class discussion for students to reflect on (1) what a vertical asymptote is, (2) what the vertical asymptote to $a = \frac{4.2d}{10-d}$ means in the alcohol context, (3) how they might have recognized from the equation what the behavior of this function would be as d approached 10.

Note in problem 9 how a very small change in d produces a large enough change in a so that in this situation there is a risk of death. The physiological significance of this is quite compelling; be sure students note the <u>mathematical</u> significance, that is, the concept of slope of a curve. That small changes in the independent variable produce large changes in the dependent variable and its relation to slope of a function is a precursor to the concept of the derivative. Any class discussion on this point now will help students prepare for the future study of calculus.

To graph $d = \frac{10a}{4.2+a}$ and $a = \frac{4.2d}{10-d}$ on their calculators, students will reverse their assignments of a and d to x and y. Be sure students recognize that these two functions are a pair of inverses. Students should be led to note that where the first function had a horizontal asymptote $(y = 10)$, the second function has a vertical asymptote $(x = 10)$ and to reflect on why this should be so in the context of what they know about inverse functions. (If they have not studied inverses, you can mention the word in this context and make reference to it at a later time when they do study inverses.)

At this point you might want to continue your discussion of vertical asymptotes and how to find them for more general rational functions, including problems from their text. They should understand that the techniques they learn from the book can be used on the problems in this unit and the problems from this unit should help them understand what they are finding when they work problems in their book.

11. a. $\frac{4.2d}{10-d} = 140$ if $d \approx 9.7$

b. $\frac{4.2d}{10-d} = 210$ if $d \approx 9.8$.

Problem 11 further reinforces the concept that small changes in the independent variable produce large changes in the dependent variable, but from another point of view; that is, large changes in the dependent variable produce small changes in the independent variable.

12. If $d = \frac{9.6a}{4.2+a}$ and $d = 9.5$, then $a \approx 399$ grams.
Note that here, $a = \frac{4.2d}{9.6-d}$, so the vertical asymptote is $d = 9.6$. For $d < 9.6$ but close, a is volatile!

Wrap-up: summarize the main points. Be sure students know what a rational function is; what a horizontal asymptote is both geometrically and numerically and that they have some idea how to recognize where one will be from the algebraic representation of the function; what a vertical asymptote is both geometrically and numerically, and that they have some idea how to recognize where one will be from the algebraic representation.