

Fertility and Consumption when Having a Child is a Risky Investment*

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Abstract

This paper analyzes fertility and consumption decisions when the costs of raising a child and parents' income are stochastic and correlated. We model the decision to have a child similarly to the decision to exercise an option in finance literature. We obtain several new results relative to models where children are deterministic goods and only income and substitution effects drive fertility. For example, 1) Higher child risks diminish fertility and consumption. 2) Fertility is increasing in the correlation between income and child cost shocks. 3) The sign of the correlation determines whether higher income volatility speeds up or delays fertility.

Keywords: Fertility, Real Options, Uncertainty.

JEL Classification: J13, D1

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1 Introduction

In this paper we study the consequences for fertility and consumption of taking into account that children generate risks, and how these risks interact with other risks borne by the parents. For example, sometimes children get sick, which implies money and time costs for the parents. Moreover time off from work to care for a child can affect the career paths of the parents. There are no insurance markets for many of the costs associated with a child, especially those related to the time costs for the parents. Thus, children are risky assets that are at most partially insurable, and childbearing adds another source of risk to households.

We study the problem of an infinitely-lived household who has an initial level of assets and every period receives stochastic income, which we assume exogenous for simplicity. She receives utility from consumption and can save at the risk free rate. Markets are incomplete because the risk free rate is the only available asset. The decision to have a child is like the decision to exercise an option. Every period she can have a child or postpone the decision. If she has a child then she is acquiring an irreversible, durable and non-tradable asset that gives her deterministic utility, but implies stochastic exogenous costs.¹

Our main results are the following: 1) Higher child cost volatility diminishes fertility and consumption as risk averse households are less willing to invest in riskier assets. Higher child risk results in higher precautionary savings. 2) Fertility is increasing in the correlation between income and child cost shocks. A household is reluctant to have children when positive cost shocks come together with bad income shocks (for example, households who reduce hours of work and damage their careers to care for a sick child). 3) The sign of the correlation determines whether higher income volatility speeds up or delays fertility.

These results come from the interplay of four effects. These effects differ from the income and substitution effects, which are the only effects in models where children are deterministic goods. 1) As in any real options problem, having the option to time an investment implies asymmetric convex payoffs ("in bad times do not exercise and wait for good times to come"). This makes the value of the option increasing in child cost volatility and encourages delaying fertility. 2) Higher cost volatility increases the risk of not exercising the option (the range of prices tomorrow is larger). This pushes risk averse households towards earlier fertility (exercise today at known price). 3) If the fertility option is exercised the child costs can be thought of as an income shock. Incomplete markets and preferences displaying precautionary savings

¹These are realistic assumptions given that once a child is born, the child is not easily disposable because of sentimental reasons and legal constraints. Moreover, children do not depreciate in value and it takes several years before they can live independently.

push the household to delay fertility to avoid new sources of risk. 4) When the income and cost shocks are correlated, the sign of the correlation determines if the shocks hedge or add up. Fertility speeds up when the shocks hedge each other.

Our analysis offers new insights that may help both in explaining facts and in the design of pronatalist policies. For example, Stetsenko (2010) documents that U.S. fertility changed from countercyclical to procyclical as female labor supply increased. This is consistent with our model if an increase in female labor supply implies a negative correlation between child costs and income shocks. Also, the model could have different implications for different subsets of the population. For example, perhaps most of the monetary costs of child care are covered by health insurance, but there is no insurance for the lost wages due to taking care of a sick child at home. In this case, child care costs disproportionately affect families in which the mother works. Child care costs could then have implications for married women's extensive margin labor force decision in addition to fertility decisions.

On the policy side, our theory suggests that pronatalist policies may seek to reduce uninsurable uncertainty from raising a child (for example providing child care programs), and the negative correlation between income and child costs shocks (for example State-paid leaves of absence or guarantees for workers who take time off to care for sick children).

This paper contributes to the literature on the economics of fertility. This literature has followed Gary Becker's seminal work in modeling children as deterministic durable goods (Jones et al. 2008 and Tamura 2000 survey the literature). Our contribution is to focus on the risks stemming from children themselves and their correlation with income risk. These new assumptions expand the range of forces driving fertility. In Becker's model with deterministic costs, substitution and income effects are the only channels. Here we add the new channels discussed above.

This paper is theoretical, given the lack of closed form results we solve the model numerically, check the robustness of the results, and provided comparative statics exercises to understand the effects of each parameter. We focus on studying the fertility decision from the perspective of the value of the fertility option. Ranjan (1999) is the only paper we are aware of that explicitly discusses having a child as an option. However in Ranjan's model, as in the rest of the literature, children are deterministic.

Our work complements studies on the effects of income uncertainty on fertility. Several empirical papers have shown that fertility responds negatively to increases in income risk.²

²Adsera and Menendez (2011) focused on Latin America; Adsera (2005, 2006 and 2011) in Southern Europe; Bhaumik and Nugent (2006) on East Germany; Mira and Ahn (2002) in a sample of OECD countries; Sommer

Conesa (2000), Choi (2011), Da Rocha and Fuster (2006) and Sommer (2011) are different quantitative models on income risk or unemployment delay fertility. In those models children are deterministic assets. Our results suggest that the combination of child risk and shocks contributing to income volatility reinforce these delays.

The paper proceeds as follows. Section 2 motivates why a child is a source of risks and how these risks correlate with income shocks. Section 3 describes the model. Section 4 discusses the results. Section 5 concludes. Details on the solution method are in an Online Appendix.

2 Motivating that Children are Risky Assets

Children generate monetary and non-monetary costs and benefits to their parents. The usual assumption in the literature is that both costs and benefits are deterministic. In this Section we motivate the main innovation of the paper; the costs of raising a child are stochastic, potentially correlated with parents' income, and partially uninsurable. For example, medical costs depend on how often a child gets sick and how severe the sickness is. The 2008 National Health Interview Survey documents a large role played by exogenous health shocks. This survey reports that annually roughly 30% of the children missed 1 or 2 school days, 30% missed 3-5 days and 10% more than 6 days. These numbers are not sensitive to differences in income, race or education of the parents. Some of these health shocks can be highly persistent, such as having a child with special needs. The Center for Disease Control and Prevention has quantified the probability distributions of having a child with special needs, and Parish et al. (2009) document wide heterogeneity in the costs of these children.^{3, 4} Education costs can also vary widely across children depending on child's ability.

In the U.S. and in many other countries, most of the time costs for the parents are non-insurable.⁵ For example, most child care centers ask the parents to take their child home when the child gets sick.⁶ The National Association for Sick Child Daycare says that there is a large

(2011) in the U.S. Kreyenfeld (2005) provides a survey.

³<http://www.cdc.gov/ncbddd/birthdefects/data.html>

⁴Among these families with positive spending, 30% had expenses between \$250 and \$500, and 34% had expenses of more than \$500. Twenty-seven percent had expenditures from 5.6% to 25.8% of their total household income. Part of the variation in amount of insurance against shocks is associated with variation across states in the income eligibility guidelines for Medicaid and SCHIP.

⁵Moreover, most health insurance plans have deductibles and co-payments. Thus, even if the parents buy health insurance a sequence of bad health costs can be costly.

⁶Hotz and Miao (2011) report that in the spring of 2005, 7.2 million children under the age of 5 (36.9%) were in some form of non-relative care, of which 4.6 million (23.3%) were in some form of organized child care. Among employed mothers with a child under 5, 5.9 million of their children (52.1%) were in non-relative child care, of which 3.6 million (31.9%) were in an organized care facility.

unmet need for child care of sick children. They estimate that each day more than 350,000 children younger than 14 years of age, with both parents working, are too sick to attend school or child care. And that working mothers are absent from their jobs from 5 to 29 days per year caring for ill children.⁷

Shocks to the cost of raising a child can be correlated with parents' income shocks. For example, because child health shocks force parents to divert time from work, income from hours not worked is lost, and time off from work to care for the child may harm the parents' career path. Thus, in this case an increase in children's cost coincides with a reduction in parents' income (income and child shocks have negative correlation). It may also happen that the shocks hedge each other. For example, parents of child athletes receive negative income shocks if they sacrifice their careers to travel with their children (as it is often the case with tennis players), but this income reduction may come with less costs of raising the child as the child earns money for the family (positive correlation between parents' income and child costs). Or, as another example, eligibility guidelines for public subsidies may imply that an increase in child costs allows the household to qualify for public support, thus generating positive correlation between child costs and family income.

The previous empirical facts suggest that children are sources of risks for the parents and children's costs can be correlated with the parents' income. In the next sections we study how these assumptions may affect consumption and fertility.

3 Model

We use a discrete-time, partial equilibrium model, with an infinitely-lived household whose utility function depends on her consumption of goods (c_t) and on whether or not she has a single child (I_t). The household starts without a child at time $t = 0$ and has the option to have a child at any moment. We denote by I_t an indicator function that captures the fertility status at time t : it takes the value one if the household has had a child prior to, or at time t , while it takes the value zero if she has not. We assume that the utility function is separable into utility from goods' consumption and utility from the fertility status. We assume Constant Relative Risk Aversion for the consumption component:

$$u(c_t, I_t) = \frac{c_t^{1-\gamma}}{1-\gamma} + \alpha I_t, \quad \alpha > 0, \quad \gamma > 0, \quad (1)$$

⁷<http://www.nascd.com/>

where γ is the relative risk aversion coefficient and α is a parameter that captures the utility benefit from having a child.

Every period the household earns a stochastic labor income stream (y_t). Moreover, if she has had a child she must pay the stochastic costs (q_t) associated with child rearing. We assume that markets are not complete and the household cannot hedge against child cost or income risk because she can only transfer consumption over time via the riskless asset. We denote by r the constant one-period risk-free rate. Hence the wealth (W) of the household evolves as

$$W_{t+1} = (W_t + y_t - q_t I_t e_t - c_t)(1 + r), \quad (2)$$

where both income and costs are denominated in units of the consumption good that we use as numeraire.

We model the dynamics of the stochastic processes as first order autoregressions:

$$y_{t+1} = a_y + b_y y_t + e_{t+1}^y, \quad (3)$$

$$q_{t+1} = a_q + b_q q_t + e_{t+1}^q, \quad (4)$$

$$(e_{t+1}^y, e_{t+1}^q) \stackrel{iid}{\sim} N \left(0, \begin{pmatrix} \sigma_y^2 & \sigma_y \sigma_q \rho \\ \sigma_y \sigma_q \rho & \sigma_q^2 \end{pmatrix} \right), \quad (5)$$

where ρ denotes the correlation coefficient between shocks to income and child costs, while σ_y and σ_q denote the volatilities.

The household maximizes time-additive expected utility over consumption and her parental status, i.e. she decides two things: whether and when to have a child, and how much to consume and save every period. After she has had a child she only has to decide how much to save and consume. The problem of the household before having a child is:

$$H(W_0, q_0, y_0) = \max_{\{c_t, I_t\}} \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t u(c_t, I_t) \right] \quad (6)$$

$$s.t. \quad (2) - (5). \quad (7)$$

3.1 Solving the model

The problem has a recursive nature. Conditional on having or not having a child, the household's decisions depend on her wealth, income, and on the cost of the child. These are the state variables of the problem. We denote by $J(W, q, y)$ the value function of a household

that already has a child:⁸

$$J(W, q, y) = \max_c \left[\frac{c^{1-\gamma}}{1-\gamma} + \alpha + \beta \mathbb{E} [J(W', q', y')] \right] \quad (8)$$

$$s.t. \quad (9)$$

$$W' = (W + y - q - c)(1 + r), \quad (10)$$

$$y' = a_y + b_y y + e^y, \quad (11)$$

$$q' = a_q + b_q q + e^q. \quad (12)$$

The household without a child compares the value of having a child (the value function J), with the value of continuing without child. Her Bellman equation is:

$$H(W, q, y) = \max_I (1 - I) \left\{ \begin{array}{l} \max_c \left[\frac{c^{1-\gamma}}{1-\gamma} + \beta \mathbb{E} [H(W', q', y')] \right] \\ s.t. \\ W' = (W + y - c)(1 + r) \\ y' = a_y + b_y y + e^y \\ q' = a_q + b_q q + e^q \end{array} \right\} + IJ(W, q, y). \quad (13)$$

As in the option pricing literature, we define the continuation region as the set of realizations of the state variables in which the household is better off without children. The fertility region is the complement of the continuation region. The boundary between these regions is the critical level of costs below which the household is better off with a child, $\bar{q}(W, y)$. This decision (or fertility) boundary depends on current wealth and income:

$$\bar{q}(W, y) := \left\{ \sup q : \left\{ \begin{array}{l} \max_c \left[\frac{c^{1-\gamma}}{1-\gamma} + \beta \mathbb{E} [H(W', q', y')] \right] \\ s.t. \\ W' = (W + y - c)(1 + r) \\ y' = a_y + b_y y + e^y \\ q' = a_q + b_q q + e^q \end{array} \right\} < J(W, q, y) \right\}, \quad (14)$$

where $J(W, q, y)$ is decreasing in q . The household has a child once the stochastic cost process is equal to or smaller than $\bar{q}(W, y)$.

⁸An apostrophe identifies next-period quantities.

4 Results

In this Section we analyze the predictions of the model. Given the lack of closed form solutions, we solve the model numerically to illustrate theoretical results that cannot be proved analytically. While we choose somewhat plausible values, we invite the reader not to focus on the precise numbers but rather on the comparative statics exercises. They illustrate how changing the parameters affect the results. We refer to a lower boundary $\bar{q}(W, y)$ as lower or deferred fertility because the household will more likely have a child when $\bar{q}(W, y)$ is lower.

4.1 Parameterization

Table 1 summarizes the benchmark parameterization. This is how we selected these parameters.

Insert Table 1 about here

We assume $\gamma = 3$ as benchmark coefficient of relative risk aversion. This value is in the standard range in macro models, and in models with preferences additively separable in consumption and childbearing utility, such as Becker et al. (1990), or Jones et al. (2008). We checked thoroughly how risk aversion affects our results. We assume one period to be one year and set the subjective discount rate β to match an annual interest rate of 3%. Concerning α , the parameter which governs the deterministic utility from having a child, we set it such that the utility stream from a child equals 50% of the utility stream from consumption of a household with no fertility option. We discuss how changing α affects the results.

Concerning wealth and income, we set a_y such that the long-term mean of income is normalized to 10, and consider different wealth levels in a grid ranging from 1/2 to 6 times that value. This range gives enough room to explore the effects of wealth and precautionary savings in household's fertility decision. We assume income shocks to be fairly persistent ($b_y = 0.88$), as it is usual in the literature estimating stochastic processes for households' income. Following that literature, we set the income standard deviation σ_y to be 0.19 and then we conduct comparative statics on it.

Concerning the parameters related to the costs of a child, we quantify their level using the estimates of expenditures on children from birth through age 17 from the U.S. Department of Agriculture (Lino 2011). We choose a_q , the long-term mean of the child cost process, to match a long term child cost mean of 18% of long term mean income. For the persistence of the child cost shocks (b_q) and their volatility (σ_q), we start with an agnostic choice ($b_q = b_y, \sigma_q = \sigma_y$)

and compare how altering these parameters would alter the results. We use a broad range of parameters. We use $\rho = 0$ as benchmark correlation and discuss how altering the sign of this correlation impacts the results.

4.2 Results

We start by analyzing the influence of child cost volatility. Figure 1 reports the fertility boundary as a function of child cost volatility for different levels of risk aversion and wealth. In Panel B the household is wealthier than in Panel A.

Insert Figure 1 about here

The main two results are: i) Higher cost volatility diminishes fertility, but the effect is less pronounced for wealthier households. ii) Higher risk aversion increases fertility. These results come from the interplay of two channels:

- 1) When a child is risky, the problem is similar to a standard real options problem: having the option to time an investment implies asymmetric convex payoffs ("do not exercise in bad times and wait for good times to come"). This makes the value of the option increasing in the cost volatility for a risk-neutral household. For example, if child cost volatility was different depending on the parents' profession, then this channel pushes households with larger volatility towards delaying fertility. This channel is more complicated for a risk averse household. On one side the previous argument applies. The higher the risk aversion, the higher the value of being able to time fertility (avoid investing in bad times). However, when cost volatility increases, the risk of not exercising the option also increases. Intuitively, the household needs to choose between exercising today at price q or having the option to exercise tomorrow. When cost volatility increases, waiting for tomorrow is riskier (the range of tomorrow prices is larger), which pushes a risk averse household towards earlier exercise of the option.
- 2) Once the household has a child, she starts bearing the uninsurable risk of the costs of raising that child. Her problem becomes a standard incomplete-markets consumption problem with stochastic income, but the level of risk depends on the fertility decision (Caballero 1991 or Wang 2006). For $\rho \leq 0$, people with children have riskier incomes than people without children (if the shocks are positively correlated, the different shocks hedge one another). We can see this in equation (2) denoting $(y_t - q_t)$ as income net of children costs. If $\rho \leq 0$ then both the risk and the need for self insurance are larger. Thus

a household with precautionary savings preferences ($\gamma > 0$) will wait to have a child as she accumulates assets for precautionary reasons. Volatility under risk aversion is bad, leading to the valuation of children to be lower than if children were deterministic goods. Parents then wait for lower child costs or higher income. The precautionary savings effect is present both before and after the option exercise, but is larger after the option exercise, because the option to time fertility provides some insurance (it allows avoiding investing in bad times). Precautionary savings are increasing in risk aversion and child cost volatility, while decreasing in wealth. Higher precautionary savings push the household to reduce consumption and delay fertility.

- 3) More risk averse people have marginal utility of consumption decreasing more quickly, leading them to be more likely to sacrifice consumption for a safe flow of the utility that children provide. In the model households are risk neutral over the benefits of a child, as the benefits are a deterministic flow of utility, not consumption. If children were substitutable with consumption inside the CRRA utility function, then the utility derived from being parent can hedge fluctuations in utility from goods consumption as a child is an alternative source of utility different from goods consumption.

Channel 2 explains why the fertility boundary is decreasing in the volatility of the cost of having a child. While for small volatility Channel 1 can partially offset Channel 2. As volatility increases Channel 2 pushes the household to delay fertility. Wealthier households withhold less precautionary savings (their Channel 2 is smaller) and Panel B reflects this in a fertility boundary that is less steep as child risks increase.

Channel 1 and 3 explain why risk aversion (γ) speeds up fertility. As child cost volatility increases and risk aversion is higher the household is less prone to wait for more favorable costs conditions because there is higher cost uncertainty about tomorrow. Moreover, a child is a way to hedge future consumption uncertainty by buffering negative income shocks. A child assures a constant flow of altruistic utility that offsets low consumption levels. Higher γ also exacerbates Channel 2, thus making fertility less attractive for more risk averse households, nonetheless Channels 1 and 3 dominate.

Figure 2 reports average consumption with respect to the cost distribution. That is, weighted averages of the consumption associated with each child cost level, with weights being the unconditional probability of each cost level. Panels A and B focus on households who did not yet have a child. While Panels C and D report households with children. We compare two wealth levels.

Insert Figure 2 about here

In all four panels, consumption decreases as child cost volatility increases. And, as Channel 2 predicts, the decrease is steeper for households with children and low wealth (Panel C). Among low wealth households we know from Figure 1A that those with higher risk aversion are more likely to have children, and decrease consumption more in Figure 2A. They are substituting present consumption with the utility gained from being parents. When wealth is higher it is more difficult to see these mechanisms.

Increasing α (parental utility from a child) makes the child more attractive, which reinforces Channel 3. We see in Figure 3a that this factor produces higher fertility for any wealth level. Figure 3b shows that when α is smaller, households substitute more utility from consumption for childbearing utility. Thus, as child cost risk increases (higher cost volatility) households increase their precautionary savings. As it is expected, the increase in precautionary savings from the extra desire to have children is larger for less wealthy households (Panel A versus Panel B of Figure 3b).

Insert Figure 3a about here

Insert Figure 3b about here

Figure 4a studies the effects of child cost persistence (b_q) on the fertility decision.⁹ The effects depend on wealth levels. For low persistence (small b_q), fertility is decreasing for lower wealth households (that is, the boundary is increasing) and increasing for higher wealth households (decreasing boundary). The intuition is as follows: when b_q is low the cost process converges very quickly to the long-run mean cost level. Poor households that would have a child at costs below this long run mean become more conservative as they recognize that any low cost level will be short-lasting. A wealthy household, which can afford children at mean cost levels, will interpret the lack of cost persistence as an advantage, and can absorb short term high costs thanks to their wealth buffer, as they expect costs to decrease to the mean in a short amount of time. Thus, wealthier households take advantage of the high speed of mean reversion to have a child at higher cost levels and enjoy earlier childbearing utility.

Insert Figure 4a about here

⁹Rewriting the cost process (4) as:

$$\Delta q_{t+1} = (1 - b_q)(\bar{q} - q_t) + e_{t+1}^q$$

with $\Delta q_{t+1} = q_{t+1} - q_t$ and $\bar{q} = a_q/(1 - b_q)$ then $(1 - b_q)$ is the speed of mean reversion for child cost.

Child cost persistence (b_q) alters the relation between fertility and child risk, as shown in Figure 4b. When b_q is closer to 1 the cost level at which the fertility option is exercised matters for a long time. This reinforces the asymmetric convex payoffs discussed as Channel 1: the rewards from waiting to exercise in good times are higher because good times last longer. Thus, higher b_q delays fertility as cost volatility increases.

Insert Figure 4b about here

Higher b_q implies a substantial increase in precautionary savings. Figure 4c shows that average consumption with respect to the cost distribution decreases very quickly when persistence is high. Bad shocks last longer causing precautionary households to save more. When persistence is low, shocks do not last long and consumption before fertility is almost insensitive to cost volatility.¹⁰

Insert Figure 4c about here

Figure 5 shows an important effect of child cost shocks. As discussed in Section 2, they can hedge or reinforce shocks to income. Thus, fertility is monotonically increasing in the correlation of income and cost shocks, regardless of the household's risk aversion.

Insert Figure 5 about here

Intuitively, the household is reluctant to have children when positive child cost shocks coincide with bad income shocks. A pronatalist government may encourage fertility by altering the correlation. For example, a negative correlation may be the outcome of child illnesses having negative effects on the parents' careers. State-paid leaves in periods of high probability of child illnesses may break the correlation.

The correlation of income and cost shocks alters how income volatility affects fertility. This is studied in Figure 6.

Insert Figure 6 about here

When shocks are uncorrelated, higher income volatility defers fertility (decreasing boundary in Figure 6) because of Channel 1 and 2. Higher income volatility pushes for higher precautionary savings, and the household is reluctant to add the additional source of risk of a child to an already volatile income. Moreover, when income volatility is high, precautionary households

¹⁰The low cost consumption in Figure 4c looks flat but it is not exactly flat. Its variation is smaller than the step size of our grid. Thus, since the numerical algorithm cannot capture those small variations the consumption looks flat.

value the option to time fertility even more and are more conservative in exercising it. If the correlation is negative, the previous effects are reinforced because bad child cost shocks coincide with bad income shocks. Thus for $\rho < 0$ high income volatility decreases fertility substantially. This is consistent with the empirical literature on the effects of income uncertainty on fertility cited in the introduction.

However, when costs hedge each other, higher income volatility may push for earlier fertility. We see this in Panel A of Figure 6. The idea is that when risk rises, it increases for both parents and non-parents. For parents, consumption will fluctuate less, as child costs hedge income shocks, and encourage fertility. However, if the household is sufficiently risk averse, the Channels 1 and 2 previously discussed push for decreasing fertility even if the correlation is positive (Panel B of Figure 6). These channels are tampered if income is not very persistent. This is shown in Figure 7 that revises Panel B of Figure 6, substituting $b_y = 0.4$ for $b_y = 0.88$.

Insert Figure 7 about here

When b_y is low, income shocks do not last long. Thus, there is lower need for precautionary savings. Figure 7 shows that, in this case, when the shocks hedge each other ($\rho > 0$), higher income volatility does not seem to affect fertility. The hedging benefit from having a child offsets the push for decreasing fertility from Channels 1 and 2 discussed before. Our analysis then predicts that households with high but volatile and non-persistent income (hence very sensitive to the correlation between income and child risks) are those whose fertility rates are reduced more by a negative correlation.

So far, we have focused on comparative statics of wealth rather than on income levels. For a qualitative analysis, this does not imply any loss of generality for very persistent income process (as in the benchmark parameterization of Table 1). Figure 8 explores the effects of different levels of income persistence on fertility. Intuitively, households with bad income shocks (below average current income) have more children when income shocks are not persistent (fertility boundary decreases in b_y for low income household). The opposite trend occurs in households with good income shocks (above average current income).

Insert Figure 8 about here

Finally, Figures 9 and 10 illustrate the standard income and substitution effects well known since Becker (1960). In Figure 9, when children are more expensive (higher long-run mean of costs) the household substitutes them for goods consumption. In Figure 10, since children are

normal goods, fertility is increasing in wealth (Panel A), and income (Panel B).

Insert Figure 9 about here

Insert Figure 10 about here

5 Conclusions

In this paper we studied consumption and fertility decisions when the costs of raising a child and parents' income are stochastic and correlated. Our analysis can be extended in a number of directions. For example, for quantitative work, adding life-cycle effects is crucial as the ability to be fertile is age dependent. We also abstracted from the fact that income is endogenous and fertility alters labor supply. Introducing these extensions into the model would allow studying career and fertility choices together. Also, cross-country comparisons could test whether different policies regarding childcare risk affect fertility.

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Tables and Figures

Table 1: Benchmark parameterization

| | | |
|-----------------------|-------------------------|-----------------------|
| $r = 0.03$ | $\gamma = 3$ | |
| $\alpha = 22.54$ | $\beta = \frac{1}{1+r}$ | $\rho = 0$ |
| $a_y = 10(1 - b_y)$ | $b_y = 0.88$ | $\sigma_y = 0.19$ |
| $a_q = 1.82(1 - b_q)$ | $b_q = b_y$ | $\sigma_q = \sigma_y$ |

Table 1: *Benchmark parameterization.* The values are discussed in Subsection 4.1.

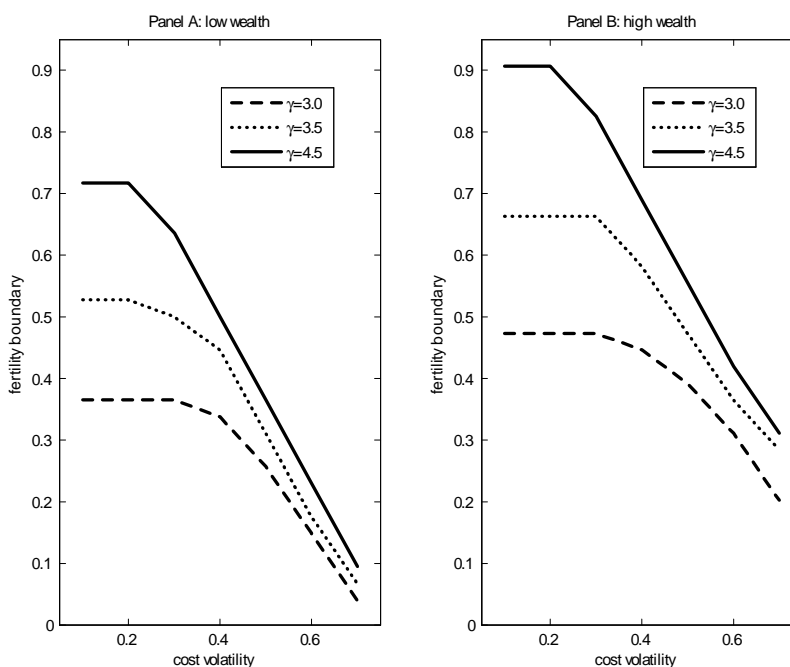


Figure 1: *Fertility as a function of cost volatility.* Panel A reports the fertility boundary (critical cost at which the household has a child) as a function of cost volatility, when household's wealth is the long-term expected income. Panel B reports the same variable when the household's wealth is 2.5 times long-term income. The solid, dashed, and dotted line correspond to a relative risk aversion coefficient γ of 3, 3.5, and 4.5, respectively. The fertility boundary is expressed as a fraction of long-term income. Both panels assume income at 65% of its long-run mean. The other parameters are as in Table 1.

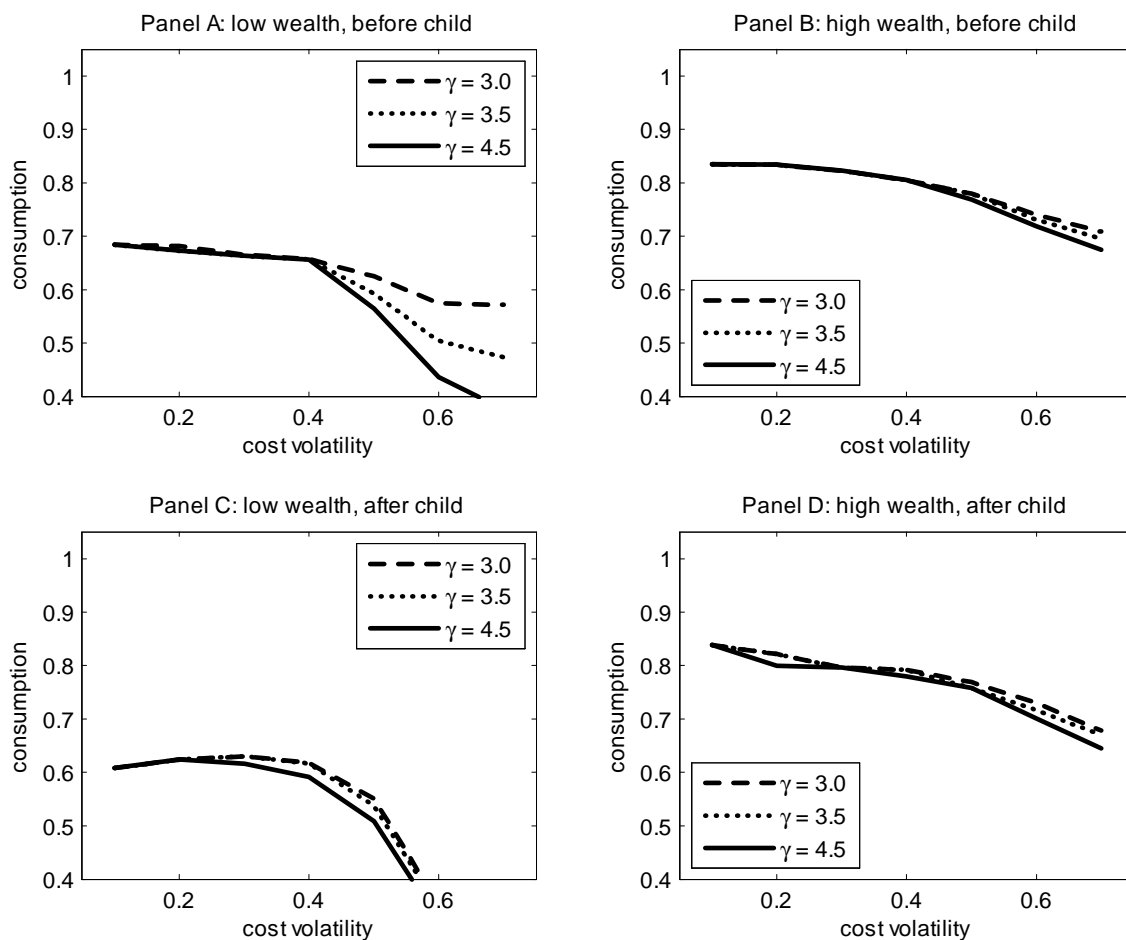


Figure 2: *Average Consumption as a function of cost volatility.* The figure reports average consumption for households who did not yet have a child (Panels A and B), and for those who already have a child (Panels C and D). The average is computed over child cost levels as discussed in the Online Appendix. In Panels A and C wealth is the income long-run mean, while in Panels B and D it is 2.5 times higher. The solid, dashed, and dotted line correspond to a relative risk aversion coefficient γ of 3, 3.5, and 4.5, respectively. Consumption is expressed as a fraction of long-term income and all panels assume income at 65% of its long-run mean. The other parameters are as in Table 1.

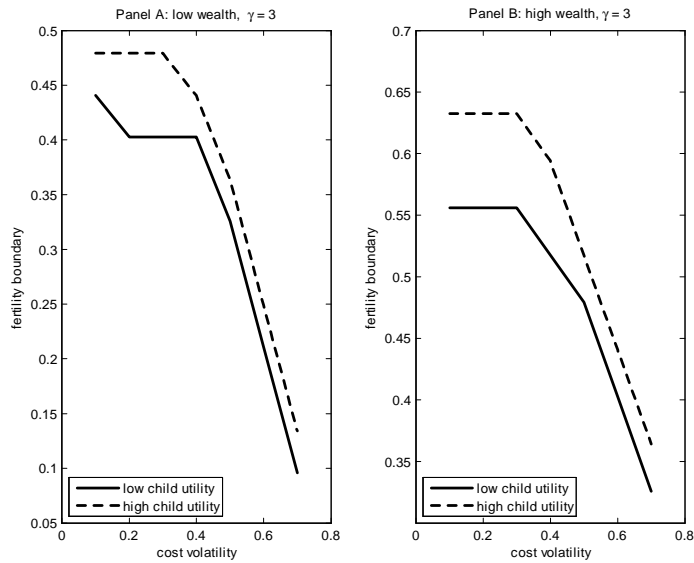


Figure 3a: *Fertility as a function of cost volatility for different levels of α .* In the solid line, α is calibrated such that the utility flow from a child equals 25% of the utility flow from consumption for a household with no fertility option. In the dashed line, α is calibrated to 30% of the utility flow from consumption if no fertility option. Wealth is the income long-run mean in Panel A, and 2.5 times higher in Panel B. The other parameters are as in Table 1.

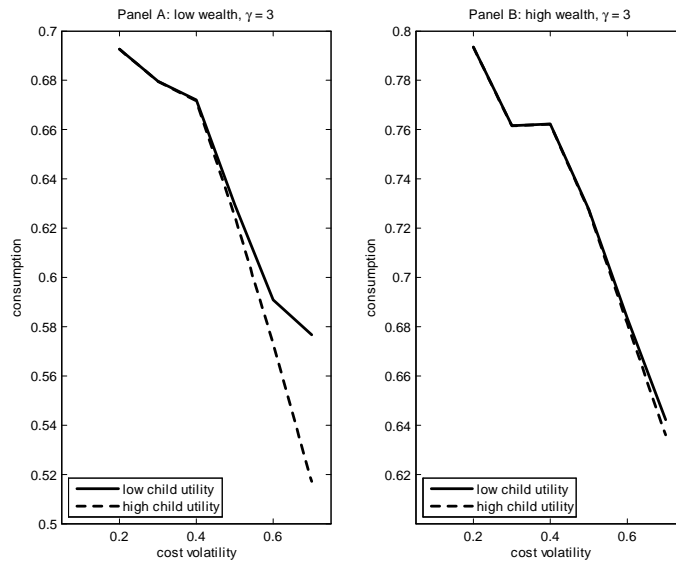


Figure 3b: *Average consumption as a function of cost volatility for different levels of α .* The figure reports average consumption (as a fraction of income's long-term mean) for households who did not yet have a child. The average is computed over child cost levels as discussed in the Online Appendix.

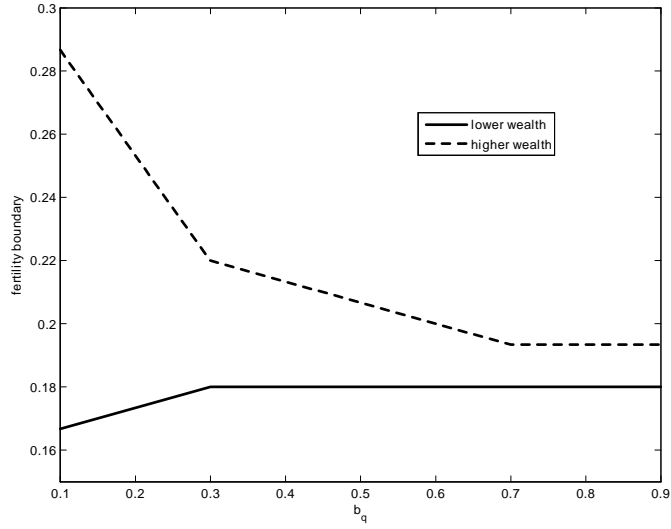


Figure 4a: *Fertility as a function of persistence of childrearing cost (b_q)*. The figure reports the fertility decision boundary (critical cost at which the household takes the fertility decision, as a fraction of income long term mean) as a function of the autoregressive parameter of the cost process, b_q , while leaving the long term mean $a_q/(1 - b_q)$ unchanged. The dashed line plots a higher level of wealth than the solid line (70% of income long-run mean versus 60%). Income is at 65% of its long-run mean. The other parameters are as in Table 1

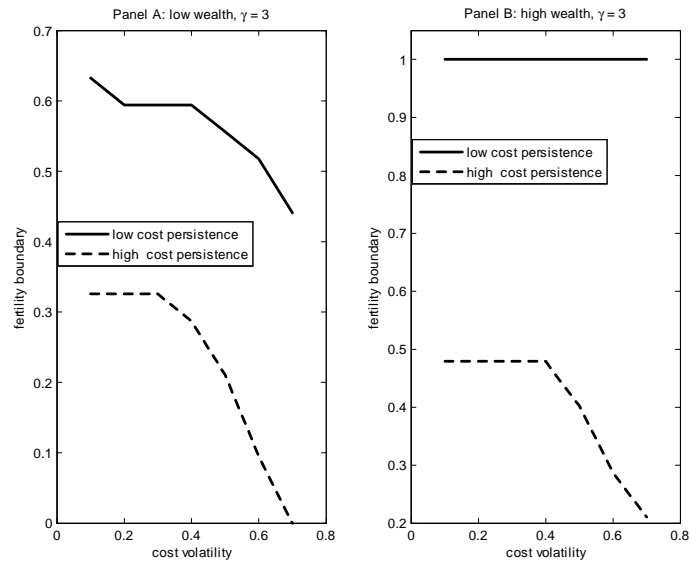


Figure 4b: *Fertility as a function of cost volatility for different levels of cost persistence.* Panel A reports the fertility decision boundary (critical cost, as a fraction of income long-term mean, at which the household has a child) as a function of cost volatility. The solid line has $b_q = 0.4$ while the dashed line $b_q = 0.88$, in both cases the cost long-run mean is unchanged. Wealth is the income long-run mean in Panel A, and 2.5 times higher in Panel B. The other parameters are as in Table 1.

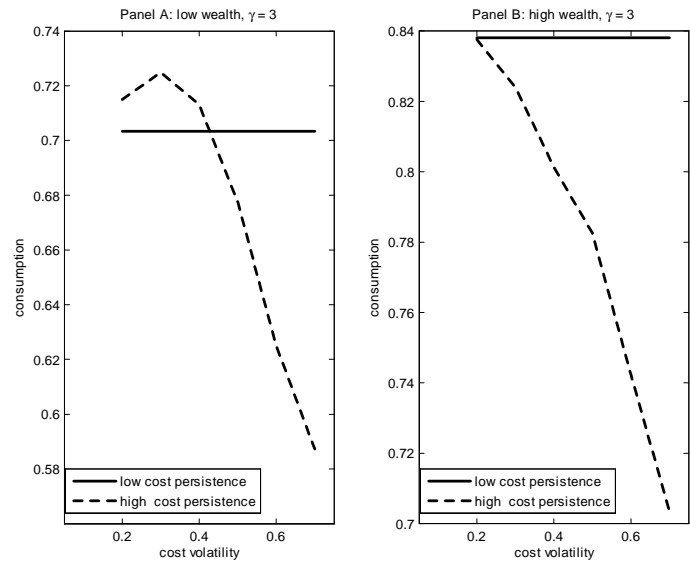


Figure 4c: *Average consumption as a function of cost volatility for different levels of cost persistence.* The figure reports average consumption (as a fraction of income's long-term mean) for households who did not yet have a child. The average is computed over child cost levels as discussed in the Online Appendix.

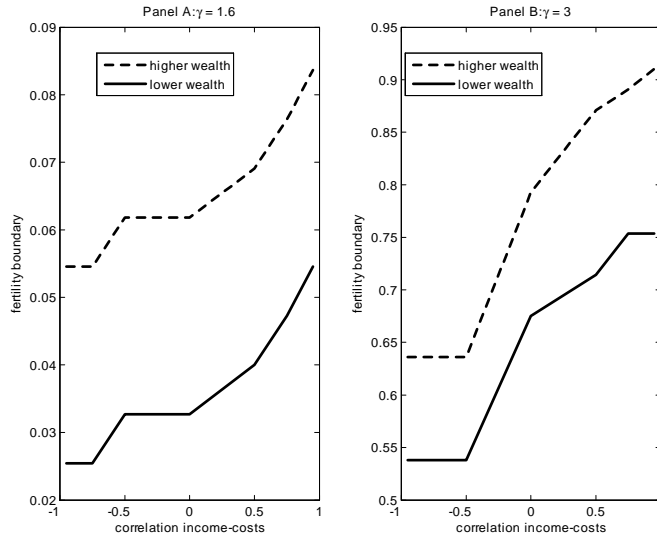


Figure 5: *Fertility as a function of the correlation between income and cost shocks.* This Figure reports the fertility decision boundary as a function of the correlation (ρ) between shocks to income and to child cost. Households's relative risk aversion is $\gamma = 1.6$ in Panel A, and 3 in Panel B. Solid lines plot current wealth at the income long-run mean, while dashed lines at 2.5 times that value. The income is at its long-run mean. The other parameters are as in Table 1.

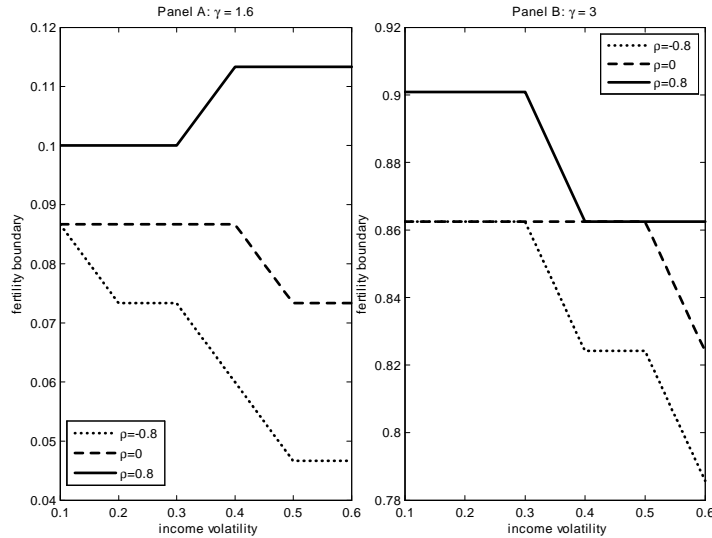


Figure 6: *Fertility as a function of the volatility of income.* Panel A reports the fertility boundary as a function of volatility of income (σ_y). Households's relative risk aversion is $\gamma = 1.6$ in Panel A and 3 in Panel B. Solid, dashed and dotted lines correspond to different levels of correlation between income and childrearing costs: $\rho = 0.8, 0,$ and $-0.8,$ respectively. In both panels, income is at its long-run mean and wealth is 2.5 times this value. The other parameters are as in Table 1.

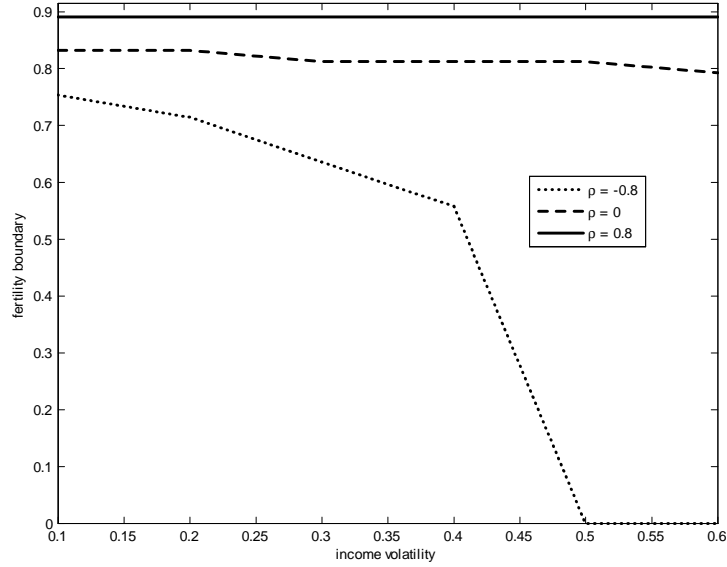


Figure 7: *Fertility as a function of the volatility of income for lower income persistence.* This figure redoes Panel B of Figure 6 when $b_y = 0.4$ instead of $b_y = 0.88$.

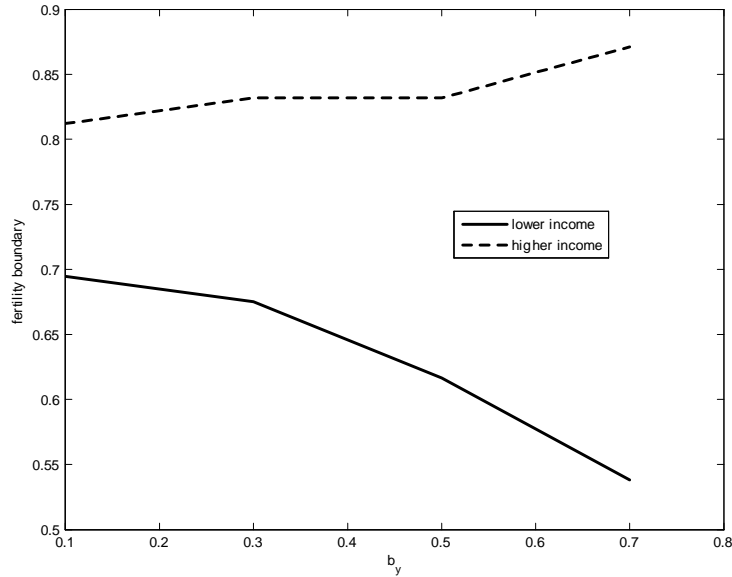


Figure 8: *Fertility as a function of income persistence.* This figure reports the fertility boundary as a function of b_y , without altering the long-term mean of income. The solid line is for income being 65% of its long-term mean and the dashed line for income being 125% of its long-term mean. Wealth is at the income long-term mean. The other parameters are as in Table 1.

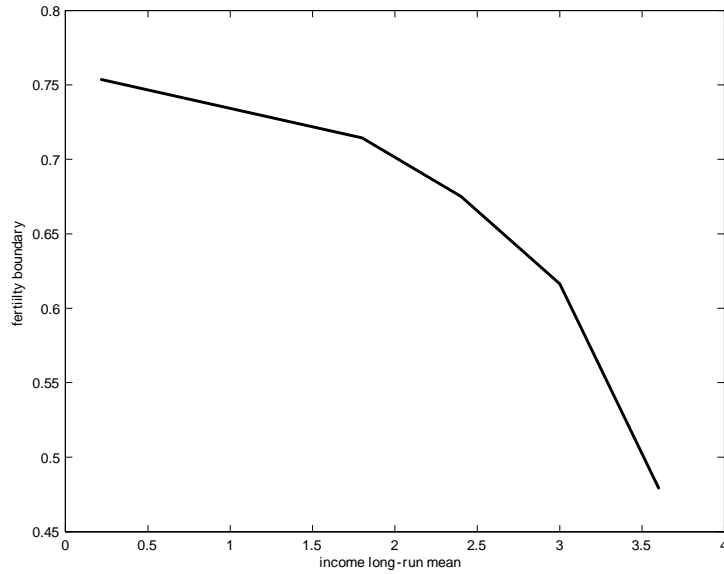


Figure 9: *Fertility as a function of the long-term mean of childrearing cost.* The figure reports the effects on the fertility boundary of altering the long-term mean of childrearing costs, while keeping unaltered the autoregressive parameter b_q . Wealth is at the benchmark income long-run mean, while income is 65% of its benchmark long-run mean. The other parameters are as in Table 1.

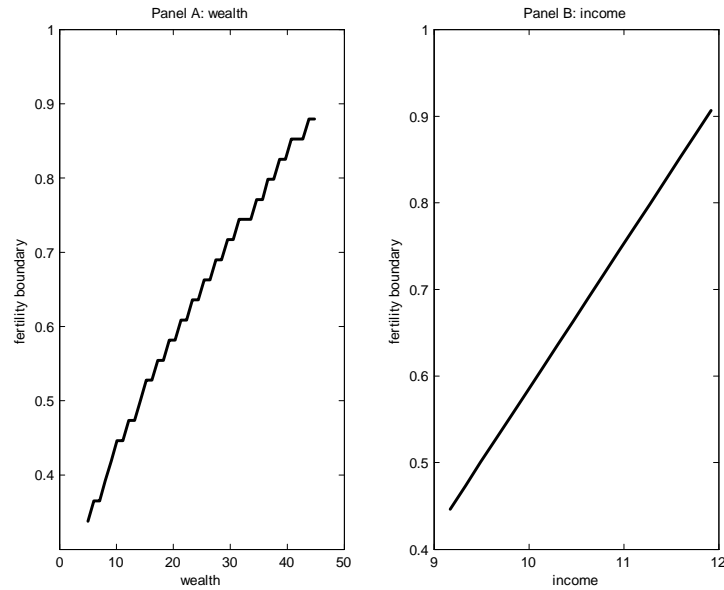


Figure 10: *Fertility as a function of income, and wealth.* This figure reports the fertility decision boundary as a function of wealth (Panel A) and income (Panel B), both expressed as fractions of income long-run mean. Parameters are as in Table 1.

Numerical Method

We discretize the state-space by setting: i) a grid of $n_W = 50$ equally distant realizations for wealth, with lower bound $W_1 = 5$ and upper bound $W_{n_W} = 60$. As discussed in Section 4, these values correspond to 0.5 and 6 times, respectively, the income long-run mean. ii) A grid of $n_q = 25$ equally distant values for costs, with lower bound $q_1 = 0$ and upper bound $q_{n_q} = 9.2$. iii) An equispaced grid of $n_y = 25$ values for income, with upper and lower bounds $y_{n_y} = 12.5$ and $y_1 = 6.5$, respectively.

The realizations of (y, q) on the grids follow a two-state Markov chain whose transition probabilities approximate the transition density implied by the VAR process (3) – (4). We compute them following the variant of Tauchen (1986) method proposed by Terry and Knotek (2010). For each realization of y , q , and current wealth on the grids, we consider a grid of possible consumption values such that next period wealth, computed according to the budget constraint (2), takes values on the same grid as current period wealth. In other words, given present income y_z , cost q_v , wealth W_i , and next period wealth W'_j , we back out the implied consumption value from the budget constraint (2), thus obtaining the consumption grid:

$$c_j(W_i, q_v, y_z) = -\frac{W'_j}{1+r} + W_i + y_z - q_v \quad j, i = 1, \dots, n_W; \quad z = 1, \dots, n_y; \quad v = 1, \dots, n_q.$$

This methodology is without loss of generality provided that the grid for wealth is dense and wide enough.¹¹

We compute the value function $J(W, q, y)$ over the 3-dimensional grid by value function

¹¹We have compared the predictions of this methodology with those obtained with the methodology described below. Both methodologies give the same results and we opted for the first one because it is computationally less intensive. This is the alternative methodology:

- 1) We set a consumption grid c_j , $j = 1, \dots, n_c$.
- 2) We determine a grid for next period wealth according to the budget constraint:

$$W'_{ijzv} = (W_i - c_j + y_z - q_v)(1+r)$$

enforcing the transversality condition by imposing the wealth lower bound $W'_{ijzv} > W_1$.

- 3) We compute the value function next period corresponding to the off-grid wealth levels by linear interpolation (weighted average of the value function at the closest wealth grid values)

$$J(W'_{ijzv}, q'_m, y'_l) = w_{jzv} J(\underline{W}'_{ijzv}, q'_m, y'_l) + (1 - w_{jzv}) J(\overline{W}'_{ijzv}, q'_m, y'_l)$$

iteration, iterating the updating rule

$$J^a(W_i, q_v, y_z) = \max_j \left[\frac{\left(-\frac{W'_j}{1+r} + W_i + y_z - q_v\right)^{1-\gamma}}{1-\gamma} + \alpha + \beta \sum_{l=1}^{n_y} \sum_{m=1}^{n_q} p(q'_m, y'_l | q_v, y_z) J^{a-1}(W'_j, q'_m, y'_l) \right] \quad (16)$$

until the convergence criterion $\max_{i,v,z} \frac{|J^a(W_i, q_v, y_z) - J^{a-1}(W_i, q_v, y_z)|}{J^{a-1}(W_i, q_v, y_z)} < 0.001$ is satisfied. We assign an arbitrary small value to the value function whenever consumption is negative. $p(q'_m, y'_l | q_v, y_z)$ are the one-period transition probabilities of the two-state Markov chain for (q, y) .

The consumption rule of a household with children is

$$c_{j^*}(W_i, q_v, y_z) = -\frac{W'_{j^*}}{1+r} + W_i + y_z - q_v, \quad (17)$$

where j^* is the index achieving the maximum in (16) at the last iteration before convergence.

We compute the value function $H(W, q, y)$ over the 3-dimensional grid by value function iteration of

$$H^a(W_i, q_v, y_z) = \max \left[\max_j \left[\frac{\left(-\frac{W'_j}{1+r} + W_i + y_z\right)^{1-\gamma}}{1-\gamma} + \beta \sum_{l=1}^{n_y} \sum_{m=1}^{n_q} p(q'_m, y'_l | q_v, y_z) J(W'_j, q'_m, y'_l) \right], \right. \\ \left. \max_j \left[\frac{\left(-\frac{W'_j}{1+r} + W_i + y_z\right)^{1-\gamma}}{1-\gamma} + \beta \sum_{l=1}^{n_y} \sum_{m=1}^{n_q} p(q'_m, y'_l | q_v, y_z) H^a(W'_j, q'_m, y'_l) \right] \right], \quad 0 \leq t < T. \quad (18)$$

We use as convergence criterion $\max_{i,v,z} \frac{|H^a(W_i, q_v, y_z) - H^{a-1}(W_i, q_v, y_z)|}{H^{a-1}(W_i, q_v, y_z)} < 0.001$.

The consumption rule for the household without children is

$$c_{\hat{j}}(W_i, q_v, y_z) = \begin{cases} -\frac{W'_{\hat{j}}}{1+r} + W_i + y_z & \text{if } I(W_i, q_v, y_z) = 1 \\ -\frac{W'_{\hat{j}}}{1+r} + W_i + y_z & \text{if } I(W_i, q_v, y_z) = 0 \end{cases} \quad (19)$$

where \hat{j} is the index achieving the maximum in (18) at the last iteration before convergence.

where $\underline{W}'_{ijzv} \equiv \max_j W_j \leq W'_{ijzv}$, $\overline{W}'_{ijzv} \equiv \min_j W_j \geq W'_{ijzv}$ and

$$w^{jzv} = \frac{\overline{W}'_{ijzv} - W'_{ijzv}}{W'_{ijzv} - \underline{W}'_{ijzv}}$$

4) We iterate the updating rule

$$J^a(W_i, q_v, y_z) = \max_j \left[\frac{c_j^{1-\gamma}}{1-\gamma} - \frac{1}{1-\alpha} + \beta \sum_{l=1}^{n_y} \sum_{m=1}^{n_q} p(q'_m, y'_l | q_v, y_z) J^{a-1}(W'_{ijzv}, q'_m, y'_l) \right] \quad (15)$$

The notation $I(W_i, q_v, y_z)$ emphasizes that the fertility decision depends on the current values of wealth, cost and income on the grids. The fertility boundary, expressed as a critical level of cost given income and wealth, is:

$$\bar{q}(W_i, y_z) := \max \left\{ q_v : \max_j \left[\frac{(-\frac{W'_j}{1+r} + W_i + y_z)^{1-\gamma}}{1-\gamma} + \beta \sum_{l=1}^{n_y} \sum_{m=1}^{n_q} p(q'_m, y'_l | q_v, y_z) H(W'_j, q'_m, y'_l) \right] < \right. \\ \left. \max_j \left[\frac{(-\frac{W'_j}{1+r} + W_i + y_z)^{1-\gamma}}{1-\gamma} + \beta \sum_{l=1}^{n_y} \sum_{m=1}^{n_q} p(q'_m, y'_l | q_v, y_z) J(W'_j, q'_m, y'_l) \right] \right\}, \quad 0 \leq t < T. \quad (20)$$

In the pictures reported in the manuscript, average consumption is computed as the unconditional expectation with respect the stationary marginal distribution of childrearing cost:

$$E [c(W, y, q)] = \int_{\mathbb{R}} c(W, y, q) f_{\infty}(q) dq, \quad (21)$$

where $f_{\infty}(q)$ is the Gaussian density with mean $\frac{a_q}{1-b_q}$ and standard deviation $\frac{\sigma_q}{\sqrt{1-b_q^2}}$. For households with child $c(W, y, q)$ is (17), while for households without child $c(W, y, q)$ is (19).