Lending Standards and Countercyclical Capital Requirements under Imperfect Information*

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Abstract

We propose a quantitative model of lending standards with two reasons for inefficient credit: lenders’ moral hazard from deposit insurance or government guarantees, and imperfect information about the persistence of asset price growth, which generates incorrect but rational beliefs in the lenders. We calibrate the model to match recent credit boom-bust episodes. Then we study which patterns of real estate price growth and banks’ beliefs could serve as early warning indicators of a crisis. Finally, we propose a Value-at-Risk (VaR) rule to implement the capital requirements. The VaR framework ensures that the probability of banks not having enough equity to cover their losses is maintained at a certain level. Capital requirements should be state-contingent and lean against lenders’ beliefs by tightening after periods of asset price growth. However, the relationship between asset price growth and financial risk is not monotone and this should be integrated in the setting of the capital requirements and early warning indicators.

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1 Introduction

Lax lending standards are usually blamed for over-exposing banks to risk.\(^1\) In this paper we propose a model of lending standards and two reasons why lending standards may be inefficient. Then we show that the model can replicate empirical patterns of credit booms and busts, use the model to conduct a quantitative study of early-warning risk indicators and analyze a Value-at-Risk (VaR) rule to implement countercyclical capital requirements. We show that capital requirements should be state-contingent and lean against lenders’ beliefs by tightening after periods of asset price growth. However, the relationship between asset price growth and financial risk is not monotone, and this should be integrated in the setting of the capital requirements and use of early-warning indicators. We apply our model to the countercyclical capital buffer (CCB) proposed by Basel III for banks, but our model would also apply to other financial institutions that are now subject to capital requirements, such as mutual funds or broker dealers.\(^2\)

The first reason lending standards are inefficient in the model is a truncation in lenders’ return function such that lenders can take on leverage, but their maximum loss is their initial capital. This may be due to deposit insurance, government guarantees or limited liability. Each of these generates moral hazard on the part of the lenders and lax lending standards at all stages of the cycle relative to those of a regulator who fully absorbs all losses in excess of banks’ equity.

The second friction leading to inefficient credit standards is imperfect information about the persistence of asset price growth. That is, asset prices are random and lenders cannot perfectly anticipate how persistent the prices will be. To make decisions, they form rational expectations using Bayes’ rule. Lenders may have episodes of incorrect but rational beliefs in

\(^1\)For example, Dell’Ariccia et al. (2012), Demyanyk and Van Hemert (2011), Favilukis et al. (2012), Keys et al. (2010 and 2012) and Maddaloni and Peydro (2011) provide evidence of lax standards before the recent crisis. Corsetti et al. (1999) blame them for the Asian crises of the late 1990s.

\(^2\)See Kramer et al. (2013) for a survey of capital rules on mutual funds, and Sacks (2013) for a study of SEC rules for broker-dealers.
which optimistic (pessimistic) expectations lead them to take on too much (little) risk. Surveys of lending standards show that banks’ expectations about economic activity and asset prices are among the main drivers of lending standards (ECB 2013).

The interaction of moral hazard and imperfect information reinforces the need for regulation, as a regulator exposed to covering large depositor or bank creditor losses in the event that bank beliefs about asset price growth are wrong will be more cautious than banks operating under limited liability.

In our model there are lenders and heterogeneous borrowers who borrow to invest in projects whose returns depend on exogenous asset price growth. Lenders select their credit standards to ensure they only give credit to investors with a minimum level of idiosyncratic characteristics. For example, through adequate screening the lender can ensure it lends only to borrowers with a minimum skill level, credit score or past success record. Lending standards change with expectations of asset price growth. When lenders expect growth to be high, all borrowers will be more profitable (bad investments are less bad in an environment of expected price increases) and lenders relax their standards to save on screening costs and maximize their chances of giving credit.

Capital requirements operate via two channels. First, higher requirements reduce leverage, thus limiting the expansion of banks’ balance sheets during the credit boom and ensuring banks can absorb more losses in a downturn. Second, capital requirements affect the incentives behind banks’ lending standard decisions via two opposing mechanisms. On one side, when capital requirements go up, banks’ leverage and profits per unit of capital go down. Thus, there is less incentive for banks to pay the cost of implementing high lending standards. On the other side, because capital is more expensive than debt, when capital requirements go up the cost of

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3We are trying to capture the recent experience of Deutsche Bank in Spain. In 2007, its research department issued a report stating that "Spain’s economic success over the past years has been most impressive... GDP growth is likely to remain above the euro-area average of just below 2% for several more years, allowing Spain to climb past Italy and Germany in the rankings of GDP per capita by 2020" (Deutsche Bank 2007). What happened in the following years suggests that Deutsche Bank had incorrectly overestimated the persistence of Spanish growth and had over lent.
banks’ funds go up and banks need to raise their standards and be pickier to remain profitable. This last mechanism is quantitatively the strongest in our calibrated model.

Our model could be calibrated to any asset price or income growth process. Due to the large role played by housing in the recent crisis, we calibrate it to match long-term averages of housing prices and U.S. banking data. We show that it can replicate patterns of recent credit boom episodes documented by Elekdag and Wu (2011).

We then study which patterns of real estate price growth and banks’ beliefs could serve as early-warning indicators of a crisis. We find a non-linear relationship between real estate price growth, banks’ optimism and the risk of bank losses that are in excess of banks’ equity. There are two opposing forces at work. Higher real estate price growth makes banks optimistic about the persistence of price growth, causing them to lower lending standards and become more exposed to risks. But these risks are also smaller because it is usually the case that rational banks are more optimistic in times when it is less likely a bad shock will happen. Thus, the maximum risks arise during price booms that occur in a middle ground. They are large enough to generate optimistic bank beliefs, but not large enough such that the likelihood of a bad shock is small. This middle ground in our calibrated model means two years of 5% price growth.

Finally, we analyze a VaR rule to implement the CCB. That is, under VaR the regulator adjusts capital requirements to ensure the probability that banks do not have enough equity to cover a given percentage of their losses is fixed at a certain level. The VaR rule implied by the model says that the regulator should increase the CCB when higher real estate prices lead to higher risk. Again, however, this relationship is not monotone. Higher prices lead banks to relax standards, thus building risk. However, if the price growth is very large, in our rational model it is very unlikely that this comes from bad fundamentals, thus a hard landing is less likely because the risk of a bad shock is smaller. Overall, we find that usually the first force dominates and more optimism means more risk for the regulator. Thus, we find that optimal regulation should lean against banks’ beliefs, tightening in periods of optimism after increased
real estate price growth, and relaxing in periods of pessimism after price downturns. Although, we do find the rule should be applied non-linearly.

The structure of the paper is the following. Section 2 reviews the related literature. Section 3 presents the model. Section 4 characterizes the lending standards decision. Section 5 calibrates the model and studies its quantitative properties. Section 6 analyzes which patterns of real estate growth induce larger financial risks. Section 7 contains the VaR implementation of the CCB. Section 8 concludes. The Appendix defines the variables used to calibrate the model and contains the numerical algorithm.

2 Related Work

Our paper is related to several literatures. In terms of objectives, it complements a growing literature that studies the design of countercyclical capital regulation. Recent examples include Aliaga-Díaz and Olivero (2011), Aliaga-Díaz et al. (2011), Angeloni and Faia (2013), Gersbach and Rochet (2012), Malherbe (2013), Martinez-Miera and Suarez (2012), Repullo and Suarez (2013), and Repullo (2013), among others. We believe that we contribute to this literature through the mechanisms in our model, the model’s quantitative implications which allow us to study counterfactuals, and the model’s applications to studying early-warning indicators of risk and designing rules for the state-contingent capital requirements.

First, in terms of the mechanisms of our model, we analyze the use of capital regulation as a macroprudential tool to dissuade banks from taking on excessive risk (lax lending standards) during the build-up phase of the cycle. In this regard, the message of our paper is related to papers that have discussed the connection between capital requirements and bank incentives (for example, Allen et al. 2011, Dell’Ariccia and Marquez 2006, Di Iasio 2013, Holmström and Tirole 1997, Koehn and Santomero 1980, and Mehran and Thakor 2011). Our model does not have the elegant closed form results of these papers, but in exchange it allows us to study
quantitatively the interaction between limited liability and the way rational banks form beliefs in an environment of imperfect information. This process of bank belief formation has not been studied much in the banking literature.

Also in terms of the model, we propose a new way of modeling lending standards. In our model tighter standards means that banks are pickier and raise the threshold that qualifies a borrower for credit. Most of the literature models lending standards as a creditworthiness test, for example Broecker (1990), Gorton and He (2008), Ruckes (2004) or Thakor (1996). That is, tighter standards means that the banks screen more and are more likely to discover the true type of their borrowers. In those models the income threshold to qualify for credit does not change. Instead, the amount of effort to discover the type of the borrower changes. Thus, lax standards in our model captures something different. In a model of lending standards as a creditworthiness test, lax standards means banks qualify more borrowers because they did not screen them enough to discover they were bad. In our model, lax standards means banks give credit to lower quality borrowers even if they know the income of the borrower.\(^4\)

Second, we analyze house price growth as an early-warning indicator of increased banking system risk. Implementing the CCB requires that regulators identify data-based indicators that illustrate when credit growth is excessive. Basel III proposes the deviation from trend in the credit-to-GDP ratio as a primary indicator. However, Edge and Meisenzahl (2011) and Repullo and Saurina (2011) present some drawbacks to reliance on this credit-to-GDP gap.\(^5\) Using data from the EU, recent empirical work by Behn et al. (2013) shows that using equity prices, house prices and banking sector variables in addition to aggregate credit variables improves the predictive power of CCB early-warning models. Smith and Weiher (2012) also argue that

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\(^4\)Our modeling assumption is inspired by the new literature on trade (Melitz 2003, Eaton and Kortum 2004). In new trade models, only the most productive firms decide to export, and in our model, the banks decide who are the most productive borrowers qualified for a loan.

\(^5\)Edge and Meisenzahl (2011) find that real-time estimates of the credit-to-GDP gap differ from final estimates and that these differences can be quite large. The authors argue that regulatory reliance on a real-time credit-to-GDP gap can induce policy action when final estimates of the gap would not, generating an unnecessary drag on the economy. Repullo and Saurina (2011) argue that use of the credit-to-GDP gap may in fact worsen the pro-cyclicality of capital regulation because for many countries the variable is negatively correlated with GDP growth.
a key risk driver is the deviation of the house price index from its trend (a variable that may capture over-optimistic expectations) and propose a methodology to implement countercyclical capital requirements based upon this variable. Their analysis is empirical, while ours employs a calibrated model.\textsuperscript{6}

Third, we propose a VaR rule to set capital requirements. Basel III does not provide much guidance on how to implement changes in the CCB other than some thresholds related to the credit-to-GDP gap. In that regard we contribute to the debate on what rules to follow to apply macroprudential policy tools. Our use of a VaR approach to design the regulatory capital requirements complements models where individual banks use a VaR framework to determine their desired capital levels, such as Di Iasio (2013), Gordy (2003) or Shin (2012). The VaR framework captures the risks for regulators from imperfect information well and shows that the relationship between asset price growth and financial risk is not monotone.

3 Model

In every period $t$ there is a continuum of mass one of borrowers and another continuum of financial institutions. We will refer to the financial institutions as banks, although there is nothing in the model that distinguishes them from private equity funds, mutual funds, broker dealers or some other type of financial institution that lends or invests. Our banks can be thought of as a representative bank because we abstract from strategic interactions between them.

Borrowers borrow from the banks and invest in projects whose return depends on asset price growth. This price growth is exogenous in the model and subject to persistent and non-persistent shocks. Only the sum of these two shocks is observable, and banks solve a signal extraction problem to infer how persistent asset prices will be.

\textsuperscript{6}Some countries, such as Switzerland and Norway, are using real estate price growth as an early-warning indicator although their methodologies are not described in detail (Swiss National Bank 2013, Olsen 2013).
We simplify the model on the borrowers’ side and on the price of credit to focus on the banks’ lending standards decisions. The endogenous variables of our interest are lending standards, the amount of credit, borrowers’ output, non-performing loans, delinquency rates, the quality of banks’ portfolios, and banks’ return on equity.

3.1 Borrowers and Lending Standards

Borrowers are heterogeneous in the parameter $\omega$ and if they qualify for credit they receive the amount $L_t$. A borrower of type $\omega$ who invests $L_t$ dollars in a project receives earnings of

$$y(\omega, \frac{p^h_t}{p^h_{t-1}}, L_t) = \frac{p^h_t}{p^h_{t-1}} \omega^\alpha L_t,$$

(1)

where $\frac{p^h_t}{p^h_{t-1}}$ is aggregate asset price growth. For example, we can think of the borrowers as investors buying $\frac{L_t}{p^h_{t-1}}$ units of real estate at the start of the period, then they sell those units at the end of the period receiving as proceeds those units times the current real estate price $\left(\frac{p^h_t}{p^h_{t-1}}\right)$. The term $\omega^\alpha$ captures the idiosyncratic characteristics of the project or investor. This implies that for the same level of price growth $\left(\frac{p^h_t}{p^h_{t-1}}\right)$ and investment $(L_t)$, some investors are more profitable than others.

We assume that $\omega$ is distributed following a Pareto distribution with support $[M, \infty)$ and distribution function $G(\omega) = 1 - (\frac{M}{\omega})^\mu$, where $\mu > 0$ is the shape parameter. As $\mu$ increases, the dispersion of $\omega$ decreases and is increasingly concentrated towards the lower bound $M$. The Pareto assumption fits quite well firm-level data about the size and productivity distribution of firms (Ghironi and Melitz 2005) as well as households’ wealth distribution.

We assume that each bank randomly meets one borrower, observes the borrower’s idiosyncratic type $\omega$, and chooses lending standards to weed out the bad ones. To avoid modeling competition between banks, we assume that borrowers cannot shop around at different banks. We denote by $\pi_t \in [0, \infty)$ the bank’s lending standards. A bank with lending standards $\pi_t$
denies credit to any borrower with $\omega < M + \pi_t$, where $M$ is the lower bound of the Pareto distribution of $\omega$. Figure 1 plots the distribution of $\omega$ and the lending standards cutoff. Only borrowers to the right of $M + \pi_t$ receive credit. As $\pi_t$ increases, lending standards are higher and the bank is more selective when lending because the bank has increased the minimum cutoff to give credit.

Higher lending standards also imply that it is less likely the bank will meet with a borrower worthy of receiving credit, because the probability $\Pr(\omega > M + \pi_t)$ is decreasing in $\pi_t$ (holding everything else constant). Moreover, we assume that implementing higher lending standards is more costly than having lax standards (for example, due to increases in loan officers’ time and the costs of analyzing the borrower and her project). We assume that the cost of implementing lending standards $\pi_t$ when lending $L_t$ is $C(\pi_t) L_t$. In the calibrated model we work with the function

$$C(\pi_t) = \xi \pi_t$$ (2)

where $\xi$ is a parameter.

We make some assumptions to avoid equilibria in which bad borrowers who know they would not qualify for credit (they have $\omega < M + \pi_t$) do not apply for credit, causing banks to not need to spend resources on implementing standards. Ruckes (2004) discusses different assumptions that give the same result. For example, assuming borrowers do not know their types is equivalent to assuming good borrowers cannot signal their types and every borrower applies for credit because she obtains a non-verifiable control rent. Either of those assumptions, together with assuming that borrowers do not have any initial capital and cannot save, give us that borrowers always apply for the maximum credit.

To focus on the quantity of credit instead of on the price of credit, we assume that a fraction $\kappa \geq 0$ of the project’s output, $y(\omega, \frac{y_t}{p_{t-1}}, L_t)$, goes to the borrower and the remaining
fraction, \((1 - \kappa)\), goes to the lender. The parameter \(\kappa\) controls how the surplus from the lending relationship is split between the lender and the borrower. It is the equivalent of the Nash Bargaining parameter common in search models and we calibrate it to match data on interest rates. We are assuming state contingent payoffs to the lender. This can be justified if we think of the lenders as banks using debt contracts including many covenants that make the contract state contingent. We could also think of them as large banks or non-bank financial intermediaries investing through ways other than standard debt contracts. Boot and Thakor (2010) document large increases in financial intermediation through equity instruments. Several recent models of banks, such as Bocola (2013), Gertler and Kiyotaki (2010) or Dedola et al. (2013), also use equity contracts for simplicity. This assumption does not alter our results. The two frictions that we study would also lead to inefficient lending standards if payoffs to the lender were the same as those in a standard debt contract, that is, only state contingent in case of borrower's default.\(^7\)

### 3.2 Imperfect Information

Asset price growth \(\left( \frac{p_t^h}{p_{t-1}^h} \right)\) is exogenous and stochastic. It is unknown at the time of decision-making in period \(t\), but is observed at the end of the period. To capture imperfect information, we assume that \(\left( \frac{p_t^h}{p_{t-1}^h} \right)\) is the sum of two unobservable parts, both with permanent effects, but one part is persistent while the other is not:

\[
\frac{p_t^h}{p_{t-1}^h} = \exp (z_t + \eta_t) \tag{3}
\]

\(^7\)This result is available in an Appendix upon request.
where \( z_t \) is the persistent part that follows a two state Markov chain. That is, prices can have high or low growth \( z_t = \{ z^L, z^H \} \), with \( z^L < z^H \), and transition matrix

\[
P = \begin{bmatrix}
P_{LL} & P_{LH} \\
P_{HL} & P_{HH}
\end{bmatrix}
\]

The non-persistent part \( \eta_t \) is an i.i.d. Normal shock with mean \( -\frac{\sigma_\eta^2}{2} \) and variance \( \sigma_\eta^2 \). This assumption for the mean of \( \eta_t \) ensures that, conditional on \( z_t \), \( \frac{\nu^h_t}{p^h_{t-1}} \) follows a lognormal distribution whose conditional mean is \( \exp(z_t) \). We will refer to the \( \eta_t \) shock as a noise shock because it prevents banks from perfectly observing \( z_t \) and it is a shock to which agents should not react because it is i.i.d.

Banks must make period \( t \) decisions before price growth is known, so they form expectations about it from their past observations (we denote by \( \Theta_{t-1} \) the information set known at the start of the \( t \) period). They do so by forecasting the unobservable state of the persistent part, \( z_t \), from past observations of \( \frac{\nu^h_t}{p^h_{t-1}} \) using a Bayesian filter. We denote by \( p_{t-1} \equiv \Pr(z_t = z^H|\Theta_{t-1}) \) the belief or prior of \( z_t \) being in the high state in period \( t \).

Banks start period \( t \) with a prior \( p_{t-1} \) and base their period \( t \) decisions on this prior. Once \( \frac{\nu^h_t}{p^h_{t-1}} \) is observed at the end of period \( t \), agents compute their posterior beliefs about the state of the persistent component, \( \Pr(z_t = z^i|\Theta_t) \), using the Bayesian filter

\[
\Pr(z_t = z^i|\Theta_t) = \frac{f\left(\frac{\nu^h_t}{p^h_{t-1}} | z_t = z^i\right) \Pr(z_t = z^i|\Theta_{t-1})}{f\left(\frac{\nu^h_t}{p^h_{t-1}} | z_t = z^j\right) \Pr(z_t = z^j|\Theta_{t-1}) + f\left(\frac{\nu^h_t}{p^h_{t-1}} | z_t = z^i\right) \Pr(z_t = z^i|\Theta_{t-1})}, \quad i = H, L
\]

where the conditional density \( f\left(\frac{\nu^h_t}{p^h_{t-1}} | z_t = z^i\right) \) is the normal probability density

\[
f\left(\frac{\nu^h_t}{p^h_{t-1}} | z_t = z^i\right) = \frac{1}{\sigma_\eta \sqrt{2\pi}} \exp\left(-\frac{1}{2\sigma_\eta^2} \left( \frac{\nu^h_t}{p^h_{t-1}} - z^i + \frac{\sigma_\eta^2}{2} \right)^2 \right)
\]

(4)
Banks form next period’s prior $p_t$ by updating the posterior with the transition matrix $P$

$$p_t = \Pr(z_{t+1} = z' | \Theta_t) = \Pr(z_t = z' | \Theta_t)P_{ii} + \Pr(z_t = z' | \Theta_t)P_{ji}$$  \hspace{1cm} (6)

This is the prior used to make decisions in period $t + 1$.

We will use the notation $E_{t-1}(.)$ to denote the expectation over $\frac{p^h_t}{p^h_{t-1}}$ conditional on the information known at the start of the period. That is, the expectation of the proceeds from the project conditional on information at the start of period $t$ is:

$$E_{t-1} \left[ y(\omega, \frac{p^h_t}{p^h_{t-1}}, L_t) \right] = E_{t-1} \left[ \frac{p^h_t}{p^h_{t-1}} \right] \omega^\alpha L_t = \right.$$

$$= \left[ p_{t-1} \left( \exp \left( z^H \right) \right) + (1 - p_{t-1}) \left( \exp \left( z^L \right) \right) \right] \omega^\alpha L_t$$  \hspace{1cm} (7)

### 3.3 Banks

In every period $t$ there is a continuum of mass one of risk neutral banks. Banks can fund their loans with their own equity, $K_t$, whose gross cost we assume to be $R_E^t$, or with deposits or borrowings, $B_t$, that cost $R_B^t$. Banks are subject to a capital requirement, $\gamma \geq 0$, such that

$$L_t \leq B_t + K_t$$  \hspace{1cm} (8)

$$K_t \geq \gamma L_t$$  \hspace{1cm} (9)

We assume that

$$R_E^t \geq R_B^t.$$  

That is, the cost of equity is larger than the cost of debt, for example because equity holders face the risk of losing their investment while the debtholders are deposit insured or the government provides a guarantee. This assumption is important because as the capital requirement increases, banks’ cost of funds increases as well, since banks are financing a larger share of their
loans with equity.\textsuperscript{8}

Each bank lives for one period, meets one borrower and chooses its lending standards \( \pi_t \). If the borrower does not satisfy the lending standards \( (\omega < M + \pi_t) \), then the bank does not lend and sits on its capital. If the borrower satisfies the standards \( (\omega \geq M + \pi_t) \) then the bank lends the amount \( L_t \). At the end of the period \( \frac{p^h_t}{p^t_{t-1}} \) is realized, the return from the project is observed, split between the bank and its borrower, and the borrower and the bank separate. With the proceeds received, the bank pays its debtholders and the cost of implementing lending standards. Any remaining proceeds then go to shareholders. Limited liability implies the net profits can never be negative, as shareholders are not asked to inject more capital to cover losses. The payoff for the bank from lending is

\[
\left( (1 - \kappa) \gamma(\omega, \frac{p^h_t}{p^t_{t-1}}, L_t) - R_t B_t - C(\pi_t) L_t \right)^+
\]

where the notation \((x)^+\) stands for the maximum operator, \( \max(x, 0) \).

For a given \( K_t \) and \( p_{t-1} \) the bank chooses lending standards to maximize expected shareholders’ value at the end of the period. Since lending is risky, we assume that profits from the risky activity are discounted using the cost of equity while those from not taking risk are discounted at the deposit rate. The banks take expectations over both \( \omega \) (because the bank does not know which type of borrower it will meet) and \( \frac{p^h_t}{p^t_{t-1}} \). Banks start period \( t \) with a prior, \( p_{t-1} \), inherited from the posterior of the previous cohort of bankers according to equation (6).\textsuperscript{8} As discussed by Admati et al. (2013), it may be that higher capital requirements lower the cost of equity. Our set up is a partial equilibrium model and cannot capture that effect. This should not invalidate our analysis if the cost of bank equity remains higher than the cost of raising debt or deposits for banks, which would probably be the case if the deposits are insured while equity is more risky.
The bank solves:

$$\max_{\{\pi_t, L_t\}} \int_M^{M+\pi_t} \frac{1}{R_t^B} K_t dG(\omega) + E_{t-1} \left( \int_M^{\infty} \frac{1}{R_t^E} \left( (1 - \kappa) y(\omega, \frac{p^h_t}{p^h_{t-1}}, L_t) - R_t^B B_t - C(\pi_t) L_t \right)^+ \right) dG(\omega)$$

(10)

s.t. (8) and (9)

where \([M, M + \pi_t]\) is the region where the banks are not lending \((L_t = 0)\).\(^9\)

Given banks’ linear utility, if the borrower is considered worthy of receiving credit, the bank will always try to give her the maximum credit possible. Thus, equations (8) and (9) would hold with equality. However, given that not all banks are giving credit, even for linear utility banks, the model can match the empirical fact that the banking system holds capital above the regulatory minimum, a fact discussed by Allen et al. (2011) among others.

We denote by \(P(\omega, \frac{p^h_t}{p^h_{t-1}}, K_t, \pi_t)\) the net profits at the end of the period for a bank with capital \(K_t\) and lending standards \(\pi_t\), matched with a borrower of type \(\omega\) when asset price growth is \(\frac{p^h_t}{p^h_{t-1}}\)

$$P(\omega, \frac{p^h_t}{p^h_{t-1}}, K_t, \pi_t) = \begin{cases} (1 - \kappa) y(\omega, \frac{p^h_t}{p^h_{t-1}}, L_t) - R_t^B B_t - C(\pi_t) L_t \overline{\text{if } \omega \geq M + \pi_t}\, \\
K_t \overline{\text{if } \omega < M + \pi_t} \end{cases}$$

(11)

We assume that at the end of each period, banks’ net cash flows are aggregated to form next period’s aggregate capital \(K_{t+1}\), which will be evenly split among the next cohort of banks.\(^10\)

$$K_{t+1} = \int_M^{\infty} P(\omega, \frac{p^h_t}{p^h_{t-1}}, K_t, \pi_t) dG(\omega)$$

(12)

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\(^9\)To ensure the objective function is finite, we require that \(\alpha < \mu\).

\(^10\)This assumption is without loss of generality since it is the leverage ratio \(\left(\frac{K_t}{L_t}\right)\) and not the level of capital which affects the choice of lending standards. That is, in our model small banks and large banks behave the same way if both have the same leverage ratio.
4 The Lending Standards Decision

The bank chooses its lending standards, $\pi_t$, by solving equation (10), where the notation $(x)^+$ stands for the maximum operator, $\max(x, 0)$. The bank’s problem can thus be broken into two separate cases, one in which the bank lends to borrower types that are expected to default ($\max(x, 0) = 0$) and one in which the bank does not lend to borrower types that are expected to default ($\max(x, 0) = x$). We define the objective function for the bank when there is expected default as:

$$W(\pi_t) = \int_M^{M+\pi_t} 1 \frac{1}{R^B_t} K_t dG(\omega) + E_{t-1} \left( \int_{\pi_t}^{\infty} \frac{1}{R^E_t} (1-\kappa)\gamma(\omega, \frac{p^h_t}{p_{t-1}}, L_t) - R^B_t B_t - C(\pi_t) L_t \right) dG(\omega)$$

where $\hat{\omega}(\pi_t) \geq M + \pi_t$ is the threshold borrower expected to generate a lending payoff for the bank of zero. That is, $\hat{\omega}(\pi_t)$ satisfies:

$$(1-\kappa)E_{t-1} \left[ \frac{\hat{\omega}(\pi_t)}{p^h_t} \right]^\alpha L_t - R^B_t B_t - C(\pi_t) L_t = 0$$

Likewise, we define the objective function for the bank when there is no expected default as:

$$V(\pi_t) = \int_M^{M+\pi_t} 1 \frac{1}{R^B_t} K_t dG(\omega) + E_{t-1} \left( \int_{\pi_t}^{\infty} \frac{1}{R^E_t} (1-\kappa)\gamma(\omega, \frac{p^h_t}{p_{t-1}}, L_t) - R^B_t B_t - C(\pi_t) L_t \right) dG(\omega).$$

where, implicitly, $M + \pi_t > \hat{\omega}(\pi_t)$. The optimal choice of lending standards in each of these
cases can be defined as follows:

\[ \pi_t^* = \arg \max_{\pi_t} W(\pi_t) \tag{16} \]

and

\[ \pi_t^{**} = \arg \max_{\pi_t} V(\pi_t). \tag{17} \]

When there is lending with expected default, the following first-order condition implicitly defines \( \pi_t^* \) as long as \( \triangledown (\pi_t^*) \geq (M + \pi_t^*) \):

\[ \frac{1}{R^E_t} \left( \frac{1}{\gamma} \right) C''(\pi_t^*) M^\mu (\triangledown (\pi_t^*))^{-\mu} = \frac{1}{R^B_t} \mu M^\mu (M + \pi_t^*)^{-\mu - 1}. \tag{18} \]

This condition equates the discounted marginal revenue from lending to types above \( \triangledown (\pi_t^*) \) (the left-hand side of equation 18) with the discounted marginal opportunity cost from lending to the cutoff type, \( M + \pi_t \) (the right-hand side of equation 18). There are two important relationships that can be observed in this optimality condition. First, because the marginal benefit of lending falls as the capital requirement rises, a higher capital requirement causes banks to want to lend less. Hence they raise their lending standards to lend to fewer borrowers. Second, as banks’ beliefs about real estate price growth deteriorate, banks will want to seek out higher quality borrowers and will raise their lending standards in response.

For the case of lending without expected default, the first-order condition that implicitly defines \( \pi_t^{**} \) is:

\[ \frac{1}{R^E_t} \left( \frac{1}{\gamma} \right) \left[ (1 - \kappa) E_{t-1} \left( \frac{\phi^t}{\phi^t_{t-1}} \right) (M + \pi_t^{**})^\alpha - R^B_t (1 - \gamma) - C(\pi_t^{**})^{\mu} + \right] \left( \frac{1}{\mu} \right) (M + \pi_t^{**}) C'(\pi_t^{**}) = \frac{1}{R^B_t}. \tag{19} \]

as long as \( (M + \pi_t^{**}) > \triangledown (\pi_t^{**}) \). In this case, the first-order condition equates the discounted marginal revenue from lending to the cutoff type \( M + \pi_t^{**} \) (the left-hand side of equation 19) with the discounted opportunity cost from lending to the cutoff type (the right-hand side of...
equation 19). Again, we see that the marginal benefit of lending falls as the capital requirement rises, implying banks will want to raise their standards to lend to fewer borrowers. Also, more pessimistic beliefs about real estate price growth will again lead to banks to raise their lending standards.

Banks choose the level of lending standards that maximizes value over these two cases. That is, they will choose $\pi_t$ such that:

$$
\pi_t = \arg \max_{\{\pi_t^*, \pi_t^{**}\}} \{W(\pi_t^*), V(\pi_t^{**})\}. 
$$

Lending with expected default ($\pi_t = \pi_t^*$) is costly as banks lose equity on the defaulted borrowers. Banks will only choose to do so if the marginal costs of implementing standards such that $\pi_t = \pi_t^{**}$ are too high.

## 5 Quantitative Properties

In this section first we calibrate the model to match long-term averages of real estate prices and financial data series. Then, we use recent data on credit booms and busts to illustrate how imperfect information can generate rational credit booms and busts in the model.

### 5.1 Calibration

We calibrate one period in the model to be one year and set the exogenous cost of debt, $R_t^B$, to 2% which is the standard risk free rate in most macroeconomic calibrations. We set the cost of bank equity, $R_t^E$, to 7% following Damodaran (2012). We assume a capital requirement $\gamma$ of 4%, which was the Tier 1 capital requirement under Basel I.

We follow Ceron and Suarez (2006) to parameterize the stochastic process of equation (3). They use Hamilton (1989) methodology to estimate a two-state Markov switching process
for housing prices using inflation-adjusted residential property price index data from fourteen developed countries. We use their estimates for $P_{LL}$, $P_{HH}$ and for the ratio between the good and the bad persistent state, $\frac{1+z_H}{1+z_L}$. This ratio pins down $z_L$ once we set $z_H$, which becomes our scale parameter for the real estate price shock and is selected as explained below.

We select the remaining seven parameters (borrowers’ fraction of output $\kappa$, borrowers’ technology $\alpha$, screening cost function $\xi$, the real estate price shock in the high state $z_H$, the volatility of the i.i.d shock $\sigma_\eta$, and the parameters of borrowers’ distribution $M$ and $\mu$) for the model to jointly match the following facts for 1985-2006: 11 1) Average fraction of reserves held in U.S. banks’ asset portfolios (6.1%) (series: CASACBM027SBOG and TLAACBM027SBOG from the FRED database); 2) Average real return on equity (13.1%) for U.S. banks (series: USROE from the FRED database); 3) Average real return on assets (1.1%) for U.S. banks (series: USROA from the FRED database); 4) Delinquency rate to match the historical default rates computed by Moody’s (8.1%) for B/B borrowers, which is the rating for the majority of defaulting borrowers one year before default (Moody’s 2002); 5) Average credit-to-GDP ratio (81.1%) for Ireland, Spain, the U.K. and the U.S. from Beck et al. (2009), which we proxy by the ratio of borrower credit to borrower output in the model; 6 and 7) A 5% likelihood of noise shocks implying losses larger than 82% of bank capital. This last fact is based on the estimation by the Basel Committee on Banking Supervision (2010b) that there is a 5% probability of a member country facing a crisis in a given year, and that the last two U.S. financial crises (the Savings and Loan Crisis of 1988 and the recent crisis) cost regulators on average 82% of banks’ capital as we discuss in Section 7. Table 1 contains the parameters that we obtain, Table 2 reports how well the model matches the targets.

Our calibration implies a real estate price growth rate of 5% if the persistent component of price growth is high and a -2% growth rate if it is low. Likewise, the invariant distribution of our

---

11 The Appendix defines the model counterparts to the facts. It also discusses the numerical algorithm.
calibrated Markov transition matrix $P$ implies that in the long-run, the persistent component of price growth is high 40% of the time low 60% of the time.

Figure 2 illustrates how banks update their beliefs given our calibration. It plots how, for a given prior, observations of price growth lead to new posteriors.\textsuperscript{12} Three facts are interesting. First, posterior beliefs are increasing in the asset price growth rate. That is, when banks see a higher price growth rate, they attribute part of it to a higher likelihood that the persistent state of price growth is $z^H$. In other words, they always attribute part of the growth to fundamentals. Second, a larger number of low posterior beliefs arise when the prior itself is also low. Similarly, we observe a larger number of high posterior beliefs when the prior itself is also high. That is, given our calibration, banks need to see large changes in price growth in order to drastically update their beliefs. Third, it is possible, given different priors, that different observations of house price growth can lead to the same posterior belief.

Insert Figure 2 about here

Figure 3 shows why beliefs matter. It plots banks’ lending standards and total credit for different levels of the prior $p_{t-1}$. Optimism leads to more credit.

Insert Figure 3 about here

\section*{5.2 Rational Credit Booms and Busts}

A model with imperfect information can easily generate boom-bust patterns. In this section we show so. Moreover, we test the ability of the calibrated model to match data. Elekdag and Wu (2011) study data from 1960-2010 and identify 99 credit booms across 21 advanced and

\textsuperscript{12}It presents a scatter plot of the transition of beliefs for a time series of 10,000 periods. The time series starts with $z_t$ at its long-run mean and with banks’ prior $p_t$ consistent with that mean.
43 emerging economies.\textsuperscript{13} Figure 4A shows the typical evolution of these booms over five-year windows centered at their peaks. The Figure distinguishes between different kinds of booms (worst booms, abrupt booms and smooth booms) depending on how severe the following crises were.

We ask what would happen in the model if we input a pattern of house price growth \( \left\{ \frac{p^h_t}{p^h_{t-1}} \right\} \) driven by noise shocks \( \eta_t \) that allows the model to generate a credit boom matching the data reported in Figure 4A. When \( \left\{ \frac{p^h_t}{p^h_{t-1}} \right\} \) increases, then banks’ beliefs rise (Figure 4C) and lending standards fall as shown in Figure 4D. This leads to the credit increases observed in Figure 4A. Return on assets in Figure 4E is initially high because banks are lending more and the noise shocks \( \eta_t \) are positive. However, when the noise shocks disappear, the fraction of non-performing loans rises (Figure 4F), bank profits fall (Figure 4E), and banks suddenly readjust expectations about the persistent part of the aggregate component of income (Figure 4C). The combination of banks tightening their lending standards and bank losses lowering banks’ available capital leads to a severe contraction in credit, generating a bust.

Interestingly, the model predictions seem to be in line with the data. This motivates us to use the model for the policy applications of the next sections.

6 Early Warning Indicators

Implementation of the Basel III countercyclical capital buffer requires national regulators to identify data-based early-warning indicators of excessive credit growth. In this section, we use our model framework to investigate which patterns of banks’ beliefs and real estate price

\textsuperscript{13}A credit boom is defined as an episode when the cyclical component of real credit is larger than 1.55 times its standard deviation.
growth may induce damaging credit booms. We measure risk by the size of losses in excess of bank capital and by the likelihood of such losses. Interestingly, we find that with rational agents the more dangerous patterns are not those of maximum optimism, even if standards are monotonically decreasing in optimism.

For a given realization of price growth, \( p_{h_{t-1}} \), and banks’ lending standards, \( \pi_t \), we can define with the function \( \bar{\omega} \left( \frac{p_h}{p_{h_{t-1}}}, \pi_t \right) \) the borrower type receiving credit such that banks make losses in excess of their capital for all financed borrowers whose idiosyncratic component of income \( \omega \) was lower than \( \bar{\omega} \left( \frac{p_h}{p_{h_{t-1}}}, \pi_t \right) \), that is,

\[
(1 - \kappa) y(\bar{\omega}, \frac{p_h}{p_{h_{t-1}}}, L_t) - R_t^B B_t - C(\pi_t) L_t - K_t = 0
\]

or

\[
\bar{\omega} \left( \frac{p_h}{p_{h_{t-1}}}, \pi_t \right) = \left[ \frac{\gamma + R_t^B (1 - \gamma) + C(\pi_t)}{(1 - \kappa) \left( \frac{p_h}{p_{h_{t-1}}} \right)} \right]^{1/\alpha}.
\] (21)

The losses of the banking system in excess of bank capital are the sum of the losses on all financed borrowers (\( \omega > M + \pi_t \)) whose type is below \( \bar{\omega} \left( \frac{p_h}{p_{h_{t-1}}}, \pi_t \right) \). We define those losses as

\[
\Omega_t = - \int_{M + \pi_t} \left[ (1 - \kappa) y(\omega, \frac{p_h}{p_{h_{t-1}}}, L_t) - R_t^B B_t - C(\pi_t) L_t - K_t \right] dG(\omega)
\] (22)

where we multiply by a negative sign to have a positive value for the losses.

The size of bank losses depends on both how bad the price growth shock is, \( \frac{p_h}{p_{h_{t-1}}} \), and on banks’ lending standards, \( \pi_t \). From Figure 3 we know that lending standards are a decreasing function of beliefs. In Figure 5 we plot the probability of observing different losses (\( \Omega_t \)) in excess of bank capital for different prior beliefs. Specifically, we set \( \Omega_t \) to 50%, 65%, 75%, or 100% of banks’ beginning-of-period capital, compute \( s^* \) which is the size of the aggregate shock.
bad enough to generate such losses, and then plot the probability of observing a shock worse than \( s^* \).

Insert Figure 5 here

In each of the four panels of Figure 5, we see that this probability is mostly increasing in the prior, signifying that the likelihood of observing a crisis rises with bank optimism, as lending standards are decreasing in bank optimism (Figure 3) and there is more lending. However, the probabilities are non-monotonic in \( p_t \) because for very high \( p_t \), even if the banks have very low lending standards, it is very unlikely to see a shock bad enough to generate 50%, 65%, 75%, or 100% bank losses.

Thus, the non-monotonicity illustrates two forces affecting the regulator’s potential losses: as \( p_t \) increases banks are more exposed to risks because their standards are lower, but these risks are also smaller because rational banks have larger \( p_t \) when it is less likely that a bad shock happens. Over most of the \( p_t \) range, more optimism means more risk for the regulator. In other words, the more dangerous times are times of optimism where there are doubts about the strength of the fundamentals.

Now that we have examined how the risk of regulator losses changes with the prior, we turn to how the risk responds to different sequences of real estate price growth. In Figure 6, we plot how the size of regulator losses changes for three scenarios of real estate price growth: a sequence of two periods of 2%, 5% and 8% growth respectively. We specifically examine the size of regulator losses that occur with 2% probability. In all three scenarios, the starting house price growth rate is at the mean of the invariant distribution of the stochastic process in equation (3) and banks have the prior \( p_t \) consistent with that mean.

Insert Figure 6 here

In Panel A, we find that real estate price growth around 2% generates an increasing risk of
regulator losses. This level of real estate price growth causes banks to slowly update their beliefs and take on more risk. Panels B and C, however, show that the pattern of risk is non-monotonic. The size of potential losses is highest once the first real estate price growth shock is observed. At faster rates of real estate price growth, banks update their beliefs relatively quickly. Hence, more risk arises in the banking system after the initial shock. However, upon observing the second shock, it is increasingly likely that the housing market is actually in the high growth state and less likely it will see a housing price growth shock bad enough to generate large losses, so risk falls in the second period. That is, as in the previous figure, there is a trade-off between the risk generated by laxer lending standards associated with higher growth, and the fact that if the growth is very high, then it is very unlikely not to come from good fundamentals. Weighting these two channels gives as a result that the sequence of 5% real estate price growth (Panel B) induces the most risk.

Figure 7 redoes Figure 6 but focuses on the probability of a crisis (defined as regulator losses of 100% or more of bank capital) for the same three scenarios of real estate price growth. We observe patterns of regulator risk similar to those discussed in Figure 6. In Panel A, the probability of a crisis is increasing for real estate price growth around 2%, due to a gradual updating of bank beliefs. Panels B and C again display a non-monotone response to real estate price growth rates of 5% and 8% respectively. The risk of crisis is again at a maximum for the sequence of 5% real estate price growth (Panel B).

Insert Figure 7 here
7 A Value-at-Risk Macroprudential Regulator

In this Section we analyze a Value at Risk (VaR) framework to set capital requirements and leverage ratios. VaR is a tool commonly used to assess the risk of a portfolio. It measures the minimum potential loss of a portfolio for a given confidence interval. For example, a VaR of $100 with a 98% confidence level means that there is a 2% chance the portfolio will lose more than $100 over a specified period.

We propose that the regulator sets the capital requirement, $\gamma$, using a VaR criteria such that losses ($\Omega_t$) in excess of $x\%$ or more of banking system capital occur with confidence level $1 - \rho$. That is, the regulator chooses the capital requirement such that

$$\Pr \left( \Omega_t > \left( \frac{x}{100} K_t \right) | p_t \right) = \rho. \quad (23)$$

Losses ($\Omega_t$) are defined as in equation (22). The fact that banks are leveraged institutions imply that losses could exceed all banking capital.

The conditional expectation in (23) reflects our assumption that the regulator computes the probabilities over $\frac{p_t}{p_{t-1}}$ conditional on the same prior, $p_t$, that the banks are using to choose their lending standards. In other words, we do not assume that the regulator has different information about the state of the economy than private banks have. It would be easy to incorporate the case in which the regulator has different information. In that case the capital requirement would also change to encourage banks to behave according to the regulator’s priors.

---

14 The Basel Committee seems to reason with a VaR framework. The Committee discussed that the goal of regulation is to reduce from 5% to 1%, 2% or 3% the probability that a country faces a crisis in any given year (Walter 2011).

15 We can motivate this assumption using the fact that regulators, via deposit insurance or government guarantees, often cover losses in excess of bank equity. Similarly, we could motivate it in the case of a regulator that does not like banks defaulting on their creditors.

16 Laeven and Valencia (2012) estimate that the fiscal cost of the U.S. Savings and Loan crisis in 1988 was 3.7% of GDP and the cost of the recent U.S. crisis was around 4.5% of GDP. These costs include bank recapitalizations and other outlays related to restructuring the financial sector, but do not include asset purchases. Using that the ratio of equity to GDP in 1988 and 2008 was 3.5% and 7.7% respectively, we can convert the data on losses to a percentage of bank equity. We obtain that the fiscal costs of those crises were between 50% and 100% of bank capital. This is the range we will use for $x\%$. 

24
Table 3 compares how different capital requirements would affect regulator losses for VaR confidence levels of 98%, 95% and 90%.\textsuperscript{17} The first row of the table reports the regulator losses for each VaR confidence level for our benchmark calibration of a 4% capital requirement.

Insert Table 3 here

As the capital requirement rises, the size of potential regulator losses falls for each VaR confidence level. There are two channels through which the capital requirement operates. First, it operates on a quantity dimension. Raising the capital requirement lowers the amount of leverage banks can use to finance a loan. In the event of a bad real estate price shock, this means that capital is thus able to absorb more of the losses. Second, the capital requirement operates on a price dimension. Because we assumed that for banks raising capital is a more expensive form of finance than borrowing ($R_t^E \geq R_t^B$), when higher capital requirements force banks to finance a larger set of their lending with equity then banks need to raise their lending standards to lend to borrowers who are more profitable. That is, when banks’ costs increase, banks need to be more selective in terms of to which investors to lend.

Figure 8 plots the optimal capital requirement as a function of bank beliefs for a target losses of 75% and 100% to ensure that the probability of observing a crisis is fixed at 2%\textsuperscript{18}.

Insert Figure 8 here

Figure 8 shows that capital requirements should lean against bank beliefs. In general, the VaR regulator should raise capital requirements when the banks are more optimistic and therefore more exposed to bad shocks. When the banks are more pessimistic and less exposed, they need lower capital requirements. However, rational banks usually have optimistic beliefs when it

\textsuperscript{17}To generate the table, banks’ prior is set to 0.75, and regulator losses are computed at the level of house price growth such that an equivalent or worse loss occurs 2%, 5% or 10% of the time.

\textsuperscript{18}When discussing Basel III, the Basel Committee claims that there is a 5% probability of a Basel Committee member country facing a crisis in any given year and discusses reducing this probability by 1%, 2% or 3% (Basel Committee on Banking Supervision 2010a).
is less likely that a bad shock will happen. This force generates a non-monotone response of capital requirements.

Figure 8 also shows the effect of the regulator’s loss tolerance. When the tolerance switches from 75% to 100% of bank capital, the regulator is allowing larger losses to happen within its 98% confidence interval. Because these 100% losses happen less frequently in general, the capital requirements needed to combat them are lower than those required to combat losses of 75%.

To illustrate the link between house price growth and VaR policy based upon bank beliefs, we plot in Figure 9 the VaR capital requirement corresponding to the house price growth scenarios we presented in Section 6: a sequence of two periods of 2%, 5% and 8% growth respectively.

In each panel of Figure 9, the VaR capital requirement very closely follows the pattern of risk observed in the corresponding panels of Figures 6 and 7. In Panel A, the gradual increase in bank risk in response to 2% house price growth is met with a gradual increase in the capital requirement. In Panels B and C, the non-monotone response of risk means that the VaR capital requirement is non-monotone as well. As before, this non-monotonicity arises because optimistic banks lower their lending standards and become more exposed to bad shocks, but the risk of those bad shocks is smaller since rational bankers only become optimistic when it is less likely that a bad shock occurs. We also observe in Figure 9 that a house price growth rate of 5% (Panel B) requires a higher VaR capital requirement than a house price growth rate of 8% (Panel C). This occurs because risk is higher in the case of a 5% house price growth rate.
8 Conclusions

This paper proposed a quantitative model of lending standards with two frictions generating inefficient credit: 1) lenders’ moral hazard from limited liability (that can also be interpreted as deposit insurance or government guarantees), and 2) imperfect information about the persistence of real estate price growth, which generates the possibility of rational mistakes. We studied which patterns of real estate price growth and banks’ beliefs could serve as early warning indicators of a crisis. With rational agents, even if lending standards are monotonically decreasing in optimism, the more dangerous booms are not monotonically increasing in optimism. When the banks are more optimistic they are more exposed to bad shocks. However rational banks usually have optimistic beliefs when it is less likely that a bad shock happens.

Finally, we proposed a Value at Risk (VaR) rule to implement countercyclical capital requirements. Capital requirements can incentivize banks to pick the socially optimal level of lending standards. Capital requirements should be state-contingent and lean against lenders’ beliefs by tightening in periods of price optimism. However, they should be not monotone, as risk is not monotone in beliefs.

Future extensions of this paper may include bringing the model into a general equilibrium setting, analyzing the welfare implications of the VaR rule, and studying cases in which banks’ beliefs could be irrational and diverge from those of a rational regulator.
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Appendix

A. Model Definitions

We define the delinquency rate in the model as the fraction of borrowers receiving credit
who cannot repay the principal of the loan

\[
\hat{\omega} = \frac{\int_{M+\pi}^{\infty} g(\omega) d\omega}{\int_{M+\pi}^{\infty} g(\omega) d\omega},
\]

where \(\hat{\omega}\) is defined as the borrower type who is just able to repay the loan principal

\[
(1 - \kappa)g(\hat{\omega}, \frac{p^h_t}{p_{t-1}^h}, L_t) - L_t = 0.
\]

We define return on equity and return on assets as

\[
ROE_t = \frac{K_{t+1} - K_t}{K_t},
\]

\[
ROA = \frac{K_{t+1} - K_t}{\text{assets}} = ROE \ast \frac{K_t}{\text{assets}},
\]

with

\[
\text{assets} = \int_{M}^{M+\pi} K_t g(\omega) d\omega + \int_{M+\pi}^{\infty} L_t g(\omega) d\omega.
\]
We define the investor credit-to-output ratio as total credit over total output

\[
\frac{\int_{M+\pi}^{\infty} L_t g(\omega) \, d\omega}{\int_{M+\pi}^{\infty} y(\omega, \frac{\phi}{p_t^{-1}}, L_t) g(\omega) \, d\omega}.
\]

(29)

We define the fraction of reserves in the representative bank’s asset portfolio as total reserves divided by total assets

\[
\frac{\int_{M}^{M+\pi} K_t g(\omega) \, d\omega}{\text{assets}}.
\]

(30)

Lastly, we define regulator losses in equation (22).

**B. Numerical Algorithm**

1. Initialize the prior and level of capital. We set the initial prior, \( p_t \), to its invariant distribution value of 40%. Next, we normalize starting capital, \( K_0 \), to one. This assumption is without loss of generality because in the model it is the leverage ratio, and not the absolute level of capital, which affects the choice of lending standards, \( \pi_t \).

2. Compute expected house price growth, \( E_{t-1} \left[ \frac{\phi}{p_t^{-1}} \right] \), as described in equation (7).

3. Compute the optimal choice of lending standards using equations (18) – (20).

4. Draw a house price growth shock from the stochastic process described in Section 3.2.\(^{19}\)

5. Using the optimal \( \pi_t \) from step 3 and the value of the house price growth shock, compute the model values described in Appendix A.

\(^{19}\)For the purposes of calibrating the model and reporting comparative statics, the house price growth shock was set to its long-run mean.
6. Use the house price growth shock to compute banks’ updated beliefs (equations 4 – 6).

Repeat steps 2-6.
### Tables

**Table 1: Parameters**

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<tr>
<th>Parameter</th>
<th>Value</th>
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<td>$R^E_t$</td>
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</tbody>
</table>

Note: For calibration details see Section 5.1

**Table 2: Calibration**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Data</th>
<th>Model</th>
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</thead>
<tbody>
<tr>
<td>1) Reserves as fraction of banks’ asset portfolios</td>
<td>6.1%</td>
<td>5.8%</td>
</tr>
<tr>
<td>2) ROE</td>
<td>13.1%</td>
<td>13.2%</td>
</tr>
<tr>
<td>3) ROA</td>
<td>1.1%</td>
<td>1.3%</td>
</tr>
<tr>
<td>4) Delinquency rate</td>
<td>8.1%</td>
<td>7.7%</td>
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<tr>
<td>5) Credit-to-output ratio</td>
<td>81.1%</td>
<td>82.1%</td>
</tr>
<tr>
<td>6) Probability of crisis</td>
<td>5%</td>
<td>5%</td>
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<tr>
<td>7) Losses during crisis (as a % of banks’ capital)</td>
<td>82%</td>
<td>86%</td>
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Note: For calibration details see Section 5.1
### Table 3: Regulator Losses, the Capital Requirement and Value-at-Risk

<table>
<thead>
<tr>
<th>Value-at-Risk Confidence Level:</th>
<th>98%</th>
<th>95%</th>
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<tr>
<td>4.0%</td>
<td>125%</td>
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<td>6.0%</td>
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<td>48%</td>
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</table>

Note: This table computes the losses as a percent of total bank equity that the regulator could suffer. For more details, see Section 7.
Figure 1: Distribution of Borrowers and Lending Standards. This picture plots the Pareto distribution of borrowers’ idiosyncratic characteristics ($\omega$) and how lending standards ($M + \pi$) are modeled as a cut-off such that no borrower below it qualifies for credit.
Figure 2: Transition of Beliefs for Different Observations of House Price Growth. This figure plots banks’ posterior belief about being in the high state of the economy as a function of banks’ prior and different observations of house price growth. The data come from simulating the stochastic process described in Section 3.2 for 10,000 periods.
Figure 3: Countercyclical Lending Standards and Procyclical Credit. This Figure plots lending standards and credit as a function of the prior about the persistent part of price growth, $z_t$. 
Figure 4: Credit Booms in the Model and in the Data. This Figure plots empirical patterns documented by Elekdag and Wu (2011) and a model-simulated credit boom-bust. Model computations are explained in Section 5.
Figure 5: Banks’ Beliefs and Probability of Regulator Losses. This figure plots the probability that the regulator experiences losses of at least 50%, 65%, 75%, or 100% of banking system capital as a function of the prior about the persistent component of house price growth.
Figure 6: House Price Dynamics and Regulator Losses. This figure plots regulator losses as a percentage of banking system capital that occur 2% of the time for different house price growth rates.
Figure 7: House Price Dynamics and Probability of Crisis. This figure plots the probability of regulator losses of 100% or more of banking system capital for different house price growth rates.
Figure 8: Capital Requirements and Banks’ Beliefs. This figure plots, as a function of the prior about the persistent part of price growth, the capital requirement such that the probability of the regulator losing more than 75% or 100% of banking system capital is fixed at 2%.
Figure 9: House Price Dynamics and Value-at-Risk Capital Requirements. This figure plots the Value-at-Risk capital requirement such that the probability of the regulator losing more than 75% or 100% of banking system capital is fixed at 2% for different house price growth rates.