Demand for Collateral and Foreign Holdings of U.S. Assets*

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Abstract

This paper proposes a new explanation for why foreigners hold assets as U.S. Treasuries: a demand by foreign borrowers interested in their collateral properties. First, we document the widespread use by foreigners of U.S. Treasuries as collateral. Second, we solve a set of models to disentangle the demand for collateral from the demand for safe assets. We show that foreign holdings of U.S. Treasuries are smaller when foreigners have access to repo markets, even if these markets are based on U.S. collateral. This result happens because repo markets crowd out demand for U.S. Treasuries as storage of value. Finally, we discuss a policy implication: a subsidy on repo borrowings, or on purchases of U.S. Treasuries, can alleviate the inefficiency arising from the shortages of collateral.

Keywords: Collateral, Capital flows, Foreign investment, Financial development.

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1 Introduction

Financial institutions extensively employ collateral in lending and borrowing, in derivatives markets and in payment and settlement systems. Central banks require collateral in most of their refinancing and other credit operations. However, few assets have good collateral properties, especially for international transactions. U.S. Treasuries have never defaulted and have a highly liquid secondary market. These two reasons make them the assets more widely accepted as collateral.\(^1\)\(^2\) On the other extreme, developing countries lack assets with good collateral properties. They are either too volatile (Aguiar and Gopinath 2007), or have legal systems that make them very costly to repossess the collateral and defend creditors rights (World Bank 2010).

In this paper we propose demand for collateral as a reason why foreigners buy U.S. assets. First, we provide anecdotal evidence to support this claim. Second, we study a set of models that show conditions for equilibria with a foreigner buying U.S. assets to use them as collateral to borrow from another foreigner.

Moreover, we obtain a result potentially interesting to understand the dynamics of foreign holdings of U.S. Treasuries: foreign holdings are larger when foreigners do not have access to repo markets, even if these markets use U.S. Treasuries as collateral. This result happens because repo markets crowd out demand for U.S. Treasuries as storage of value. We believe that this result may be especially relevant for Asia. Asian countries are major holders of U.S. Treasuries, and their domestic repo markets are underdeveloped.\(^3\)

There is a lack of data, and thus empirical work, on international securitized lending. We relied directly on sources from the financial industry to write Section 2. It provides evidence supporting the widespread use by foreigners of U.S. Treasuries as collateral. It also shows that cross-border collateralized lending exists, probably in large amounts. We could not get better data, thus the rest of our paper is theoretical. Several international organizations, mostly from

\(^1\)The chief investment officer of BlackRock explained in the following way why U.S. Treasuries are good collateral: "The U.S. Treasury market is a $13 trillion market and a lot of the reason it is used as a form of collateral is not just the AAA rating, or what was a AAA rating, but the incredible liquidity and because it has become such an accepted form of payment and collateral. A one-notch downgrade by one of the three agencies won’t change that dynamic in any significant form.” (Rieder 2011)

\(^2\)Bartolini et al. (2010) and Krishnamurthy and Vissing-Jorgensen (2010) provide evidence that Treasuries command a collateral premium in U.S. markets.

\(^3\)The 2010 report of the Executives’ Meeting of the East Asia-Pacific Central Banks (EMEAP 2010) discusses this problem. For example, in Malaysia, 89 percent of money market transactions in 2009 were concentrated in uncollateralized interbank lending; in 2009 repo transactions represented only 5 percent of total interbank transactions in Indonesia, 14 percent in Thailand and are insignificant in the Philippines; in Hong Kong, Korea, New Zealand and Singapore while turnover has increased in domestic repo markets, financial institutions remain mostly dependent on foreign exchange swaps and uncollateralized interbank markets.
the financial industry, have study groups on cross-border collateral arrangements. We hope they may also increase data availability.  

In the model there are two agents with changing endowments over time that can buy U.S. Treasuries; either to save for future periods (a demand to store value à la Caballero et al. 2008), or to use U.S. assets to borrow against them. We think of the agents as financial institutions in different countries, or inside the same foreign country. They can be from either emerging economies that lack assets accepted as collateral for international borrowing, or from advanced economies that may have those assets but not in enough quantities for all their borrowing needs.

We study equilibria in which all borrowing and lending must be via repo markets and only U.S. Treasuries can serve as collateral. We discuss two versions of the model. To simplify, in both versions the U.S. returns are taken as exogenous. However, while in the first version the returns are deterministic, in the second version they are stochastic (for example because foreigners are subject to exchange rate or interest risk). We show that in both versions there are equilibria with foreign demand for U.S. assets for collateral reasons. The first version allows for closed form solutions: when there is overcollateralization the foreigner who wants to save buys Treasuries to store value; when there is undercollateralization, the foreigner who wants to borrow buys Treasuries to use them as collateral for repo borrowing (the purchase of U.S. assets looks like the purchase of a "permit to borrow", as the collateral friction is weaker "the borrowing permit is less needed" and demand for U.S. collateral vanishes).

A shortcoming of the model with deterministic returns of the U.S. asset is that in equilibrium there are no repo markets if there is overcollateralization. In Section 2.2 we provide empirical support for both under and overcollateralization. Thus, the goal of the second version of the model is to show that if returns are stochastic there is equilibria with demand for collateral and repo markets even if there is overcollateralization. The reason is that heterogeneity in endowments together with CRRA preferences imply different willingness to take risks. Thus, even with overcollateralization there is a foreigner buying the U.S. asset to borrow against it.

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4See Rusen (2009) for a survey of initiatives by the International Swaps and Derivatives Association (ISDA), the Giovannini Working Group and the International Institute for the Unification of Private Law (UNIDROIT). The Bank of International Settlements (BIS) also had a working group who published a report describing existing institutional cross-border collateral arrangements (BIS 2006).

5All insights from the paper apply to a model where borrowing and lending comes from heterogeneity in investment opportunities.

6We do not provide micro-foundations of the existence of the collateral constraint. Enforcement problems, limited commitment or moral hazard are standard reasons to motivate collateral constraints in deterministic and stochastic models.

7To derive analytical results we work in model with deterministic endowments. This assumption is not essential for these results. For example, in Appendix II we show a model with fully idiosyncratic uncertain endowments à la Aiyagari (1994) and Huggett (1993) which gives the same results numerically.
In this case, the foreigner more willing to take risks.

We show that in both versions of the model having access to repo markets reduces the aggregate holdings of U.S. assets. Even if these assets are the collateral needed in repo markets. Due to repo markets replacing U.S. Treasuries as a storage of wealth, savers switch from Treasuries to repos. Thus, our model suggests that the development of foreign repo markets will crowd out foreign demand for U.S. Treasuries, even if these assets keep their privileged status as collateral.

In Section 5 we study a policy implication of the model. The collateral margin is a friction that reduces welfare. It prevents the foreign countries from achieving risk sharing between them. We evaluate two types of short term policies that allow the achievement of full risk-sharing: a subsidy on repo borrowings that is inversely proportional to both the collateral margin and to the yield of U.S. Treasuries. Or a subsidy on the price of U.S. Treasuries.

The paper contributes to three literatures:

1) We contribute to the literature on why foreigners hold U.S. assets. Explanations for official public holdings usually focus on precautionary and mercantilist motives (Dominguez 2010 provides a summary). Forbes (2010) surveys the literature on private-sector holdings and evaluates different theories. She shows that despite strong theoretical support, diversification motives appear to have little impact on patterns of foreign investment. Trade with the U.S. and lack of capital controls explain holdings. But, her strongest and most consistent result is that a country’s level of financial development (measured by the size of the country’s stock and bond markets) is an important explanation of its share of investment in both U.S. equity and debt markets. Caballero et al. (2008) and Mendoza et al. (2009) provide the existing theoretical link between financial underdevelopment and foreign holdings of U.S. Treasuries. In Caballero et al. (2008) the demand for U.S. assets is driven by a shortage of safe assets in emerging economies. In Mendoza et al. (2009) foreigners by U.S. assets because they provide insurance and risk sharing. Our paper proposes demand for collateral as an alternative and complementary explanation which, to our knowledge, nobody has explored. Only Caballero (2006) mentioned it.

2) Several studies have documented that institutional differences help explain differences on international capital flows. For example, Gelos and Wei (2005) show this for both government and corporate transparency; Ferreira and Matos (2008) for disclosure standards and Leuz et al. (2009) for outsider protection. Our paper provides theoretical support for institutional differences related to the supply of assets with collateral properties.

3) Our policy exercises complement the recent literature on capital controls (Korinek 2011
is a survey). This literature focuses on pecuniary externalities via borrowers who do not internalize the effects of their borrowing on asset prices. The policy recommendations are taxes on international borrowing or on capital flows. Our model shows a different mechanism implying the opposite policy recommendation, the capital flows alleviate the inefficiency arising from the shortages of domestic collateral.

The rest of the paper is organized as follows. Section 2 provides empirical evidence on the use by foreigners of U.S. assets as collateral. Section 3 describes the deterministic model and characterizes its analytical results. Section 4 solves numerically the stochastic version for which we do not have analytical results. Section 5 discusses some policy implications. Section 6 concludes. Appendix I contains the proofs. Appendix II describes a model in which income is stochastic and the predictions of Section 3 also hold.

2 Empirical evidence

2.1 The International Use of U.S. Treasuries as Collateral

We describe below some evidence supporting the widespread use by foreigners of U.S. Treasuries as collateral. There are neither official statistics on the overall size of repo markets, nor on the collateral used in international collateralized lending transactions. We rely on estimates from professional practitioners:

a) U.S. assets are widely used in European Repo markets.\(^8\) According to the 2010 ICMA survey, 3.1% of the collateral used was explicitly declared as American and 22.8% of it as "from other OECD countries", the U.S. being one of them (ICMA 2010).\(^9\) It seems reasonable to believe that a large part of the "other OECD collateral" must be U.S. Treasuries because 28.3% of the total Euro-repos were in dollars, 37.3% involved a party not from the Eurozone, and U.S. Treasuries and U.S. agency obligations accounted for approximately 80% of U.S. tri-party repo collateral (Copeland et al. 2011).\(^10\)

b) The Asian collateralized financing market is split into four categories: 1) Equity financing using local market equities; 2) G10 bond repo with Central Banks, sovereign wealth funds and

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\(^8\)European repo markets hit a peak of EUR 6,775 billion in June 2007, then dropped to a low in December 2008 of EUR 4,633 billion, and have increased again to EUR 5,908 billion in December 2010 (ICMA 2010, BIS 2010).

\(^9\)In the ICMA survey “other OECD countries” are Australia, Canada, Chile, Iceland, Korea, Mexico, New Zealand, Norway, Switzerland, Turkey and the U.S.

\(^10\)In addition to the tri-party repo market there is a “bilateral repo market,” which does not settle on the books of a third-party agent, but also may use Treasuries as collateral.
commercial banks; 3) Asian Corporate bond (USD-denominated) repo; and 4) Local currency
Government bond market financing and monetary policy-based liquidity management. U.S.
issued assets (Treasuries, Agencies and Mortgage Backed Securities) are a reasonable portion of
the G10 bond repo only. USD-denominated assets make up almost the entire Asian Corporate
Bond sector, but these are Asian company issued bonds. G10 government bond repo is probably
around 25-50% of the total Asian collateralized financing market.\footnote{We are grateful to Edward Gildar (Head of Global Finance Desk, Asia ex-Japan, Citi) for these estimations.}

c) Anecdotal evidence shows that many foreign financial institutions borrow or lend against
U.S. Treasuries in U.S. repo markets.\footnote{Gorton and Metrick (2011) estimated the pre-crisis U.S. repo markets to be about the same size, or larger, than the U.S. banking system of $10 trillion.} For example, European commercial banks (Gore 2011),
British hedge funds (FSA 2011), Costa Rica’s Central Bank (Monge-Naranjo 2011) or China’s
State Administration for Foreign Exchange (Reuters 2011).\footnote{We are grateful to Gary Gorton for this suggestion.}

d) Around 85% of the participants in the global OTC derivatives industry, including Chinese
banks (Lee 2009), declare to accept U.S. Treasuries as collateral (ISDA 2000).

e) Several foreign Central Banks and financial authorities from different regions of the world,
accept U.S. Treasuries as collateral for borrowing transactions. Examples include, among others,
the Bank of England, the Bank of Canada, the Korea Securities Depository or the Hong Kong
Futures Exchange.\footnote{The G10 central banks have an “à la carte approach”, under which each central bank decides independently
whether and, if so, under what circumstances, to accept cross-border collateral (BIS 2006).}

f) Informal groups of private creditors on the international stage, as the London or Paris
Clubs, ask for the debt to be collateralized by U.S. Treasuries (Central Bank of Nigeria 2007).
It happens similarly with the Brady bonds (Salomon Smith Barney 2000).

2.2 Over or Under Collateralization

A key element of our model is the exogenous haircut on U.S. Treasuries. We will show
that demand for collateral may happen both when there is overcollateralization and undercol-
lateralization.

As detailed in Copeland et al. (2010), in U.S. repo markets the haircut is typically based
on several factors: the historical price volatility of the asset, the loan term, and on who is
the dealer. In the U.S. tri-party repo market in May 2011 the median haircut applied to U.S.
Treasuries was 2%. Thus, at the level of the individual repo transaction, overcollateralization
seems to be the norm.

However, undercollateralization may also happen: 1) Repo borrowers may negotiate a repo which is under-collateralized when the value of the security is expected to rise (Garbade 2006). 2) Most large banks, hedge funds and prime brokers rehypothecate their collateral, i.e., they re-poste it as collateral in another transaction potentially with a different counterparty (Deryugina 2009). Rehypothecation implies that at the aggregate level there is undercollateralization. 3) Fractional-reserve banking and OTC derivatives are markets that operate with undercollateralization.\footnote{Rehypothecation is very important in repo markets, it is not something that started over the late credit cycle (Johnson 1997, Singh 2010).}

\section{A Deterministic Model}

\subsection{Setup}

There are two agents who differ in their endowments of a single good. As in Woodford (1990), in odd periods agent 1 receives high endowment ($\overline{e}_1$) and agent 2 low endowment ($\underline{e}_2$); in even periods the pattern is reversed. Thus, the agents have an incentive to borrow or lend among themselves to smooth consumption. They can do so at repo rate $R_t$, but they are subject to a collateral constraint with an exogenous collateral margin ($m$). Their borrowings today ($b_{i,t+1} < 0$) need to be collateralized by holdings of U.S. assets, that we denote by $k_i$. The U.S. asset has an exogenous pay off of $R_t^{US}$ and costs today one unit. It cannot be short-saled. It has value only as store of value or as collateral.

Each agent chooses $\{c_{i,t}, k_{i,t+1}, b_{i,t+1}\}$ to solve

$$\max_{\{c_{i,t}, k_{i,t+1}, b_{i,t+1}\}} \sum_{t=0}^{\infty} \beta^t u(c_{i,t})$$

subject to her borrowing constraint

$$c_{i,t} + k_{i,t+1} + b_{i,t+1} \leq e_{i,t} + R_{t-1}b_{i,t} + R_t^{US}k_{i,t},$$

and the no-short selling constraint

$$k_{i,t+1} \geq 0.$$
and to the collateral constraint

\[ R_t b_{i,t+1} \geq -m R_t^{US} k_{i,t+1} \]  

(4)

**Definition 1** For exogenous \( \{ R_t^{US}, e_{i,t} \} \) a competitive equilibrium is a sequence of Repo interest rates \( \{ R_t \} \) and allocations \( \{ b_{i,t+1}, c_{i,t}, k_{i,t} \} \) such that both agents maximize and the Repo Market clears

\[ b_{1,t+1} + b_{2,t+1} = 0 \]  

(5)

### 3.2 Analytical Results

There are two types of equilibria as a function of \( m \). Basically, for \( m < 1 \) the equilibria display demand for U.S. assets to store wealth, while for \( m > 1 \) the demand for U.S. assets is purely driven by a demand for collateral. We will refer to agents with high endowment today \((\bar{e})\) as the rich, the others as the poor. We denote \((c_{it}, k_{i,t+1}, b_{i,t+1}) = (\bar{c}, \bar{k}, \bar{b})\) if agent \( i \) is rich (high endowment) at time \( t \), and \((c, k, b)\) if agent \( i \) is poor (low endowment).

#### 3.2.1 Case 1: \( m=0 \)

When \( m = 0 \) then \( b_{i,t+1} = 0 \) and no borrowings are possible. The equilibrium displays demand for U.S. assets to store wealth. This is basically the story of Caballero et al. (2008). Emerging economies do not have safe assets to store wealth. Thus, they buy U.S. assets to store their savings. We conjecture, and prove in Proposition 1, that the equilibrium is of the form:

\[ \bar{c} = \bar{e} - k \]  

(6)

\[ c = e + R^{US} k. \]  

(7)

That is, when the agent is rich she saves by buying \( k_{it} = k \) units of U.S. assets. When she is poor she consumes all her endowment plus the savings stored in U.S. Treasuries. She does not buy any additional U.S. asset, that is, for the poor \( k_{it} = 0 \).

**Proposition 1** Suppose

\[ \beta R^{US} u' e^{\epsilon} \leq \beta R^{US} u'(e) \]  

then there exists an equilibrium under the form (6)-(7).
Proof. See Appendix I

3.2.2 Case 2: m > 0

The case with \( m > 0 \) potentially allows for demand for collateral. Proposition 2 shows that for \( m < 1 \) the model is equivalent to the case with \( m = 0 \), the U.S. asset has no collateral value but only storage value. When \( m > 1 \) there is equilibria in which demand for U.S. assets is completely driven by demand for collateral. We will show by construction the existence of a \( m > 1 \) stationary equilibrium with the following form:

1) Rich agents do not buy U.S. assets, they lend in the repo market:

\[
\bar{c} = \bar{e} - m \frac{R^{US}}{R} k - mR^{US}k + R^{US}k
\]  

(9)

2) Poor agents buy U.S. assets and use them as collateral to borrow:

\[
\underline{c} = \underline{e} + m \frac{R^{US}}{R} k - k + mR^{US}k
\]  

(10)

That is, when the agent is rich she saves by investing \( b = mR^{US}k \) in the repo market. She receives high endowment \( \bar{e} \) and the return \( R^{US}k \) from past holding of U.S. Treasuries, but at the same time has to pay \( mR^{US}k \) from past borrowing in the repo market. When the agent is poor, she borrows \( b \) in the repo market but she has to buy \( k \) units of U.S. assets to use as collateral. She also receives low endowment and the payoff from past repo holdings.

Proposition 2 If \( m > 1 \) then in equilibrium there is an active repo market with \( R > R^{US} \).

If \( m \leq 1 \) then for any equilibrium, we have the same consumption allocation of the case with \( b_{i,t+1} = 0 \) for all \( i \). In a stationary equilibrium of the form described above for \( m > 1 \), the rate of return \( R \) is an increasing function in \( m \). Moreover, the non-U.S. economies cannot achieve full risk-sharing, i.e., \( \underline{c} \neq \bar{c} \)

Proof. See Appendix I

Using the expression for \( R \) in (22) and the first order for rich agents (18) implies an equation that determines \( k \), as a result, \( \bar{c} \) and \( \bar{c} \). For example, under iso-elastic utility function \( u'(c) = c^{-\sigma} \) equation (18) becomes

\[
\frac{\bar{c}}{\bar{e}} = (\beta R)^{1/\sigma}
\]
and using (10) and (9)
\[
\frac{\tau - m \frac{R_{US}}{R} k - m R_{US} k + R_{US} k}{\epsilon + m \frac{R_{US}}{R} k - k + m R_{US} k} = (\beta R)^{1/\sigma}.
\]
So,
\[
k = \frac{\tau - (\beta R)^{1/\sigma} \epsilon}{(\beta R)^{1/\sigma} \left(m \frac{R_{US}}{R} - 1 + m R_{US}\right) + m \frac{R_{US}}{R} + m R_{US} - R_{US}}.
\]
(11)
Thus, when \(m > 1\) demand for U.S. assets is completely driven by demand for collateral.

### 3.3 Numerical Example

The model is too stylized for a full quantitative analysis, but a numerical example allows to illustrate some qualitative implications. To do so we assume the standard CRRA
\[
u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}
\]
with coefficient of risk aversion \(\sigma\) equal to 3 and discount factor \(\beta\) equal to 0.95. For the exogenous yield \(R_{US}\) we assume \(R_{\beta} = \frac{1}{1.1}\). For the endowment parameters, we set \(e_{L} = 10\) and \(e_{H} = 100\).

Figure 1 illustrates the different types of equilibria as a function of the collateral requirement \((m)\). We let \(m\) to vary from 0 to 10. The model generates the same maximum demand for U.S. Treasuries for two very different reasons (Panel A). For equilibria with overcollateralization \((m \leq 1)\) the demand for Treasuries is driven by a demand to store value. For undercollateralization \((m > 1)\) it is driven by demand for collateral. As \(m \to \infty\) the collateral friction is weaker and demand for U.S. collateral vanishes. Moreover, markets are more complete as \(m \to \infty\), thus the consumptions of the rich and poor agents converge to the efficient perfect insurance case (Panel B). Countries with low \(m\) have more depressed domestic interest rates (Panel C), the need to post U.S. collateral hinders the ability of borrowers to remunerate more the lenders.

Insert Figure 1 about here

Figure 2 illustrates that economies with domestic repo markets hold less U.S. Treasuries. Figure 2 plots foreign demand for U.S. Treasuries as a function of the desire for borrowing, the larger is \((e_{H} - e_{L})\) the more uneven consumption and higher the demand to smooth. It
shows two different equilibria, one in which there are no repo markets \((m = 0.5)\) and demand for storage drives foreign purchases of U.S. Treasuries. And another in which repo markets are open \((m = 2)\) and only demand for collateral drives foreign Treasuries holdings. The presence of repo markets crowds out demand for Treasuries as a store of value. Thus, even if the repo market demands U.S. collateral, foreign holdings of U.S. assets are smaller when repo markets are available.

Insert Figure 2 about here

4 A Stochastic Model

In the previous model with deterministic returns of the U.S. asset in equilibrium there are no repo markets if there is overcollateralization \((m < 1)\). In Section 2.2 we provided empirical support for both under and overcollateralization. Now we show that if returns are stochastic we also have equilibria with demand for collateral and repo markets even if there is overcollateralization.

For simplicity we assume a 2-period economy with two states of the world in the second period (good, G, or bad, B) with probabilities \(\pi_s\), and only one state in the first period. The return on U.S. Treasuries in state \(s\) is \(R^{US} (s)\) and its price in the first period is 1. The return is higher in the good state than it is in the bad state, that is, \(R^{US} (G) > R^{US} (B)\).

Agent \(i\) has endowments \(e^i_1\) and \(e^i_2 (s)\). Her goal is to maximize the two period utility

\[
  u \left( c^i_1 \right) + \beta \sum_{s \in \mathcal{S}} \pi_s u \left( c^i_2 (s) \right)
\]

The budget constraints in each period are

\[
  c^i_1 + k^i_1 + b^i \leq e^i_1 \\
  c^i_2 (s) = e^i_2 (s) + R^{US} (s) k^i_1 + Rb^i
\]

And the collateral constraint that assures no default is

\[
  Rb^i \geq -R^{US} (B) k^i_1 \quad (13)
\]

Thus, the maximum borrowings that an agent can do is the value of her Treasury holdings in the worst state of the world. Given that \(R^{US} (B) < 1\) we have overcollateralization, i.e. \(m < 1\)
in equation (4). As we assumed before, the agent can never short treasuries

\[ k_i^i \geq 0 \]

The stochastic model does not have analytical solutions thus we solve the model numerically.\textsuperscript{17} We focus on results that are robust to different parameterizations. As we did in Section 3.3, in Figure 3 we plot the competitive equilibrium as a function of the difference in the endowments of the agents (we increase the endowment of agent 2 by the same amount in all periods and states of the world\textsuperscript{18}).

Insert Figure 3 about here

Agent 1 is poorer, what given the CRRA makes her more risk averse than agent 2. Thus she saves more than agent 2. We can see this in Panel C, that plots the case with no repo markets. In that case agents are in autarky and can only save (not borrow) via U.S. Treasuries. Agent 1 buys more U.S. Treasuries than agent 2.\textsuperscript{19} When repo markets are available (Panel D) agent 1 holds less Treasuries than agent 2. U.S. Treasuries are risky and agent 1 switches her savings towards repos (Panel B). Agent 2, who is richer and thus less risk averse, buys U.S. Treasuries and uses them as collateral to borrow from Agent 1 (by equilibrium condition ?? agent 2 repo borrowings are the negative of Panel B). Panel A shows the aggregate effects: when the foreign economy has repo markets the demand for U.S. Treasuries decreases.

5 A Policy Application

In this section we explore how a subsidy on repo borrowings, or on purchases of U.S. Treasuries, can achieve perfect risk sharing. We looked only whether we have full risk sharing, but not at which level we have it. The reason is because we have heterogenous agents, thus to specify a social welfare function we should weigh arbitrarily the different agents.

\textsuperscript{17}The parameterization assumes the same likelihood of the states of the world (\( \pi_s = \frac{1}{2} \)), discount factors and exogenous returns satisfying \( \beta R^{US} (G) = 1 \) and \( \beta R^{US} (B) = 0.1 \).

\textsuperscript{18}We fix the endowments of agent 1 at \( e^1_1 = 10, e^1_2 (G) = e^1_2 (B) = 5 \).

\textsuperscript{19}The holdings of agent 1 are flat in Panel C because when there are no repo markets changes in the endowment of agent 2 have no effect on agent 1.
5.1 Tax or Subsidize Repo Borrowings

The setup is the same of Section 3 but now the Government can use lump sum taxes (subsidies) $T_{i,t}$ to finance a tax (subsidy) on repo borrowing ($\tau$). Thus, now the agents maximize preferences (1) subject to

$$c_{i,t} + k_{i,t+1} + b_{i,t+1} \leq e_{1,t} + R_{t-1}b_{i,t} + R_{t-1}^{US}k_{i,t} - T_{i,t} \text{ if } b_{i,t+1} \geq 0$$
$$c_{i,t} + k_{i,t+1} + (1 + \tau)b_{i,t+1} \leq e_{1,t} + R_{t-1}b_{i,t} + R_{t-1}^{US}k_{i,t} - T_{i,t} \text{ if } b_{i,t+1} < 0,$$

to the no-short selling constraint (3), and to the collateral constraint (4).

Definition 2 For exogenous $\{R_{t}^{US}, e_{i,t}, T_{i,t}\}$ a competitive equilibrium is a sequence of repo interest rates $\{R_{t}\}$ and allocations $\{b_{i,t+1}, c_{i,t}, k_{i,t}\}$ such that both agents maximize, the repo market clears and transfers are self-financing

$$b_{1,t+1} + b_{2,t+1} = 0$$
$$\sum_{i} (T_{i,t} + \tau b_{i,t+1}1_{b_{i,t} < 0}) = 0$$ (14)

We can show that there is a system of taxes and transfers that can achieve full risk-sharing among foreigners. The borrowers should receive a subsidy in the repo rates they have to pay back. This subsidy should be inversely proportional to $m$ and the yield of U.S. Treasuries.$^{20}$

Proposition 3 Let $\tau^{*} = \frac{1}{m} \left( \frac{1}{R_{t}^{US}} - 1 \right)$, then the foreign economies achieve full risk-sharing.

Proof. See Appendix I $\blacksquare$

5.2 Tax or Subsidize Foreign Holdings of U.S. Treasuries

The setup is the same of Section 3 but now the Government can use lump sum taxes (subsidies) $(T_{i,t})$ to finance a tax (subsidy) $(\tau)$ on capital flows with the U.S. Thus, now the

$^{20}$The set of possible transfers allows us to choose the level of consumption at which we have full-risk sharing. For example, when $R_{t}^{US} \leq 1$, $\tau = c$ can be anywhere between 0 and $\frac{e_{1,t}}{e_{2,t}}$. For $R_{t}^{US} > 1$ that level also depends on the initial level of U.S. asset holding in the economy.
agents maximize preferences (1) subject to

\[
\begin{align*}
    c^F_{i,t} + (1 - \tau) k_{i,t+1} + b_{i,t+1} &\leq e_{1,t} + R_{i-1} b_{i,t} + R_{US}^t k_{i,t} - T_{i,t} & \text{if } b_{i,t+1} \geq 0 \\
    c^F_{i,t} + (1 - \tau) k_{i,t+1} + b_{i,t+1} &\leq e_{1,t} + R_{i-1} b_{i,t} + R_{US}^t k_{i,t} - T_{i,t} & \text{if } b_{i,t+1} < 0,
\end{align*}
\]

to the no-short selling constraint (3), and to the collateral constraint (4).

**Definition 3** For exogenous \( \{ R_i^t, e_{i,t}, T_{i,t} \} \) a competitive equilibrium is a sequence of repo interest rates \( \{ R_t \} \) and allocations \( \{ b_{i,t+1}, c_{i,t}, k_{i,t} \} \) such that both agents maximize, the repo market clears and transfers are self-financing

\[
\begin{align*}
    b_{1,t+1} + b_{2,t+1} &= 0 \\
    \sum_i (T_{i,t} - \tau k_{i,t+1}) &= 0.
\end{align*}
\]

We can show that a subsidy to the price of the U.S. Treasuries would allow to achieve full risk-sharing among the non-U.S. economies. This subsidy should be such that the price of U.S. assets faced by the borrowers is \( \beta \).

**Proposition 4** Let \( \tau^* = 1 - \beta R^{US} \), then the economy achieves full risk-sharing.

**Proof.** See Appendix I □

6 Conclusion

In this paper we proposed a potentially new explanation to foreign demand for U.S. Treasuries: a demand for collateral. First, we documented that foreigners use U.S. Treasuries as collateral outside the U.S. Second, we solved a model to highlight the mechanism and disentangle the demand for collateral from the demand for safe assets. Moreover, we showed that countries without repo markets buy more U.S. Treasuries.
References


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Appendix I: Proofs

Proof of Proposition 1. We need to verify the optimality of the first-order conditions on the asset holding of the consumers. For the rich agent

\[
\frac{1}{R_{US}} u' (\bar{e} - k) = \beta u' (\bar{e} + R_{US} k) \tag{15}
\]

and we can define

\[
\Delta (k) = 1 - \beta R_{US} \frac{u' (\bar{e} + R_{US} k)}{u' (\bar{e} - k)}.
\]

Condition (8), plus preferences satisfying Inada conditions, imply that

\[
\Delta (0) \leq 0 \quad \text{and} \quad \Delta \left( \frac{\bar{e} - \bar{e}}{1 + R_{US}} \right) \geq 0.
\]

Thus, there exists \( k^* \in (0, \frac{\bar{e} - \bar{e}}{1 + R_{US}}) \) such that \( \Delta (k^*) = 0 \), that is, the Euler condition for the rich agent is satisfied.

For the poor agent the non-short selling for U.S. asset is binding:

\[
\frac{1}{R_{US}} u' (\bar{e} + R_{US} k) \geq \beta u' (\bar{e} - k).
\]

The Euler condition for the poor agent is also satisfied because \( \beta R_{US} \leq 1 \).

Proof of Proposition 2. Let \( \lambda_{i,t} \) denote the Lagrange multiplier on the collateral constraint (4) and \( \mu_{i,t} \) denote the Lagrange multiplier on the no-short selling constraint (3). The first-order condition with respect to \( k_{i,t+1} \) implies

\[
-u' (c_{i,t}) + \beta R_{US} u' (c_{i,t+1}) + \mu_{i,t} + m R_{US} \lambda_{i,t} = 0. \tag{16}
\]

The first order condition with respect to \( b_{i,t+1} \) implies

\[
-u' (c_{i,t}) + \beta R_{t} u' (c_{i,t+1}) + R_{t} \lambda_{i,t} = 0. \tag{17}
\]

And the complementarity-slackness conditions are

\[
\lambda_{i,t} \left( R_{t} b_{i,t+1} + m R_{US} k_{i,t+1} \right) = 0
\]

\[
\mu_{i,t} k_{i,t+1} = 0.
\]

The analysis of equilibria will involve working with the multipliers as in Cao (2011). For
example, the pricing of the domestic bond departs from the present discounted value of payoffs by the multiplier on the collateral constraint: \( 1 = R_t \frac{\beta u'(c_{i,t+1})}{u'(c_{i,t})} + R_t \frac{\lambda_t}{u'(c_{i,t})} \).

If \( m > 1 \), (17) and (16) imply

\[
\frac{1}{R_t} u'(c_{i,t}) = \beta u'(c_{i,t+1}) + \mu_{i,t} + m \lambda_{i,t} \geq \beta u'(c_{i,t+1}) + \lambda_{i,t} \geq \frac{1}{R_t} u'(c_{i,t})
\]

so \( R_t \geq R_t^{US} \) with strict inequality if \( \lambda_{i,t} > 0 \).

If \( m \leq 1 \): Suppose, by contradiction, that there is an active repo market in equilibrium. Then there must be \( i \) such that \( b_{i,t} > 0 \). So \( \lambda_{i,t} = 0 \). (17) and (16) imply

\[
\frac{1}{R_t} u'(c_{i,t}) = \beta u'(c_{i,t+1}) + \mu_{i,t} \geq \beta u'(c_{i,t+1}) = \frac{1}{R_t} u'(c_{i,t})
\]

so \( R_t \geq R_t^{US} \). Now, there must be an \( i \) such that \( b_{i,t} < 0 \). The collateral constraint (4) implies that \( k_{i,t+1} > 0 \) or \( \mu_{i,t} = 0 \). So

\[
\frac{1}{R_t} u'(c_{i,t}) = \beta u'(c_{i,t+1}) + m \lambda_{i,t} \leq \beta u'(c_{i,t+1}) + \lambda_{i,t} \leq \frac{1}{R_t} u'(c_{i,t}),
\]

or \( R_t \leq R_t^{US} \). Thus, the two inequalities imply \( R_t = R_t^{US} \). Consequently \( \lambda_{i,t} = 0 \) for the agents who are borrowing, i.e. their collateral constraints are not binding. Now for agents with \( b_{i,t+1} > 0 \) we can replace their repo lending by holdings of U.S. Treasuries. For agents with \( b_{i,t} < 0 \), we can replace \( k_{i,t+1} \) by \( k_{i,t+1} + \frac{R_t}{R_t^{US}} b_{i,t+1} \) (>0 due to the collateral constraint) and reduce \( b_{i,t+1} \) to zero. This gives us an equivalent allocation with no repo lending. Notice that the total holdings of U.S. Treasuries are unchanged.

Given that the rich agents are not borrowing in the repo market, their collateral constraints are not binding. The first-order condition (17) implies

\[
\frac{1}{R} u'(\bar{c}) = \beta u'(\bar{c}) \tag{18}
\]
Now, the poor agents borrow in the repo markets. Because they have to buy the U.S. asset to use as collateral, the no short selling constraint (3) is not binding, that is $\mu_{i,t} = 0$ in (16). The first order conditions (17) and (16) become

$$-\frac{1}{R} u'(\xi) + \beta u'(\bar{x}) + \lambda = 0$$

and

$$-\frac{1}{R^{US}} u'(\xi) + \beta u'(\bar{x}) + m\lambda = 0. \tag{20}$$

Multiplying the first equation by $m$ and subtracting it from the second equation imply

$$-\left(\frac{m}{R} - \frac{1}{R^{US}}\right) u'(\xi) + \beta (m - 1) u'(\bar{x}) = 0.$$

Combining this equation with equation (18) implies

$$\frac{1}{R} \left(\frac{m}{R} - \frac{1}{R^{US}}\right) = \beta^2 (m - 1). \tag{21}$$

This a quadratic equation in $R$ and gives a unique solution for $R$ as a function of $m$ and $R^{US}$

$$R = m R^{US} \frac{2}{\sqrt{1 + 4m (m - 1) \beta^2 (R^{US})^2 + 1}}. \tag{22}$$

If $\xi = \bar{x}$ then equation (18) implies $R = \frac{1}{\beta}$. This result and equation (21) imply $R^{US} = \frac{1}{\beta}$. However, $R = R^{US}$ contradicts (19) and (20) for $m > 1$. \qed

**Proof of Proposition 3.** As in Section (3.2.2), the system that determines interest rate and allocations is the bond Euler equation for the rich

$$\frac{1}{R} u'(\bar{x}) = \beta u'(\xi),$$

and those of the poor

$$-\frac{1 + \tau}{R} u'(\xi) + \beta u'(\bar{x}) + \lambda = 0 \tag{23}$$

$$-\frac{1}{R^{US}} u'(\xi) + \beta u'(\bar{x}) + m\lambda = 0. \tag{24}$$
Combining the last two equations yields

\[- \left( \frac{m(1 + \tau)}{R} - \frac{1}{RUS} \right) u'(\xi) + \beta (m - 1) u'(\bar{\xi}) = 0.\]

Now in order to obtain full risk sharing ($\bar{c} = \xi$) we need

\[R = \frac{1}{\beta}\]

\[\left( \frac{m(1 + \tau)}{R} - \frac{1}{RUS} \right) = \beta (m - 1)\]

The two equations yield $\tau^*$. ■

**Proof of Proposition 4.** Similar to the proof of Proposition 3 but replacing (23) and (24) by

\[- \frac{1}{R} u'(\xi) + \beta u'(\bar{\xi}) + \lambda = 0\]

\[- \left( 1 - \tau \right) \frac{1}{RUS} u'(\xi) + \beta u'(\bar{\xi}) + m \lambda = 0\]

■

**Appendix II: Stochastic Income**

Consider the endowment economy of Section 3. Now we populate the economy with a continuum of agents, each one facing idiosyncratic endowment shocks

\[e_{i,t} = \begin{cases} \bar{\xi} & \text{with probability } p \\ \xi & \text{with probability } 1 - p \end{cases}\]

Agent $i$ solves

\[
\max_{c_{i,t},k_{i,t+1},b_{i,t+1}} \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t u(c_{i,t}) \right]
\]

subject to

\[c_{i,t} + \frac{1}{RUS} k_{i,t+1} + \frac{1}{R} b_{i,t+1} \leq e_{i,t} + b_{i,t} + k_{i,t}\]
and

\[ k_{i,t+1} \geq 0 \]
\[ b_{i,t+1} \geq -mk_{i,t+1}. \]

In the case of over-collateralization, that is \( m > 1 \), we need to impose the additional natural borrowing limit. For example, in a stationary equilibrium in which \( R_t = R \) is constant over time, we must impose

\[ b_{i,t+1} + k_{i,t+1} \geq -\frac{\epsilon}{R^{US} - 1}. \]

The numerical solution of this model gives the same qualitative results of the deterministic model of Section 3.
Figure 1. Comparative Statics as a function of the Collateral Margin when the Return on U.S. Treasuries is Deterministic. This figure plots foreign holdings of U.S. Assets, foreign consumption and interest rates in repo markets as a function of the collateral margin.
Figure 2. Demand for U.S. Treasuries and Repos when the Return on U.S. Treasuries is Deterministic. This figure plots foreign holdings of U.S. Assets and volume in repo markets as a function of the difference in endowments of the agents.
Figure 3. Demand for U.S. Treasuries and Repos when the Return on U.S. Treasuries is Stochastic. This figure plots different variables of the stochastic model of Section 4 as a function of the difference in endowments of the agents.