Understanding Why High Income Households Save More Than Low Income Households

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Abstract

We use a calibrated life-cycle model to evaluate why high income households save as a group a much higher fraction of income than do low income households in US cross-section data. We ¯nd that (1) age and relatively permanent earnings di®erences across households together with the structure of the US social security system are su±cient to replicate this fact, (2) without social security the model economies still produce large di®erences in saving rates across income groups and (3) purely temporary earnings shocks of the magnitude estimated in US data alter only slightly the saving rates of high and low income households.

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1 Introduction

A famous problem in the consumption and saving literature is to ¯nd a microeconomic explanation for three stylized facts of US data: (i) the aggregate saving rate is roughly constant over long time periods, (ii) household saving rates increase strongly with household income in cross-section data and (iii) income inequality does not increase over long time periods.[1] Duesenberry (1949), Modigliani and Brumberg (1954) and Friedman (1957) all attempted to provide a theory of consumption and saving behavior that was qualitatively consistent with these facts. The standard theory that present-day economists use for consumption and saving problems is a result of this work as well as subsequent theoretical work that provided an expected utility maximization foundation to unify the Modigliani-Brumberg and Friedman theories. This theory is sometimes referred to as the life-cycle/permanent-income theory of household consumption and saving behavior.

In this paper we return to the stylized facts that originally motivated the theoretical work on consumption and saving behavior. In particular, we focus on the quantitative magnitudes of the relationship between household saving rates and household income in cross-section data. Figure 1 summarizes the ¯ndings of Kuznets (1953) and Projector (1968) for this stylized fact when one averages the cross-section saving rates across the years of their data sets. Figure 1 shows that households with annual income levels below one half of mean income in the economy dissave as a group, whereas households with annual incomes of three or more times mean income save as a group in excess of 20 percent of income.

[Insert Figure 1 Here]

[1] Friedman (1957, Ch. 4) states this problem. At the time that Friedman wrote, Kuznets (1952) documented the ¯rst fact, Brady and Friedman (1950) and Kuznets (1953) documented the second fact and Kuznets (1953) and Goldsmith et al (1954) documented the third fact. One may question whether these facts have continued to hold. Williamson and Lindert (1980, Figure 4.3) is consistent with a general decrease in measures of income inequality in this century together with increases and decreases over shorter periods. Gottschalk and Smeeding (1997) review work documenting increased income inequality in the last two decades. Evidence on the second fact is discussed in Section 2. Browning and Lusardi (1996), Carroll and Summers (1996) and Gokhale, Kotliko® and Sabelhaus (1996) document the decrease in aggregate saving rates in the last two decades.

We pose the following questions related to Figure 1. Are such large differences in saving rates puzzling relative to standard theory or are they an implication of an empirically-specified version of the standard theory? If the latter possibility turns out to be the case, then which features of the standard theory seem to be central in replicating the observations? We are motivated to pose these questions for two main reasons. First, these questions have never been answered despite the fact that beginning economics students are taught that the life-cycle/permanent-income theory provides a qualitative explanation for the pattern in Figure 1.[2] Second, an answer would be quite useful in the development of empirically-oriented models of distribution. Such models are essential in developing answers to a wide variety of policy questions. Potential policy questions range from the present-day concerns with the magnitude and nature of the effect of Individual Retirement Accounts on saving to the older concerns with whether income can or should be redistributed across households so as to smooth out business-cycle fluctuations or to hasten economic growth. To have any confidence in using such models to analyze these types of policy questions one should first know whether these models can replicate some of the stylized facts of the distribution of consumption, income, wealth and saving.

There are many possible sources of saving rate heterogeneity in actual economies even holding age constant. A short, but far from exhaustive, list would include (1) permanent and temporary differences in earnings, (2) heterogeneity in preferences arising from differences in discount factors, bequest motives and mortality rates, (3) health shocks, (4) heterogeneity in investment opportunities across households and (5) government tax and transfer

[2] Three plausible reasons for why these questions have not been addressed are as follows. First, until the 1970's the consumption and saving literature focused on estimating the consumption function. Second, since the 1970's the literature has focused on estimating the Euler equation produced by modern versions of the standard theory. Thus, the role of simulation as a means of characterizing the implications of theory has been underemphasized. Third, until recently the methods needed to simulate versions of the standard theory with earnings uncertainty, a realistic social security system and with sufficient heterogeneity to match to the data were not widely known to economists nor were sufficiently powerful computers widely available.

programs.[3] [4]

Our methodology for addressing the questions posed above is to start o® with a small number of features from the above list that can be incorporated within the standard theory and that can generate saving rate heterogeneity as well as the large di®erences in income observed in the data. In particular, we choose to focus on earnings di®erences across agents as the sole exogenous source of heterogeneity across agents of a given age. Unlike a number of the possible sources of saving rate heterogeneity listed above, earnings di®erences are directly measurable even though they may be generated by deeper considerations that we abstract from at this stage of research.

As a modeling strategy, we focus on steady-state equilibria of the model economies. By focusing on steady states the model economies will, by construction, produce (i) a constant aggregate saving rate, (ii) a constant relationship between saving rates and multiples of mean income in cross-section data and (iii) a constant income distribution up to a scale factor accounting for growth in output per person. Since the model economies are calibrated to approximate the US capital-output ratio, the models will match the US aggregate saving rate for the savings concept that corresponds to the notion of physical capital used to measure the capital-output ratio. By calibrating

[3] A separate class of explanations follows the work of Kaldor (1956) and Pasinetti (1962). These theories directly assume di®erences in saving rates. As this is the basic fact to be explained, such theories seem to be uninteresting for the questions that we pose.

[4] Some of the literature related to these sources of saving rate heterogeneity are as follows: (1) Duesenberry (1949, pp.76-89), Modigliani and Brumberg (1954, pp.404-25) and Friedman (1957, pp. 39-40) suggest that uninsured, temporary shocks are responsible for the saving rate pattern documented in Figure 1. (2-3) Lawrance (1991) estimates that discount factors for households within an age group are increasing in measures of household labor income and education. Diamond and Hausman (1984) suggest that \individual differences" are important in understanding actual saving patterns. Rust and Phelan (1997) estimate that mortality rates at di®erent ages di®er substantially by reported health and marital status and that health expenditures are characterized by small probabilities of very large health expenditures. (4) Investment opportunities of entrepreneurs are generally thought to be unavailable to the general population. Thus, the saving incentives to entrepreneurs could di®er dramatically from those of non-entrepreneurs. (5) Social security programs o®er dramatically di®erent returns to one and two-earner households and to households with di®erent earnings levels. Huggett (1996) calculates that stylized versions of the US social security system are consistent with very high wealth accumulation by high income households and very low wealth accumulation by low income households. Hubbard, Skinner and Zeldes (1995) calculate that means-tested transfer programs can strongly in°uence the saving behavior of households with low earnings levels.

the earnings process in the model economies to match a number of features of the US earnings distribution, the models will produce a substantial amount of income heterogeneity. The remaining issue is then whether or not the models successfully replicate the saving rate pattern documented in Figure 1.

Within the model economies investigated, there are several reasons for why saving rates will differ for households at different multiples of mean income. First, households will differ in age in cross section. Middle-age households will tend to save at higher rates than either young or old households as income is humped shaped over the life cycle. As high income households will tend disproportionally to be in the middle of the life cycle, high income households will as a group save at higher rates than low income households. Second, both temporary as well as permanent earnings shocks could be important. Following the standard intuition (see Vickery (1948), Modigliani and Brumberg (1954) and Friedman (1957)), an uninsured, earnings shock that is purely temporary will be largely saved if positive and dissaved if negative. Thus, the saving rates of high income households will be higher than low income households in the same age group due to a composition effect. High income households will be composed of a higher fraction of high temporary shock households, whereas exactly the opposite composition effect will hold for low income households. This intuition could also hold for the case of permanent shocks. The reason is that, with a retirement period in which labor earnings are zero, a shock that increases earnings in all periods over the remainder of one's working life is still temporary when viewed over one's entire lifetime. Third, social security could be important. One key feature of the US social security system is that annual benefits are not proportional to social security taxes paid. Thus, households with a permanently high earnings level will save at a higher rate before retirement than will households in the same age group with a permanently lower earnings level as the annual retirement and health benefits provided by social security are a small fraction of the earnings of very high earners but a much higher fraction of the earnings of low earners.

We find that empirically-specified model economies with the features described above imply the type of saving behavior documented in Figure 1. Thus, low income households as a group dissave, high income households as a group save at high rates and saving rates tend to increase with income even into the extreme upper tail of the income distribution. The key features of the model economies that produce this behavior are age di®erences across households, relatively permanent di®erences in earnings across households and the structure of the social security system. We note that a speci¯c pattern of earnings shocks is not essential to produce this result. Indeed, we ¯nd that purely temporary earnings shocks of the magnitude estimated in US data have only a modest contribution to decreasing the saving rate at low income multiples and increasing the saving rate at high income multiples. This is surprising as the standard explanation for the saving rate pattern documented in Figure 1 is temporary shocks.[5] Lastly, we ¯nd that in the absence of a social security system the model economies produce a saving rate pattern roughly similar to that in Figure 1. This is mostly due to di®erences in saving rates across age groups rather than the saving rate di®erences within age groups that were key in the economies with a social security system.

This paper is organized in six sections. Section 2 reviews evidence for the cross-section saving fact discussed in the introduction. Section 3 describes the model economies. Section 4 describes how the parameters of the model economies are selected so that the model economies are realistic descriptions of the US economy along some dimensions. Section 5 answers the questions posed in the introduction. Section 6 concludes.

2 Saving Rates and Income in the US

We brie°y review some of the evidence documenting the stylized fact that saving rates increase with the level of income in US cross section data. This relationship has been documented in numerous studies including Brady and Friedman (1950), Fisher (1952), Kuznets (1953), Friend and Schor (1959), Projector (1968), Avery and Kennickell (1991), Bosworth et al (1991) and Sabelhaus (1993). These studies construct measures of annual household

[5] For statements of this view one can read Duesenberry (1949, pp. 76-89), Modigliani and Brumberg (1954, pp. 404-25) and Friedman (1957, pp. 39-40). For instance, Friedman states: \...many people have low incomes in any one year because of transitory factors and can be expected to have higher incomes in other years. Their negative savings are ¯nanced by large positive savings in years when their income is abnormally large, and it is these that produce the high ratios of savings to measured income at the upper end of the measured income scale." Given our results, we conclude that if this view is to prove correct then the transitory factors that really matter must e®ect something other than earnings.

income and net saving from cross-sectional household surveys.

Consider the results obtained by Kuznets (1953). The data that he considers come from various surveys conducted between 1929 and 1950. For each year, households are separated into di®erent income groups. For a given income group the saving rate is calculated by dividing total saving in the group by total income in the group. The results of this analysis are presented in Table 1. The averages of these data across the survey years were previously displayed in Figure 1. We observe that the saving rates are typically negative for households with income levels below one-half of mean income. The saving rates increase nearly monotonically as the multiple of mean income increases. For households with income multiples of three or more times mean income the saving rate exceeds 20 percent. It is interesting to note that these patterns occur in the individual years examined and therefore the pattern in Figure 1 is not solely the result of time averaging the data.

Table 1: Saving Rates at Multiples of Mean Income: US 1929-1950

Income Multiple	1929	1935	1941	1942	1945	1946	1947	1948	1949	1950
.25	-30.4	-32.1	-15.6	-25.1	4.9	-9.3	-14.8	-22.2	-31.1	-15.9
.50	-1.3	-7.4	0.2	-0.1	7.9	1.9	1.4	-1.3	-5.7	-0.8
.75	8.1	-1.5	5.3	8.3	10.7	7.0	4.6	3.2	-0.6	3.9
1.0	11.6	3.5	5.0	10.9	12.9	10.8	7.0	6.4	5.0	7.4
1.5	16.3	9.4	10.7	15.9	15.7	15.9	10.2	10.8	11.2	12.1
2.0	19.5	14.1	13.9	18.2	19.6	19.7	14.0	14.0	15.6	15.4
3.0	23.6	21.9	19.3	22.7	28.6	24.9	21.5	18.5	21.8	20.2
4.0	29.0	27.2	24.8	27.2						
7.0	37.0	37.5								
10.0	38.5	39.8								
25.0	43.1	49.2								

Source: Kuznets (1953, Table 48)

It is interesting to compare the saving rates at di®erent income multiples found by Kuznets to those in more recent data. For example, Projector (1968, Table 4) presents results from several surveys in the 1960's following the procedure in Kuznets (1953). The averages across years of Projector's studies in the 1960's were previously displayed in Figure 1. As Figure 1 shows, the ¯ndings are qualitatively and quantitatively similar to those in the Kuznets study. As in the Kuznets study, saving rates also tend to increase monotonically with income in the individual years examined. Note in Figure 1 that Projector does not have data for income multiples of 4.0 or higher.

More recently, Bosworth et al (1991, Table 5) have analyzed survey data from the 1960's, 1970's and the 1980's. They ¯nd that household saving rates are negative for the lowest income quintile and tend to increase for higher income quintiles in all the survey years they examine. As the average income of households in the highest income quintile is only about two times average income, their results do not allow a detailed analysis of saving rates for households in the extreme upper tail of the income distribution.

We wish to draw attention to one additional feature of the results presented in Bosworth et al (1991). In some surveys the authors calculate saving as the change in wealth across time periods, while in other surveys saving is calculated as income less consumption. They find that both the saving rates of low income quintiles is lower and that the saving rates of high income quintiles is higher when saving is calculated using income less consumption instead of using the change in wealth. Sabelhaus (1993) shows that this same result occurs even when saving is calculated by both methods within the same data set.[6]

3 The Economies Investigated

3.1 The Environment

We consider an overlapping generations economy.[7] Each period a continuum of agents are born. Agents live a maximum of N periods and face a probability s_j of surviving up to age j conditional on surviving up to age $j-1$. The population grows at a constant rate n. These demographic patterns are stable so that age j agents make up a fraction μ_j of the population at any point in time.[8] All agents have identical preferences over consumption that are given by the following utility function:

$$E\left[\sum_{j=1}^{N} \beta^j \left(\prod_{i=1}^{j} s_i\right) u(c_j)\right]$$

The period utility function $u(c)$ is of the constant relative risk aversion class, where σ is the coefficient of relative risk aversion and $(1/\sigma)$ is the intertemporal elasticity of substitution.

[6] The data set is the 1989 Consumer Expenditure Survey

[7] The modeling framework used here is similar to that used by Imrohoroglu et al (1995) and Huggett (1996).

[8] The weights μ_j are normalized to sum to 1, where $\mu_{j+1} = (s_{j+1}/(1+n))\mu_j$.

$$u(c) = c^{(1-\sigma)}/(1-\sigma)$$

An agent's labor endowment in efficiency units is given by a function $e(z, j)$ that depends on the agent's age j and on an idiosyncratic labor productivity shock z. The shock z lies in a finite set Z and at birth is distributed across agents according to a distribution $\pi(z)$. After birth the shock follows a Markov process. Labor productivity shocks are independent across agents. This implies that there is no uncertainty over the aggregate labor endowment even though there is uncertainty at the individual agent level. The function $e(z, j)$ is described in detail in section 4.

At any time period t there is a constant returns to scale production technology that converts capital K and labor L into output Y. The technology improves over time because of labor augmenting technological change. The technology level A_t grows at a constant rate, $A_{t+1} = (1+g)A_t$. Each period capital depreciates at rate δ.

$$Y_t = F(K_t, L_t A_t) = AK_t^{\alpha}(L_t A_t)^{1-\alpha}$$

3.2 The Arrangement

We consider an arrangement where in each period t an age j agent with idiosyncratic shock z and average past earnings $\bar{e}$ chooses consumption c_t and risk-free asset holdings a_{t+1}. The period budget restriction for such an agent is then

$$\begin{aligned} & c_t + a_{t+1} \leq a_t(1 + r_t(1-\tau)) + (1-\theta-\tau)e(z, j)w_t + T_t + b_t(\bar{e}, j) \\ & c_t \geq 0,\ a_{t+1} \geq \underline{a}_{t+1} \text{ and } a_{t+1} \geq 0 \text{ if } j = N \end{aligned}$$

In the above budget constraint resources are derived from asset holdings a_t, labor endowment $e(z, j)$, a lump-sum transfer T_t and a social security benefit $b_t(\bar{e}, j)$. Assets pay a risk-free return r_t and labor receives a real wage w_t. Agents are allowed to borrow up to a credit limit $\underline{a}_{t+1}$ in period t. In addition, if an agent survives up to the terminal period ($j = N$), then asset holdings must be non-negative.

There are income and social security taxes in the model economies. Capital and labor income are taxed at the income tax rate τ. Labor income is also subject to a social security tax θ. The social security benefit $b_t(\bar{e}, j)$ is allowed to depend on the agent's age j as well as on an average of past earnings $\bar{e}$. In section 4 this function is specified so as to capture a number of features of the way that benefits are related to both age and earnings history in the US social security system.

For computational purposes we transform variables so as to remove the effects of growth. These transformations are as follows:

$$\hat{a}_t = a_t/A_t, \ \hat{c}_t = c_t/A_t, \ \hat{T}_t = T_t/A_t, \ \hat{b}(\bar{e}, j) = b_t(\bar{e}, j)/A_t, \ \hat{\underline{a}}_t = \underline{a}_t/A_t$$

$$\hat{K}_t = K_t/L_tA_t, \ \hat{L}_t = L_t/L_t, \ \hat{G}_t = G_t/L_tA_t, \ \hat{w}_t = w_t/A_t, \ \hat{r}_t = r_t$$

With these transformations in mind we now describe an agent's decision problem in the language of dynamic programming. At a point in time an agent's state is denoted $x = (\hat{a}, z, \bar{e})$, where $\hat{a}$ is (transformed) asset holdings carried into the period, z is the labor endowment shock and $\bar{e}$ is an average of the agents past labor earnings.[9] Optimal decision rules are functions for consumption $c(x, j)$ and asset holdings $a(x, j)$ that solve the following dynamic programming problem, given that after the terminal period N the value function is set to zero, $V(x, N+1) = 0$.

$$V(x, j) = \max_{(\hat{c}, \hat{a}')} u(\hat{c}) + \beta(1+g)^{(1-\sigma)} s_{j+1} E[V(\hat{a}', z', \bar{e}', j+1) | x]$$

subject to

$$(1) \ \hat{c} + \hat{a}'(1+g) \leq \hat{a}(1 + \hat{r}(1-\tau)) + (1 - \tau - \theta) e(z, j)\hat{w} + \hat{T} + \hat{b}(\bar{e}, j)$$
$$(2) \ \hat{c} \geq 0, \hat{a}' \geq \hat{\underline{a}} \text{ and } \hat{a}' \geq 0 \text{ if } j = N$$
$$(3) \ \bar{e}' = G(\bar{e}, e(z, j)\hat{w}, j)$$

Note that the period budget constraint in the dynamic programming problem is essentially the same as the budget constraint written in terms of untransformed variables. The key differences are that time subscripts are dropped and a term $(1+g)$ is added. Time subscripts are dropped as we focus on steady-state equilibria where transformed factor prices are constant over

[9] We focus on steady states where transformed prices are constant over time and where the distribution of agents over individual states is time invariant. Thus, we do not explicitly include either time or this distribution into a description of an agent's state.

time. The additional term $(1+g)$ appears in the objective of the dynamic programming problem due to the transformation of variables. The credit limit $\underline{a}$ appears without a time subscript. This is because we focus on credit limits that are always proportional to the current wage rate. The third restriction in the dynamic programming problem is the law of motion for the average of past earnings $\bar{e}'$. This law of motion will later be specified to approximate how average indexed earnings are calculated in the US social security system.

3.3 Equilibrium

To state the equilibrium concept, some way of describing heterogeneity in the economy at a point in time is needed. At a point in time agents are heterogeneous in their age j and their individual state x. A probability measure ψ_j defined on subsets of the individual state space will describe the distribution of age j agents over states x. So let $(X, B(X), \psi_j)$ be a probability space where $X = [\underline{a}, \infty) \times Z \times [0, \infty)$ is the state space and $B(X)$ is the Borel σ-algebra on X. Thus, for each set B in $B(X)$, $\psi_j(B)$ is the fraction of age j agents whose individual states lie in B as a proportion of all age j agents. These agents then make up a fraction $\mu_j \psi_j(B)$ of all agents in the economy, where μ_j is the fraction of age j agents in the economy. The distribution of age 1 agents across states is determined by the exogenous initial distribution of labor productivity shocks $(\pi(z))$ since all agents begin life with no assets. The distribution for agents age $j = 2, 3, ..., N$ is then given recursively as follows:

$$\psi_j(B) = \int_X P(x, j-1, B)\, d\psi_{j-1}$$

The function $P(x, j, B)$ is a transition function which gives the probability that an age j agent transits to the set B next period, given that the agent's current state is x. The transition function is determined by the optimal decision rule on asset holding and by the exogenous transition probabilities on the labor productivity shock z.[10]

[10] The transition function is $P(x, j, B) = \mathrm{Prob}(\{z' : (a(x, j), z', \bar{e}') \in B\} \mid z)$, where the relevant probability is the conditional probability that describes the behavior of the Markov process z.

We focus on steady-state equilibria. In a steady state the transformed capital and labor inputs, transfers and government consumption are constant over time. Thus, without the transformation these variables all grow at constant rates. In steady state the distribution of agents across states is stationary or unchanged over time when stated in terms of transformed variables.

Definition : A steady-state equilibrium is $(c(x;j),\ a(x;j),\ \hat{w},\ \hat{r},\ \hat{K},\ \hat{L},\ \hat{G},\ \hat{T},\ \hat{b}(\hat{e};j),\ \theta,\ \tau)$ and distributions $(\psi_1;\psi_2;\ldots;\psi_N)$ such that

1. $c(x;j)$ and $a(x;j)$ are optimal decision rules.
2. Competitive Input Markets: $\hat{w} = F_2(\hat{K};\hat{L})$ and $\hat{r} = F_1(\hat{K};\hat{L}) - \delta$
3. Feasibility:
   (i) $\sum_j \mu_j \int_X (c(x;j) + a(x;j)(1+g))d\psi_j + \hat{G} = F(\hat{K};\hat{L}) + (1-\delta)\hat{K}$
   (ii) $\sum_j \mu_j \int_X a(x;j)d\psi_j = (1+n)\hat{K}$
   (iii) $\sum_j \mu_j \int_X e(z;j)d\psi_j = \hat{L} = 1$
4. Distributions are Consistent with Individual Behavior:
   $\psi_{j+1}(B) = \int_X P(x;j;B)d\psi_j$ for $j = 1;\ldots;N-1$ and for all $B \in B(X)$:
   $\psi_1(B) = \sum_{z \in Z} 1_{\{(0;z;0) \in B\}}\pi(z)$
5. Government Budget Constraint: $\hat{G} = \tau(\hat{r}\hat{K} + \hat{w}\hat{L})$
6. Social Security Benefits Equal Taxes: $\theta\hat{w}\hat{L} = \sum_{j=R}^{N} \mu_j \int_X \hat{b}(\hat{e};j)d\psi_j$
7. Transfers Equal Accidental Bequests:
   $\hat{T} = [\sum_j \mu_j(1 - s_{j+1}) \int_X a(x;j)(1 + \hat{r}(1-\tau))d\psi_j]/(1+n)$

A brief discussion of the equilibrium concept is in order. Equilibrium condition 1 says that agents optimize. Condition 2 says that factor prices equal marginal products. The first feasibility condition is that aggregate consumption, asset holding and government consumption equals the current output plus the capital stock after depreciation. Note that the term $(1 + g)$ appears in this expression so that next period asset holdings are corrected for next periods technology level. The other feasibility conditions are that asset holdings are sufficient to keep the capital stock constant after adjusting for population growth and technological change and that the labor input per capita is normalized to equal 1. Equilibrium conditions 5 and 6 say that income taxes collected are sufficient to pay for government consumption and that social security taxes are sufficient to cover the benefits paid to agents who are past the retirement age. In this formulation social security is funded on a pay-as-you-go basis. The remaining equilibrium condition is that lump-sum transfers equal accidental bequests. This way of treating accidental bequests, while not a realistic feature of US estate taxation policy, serves to highlight the savings variability that is due to the structure of earnings.

4 Calibration

4.1 Parameters of the Model Economies

Table 2: Model Parameters

β	σ	A	α	δ	g	N	R	s_j	n	$\underline{\hat{a}}$
1.011	1.12	.89594	.36	.06	.018	81	46	US 1990	.012	$0, -\hat{w}$

The preference parameters $(\beta; \sigma)$ are set using a model period of one year. The value of the discount factor β as well as the parameter σ governing intertemporal substitution and risk aversion are set equal to the estimates in Hurd (1989). Hurd's estimate of σ is in the range of previous estimates from the microeconomic literature that are reviewed by Auerbach and Kotlikoff (1987) and Prescott (1986).[11] Hurd's estimate of the discount factor exceeds 1. Some economists may regard a discount factor greater than 1 with suspicion. However, it should be noted that, in contrast to infinitely-lived agent models, the overlapping generations model does not impose any theoretical restriction on the value of the discount factor. Thus, the value of this parameter is an empirical question. We note that the value of the discount factor in Table 1 together with the discounting due to increasing mortality rates imply a hump-shaped profile of consumption over the life cycle in the model economies we investigate. Attanasio (1994) estimates that in US household data consumption displays a hump-shaped pattern over the life cycle.

The technology parameters $(A; \alpha; \delta; g)$ are set as follows. The technology level A is normalized so that the wage equals 1.0 when the capital-output ratio is 3.0 and the labor input is normalized to equal 1.0. Capital's share of output α is the estimate in Prescott (1986). The depreciation rate δ is set equal to the estimate in Stokey and Rebelo (1995). The rate of technological

[11] See Rios-Rull (1996) for an analysis of the importance of this parameter in producing realistic capital-output ratios in life-cycle models.

progress g is set to match the US growth rate of output per capita from 1950-92 as reported in the Economic Report of the President (1994).

The demographic parameters $(N; R; s_j; n)$ are set using a model period of one year. Thus, agents are born at a real-life age of 20 (model period 1) and live up to a maximum real-life age of 100 (model period 81). Agents receive retirement benefits at a real-life age of 65 or in model period 46. Thus, we set $R = 46$. The survival probabilities s_j are the actual survival probabilities for men in 1990 as reported in Social Security Administration (1992). The growth rate of the population n is set to equal the average population growth rate in the US from 1950-92 as reported in the Economic Report of the President (1994, Table B32).

The income tax rate (τ) is set to match the average share of government consumption in output. The measure of government consumption is federal, state and local government consumption as reported in the Economic Report of the President (1994, Table B1). As the average ratio was .195 from 1959-93 the tax rate is set at $\tau = .195/(1 - \delta(K/Y))$. The tax rate is greater than .195 as capital income is taxed only after subtracting depreciation.

The credit limit $\underline{a}$ is set at 0 and for comparison purposes at $-\bar{w}$. A credit limit of 0 means that agents cannot borrow, whereas a credit limit of $-\bar{w}$ means that agents can borrow up to one years average earnings in the economy.

4.2 The Structure of Earnings

We consider four models of earnings that differ in stochastic structure. In each of these models earnings are the product of a common real wage per efficiency unit of labor and an agent's labor endowment in efficiency units. We denote y_j and $\bar{y}_j$ the log labor endowment of a specific age j agent and the mean log labor endowment of all age j agents. In model 1 all age j agents are identical and receive the mean log endowment of age j agents. In model 2 we allow agents to differ in log labor endowment at birth. As an agent ages his/her log labor endowment at birth is increased or decreased by the change in mean log labor endowment of the agents in the same age cohort. In model 3 agents differ in log labor endowment $(y_1 = \gamma_1)$ at birth. In addition, each agent receives an idiosyncratic shock ϵ_1 to log labor endowment in each subsequent period. Labor endowments exhibit regression to the age-specific mean at rate γ. Since the regression to the mean parameter will be close to

1, these shocks will be largely permanent shocks. In model 4 an agent's log labor endowment is given by the process in model 3 plus a purely temporary shock ε_2 each period. Each of models 1-4 are described in Table 3 below. Since all the random variables $(y_1, \varepsilon_1, \varepsilon_2)$ are normally distributed, labor endowment is lognormally distributed within an age group.[12]

Table 3: Labor Endowment Process

Model	$\sigma^2_{y_1}$	γ	$\sigma^2_{\varepsilon_1}$	$\sigma^2_{\varepsilon_2}$
Model 1: $y_j = \bar{y}_j$				
Model 2: $y_j = y_{j-1} + (\bar{y}_j - \bar{y}_{j-1})$	.45			
Model 3: $y_j = \gamma_j = \bar{y}_j + \gamma(\gamma_{j-1} - \bar{y}_{j-1}) + \varepsilon_{1j}$	.24	.985	.02	
Model 4: $y_j = \gamma_j + \varepsilon_{2j}$	.24	.985	.02	.01

The parameters of the models are set as follows. First, the values of the mean log labor endowment are selected to match the US cross-sectional age-earnings profile. The values of $\bar{y}_j$ across all four earnings specifications are identical up to a proportional shift. The role of the proportional shift is to make aggregate labor endowment the same across models. The US profile is given in Figure 2.[13]

[Insert Figure 2 Here]

Next, we set all variances and the regression to the mean parameter. In model 2 we set $\sigma^2_{y_1}$ so that the model reproduces the earnings Gini coefficient for the working-age population in the US. Henle and Ryscavage (1980) calculate that the earnings Gini for men averaged .42 in the period 1958-77. When we set $\sigma^2_{y_1} = .45$, the model Gini coefficient within each age group is .36 and the overall Gini for the population under age 65 is .42.

In models 3 and 4 we set the parameters according to the following procedure. First, we set $\sigma^2_{\varepsilon_1}$ and $\sigma^2_{y_1}$ to match estimates of the variance of persistent shocks to log earnings and to match estimates of the earnings inequality

[12] In models 1-3 the labor endowment function is then $e(z, j) = \exp(z + \bar{y}_j)$, where in models 1-3 z is defined as 0, $(y_1 - \bar{y}_1)$ and $(y_j - \bar{y}_j)$ respectively. In model 4, $e(z, j) = \exp(z_1 + z_2 + \bar{y}_j)$ and $z = (z_1, z_2) = ((\gamma_j - \bar{y}_j), \varepsilon_{2j})$.

[13] We multiply median earnings of men in cross-section data in 1990 by labor force participation rates in 1990 for each age group. The earnings data are from Social Security Bulletin (1995, Table 4.B6), whereas participation rates are from Fullerton (1992).

among young people. Second, we set the regression to the mean coefficient γ to match the US earnings Gini. Hubbard et al (1995) estimate that the variance of persistent shocks to log earnings range from .016 for households with a college education to .033 for households with at most a high school education. Our choice of this variance ($\sigma^2_{\epsilon_1} = .02$) implies that a one standard deviation shock increases or decreases earnings by about 15 percent. Our choice of $\sigma^2_{y_1} = .24$ implies that the earnings Gini of the youngest agents in our model economy equals .27. This is in close agreement with the estimates of Lillard (1977) and Shorrocks (1980) who estimate Gini coefficients of .254 and .268 respectively. When we set the regression to the mean parameter $\gamma = .985$ the overall Gini in the model economy equals .42. We note that this value implies slightly less regression towards the mean than the estimates of Hubbard et al (1995) indicate.

The final parameter that we set is the value of the variance of the purely temporary shock to log earnings $\sigma^2_{\epsilon_2}$. We set this variance to .01 in our baseline calculations. This means that a one standard deviation shock increases or decreases earnings by about 10 percent. There are a number of studies that estimate the magnitude of the stochastic component of log earnings variance that is purely temporary. For example, the estimates of this variance in Carroll (1992), Carroll and Samwick (1997) and Storesletten et al (1997) are .027, .044 and .017 respectively. As it is difficult to say what part of these estimates corresponds to measurement error and what part corresponds to actual random temporary variation experienced by households, we consider a smaller variance ($\sigma^2_{\epsilon_2} = .01$) in our baseline calculations and examine sensitivity by also considering much larger values ($\sigma^2_{\epsilon_2} = .04$ and $.09$).[14]

4.3 Social Security

We set social security benefits and the law of motion of average earnings as follows:

[14] We make a finite approximation to each model. In model 2 the shock z takes on 21 values between $-5\sigma_{y_1}$ and $5\sigma_{y_1}$. In models 3 and 4 the permanent shock takes on 21 values from $-6\sigma_{y_1}$ to $6\sigma_{y_1}$. The temporary shock takes on 3 values between $-2.0\sigma_{\epsilon_2}$ and $2.0\sigma_{\epsilon_2}$. Shocks are evenly spaced over these intervals. Transition probabilities are calculated by integrating the area under the normal distribution, conditional on the value of the state.

$$\hat{b}(\hat{e}, j) = \begin{cases} 0 & j < R \\ b + b(\hat{e})/(1+g)^{j-R} & j \geq R \end{cases}$$

and

$$\hat{e}' = \begin{cases} (\hat{e}(j-1) + \min\{e(z,j)\hat{w}, e_{max}\})/j & j < R \\ \hat{e} & j \geq R \end{cases}$$

In this specification benefits are paid beginning at a 'retirement age' R, which we set to $R = 46$ (a real-life age of 65). At a point in time all agents past the retirement age receive the common benefit b in addition to an earnings-related benefit $b(\hat{e})$. The earnings-related benefit is paid out as a constant real annuity. As we transform variables by dividing by the technology level, the extra term $(1+g)^{j-R}$ appears in the denominator even though this component of benefits is constant in real terms for a given person after retirement.

We calibrate the common benefit b based on the hospital and medical component of social security benefits. These benefits are paid to all qualifying members regardless of earnings history. Over the period 1990-94 the hospital and medical payment per retiree averaged 7.72 and 4.70 percent of US GDP per person over age 20.[15] Thus, we set $b = 0.1242 \times (Y)$, where Y is GDP per capita in the model economy.

We calibrate the earnings-related component of benefits using the rules that determine the old-age component of social security benefits. In the US the monthly old-age benefit payment is determined by applying a benefit formula to a person's average indexed monthly earnings (AIME). The variable AIME is an average of a specified number of the highest indexed earnings years that is converted into a monthly basis.[16] Earnings in a given year are indexed so that mean earnings in all years are equal after indexing. In the calculation of the average only earnings up to some maximum earnings level are used. In the model economies we calculate average indexed earnings by averaging all years of earnings rather than just the 35 highest years. In the

[15] The data for benefit payments comes from the Statistical Supplement of the Social Security Bulletin (1996, Tables 8.A1 and 8.A2), whereas the population and GDP data come from the Economic Report of the President (1996), tables B1 and B30.

[16] The current averaging period is 35 years.

model economies the maximum earnings that goes into the calculation of the average is denoted e_{max}. We set this at 2:47 times average earnings. This number is the maximum creditable earnings in the US social security system 1990-94 as a fraction of average earnings.[17]

In the US, old-age bene¯ts are a concave function of a person's AIME. In 1994, the old-age bene¯t equaled 90 percent of the ¯rst \$422 of AIME, 32 percent of the next \$2123 of AIME plus 15 percent of AIME over \$2545. The dollar amounts at which the percentages change are called bend points. We set old-age bene¯ts in the model economies by applying the US formula given above to an agent's average indexed earnings $\acute{e}$ at retirement. This requires determining the bend points in the model economy. We set bend points in the model economy equal to the observed bend points in the US economy over the period 1990-94. The two bend points averaged .20 and 1.24 times average earnings.[18] We note that, after amendments to the Social Security legislation in 1977, bendpoints have been automatically increased in proportion to the average wage levels. As we specify how bene¯ts are determined by past earnings, the social security tax rate is endogenously determined within the model to satisfy condition 6 in the de¯nition of equilibrium.

5 Results

This section is organized in three subsections. First, some of the general features of the model economies are documented. Second, the main results of the paper are presented and discussed. Third, some additional questions are addressed to provide further perspective on the model economies as a description of cross-sectional saving behavior. All of the details of how the results reported here are computed are described in the Appendix.

5.1 General Features of the Model Economies

Before discussing the properties of the model economies, we state our measures of wealth and saving. The concept of household wealth we use in the model economies at a point in time t is simply net asset holdings, a_t. This

[17] Social Security Handbook (1995) and Social Security Bulletin (1993, 1994) for average earnings.

[18] Social Security Bulletin (1993, 1994).

choice reflects the fact that the concept of wealth typically measured in the US data is one that includes neither social security wealth nor the value of human capital. The notion of saving used is then simply the change in wealth holding across a period. Thus, saving for a particular household is $a_{t+1} - a_t$. Given the budget constraint, the saving of an age j household in state $x = (a_t; \epsilon; z)$ can also be calculated as after-tax income plus transfers less consumption:

$$a_{t+1} - a_t = a_t r_t (1 - \tau) + (1 - \theta - \tau) e(z; j) w_t + T_t + b(\epsilon; j) - c_t$$

This measure of saving is equal at the aggregate level to both economy-wide net saving (S) and private saving. This is due to the fact that public saving is always equal to zero.

Table 4: Descriptive Statistics

	K/Y	S/Y	r	Transfer Wealth (%)	Income Gini	Percentage of Income Received by the Top 1% 5% 10% 20%
Model 1						
$\underline{a} = 0$	3.00	0.090	6.0	64.6	0.23	1.5 7.7 15.3 30.0
$\underline{a} = -\bar{w}$	2.93	0.086	6.3	67.8	0.23	1.5 7.8 15.3 30.3
Model 2						
$\underline{a} = 0$	3.05	0.092	5.8	63.7	0.42	5.8 19.3 30.8 47.6
$\underline{a} = -\bar{w}$	2.96	0.089	6.1	66.0	0.43	5.9 19.6 31.3 48.3
Model 3						
$\underline{a} = 0$	3.11	0.094	5.5	60.3	0.44	6.4 20.7 32.5 49.5
$\underline{a} = -\bar{w}$	3.03	0.091	5.9	62.6	0.45	6.5 21.0 33.0 50.1
Model 4						
$\underline{a} = 0$	3.12	0.094	5.5	60.1	0.44	6.4 20.7 32.6 49.6
$\underline{a} = -\bar{w}$	3.04	0.091	5.8	63.1	0.45	6.5 21.0 33.1 50.2

Table 4 summarizes a number of the aggregate properties of the model economies. The reader will recall that these economies differ in the stochastic structure of labor earnings as described in section 4.2. Table 4 shows that all of the model economies produce similar values in steady state for the capital-output ratio (K/Y), the saving-output ratio (S/Y) and the net return to capital (r). The annual net return is about 6% before tax and about 4.5%

after tax as the income tax rate in each of the model economies is slightly more than 20%.[19]

For many economists, a plausible model of wealth accumulation must agree with the fact that a substantial fraction of wealth accumulation can be attributed to the receipt of intergenerational transfers. For this reason Table 4 documents the share of transfer wealth in aggregate wealth in the model economies by applying the accounting framework developed in Kotliko® and Summers (1981) to separate total wealth into life-cycle and transfer wealth components. Transfer wealth in the model economies is equal to the current value, accumulated using the after-tax interest rate, of all past intergenerational transfers received by currently living agents. Kotliko® and Summers (1981) calculate that for the US economy transfer wealth makes up 81 ¡ 132 percent of total wealth. Gale and Scholz (1994) estimate that transfer wealth is at a minimum between 52 and 64 percent of total wealth, depending on whether or not college expenses are considered as part of transfer wealth. While we do not attempt to o®er a complete model of all the distinct sources of intergenerational transfers, we note that in our model economies intergenerational transfers coming from accidental bequests produce a signi¯cant amount of transfer wealth.[20]

Table 4 also describes the income distribution properties of the model economies. To take the predictions of a model economy seriously for the cross-section saving fact that is the focus of this paper, a model must have agents with income multiples comparable to those in the data. Model 1 can be criticized in this regard, but the criticism does not extend to models 2-4 where agents di®er in earnings abilities within an age group. It would also be desirable for the model economy to have a similar age structure and income distribution to those in the US economy. Models 1-4 all have a realistic age structure as they are calibrated to match the US population growth and mortality rates. We will now address how the income distribution properties of the model economies compare to those in the US economy.

[19] We note that the 4:5 percent after tax return to capital is precisely the value that Kotliko® and Summers (1981) calculate for the US economy.

[20] For a more detailed discussion of issues related to transfer wealth in economies of this type we refer the reader to the discussion in Huggett (1996, section 5.2). We note here that although the calculations in Huggett (1996) abstract from growth in output per capita, the calculations in Table 4 do not.

In the model economies the concept of income used to compute the properties of the income distribution is labor earnings and capital income before tax plus the value of all transfers received in a model period. A number of recent studies have calculated measures of the concentration of income such as the Gini index for the US economy using this definition of income. Using data from the Current Population Survey, Gramlich et al (1993) find that the income Gini was .42 in 1980, .46 in 1985 and .48 in 1990. Using data from the Survey of Consumer Finances, Avery and Kennickell (1993) find that the household income Gini was .46 in 1983 and .47 in 1986. These measures are similar to those produced in model economies 2-4. In addition, models 2-4 are able to approximate the percentage of income received by the top 20% of US households. Avery and Kennickell (1993) report that the top 20% received 50:1 percent of income in 1983 and 51:2 percent of income in 1986. The model economies do, however, produce lower levels of income concentration in the extreme upper tail (top 1%) than those reported by Avery and Kennickell (1993). Part of the reason for this is the fact that models of this type do not concentrate sufficient wealth in the upper tail of the wealth distribution. For a discussion of the wealth distribution properties of these model economies see Huggett (1996).

5.2 Saving Rates and Income in Model Economies

Table 5 presents the main findings of the paper. The table presents the saving implications of the model economies and for comparison purposes the averages of the US saving rates graphed in Figure 1. Saving rates at different income multiples are calculated by taking a 10 percent band around each income multiple and then dividing total saving of agents in the band by total income of agents in the band. Income is defined as earnings after social security taxes plus interest income and transfers.

Table 5: Saving Rates at Multiples of Mean Income

Income Multiple	US*	Model 1	Model 2		Model 3		Model 4	
		$\underline{a}=0$	$\underline{a}=0$	$\underline{a}=-w$	$\underline{a}=0$	$\underline{a}=-w$	$\underline{a}=0$	$\underline{a}=-w$
.25	-19.3	-	-3.6	-16.6	-5.8	-16.9	-5.8	-20.1
.50	-1.3	-11.6	-1.0	-2.0	-0.4	-2.2	-0.2	-2.5
.75	4.8	-5.2	4.7	6.6	5.6	6.2	5.3	5.7
1.0	7.9	9.8	8.1	8.7	11.0	11.7	10.8	11.1
1.5	13.0	25.8	18.0	17.6	18.1	18.2	18.0	18.1
2.0	16.5	-	21.1	19.7	22.6	22.7	22.5	22.6
3.0	22.4	-	25.8	26.1	27.4	27.5	27.9	27.9
4.0	27.1	-	27.6	27.0	30.7	30.7	31.3	31.3
7.0	37.3	-	32.0	32.8	34.9	35.2	35.8	35.9
10.0	39.2	-	27.1	28.1	32.0	31.4	33.1	32.6

* Averages from Kuznets (1953) and Projector (1968)

The findings in Table 5 are that a variety of earnings structures imply that in cross-section data saving rates are negative for low income multiples and increase as the multiple of mean income increases. In fact, saving rates increase monotonically even into the top 1 percent of the income distribution. These households have incomes of 7 times mean income. Furthermore, model economies 2-4 all produce quantitatively similar results that roughly approximate the magnitudes of the saving rates observed in US data.

These findings are quite interesting as the earnings structures differ significantly across the model economies. This suggests that features that are common to model economies 2-4 may be key to generating this stylized fact of saving behavior. Three features that are common across these model economies are (i) age differences, (ii) largely permanent differences in earnings and (iii) a social security system. In the remainder of this section we attempt to understand the results in Table 5 at a deeper level. We focus the analysis on models 2-4 as only these models produce enough income heterogeneity to match up to the US data.

5.2.1 Understanding Saving Rates in Model 2

The reader will recall that in model 2 agents differ at birth in the level of earnings and that these differences in earnings ability are preserved over the life cycle. Figure 3 graphs the saving rates across age groups for agents with di®erent earnings abilities. Figure 3 shows that agents in the top 10% of the income distribution for their age group save at higher rates before retirement than agents in the middle (50 ¡ 60%) of the income distribution or in the bottom 10% of the income distribution. Given that all agents have identical and homothetic preferences and that earnings are identical up to a proportional shift, one might have guessed that saving rates would be identical within age groups. This is not true as the US social security system gives relatively high annual bene¯ts to agents with low earnings and relatively low annual bene¯ts to agents with high earnings (see section 4.3). Thus, high earnings ability agents will save at high rates before retirement and low earnings ability agents will save at low rates before retirement. Both the borrowing constraint and the transfer due to the taxation of accidental bequests are additional reasons for saving rate heterogeneity.[21] Figure 3(a) shows that these features lead to substantially di®erent saving rates for agents in the same age group, even though these agents have identical and homothetic preferences.

[Insert Figure 3 Here]

We are now ready to describe how the behavior in model 2 aggregates to generate the results in Table 5. First, the agents at low multiples of mean income are largely the very youngest agents and also the agents just before the retirement age. This fact can be read o® of the cross-sectional age-income distribution described in Figure 3(b). These agents have zero or negative saving rates as Figure 3(a) shows. When the credit limit is set to allow borrowing these saving rates are even smaller as the young dissave. As the multiple of mean income increases the composition of the agents changes. In particular, there are more agents above age 25 when saving rates start to increase and there are fewer agents just before the retirement age. Both of these considerations dictate that the saving rate of agents in this income group should increase. At higher income multiples the composition changes to include higher fractions of middle-age agents and higher fractions of agents

[21] As we discuss in the next subsection, the structure of earnings in models 3-4 provide additional reasons for saving rate heterogeneity within age groups. Clearly, we have abstracted from a number of features of actual economies which could generate even more saving rate heterogeneity within age groups (e.g. di®erences in preferences, mortality rates, earnings pro¯les and household composition as well as other sources of shocks).

in the upper tail of the income distribution for their age group. Figure 3(a) illustrates that these e®ects lead the saving rate of higher income groups to increase.

5.2.2 Understanding Saving Rates in Models 3 and 4

We now investigate the saving behavior in the models with temporary and permanent shocks. One way to understand why the results in Table 5 for models 3-4 are so similar to those for model 2 is to produce the analogue of Figures 3(a)-(b) for models 3 and 4. We do this in Figure 4(a)-(b) which concentrates on model 4. We note that the corresponding pictures for model 3 are almost identical to those in Figure 4. A quick look at Figure 4 shows that both the age-saving rate distribution (Figure 4(a)) and the age-income distribution (Figure 4(b)) are similar to the distributions in Figure 3. Thus, it is not surprising that the saving rates at di®erent multiples of mean income are also quite similar.

One di®erence between model 2 on the one hand and model 4 on the other hand is that the spread in saving rates within age groups between high and low income households is larger in Figure 4 than in Figure 3 before the retirement age. This is due to the presence of permanent and temporary shocks. Following the standard intuition, positive temporary earnings shocks will be largely saved and negative temporary shock will be largely dissaved. Thus, within an age group, saving rates of high income groups will be higher and saving rates of low income groups will be lower provided positive shocks are concentrated in high income groups and negative shocks are concentrated in low income groups. A similar result may also hold for the case of positive and negative permanent shocks. This is the case as a retirement period with zero labor earnings implies that all earnings shocks are in a sense temporary.[22]

[Insert Figure 4 Here]

The above reasoning restates the traditional explanation for why in cross-section data high income households save at higher rates than low income households. The interested reader will ¯nd this explanation in Vickrey (1948

[22] If positive permanent shocks are more likely after a positive permanent shock, then the above intuition need not work. The earnings process considered here is mean-reverting. Thus, individuals above the mean will on average regress towards the mean.

pp. 287-288), Duesenberry (1949 pp. 76-89), Modigliani and Brumberg (1954 pp. 406-25), Friedman (1957 pp.39-40) as well as in many textbooks. It is therefore interesting to note that the results in Table 5 for models 3 and 4 indicate that the inclusion of purely temporary earnings shocks lowers the saving rate of low income groups and raises the saving rate of high income households only very slightly - an effect of a few percentage points.[23]

We note that within our model economies both parts of the traditional explanation turn out to be true: positive temporary earnings shocks are largely saved and households with positive temporary shocks are concentrated in high income groups with the opposite pattern holding for negative temporary shocks. In particular, we calculate that in model 4 the median marginal saving rate out of an increase in wealth in cross section data is about 96 percent and the distribution is skewed to the left.[24] The median marginal saving rate decreases with age when borrowing is allowed starting from 97.8 percent for agents age $20 - 30$. When borrowing is not allowed, the median marginal saving rate increases from 96.2 for agents age $20 - 30$ to 97.3 for agents age $30 - 39$ and then monotonically decreases with age. Agents age $70 - 79$ and 90+ have median marginal saving rates between $80 - 90$ percent and below 50 percent respectively, regardless of the setting of the borrowing limit.[25]

The relative unimportance of temporary shocks also holds when temporary shocks are much larger. For example, in Table 6 we calculate saving rates when we double the standard deviation of the temporary shock so that

[23] When we used a five-point approximation to the distribution of the temporary shocks instead of a three-point approximation described in section 4.2 the results were almost exactly the same. The finer approximation considers more extreme values of the temporary shock ranging from $-3\sigma_{\varepsilon_2}$ to $3\sigma_{\varepsilon_2}$ rather than from $-2\sigma_{\varepsilon_2}$ to $2\sigma_{\varepsilon_2}$ in the three-point approximation.

[24] The marginal saving rate in state $(x; j)$ is defined as $(da(x; j)^+/da)(1 + r(1 - \tau))^{-1}$. Thus, marginal saving rates are right-hand derivatives of the computed optimal decision rule with respect to asset holdings which are multiplied by $(1 + r(1 - \tau))^{-1}$ to get marginal savings out of an increase in wealth.

[25] When the borrowing limit is set to zero there is substantial dispersion in marginal saving rates among agents age $20 - 29$ as for many agents the borrowing limit binds. Marginal saving rates are zero for these agents. When borrowing is allowed, marginal saving rates are above 95 percent for essentially all agents in this age group. Median marginal saving rates are not monotone in either quintiles of the income or the wealth distribution. This is because agents with high income or wealth levels are typically older.

a one standard deviation shock raises or lowers earnings by about 20 percent (i.e. set $\sigma^2_{\epsilon_2} = .04$). The result is to lower the saving rate of low income households and raise the saving rate of high income households by only about an additional percentage point. It is only when we triple the standard deviation that temporary shocks have a quantitatively important effect on cross-section saving rates. However, note that purely temporary shocks of this magnitude are above the estimates in the literature previously cited in section 4.2. In Table 6 we have also calculated the importance of temporary shocks in another way. This alternative calculation excludes all households experiencing high or low temporary shocks. Thus, to determine the importance of temporary shocks one can simply compare saving rates at given income multiples when all households are included in the data to the saving rates that would exist when households receiving high or low temporary shocks are removed. This comparison leads to the same conclusion obtained previously: purely temporary earnings shocks only begin to matter quantitatively when these shocks are much larger than existing estimates.

Table 6: Saving Rates at Multiples of Mean Income in Model 4
Higher Variances of the Temporary Shock ($\underline{\hat{a}} = 0.$)

		Including All Agents			Excluding Agents w/ Temporary Shocks		
Income Multiple	US*	Variance ($\sigma^2_{\epsilon_2}$) .01	.04	.09	Variance ($\sigma^2_{\epsilon_2}$) .01	.04	.09
.25	-19.3	-5.8	-6.7	-8.3	-5.7	-5.6	-5.3
.50	-1.3	-0.2	-1.1	-2.8	5.2	4.8	4.6
.75	4.8	5.3	4.5	4.1	11.3	10.8	10.5
1.0	7.9	10.8	10.3	10.1	14.8	14.3	13.9
1.5	13.0	18.0	17.9	17.4	19.7	18.9	18.3
2.0	16.5	22.5	22.8	22.6	23.3	22.6	22.0
3.0	22.4	27.9	28.1	29.9	27.3	27.0	26.7
4.0	27.1	31.3	31.6	33.9	30.6	30.3	30.1
7.0	37.3	35.8	36.5	39.2	34.8	34.5	33.7
10.0	39.3	33.1	34.4	38.9	31.7	31.4	29.7

* Averages from Kuznets (1953) and Projector (1968)

5.2.3 Are Saving Rate Di®erences Across Age Groups Key?

To close this section we ask the following question. To what degree do the results in Table 5 depend on di®erences in saving rates within age groups versus simply di®erences in saving rates across age groups? To answer this question we set saving rates of all agents within an age group equal to the average for the age group, while holding the age-income distribution unchanged. This is accomplished by altering the decision rule $a(x; j)$ in an obvious way. The results are shown in Table A1 in the Appendix. The ¯ndings are that saving rates now tend to be much higher and even positive at the lowest income multiples and that saving rates at high income multiples now tend to be much smaller. Thus, it can be concluded that the features of the model economies that lead to di®erences of saving rates within an age group are quantitatively quite important in producing the results in Table 5.

5.3 Additional Questions

The previous section demonstrated that calibrated life-cycle economies which capture only a few ways in which households di®er at a point in time are able to approximate the pattern of saving rates observed in US cross-section data. Thus, the results of the previous section provide an answer to the questions posed in the beginning of the paper. Below, we pose a series of additional questions. Answers to these questions should shine some light on how seriously the models previously analyzed should be taken as a description of why saving rates increase with income in US cross-section data.

Questions:

1. Are the results sensitive to the absence of social security?
2. Are the results sensitive to the way in which accidental bequests are distributed?
3. Do the model economies match other facts of the distribution of saving?

5.3.1 Sensitivity to Social Security

In this section we address question 1 stated above. The motivation comes from three main sources. First, there have been large changes over time in US social security payments. In particular, before 1935 the US social security system did not exist. Since then the magnitude of government supported transfers has increased substantially. Second, the results in Table 5 could be quite sensitive to changes in the importance of social security payments. This is because saving rate di®erences within age groups are quantitatively important in producing these results and because social security is one of the key features causing saving rate di®erences within age groups. Third, the data from Kuznets (1953, Table 48) for 1929 show that high income households saved as a group a substantially higher fraction of income than low income households even before the US social security system was established.[26] Thus, if the results in Table 5 were particularly sensitive to changes in social security payments, then the very parsimonious explanation of the US cross-section saving fact o®ered here would be much less convincing.

In this section we eliminate all social security payments from the model economies. Thus, social security taxes and bene¯ts are set to zero. This is an extreme way of examining model sensitivity. A social security system that is intermediate between no social security system and the system analyzed in Table 5 is likely to produce properties between these extremes.

Table 7: Saving Rates at Multiples of Mean Income:
No Social Security

Income Multiple	US¤	Model 2		Model 3		Model 4	
		$\underline{a}=0$	$\underline{a}=-w$	$\underline{a}=0$	$\underline{a}=-w$	$\underline{a}=0$	$\underline{a}=-w$
.25	-19.3	-40.9	-50.6	-42.7	-54.4	-54.9	-70.0
.50	-1.3	2.0	0.4	-0.8	-2.1	-1.5	-2.9
.75	4.8	13.8	12.1	13.1	13.2	12.6	12.9
1.0	7.9	21.5	20.9	20.2	20.4	20.1	20.4
1.5	13.0	24.6	25.0	28.1	28.2	27.7	28.0
2.0	16.5	25.5	26.7	31.3	31.5	31.0	31.3
3.0	22.4	30.5	31.4	33.2	33.8	33.6	34.2
4.0	27.1	30.3	30.8	34.9	35.5	35.5	36.0
7.0	37.3	34.3	34.7	37.8	38.3	38.4	38.8
10.0	39.2	33.2	36.0	36.0	36.1	37.7	37.7

[26] Kuznets provides two separate estimates in 1929. We list in Table 1 Kuznets' preferred estimate. Both estimates reveal large di®erences in saving rates at di®erent income multiples.

* Averages from Kuznets (1953) and Projector (1968)

The ¯ndings are presented in Table 7. Table 7 shows that, without social security, saving rates still increase strongly with the multiple of mean income. Why is this so? To answer this question consider Figure 5. Figure 5(b) shows that the agents at the lowest income multiples are largely well into retirement. It is clear from Figure 5(a) that these agents are dissaving. Thus, it is clear why these models produce lower saving rates at low income multiples than in the same models with social security. It is simply that with social security the lowest income households are mainly the youngest agents, whereas in the absence of social security these agents are mainly the oldest agents. It is interesting to note that Boskin, Kotliko® and Knetter (1985) have documented that exactly this type of change in the age-income distribution occurred in the US economy in the period 1968-84. The explanation that they give is that the growing importance of social security was responsible for the shift in the age-income distribution.

[Insert Figure 5 Here]

To what degree do the results in Table 7 depend on di®erences in savings rates within age groups versus di®erences in saving rates across age groups? The answer to this question is given in Table A2 in the appendix. In this table all saving rates within an age group are set equal to the average of the group while holding the age-income distribution constant. The ¯ndings are that saving rate di®erences across age groups are the dominant cause of why saving rates increase with income in Table 7.[27]

5.3.2 Sensitivity to the Distribution of Accidental Bequests

We address the sensitivity to the passing of accidental bequests in two ways. First, in contrast to our baseline model, we allow the government to tax away all accidental bequests. Thus, agents never receive a bequest. While this is an unrealistic description of US estate taxation policy, it does address the issue of whether our previous results are being generated by the passing

[27] Computations of the importance of saving rate di®erences within age groups versus across age groups could also be done with US data. It would be interesting to see if the within-age-group e®ect for the cross-section facts has become more important as social security payments have become more important.

of bequests. The ¯ndings are that saving rates still increase with income in cross-section and that quantitatively the results are very similar to those previously presented in Table 5.

A second way of addressing sensitivity is to analyze more realistic ways of passing bequests. In our view, modeling the passing of bequests from parents to children as well as modeling the relation between earnings across generations is needed. Modeling this ¯rst point requires at a minimum that children forecast the probability distribution of future bequests. Technically, this means that the relevant state variable of children must be expanded to include the parents state variable. In general, this is a computationally unfeasible problem without substantial advances in computer technology, computational methods or problem formulation.[28] One experiment that is currently feasible is to examine only model 2, the deterministic earnings model, under the assumption that (i) parents pass along their earnings ability to their children and (ii) children of a given earnings type receive each period the average bequests of the parents of their own type. This formulation succeeds in given large bequests to high earnings agents and the opposite to low earnings agents. The results of this exercise are displayed in Figure 6. We ¯nd that saving rates still increase with income within an age group before the age of retirement and that saving rates increase roughly monotonically with income until very high income multiples even though the magnitudes of this relationship are a®ected by this alternative way of passing bequests.

[Insert Figure 6 Here]

5.3.3 Other Saving Facts

In this section we focus on whether our model economies match other facts of the US distribution of saving rates. In particular, we look at the distribution of saving rates by age and income. Figures 3-5 from our model economies all carry the prediction that (i) saving rates are hump shaped in that middle age households save at greater rates than either young or old households and (ii) before the retirement age high income households within an age group

[28]A formulation of this problem would have two opposing e®ects. First, saving rates will be lower for children with high wealth parents given the possibility of receiving a large bequest. Second, saving rates will be very high when a large bequest is received. The overall e®ect on Figure 1 is therefore not clear.

have higher saving rates than low income households whereas the opposite pattern holds after retirement.

Attanasio (1994, Table 2.10) provides evidence on both of these points using data from the 1990 Consumer Expenditure Survey. His ¯ndings for median saving rates for the second, third and fourth income quartile as well as for the whole sample are graphed in Figure 7.[29] The ¯ndings on the ¯rst point above agree with the pattern in Figures 3-5 in that saving rates by age are hump shaped. However, it is the case that saving rates for households beyond age 65 are positive rather than negative as in Figures 3-5.

[Insert Figure 7 Here]

Here we think that it is important to raise some points of measurement. As in our paper, Attanasio's measure of saving does not take account of social security or human capital. So this does not explain any di®erences. However, there are a couple of important di®erences in our saving measures. In particular, employer contributions to private pensions and the purchase of consumer durables are not considered to be saving nor is the wealth decumulation in private pensions after the retirement age considered to be dissaving in Attanasio's work. Pensions and consumer durables should be capitalized and the change in the value of these assets should be considered to be saving. This should magnify the hump in saving rates as consumer durable purchases are concentrated among the young and as the accumulation phase of pensions occurs before the retirement age and the decumulation phase occurs after retirement.[30] There is a debate in the literature on saving as to whether or not retired households dissave. The review of this literature by Hurd (1990) takes the position that the best available evidence indicates that retired US households do dissave as a group even when the wealth concept is bequeathable wealth.

The Attanasio's ¯ndings reported in Figure 7 on the second point is that within all age groups high income households save a higher fraction of income than do low income households. This is in agreement with Figures 3-5 except for households past the retirement age where the pattern is precisely the

[29] We exclude from Figure 7 the saving rates of the lowest income quartile which are always very negative.

[30] Ando and Kennickell (1987) and Bosworth et al (1991) attempted to treat pensions in this way and found that this changed saving rates as we indicate.

opposite. If the lack of strong dissaving among the retired households with high income proves to be a robust empirical ¯nding, then it seems clear to us that the model economies that we consider must be missing some key features of reality (e.g. an intentional bequest motive).[31] We note two things. First, as Attanasio measures saving as income less consumption it is the case that error in measuring income will act to overstate the saving rates of high income groups and understate the saving rates of low income groups. Probably this is part of the reason behind the very high saving rates of the highest income quartile and the very low saving rates of the lowest income quartile in all the age groups in Attanasio's work. Second, when Attanasio (1994, Table 2.12) measures saving rates by age for households with di®ering education levels it is the case that the retired households with the highest education level do dissave and they dissave a higher fraction of income than groups with lower education levels. This holds even without correcting for pensions or durables. This is the pattern that our model predicts.

6 Conclusion

The paper asks whether the large di®erences in saving rates across di®erent income groups observed in US cross-section data are puzzling relative to standard theory. The main ¯nding of this paper is that the calibrated model economies that we consider imply the type of behavior that is observed in Figure 1. The key features of the model economies that produce this savings behavior are age and relatively permanent earnings di®erences across agents together with the structure of the US social security system. We ¯nd that neither preference heterogeneity nor a speci¯c pattern of earnings shocks are essential to produce this result. In fact, we ¯nd that purely temporary earnings shocks of the magnitude estimated in US data have only a modest contribution to decreasing the savings rate at low income levels and increasing the savings rate at high income levels. This is true even when a one standard deviation temporary shock changes earnings by as much as 20%. Clearly, these ¯ndings do not imply that features that we have abstracted from, such as heterogeneity in preferences, household composition, earnings pro¯les, mortality rates and so forth are not important features of actual economies impacting saving rates. The ¯ndings only imply that such

[31] Dyn n e al 19 8) nv st ga e t e s vi gs f t e a ed n d ta l.

heterogeneity is not essential to produce the large di®erences in saving rates that are observed.

One question to ask is whether the ¯ndings described above were due mainly to features of the model economies that lead to di®erences in saving rates across age groups or to di®erences in saving rates within age groups. We ¯nd that the features of the model economies that lead to saving rate heterogeneity within an age group (e.g. social security and earnings shocks) are quite important. In light of this result, it is interesting to recall that even without any social security system the model economies still imply that high income households save at very high rates, low income households dissave and that saving rates increase monotonically with income. In this case, the role played by the fact that saving rates di®er across age groups is very important.

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7 Appendix 1

Equilibria are computed using the following algorithm:

1. Guess the value for capital $\hat{K}$, transfers of accidental bequests $\hat{T}$ and the social security tax θ.
2. Compute factor prices: $\hat{w} = F_2(\hat{K};1)$ and $\hat{r} = F_1(\hat{K};1) - \delta$. Obtain the income tax rate from conditions 5 in the definition of equilibrium. Obtain social security tax collections implied by the guess of $\hat{K}$ and θ. Parameterize the social security scheme by calculating the first component of the social security payments and the bend points for the history dependent component.
3. Calculate optimal decision rules: $a(x;j)$ and $c(x;j)$.
4. Calculate values of $\hat{K}$ and $\hat{T}$ that are implied by $a(x;j)$. Calculate aggregate social security payments.
5. If the values guessed for $\hat{K}$ and $\hat{T}$ in step 1 and the value in step 2 for social security tax collections equal the implied values in step 4 , then this is a steady-state equilibrium. Otherwise, try new values and repeat these steps.

Steps 3 and 4 above need to be explained. To carry out step 3, we work on the first order conditions of the household's decision problem:

$$U'(\hat{c}_j)(1+g) \geq \beta s_{j+1}(1+g)^{(1-\sigma)} E\left[V_1(\hat{a}', z', \hat{e}'; j+1) | x\right]$$

$$\hat{c}_j = \hat{a}(1+\hat{r}(1-\tau)) + \hat{w}e(z;j)(1-\theta-\tau) + \hat{b}(\hat{e};j) + \hat{T} - (1+g)\hat{a}'$$

$$\begin{aligned} V_1(\hat{a}', z', \hat{e}'; j+1) &= U'[\hat{a}'(1+\hat{r}(1-\tau)) + \hat{w}e(z'; j+1)(1-\theta-\tau) \\ &\quad + \hat{b}(\hat{e}'; j+1) + \hat{T} - (1+g)a(\hat{a}', z', \hat{e}'; j+1)](1+r(1-\tau)) \end{aligned}$$

The above conditions, together with the requirement that $a(x;N) = 0$ define a recursive algorithm for the computation of optimal decision rules for asset holdings at every age and state $x \in X$. With $a(x;j)$ at hand, the decision rule $c(x;j)$ is determined from the budget constraint.

To implement this algorithm on a computer, we require the first order condition to hold exactly on gridpoints defined over the state space. We put 301 evenly spaced gridpoints on the asset variable a, between 21 and 61 gridpoints on the shock z and 5 gridpoints on the social security variable é. Given that there are 81 possible periods of life, we then calculate decision rules $a(x; j)$ at over 2:5 million gridpoints. The solution to the ¯rst order condition at a particular grid point x is our approximation of $a(x; j)$. The approximation is obtained using a simple bisection procedure to solve the Euler equation.[32] Between gridpoints the decision rules are given by a linear interpolation. Thus, decision rules are piecewise-linear functions.

Step 4 requires for aggregation purposes the computation of the probability measures $\tilde{A}_1$, $\tilde{A}_2$,..,$\tilde{A}_N$. Instead, we perform the equivalent aggregation procedure through simulation. We simulate realizations of the state and the decision variables for a large number of agents over their life cycle using the computed decision rules, the law of motion for the state variables and the structure of earnings uncertainty. In particular, we simulate 20,000 agents over their life cycles in the cases of Models 2 and 3, and 40,000 agents in the case of model 4. Higher numbers of agents turned out to be irrelevant in the sense that they change neither the aggregate statistics of the model economies nor factor prices. Since the values for assets and average past earnings are not restricted to fall on gridpoints, we use linear interpolation to evaluate the decision rules o® gridpoints. Once the corresponding equilibrium factor prices are computed, we simulate again to construct a sample of saving and income in order to compute the saving facts and the distributional properties of the model economies reported in the paper.[33]

[32] See Huggett (1993) for a more detailed exposition on this algorithm applied to economies with in¯nitely-lived agents. Coleman (1990) describes a similar version of this algorithm.

[33] Here we draw 100 repeated samples of the numbers of agents described previously. For each sample we calculate the relevant facts and then average these facts across samples. We average across samples as some of the distributional facts (e.g. the saving facts reported in Table 5) are slightly sensitive to the particular sample drawn even though the individual samples contain more than a million agents.

8 Appendix 2

Table A1

Saving Rates at Multiples of Mean Income

[Equal Savings Rates within Age Groups]

Income Multiple	US¤	Model 2		Model 3		Model 4	
		$\underline{\hat{a}}=0$	$\underline{\hat{a}}=-w$	$\underline{\hat{a}}=0$	$\underline{\hat{a}}=-w$	$\underline{\hat{a}}=0$	$\underline{\hat{a}}=-w$
.25	-19.3	5.5	-7.9	10.5	0.4	10.8	-1.3
.50	-1.3	0.0	1.0	4.5	4.7	5.1	4.8
.75	4.8	8.6	10.2	11.3	11.6	11.3	11.2
1.0	7.9	10.6	10.4	15.0	15.0	14.9	14.6
1.5	13.0	18.5	17.6	18.8	18.6	18.7	18.4
2.0	16.5	19.4	17.4	20.6	20.5	20.4	20.3
3.0	22.4	21.4	21.1	21.4	21.4	21.7	21.6
4.0	27.1	21.2	20.5	22.0	22.1	22.3	22.3
7.0	37.3	24.2	24.8	22.8	23.4	23.2	23.5
10.0	39.2	25.5	26.7	23.8	23.8	24.0	24.4

* Averages from Kuznets (1953) and Projector (1968)

Table A2

Saving Rates at Multiples of Mean Income: No Social Security

[Equal Saving Rates within Age Groups]

Income Multiple	US¤	Model 2		Model 3		Model 4	
		$\underline{\hat{a}}=0$	$\underline{\hat{a}}=-w$	$\underline{\hat{a}}=0$	$\underline{\hat{a}}=-w$	$\underline{\hat{a}}=0$	$\underline{\hat{a}}=-w$
.25	-19.3	-37.3	-44.4	-35.6	-42.6	-45.8	-54.7
.50	-1.3	5.7	3.8	7.0	4.6	6.9	4.3
.75	4.8	15.2	13.0	17.6	17.0	17.4	17.1
1.0	7.9	22.1	21.2	22.5	22.2	22.6	22.2
1.5	13.0	24.2	24.0	27.4	27.3	27.1	27.1
2.0	16.5	24.6	25.2	29.0	29.1	28.8	28.9
3.0	22.4	29.2	29.5	29.9	30.3	29.9	30.3
4.0	27.1	28.6	28.8	30.5	30.9	30.5	30.9
7.0	37.3	32.5	32.9	31.5	31.8	31.4	31.8
10.0	39.2	32.9	31.4	30.6	31.2	31.1	31.6

* Averages from Kuznets (1953) and Projector (1968)