Macro II

Homework 5- Asset Pricing

1. (Lucas Tree Economy)
   (a) Compute the stock price function and the risk-free interest rate function in a one-tree version of the Lucas asset pricing model. Assume the dividends are iid and equal 1 and 2 with equal probability. Preferences are CRRA w/ risk aversion coefficient 2.0 and discount factor .96.
   (b) Define the gross realized return on stocks and bonds as \( R^s(y, y') \) and \( R^b(y, y') \), where \( y \) is the current state and \( y' \) is tomorrow’s realized state. This is just the value of the asset tomorrow plus dividend divided by the value of the asset today.

   Using this definition, calculate the conditional expected gross return \( R^i(y) = E[R^i(y, y') | y] \) in each state for \( i = s, b \). Also calculate the unconditional gross returns \( E[R^i(y)] \). To calculate unconditional returns use the unconditional (steady state) probabilities of the Markov chain on \( y \). Be sure to state these probabilities.
   (c) Offer an explanation for why the conditional or unconditional returns to stocks are higher/lower than bonds.

   [Hint: Use the fact that \( \text{COV}(x, y) = E[xy] - \overline{x}\overline{y} \)]

2. (Lucas Tree Economy with an Information Variable)

   Imagine that we add to the Lucas asset pricing model a variable \( i_t \) that offers information that may be relevant to future dividends in the sense that it may help predict future dividends. You can think of the information variable as news. The agent knows the conditional joint probability distribution \( F \) on future dividends and information. Here we assume that dividends and information follow a Markov process specified by \( F \).

   \[ F(y', i' | y, i) = \text{Prob}(y_{t+1} \leq y', i_{t+1} \leq i' | y_t = y, i_t = i) \]

   (i) Provide a definition of equilibrium for this economy. Be sure to describe the maximization problem the agent solves.

   (ii) Under what circumstances does the pricing function from this model coincide with the pricing function for the model without the information variable?

3. (Two person Lucas Tree Economy)

   Imagine that there are two people living in the Lucas Tree Economy.

   (i) Provide a definition of equilibrium for this economy. Be sure to indicate the maximization problem that each agent solves and state clearly what are your choices of state and control variables.
(ii) Under what circumstances does the pricing function from this model coincide with the pricing function for the model with only one agent?

4. (Hansen-Jaganathan Bounds)

(a) Suppose that we observe 20 realizations of the stock price in the economy described in problem 1. In these 20 observations 10 were the high price and 10 were the low price. Using this data calculate the Hansen-Jaganathan bounds. Graph this information.

[Hint: Do this by concentrating on discrete points (i.e. .90, .91, ..., 1.09, 1.10) on the mean of the stochastic discount factor. To calculate the bounds for this particular example it is not important in what order the data on prices were observed.]

(b) On the same graph plot the unconditional mean and variance of the theoretical stochastic discount factor from problem 1.