Representative Agent: Theory

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October, 2018

- **Question:** Given an economy with many agents, when does there exist a one-agent economy that produces the same equilibrium prices, given the same aggregate endowment?
- **Answer:** Constantinides (1982) Journal of Business
- We will consider his answer but restrict attention to exchange economies. His answer also applies to production economies. For both exchange economies and production economies, his answer relies on complete markets.

Answer: Main Steps

1. If $(\{c^i\}_{i=1}^I, p)$ is an Arrow-Debreu equilibrium to economy $\{U^i, e^i\}_{i=1}^I$, then $\exists \lambda^i > 0, \forall i$ such that

$$(c^1, ..., c^I) \in argmax \sum_i \lambda^i U^i(c^i) \ s.t. \ \sum_i c^i = \sum_i e^i$$

2. $\bar{U}(z) \equiv \max \sum_i \lambda^i U^i(c^i) \ s.t. \ \sum_i c^i = z$
3. Claim: If $(\{c^i\}_{i=1}^I, p)$ is an Arrow-Debreu equilibrium, then (z, p) is an Arrow-Debreu equilibrium to a one-agent economy with endowment $z = \sum_i e^i$ and preferences $\bar{U}$.

Example

1. $U^i(c) = U(c) = \log c_1 + \beta \log c_2, \ \ i = 1, 2$
2. $e^1 = (5, 10), e^2 = (10, 20)$
3. AD equilibrium is $(c^1, c^2, p) = (5, 10), (10, 20), (1, \beta/2))$

Step 1: Find (λ^1, λ^2)

1. FOC: $\lambda^i U_j(c^i) = \gamma_j \; holds \; i = 1, 2, j = 1, 2$
2. Solve for the four unknowns, given (c^1, c^2)

$$(\lambda^1, \lambda^2) = (1/3, 2/3)$$

$$(\gamma_1, \gamma_2) = (\frac{1}{3}\frac{1}{5}, \frac{1}{3}\frac{\beta}{10})$$

Step 2: Construct $\bar{U}$, given (λ^1, λ^2)

1. $\bar{U}(z) \equiv \max \sum_i \lambda^i U(c^i)$ $s.t.$ $\sum_i c^i = z$
2. FOC: $\lambda^i U_j(c^i) = \gamma_j$ $holds$ $i = 1, 2, j = 1, 2$

$$c^1 = (\frac{1}{3}z_1, \frac{1}{3}z_2) \; and \; c^2 = (\frac{2}{3}z_1, \frac{2}{3}z_2)$$

3. $\bar{U}(z) = \frac{1}{3}(\log \frac{z_1}{3} + \beta \log \frac{z_2}{3}) + \frac{2}{3}(\log \frac{2z_1}{3} + \beta \log \frac{2z_2}{3})$
4. $\bar{U}(z) = \log z_1 + \beta \log z_2 + constant$
5. $\bar{U}(z) = U(z_1, z_2) + constant$

Step 3: Claim (z, p) is AD equilibrium, given $(\bar{U}, z)$

1. $(z, p) = ((15, 30), (1, \beta/2))$
2. Suff. Cond.:

$$\bar{U}_1(z_1, z_2) = \gamma p_1 \; and \; \bar{U}_2(z_1, z_2) = \gamma p_2$$

3. Suff Cond holds for $\gamma = 1/15$

Summary

1. The example has agents with identical and homothetic utility.
2. Constatinides Theorem says that we can construct preferences $\bar{U}$ for a representative agent that produce the same equilibrium prices. Applying the Constantinides procedure to the example results in $\bar{U} = U$. The representative agent's preferences are those of either agent.
3. This result was intuitively obvious from the start for the example presented.