Neoclassical Growth Models: II

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November, 2017

Growth Model: Balanced Growth

- Neoclassical Growth Model is the workhorse model in macroeconomics.
- Solow (1956) provided a theoretical model that produced Kaldor's stylized growth facts in steady state. Kaldor's facts: 1. Y/L and K/L grow, 2. K/Y roughly constant, 3. capital's and labor's share are roughly constant, 4. return to capital has no trend.
- Solow: steady-state growth in Y/L iff technology grows at a positive, constant rate.
- Growth model analyzed earlier does not have growth in steady state. US data shows roughly constant long-run growth rate abstracting from BC fluctuations and the Great Depression.

Technological change: $F(k_t, l_t A_t)$ and $A_t = g_A^t, g_A \geq 1$

Def: A competitive equilibrium is $\{c_t, l_t, i_t, k_{t+1}, w_t, R_t\}_{t=0}^{\infty}$ s.t.

1. $\{c_t, l_t, i_t, k_{t+1}\}_{t=0}^{\infty}$ solve P1.
2. $w_t = F_2(k_t, l_t A_t) A_t$ and $R_t = F_1(k_t, l_t A_t)$ holds $\forall t$
3. $c_t + i_t = F(k_t, l_t A_t)$ holds $\forall t$

$$P1 \ \max \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \ s.t.$$

$$c_t + i_t \leq w_t l_t + R_t k_t$$

$$k_{t+1} = k_t(1-\delta) + i_t, l_t \in [0,1], \ given \ k_0$$

Def: A competitive equilibrium $\{c_t, l_t, i_t, k_{t+1}, w_t, R_t\}_{t=0}^{\infty}$ displays balanced growth provided $x_t = \bar{x} g_x^t$, for $x = (c, l, i, k, w, R)$.

What restrictions on $(g_c, ..., g_R)$ in a balanced-growth equilibrium come directly from the technology restrictions?

Answer:

1. $c_t + i_t = y_t = F(k_t, l_t A_t)$ imply $g_c = g_i = g_y$
2. $k_{t+1} = k_t(1-\delta) + i_t$ imply $g_k = g_i$
3. $y_t = F(k_t, l_t A_t)$ imply $y_t / l_t A_t = F(k_t / l_t A_t, 1)$. Thus, $g_y = g_k = g_A$ and $g_l = 1$ consistent.
4. Conclusion: $g_c = g_i = g_k = g_y = g_A$ and $g_l = 1$

Def: A competitive equilibrium $\{c_t, l_t, i_t, k_{t+1}, w_t, R_t\}_{t=0}^{\infty}$ displays balanced growth provided $x_t = \bar{x} g_x^t$, for $x = (c, l, i, k, w, R)$.

What restrictions on $(g_c, ..., g_R)$ in a balanced-growth equilibrium come from equil. cond. (2), given conclusion of previous slide?

Answer:

1. Cond. (2): $F\ CRS$ and $w_t = F_2(k_t, l_t A_t) A_t$ implies $g_w = g_A$
2. Cond. (2): $R_t = F_1(k_t, l_t A_t)$ implies $g_R = 1$

If we restrict attention to $u(c,l)$ additively separable, then what types of functional form restrictions are consistent with the balanced-growth restrictions derived so far?

$$u(c,l) = \log c + v(1-l)$$

Answer:

1. FOC: $\frac{-u_2(c_t l_t)}{u_1(c_t,l_t)} = F_2(k_t, l_t A_t) A_t$
2. FOC: $v'(1-l_t)c_t = F_2(k_t, l_t A_t) A_t$ is OK as $g_l = 1$ and $g_c = g_A$
3. FOC : $u_1(c_t, l_t) = \beta u_1(c_{t+1}, l_{t+1})(1 + F_1(k_{t+1}, l_{t+1} A_{t+1}) - \delta)$
4. FOC : $g_c = \beta(1 + F_1(k_{t+1}, l_{t+1} A_{t+1}) - \delta)$ OK as F_1 constant in BGE

Planners Problem: Balanced Growth

Analyze P1 using $u(c,l) = \log c + v(1-l)$

$$P1 \ \max \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \ s.t.$$

$$c_t + i_t \leq A_t F(\frac{k_t}{A_t}, l_t)$$

$$k_{t+1} = k_t(1-\delta) + i_t, l_t \in [0,1], \ given \ k_0$$

Transformation: $\hat{c}_t = c_t/A_t, \hat{i}_t = i_t/A_t, \hat{k}_t = k_t/A_t$

$$P1 \ \ \max \sum_{t=0}^{\infty} \beta^t [\log(A_t \hat{c}_t) + v(1 - l_t)] \ s.t.$$

$$\hat{c}_t + \hat{i}_t \leq F(\hat{k}_t, l_t)$$

$$\hat{k}_{t+1} g_A = \hat{k}_t(1 - \delta) + \hat{i}_t, l_t \in [0, 1], \ \ given \ \hat{k}_0$$

Transformation: $\hat{c}_t = c_t/A_t, \hat{i}_t = i_t/A_t, \hat{k}_t = k_t/A_t$

Factoring a constant term out of the transformed objective, we have a standard looking DP problem with the exception that there is an extra multiplicative term in the law of motion for capital:

$$v(\hat{k}) = \max \log \hat{c} + v(1-l) + \beta v(\hat{k}')\ s.t.$$

$$\hat{c} + \hat{i} \leq F(\hat{k}, l)$$

$$\hat{k}' g_A = \hat{k}(1-\delta) + \hat{i}, l \in [0,1]$$

Transformation: $\hat{c}_t = c_t/A_t, \hat{i}_t = i_t/A_t, \hat{k}_t = k_t/A_t$

Upshot:

(1) Can prove that there is a positive steady-state solution to the Planner's problem in transformed variables.
(2) In terms of untransformed variables, this implies constant growth rates. These variables are sometimes labeled the "balanced-growth path".
(3) Can extend standard lines of argument to prove that transformed capital converges monotonically to steady state at least for the case without valued leisure.
(4) Can compute (approx) solutions to competitive equil using the apparatus of Bellman's equation for the transformed planning problem. Back out untransformed variable implications.

We will compare equilibrium implications of stylized static and dynamic equilibrium models. Focus on the effects of a proportional tax rate τ on labor income.

Static Model:

$U(c,l) = \alpha \log c + (1-\alpha)\log(1-l)$
$y = F(l) = Al$

Def: A comp equil with transfers is (c, l, w, Tr) st

1. $(c,l) \in argmax\ U(c,l)\ s.t.\ c \leq wl(1-\tau) + Tr$
2. $w = F'(l)$
3. $Tr = wl\tau$
4. $c = F(l)$

Analysis: FOC and Feasibility

1. $\frac{U_2(c,l)}{U_1(c,l)} = w(1-\tau)$
2. $c(\frac{1-\alpha}{\alpha}) = F'(l)(1-l)(1-\tau)$
3. $F(l)(\frac{1-\alpha}{\alpha}) = F'(l)(1-l)(1-\tau)$
4. $l = \frac{\alpha(1-\tau)}{1-\alpha\tau}$ implies l falls as τ increases.
5. $c = A[\frac{\alpha(1-\tau)}{1-\alpha\tau}]$

Def: A comp equil with govt spending is (c, l, w, g) st

1. $(c, l) \in argmax\ U(c, l)\ s.t.\ c \leq wl(1 - \tau)$
2. $w = F'(l)$
3. $g = wl\tau$
4. $c + g = F(l)$

Analysis: FOC and Feasibility

1. $\frac{U_2(c,l)}{U_1(c,l)} = w(1 - \tau)$
2. $c(\frac{1-\alpha}{\alpha}) = F'(l)(1 - l)(1 - \tau) \Rightarrow (F(l) - \tau F'(l)l)(\frac{1-\alpha}{\alpha}) = F'(l)(1 - l)(1 - \tau)$
3. $l = \alpha$ and $c = \alpha A(1 - \tau)$

Discussion:

1. A proportional tax on labor income reduces labor and consumption when taxes support transfers. When taxes fund government spending, labor is unchanged but consumption falls. Yes, this analysis is based on specific functional forms ... .

2. We will now see if the same can be concluded when analyzed within models allowing balanced growth.

3. We will find that increasing the labor tax rate shifts the balanced-growth path for output, capital, labor and consumption downward without changing the growth rate. [case: taxes fund transfers]

Analyze same issue but $u(c,l) = \alpha \log c + (1-\alpha)\log(1-l)$ and $F(k_t, l_t A_t)$ and $A_t = g_A^t, g_A \geq 1$

Def: A competitive equilibrium is $\{c_t, l_t, i_t, k_{t+1}, w_t, R_t, Tr_t\}_{t=0}^{\infty}$ s.t.

1. $\{c_t, l_t, i_t, k_{t+1}\}_{t=0}^{\infty}$ solve P1.
2. $w_t = F_2(k_t, l_t A_t)A_t$ and $R_t = F_1(k_t, l_t A_t)$ holds $\forall t$
3. $Tr_t = w_t l_t \tau$
4. $c_t + i_t = F(k_t, l_t A_t)$ holds $\forall t$

$$P1 \ \max \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \ s.t.$$

$$c_t + i_t \leq w_t l_t (1-\tau) + R_t k_t + Tr_t$$

$$k_{t+1} = k_t(1-\delta) + i_t, l_t \in [0,1], \ given \ k_0$$

Analysis: FOC, Comp. prices and Feasibility

1. $u_1(c_t, l_t) = \beta u_1(c_{t+1}, l_{t+1})(1 + F_1(k_{t+1}, l_{t+1}A_{t+1}) - \delta)$
2. $\frac{U_2(c_t,l_t)}{U_1(c_t,l_t)} = F_2(k_t, l_tA_t)A_t(1-\tau)$
3. $c_t + k_{t+1} = F(k_t, l_tA_t) + k_t(1-\delta)$
4. Analyze balanced-growth equilibria (GBE) consistent w/ nec. cond. (1)-(3):

$$g_A = g_c = \beta(1 + F_1(\bar{k}, \bar{l}) - \delta)$$

$$\bar{c} = (\frac{\alpha}{1-\alpha})F_2(\bar{k}, \bar{l})(1-\bar{l})(1-\tau)$$

$$\bar{c} = F(\bar{k}, \bar{l}) - \bar{k}(g_A - 1 + \delta)$$

1. $g_A = g_c = \beta(1 + F_1(\bar{k}, \bar{l}) - \delta)$
2. $\bar{c} = (\frac{\alpha}{1-\alpha})F_2(\bar{k}, \bar{l})(1 - \bar{l})(1 - \tau)$
3. $\bar{c} = F(\bar{k}, \bar{l}) - \bar{k}(g_A - 1 + \delta)$

Eqn 1. pins down $\bar{k}/\bar{l}$ as F_1 is HD zero.

Eqn 2. and 3. determines $\bar{l}$: If there is a solution, then there is only one as LHS is st. increasing and RHS is st. decreasing in $\bar{l}$. Moreover, $RHS(\bar{l}, \tau)$ falls as τ increases, OTE. Thus, $\bar{l}$ falls as τ increases.

$$\bar{l}[F(\bar{k}/\bar{l}, 1) - (\bar{k}/\bar{l})(g_A - 1 + \delta)] = (\frac{\alpha}{1 - \alpha})F_2(\bar{k}/\bar{l}, 1)(1 - \bar{l})(1 - \tau)$$

Conclusion: Example when proportional labor taxes fund transfers.

1. If there is a BGE, then it has a unique $\bar{k}/\bar{l}$ ratio, independent of τ.
2. If there is a BGE then $\tau > \tau'$ implies $\bar{l}(\tau) < \bar{l}(\tau')$.
3. If there is a BGE then $\tau > \tau'$ implies $\bar{c}(\tau) < \bar{c}(\tau')$:

$$\bar{c} = \bar{l}[F(\bar{k}/\bar{l}, 1) - (\bar{k}/\bar{l})(g_A - 1 + \delta)]$$

4. Dynamic model agrees qualitatively with static model. Models differ in that the fall in consumption comes both via fall in labor AND capital in dynamic model.

Analyze case when labor income taxes fund government spending

Def: A competitive equilibrium is $\{c_t, l_t, i_t, k_{t+1}, g_t, w_t, R_t\}_{t=0}^{\infty}$ s.t.

1. $\{c_t, l_t, i_t, k_{t+1}\}_{t=0}^{\infty}$ solve P1.
2. $w_t = F_2(k_t, l_t A_t) A_t$ and $R_t = F_1(k_t, l_t A_t)$ holds $\forall t$
3. $g_t = w_t l_t \tau$
4. $c_t + i_t + g_t = F(k_t, l_t A_t)$ holds $\forall t$

$$P1 \;\; \max \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \; s.t.$$

$$c_t + i_t \leq w_t l_t (1 - \tau) + R_t k_t$$

$$k_{t+1} = k_t(1 - \delta) + i_t, l_t \in [0, 1], \;\; given \;\; k_0$$

Analysis: FOC, Comp. prices and Feasibility

1. $u_1(c_t, l_t) = \beta u_1(c_{t+1}, l_{t+1})(1 + F_1(k_{t+1}, l_{t+1}A_{t+1}) - \delta)$
2. $\frac{U_2(c_t,l_t)}{U_1(c_t,l_t)} = F_2(k_t, l_t A_t)A_t(1-\tau)$
3. $c_t + k_{t+1} + g_t = F(k_t, l_t A_t) + k_t(1-\delta)$
4. Analyze balanced-growth equil. consistent w/ nec. cond. (1)-(3):

$$g_A = g_c = \beta(1 + F_1(\bar{k}, \bar{l}) - \delta)$$

$$\bar{c} = (\frac{\alpha}{1-\alpha})F_2(\bar{k}, \bar{l})(1-\bar{l})(1-\tau)$$

$$\bar{c} = F(\bar{k}, \bar{l}) - \bar{k}(g_A - 1 + \delta) - \tau F_2(\bar{k}, \bar{l})\bar{l}$$

1. $g_A = g_c = \beta(1 + F_1(\bar{k}, \bar{l}) - \delta)$
2. $\bar{c} = (\frac{\alpha}{1-\alpha})F_2(\bar{k}, \bar{l})(1 - \bar{l})(1 - \tau)$
3. $\bar{c} = F(\bar{k}, \bar{l}) - \bar{k}(g_A - 1 + \delta) - \tau F_2(\bar{k}, \bar{l})\bar{l}$

Eqn 1. pins down $\bar{k}/\bar{l}$ as F_1 is HD zero.
Eqn 2. and 3. determines $\bar{l}$, given $\bar{k}/\bar{l}$: There is at most one solution as LHS is st. increasing and RHS is st. decreasing in $\bar{l}$.

$$\bar{l}[F(\frac{\bar{k}}{\bar{l}}, 1) - (\frac{\bar{k}}{\bar{l}})(g_A - 1 + \delta) - \tau F_2(\frac{\bar{k}}{\bar{l}}, 1)] = (\frac{\alpha}{1-\alpha})F_2(\bar{k}/\bar{l}, 1)(1 - \bar{l})(1 - \tau)$$

$$\bar{l}[F(\frac{\bar{k}}{\bar{l}},1)-(\frac{\bar{k}}{\bar{l}})(g_A-1+\delta)-\tau F_2(\frac{\bar{k}}{\bar{l}},1)] = (\frac{\alpha}{1-\alpha})F_2(\bar{k}/\bar{l},1)(1-\bar{l})(1-\tau)$$

Can show that $\bar{l}=\alpha$ implies $LHS(\bar{l}) > RHS(\bar{l})$. Conclude, $\bar{l}<\alpha$.

$$G(\tau,\bar{l}) \equiv LHS(\tau,\bar{l}) - RHS(\tau,\bar{l})$$

$G(\tau,\bar{l})=0, \bar{l}<\alpha$ and Implicit Function Theorem imply

$$\frac{d\bar{l}}{d\tau} = -\frac{G_1}{G_2} < 0$$

$$G_1 = \frac{1}{1-\alpha}F_2(\frac{\bar{k}}{\bar{l}},1)(\alpha-\bar{l}) > 0$$

$$G_2 = g_A\frac{\bar{k}}{\bar{l}}(\frac{1}{\beta}-1) + \frac{1}{1-\alpha}F_2(\frac{\bar{k}}{\bar{l}},1)(1-\tau) > 0$$

Conclusion: Example when proportional labor taxes fund government spending.

1. If there is a BGE, then it has a unique $\bar{k}/\bar{l}$ ratio, independent of τ.
2. If there is a BGE then $\tau > \tau'$ implies $\bar{l}(\tau) < \bar{l}(\tau')$ as $\frac{d\bar{l}}{d\tau} < 0$.
3. If there is a BGE then $\tau > \tau'$ implies $\bar{c}(\tau) < \bar{c}(\tau')$:

$$\bar{c} = \bar{l}[F(\bar{k}/\bar{l}, 1) - (\bar{k}/\bar{l})(g_A - 1 + \delta)]$$

4. Dynamic model implies consumption and labor fall. Fall in consumption is due to a fall in capital and labor. Static model implies that labor stays the same while consumption falls.

1. We will develop an overlapping generations (OG) version of the growth model. We will consider an example with a simple demographic structure.
2. What is especially novel about OG compared to infinitely lived agents when the technology is the same?

   * 1st Welfare Thm. no longer holds

   * Framework to discuss intergenerational transfers (e.g. modern social security systems or debt). Model implies valued fiat money is possible and allows government debt to crowd out physical capital.

   * Model allows one to highlight age implications $\cdots$ in data.

1. Time: $t = 1, 2, ...$
2. Demographics: agents live 2 model periods, population growth at rate n.
3. Population: $N_t = N_{t-1}(1+n)$ - number of agents born in period t.
4. Preferences: $U(c_{1,t}, c_{2,t})$ and $U(c_{2,0})$ for "initial old".
5. Endowments: Initial capital-labor ratio k_1 is held by initial old. Young can work 1 unit of time, whereas old cannot work.
6. Technology: $y_t = F(k_t, l_t)$ and $k_{t+1} = k_t(1-\delta) + i_t$

Notation: $c_{1,t}$ is consumption of agent age 1 who is born at time t

Notation: k_t has the interpretation of the capital-labor ratio at time t, whereas $k_{t+1}(1+n)$ is interpreted as "savings".

1. P1 : $(c_{1,t}, c_{2,t}, k_{t+1}) \in \; argmax \; U(c_{1,t}, c_{2,t}) \; s.t.$
   $c_{1,t} + k_{t+1}(1+n) \leq w_t$ and $c_{2,t} \leq k_{t+1}(1+n)R_{t+1}$
2. P2: $c_{2,0} \in \; argmax \; U(c_{2,0}) \; s.t. \; c_{2,0} \leq k_1(1+n)R_1$

3. A competitive equil. is $\{c_{1,t}, c_{2,t}, k_t, w_t, R_t\}_{t=1}^{\infty}$ and $c_{2,0}$ s.t.
   (1) $(c_{1,t}, c_{2,t}, k_{t+1})$ solves P1 for all $t \geq 1$ and $c_{2,0}$ solves P2.
   (2) $w_t = F_2(k_t, 1)$ and $R_t = 1 + F_1(k_t, 1) - \delta$
   (3) $c_{1,t} + \frac{c_{2,t-1}}{1+n} + k_{t+1}(1+n) = F(k_t, 1) + k_t(1-\delta)$

Example

1. $U(c_1, c_2) = \alpha \log c_1 + (1-\alpha)\log c_2$
2. $F(k, l) = Ak^{\beta} l^{1-\beta}$
3. Analysis:

$$U_1(c_1, c_2) = U_2(c_{1,t}, c_{2,t})R_{t+1}$$

$$\frac{\alpha}{w_t - k_{t+1}(1+n)} = \frac{(1-\alpha)R_{t+1}}{k_{t+1}(1+n)R_{t+1}}$$

$$k_{t+1} = \frac{(1-\alpha)w_t}{1+n} \; and \; c_{1,t} = \alpha w_t$$

Example

1. $k_{t+1} = \frac{(1-\alpha)w_t}{1+n}$ *and* $c_{1,t} = \alpha w_t$
2. $w_t = F_2(k_t, 1) = (1-\beta)Ak_t^{\beta}$
3. $k_{t+1} = \frac{(1-\alpha)(1-\beta)Ak_t^{\beta}}{1+n}$
4. Once one knows the behavior of the capital-labor ratio over time, then all other equilibrium variables are easy to determine.
5. Properties: (1) there is one positive capital-labor ratio k^* steady state, (2) the capital-labor ratio k_t converges monotonically to k^* from any $k_1 > 0$, (3) $k^* = [\frac{(1-\alpha)(1-\beta)}{1+n}]^{1/(1-\beta)}$ and (4) $k^* > k_{GR}$ is possible and such an allocation is Pareto inefficient.

Connection to Golden Rule:

1. Feasibility: $C_t + k_{t+1}(1+n) - k_t(1-\delta) = F(k_t, 1)$
2. $C_t = F(k_t, 1) - k_{t+1}(1+n) - k_t(1-\delta)$
3. C_t denotes aggregate consumption to labor ratio
4. The Golden Rule capital-labor ratio is the steady state k maximizing steady-stte consumption. Therefore, it is the value k_{GR} solving $k_{GR} \in \ argmax \ F(k, 1) - k(n+\delta)$

$$F_1(k_{GR}, 1) = (n+\delta) \Rightarrow k_{GR} = (\frac{\beta A}{n+\delta})^{1/(1-\beta)}$$

5. Steady states "beyond the Golden Rule" are inefficient as too much capital is being maintained

Notation: k_t, b_t are the ratios of capital and debt per young agent at time t, whereas $(T_{1,t}, T_{2,t})$ are lump-sum taxes at age 1 and 2 for agent born at time t.

1. P1 : $(c_{1,t}, c_{2,t}, k_{t+1}, b_{t+1}) \in \ argmax\ U(c_{1,t}, c_{2,t})\ s.t.$
   $c_{1,t} + k_{t+1}(1+n) + b_{t+1}(1+n) \leq w_t - T_{1,t}$ and
   $c_{2,t} \leq k_{t+1}(1+n)R_{t+1} + b_{t+1}(1+n)R_{t+1} - T_{2,t}$
2. P2:
   $c_{2,0} \in \ argmax\ U(c_{2,0})\ s.t.\ c_{2,0} \leq (k_1 + b_1)(1+n)R_1 - T_{2,0}$
3. A competitive equil. is $\{c_{1,t}, c_{2,t}, k_t, w_t, R_t, b_t, T_{1,t}, T_{2,t}, g_t\}_{t=1}^{\infty}$ and $c_{2,0}$ s.t.
   (1) $(c_{1,t}, c_{2,t}, k_{t+1}, b_{t+1})$ solves P1 $\forall t \geq 1$, $c_{2,0}$ solves P2 .
   (2) $w_t = F_2(k_t, 1)$ and $R_t = 1 + F_1(k_t, 1) - \delta$
   (3) $c_{1,t} + \frac{c_{2,t-1}}{1+n} + k_{t+1}(1+n) + g_t = F(k_t, 1) + k_t(1-\delta)$
   (4) $g_t = T_{1,t} + T_{2,t-1}/(1+n) + [b_{t+1}(1+n) - b_t R_t]$

The following proposition says that changes in government debt and taxes that finance the same government spending stream are equivalent in terms of real allocations.

Claim: If $\{c_{1,t}, c_{2,t}, k_t, w_t, R_t, b_t, T_{1,t}, T_{2,t}, g_t\}_{t=1}^{\infty}$ *is a competitive equilibrium, then* $\{c_{1,t}, c_{2,t}, k_t, w_t, R_t, \hat{b}_t, \hat{T}_{1,t}, \hat{T}_{2,t}, g_t\}_{t=1}^{\infty}$ *is also a competitive equilibrium provided:*

1. $T_{1,t} + T_{2,t}/R_{t+1} = \hat{T}_{1,t} + \hat{T}_{2,t}/R_{t+1}, \forall t \geq 1$
2. $\hat{b}_{t+1}(1+n) = b_{t+1}(1+n) + [T_{1,t} - \hat{T}_{1,t}]$
3. $b_1 = \hat{b}_1$ *and* $T_{2,0} = \hat{T}_{2,0}$

To prove the Claim it remains to show that equilibrium conditions 1 and 4 hold. Clearly, condition 2-3 hold directly from the Claim.

Equil. Condition 1:

It's sufficient to show that the feasible consumption set is unchanged. Form a present-value budget condition. Since the present value of taxes on each individual are unchanged and factor prices are unchanged the feasible consumption set is unchanged. Given choices for capital, best choices for debt for an agent born at time t under the new tax scheme equal

$$\hat{b}_{t+1}(1+n) = b_{t+1}(1+n) + [T_{1,t} - \hat{T}_{1,t}]$$

Equil. Condition 4:

Step 1: $g_t = T_{1,t} + T_{2,t-1}/(1+n) + [b_{t+1}(1+n) - b_t R_t]$
Step 2: $\hat{b}_{t+1}(1+n) = b_{t+1}(1+n) + [T_{1,t} - \hat{T}_{1,t}]$ implies
$g_t = \hat{T}_{1,t} + T_{2,t-1}/(1+n) + [\hat{b}_{t+1}(1+n) - b_t R_t]$
Step 3: $\hat{b}_t = b_t + [T_{1,t-1} - \hat{T}_{1,t-1}]/(1+n)$ implies $g_t =$
$\hat{T}_{1,t} + T_{2,t-1}/(1+n) + [\hat{b}_{t+1}(1+n) - \hat{b}_t R_t] + [T_{1,t-1} - \hat{T}_{1,t-1}] R_t/(1+n)$

Step 4: Use present-value condition in Claim to get
$g_t = \hat{T}_{1,t} + \hat{T}_{2,t-1}/(1+n) + [\hat{b}_{t+1}(1+n) - \hat{b}_t R_t]$