Macro 1: Exchange Economies

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Background

Much of macroeconomic theory is organized around growth models. Before diving into the complexities of those models, we consider exchange economies with time and uncertainty - key features of macro models.

We will present two types of market structures: time-zero markets and sequential markets.

- Time-zero markets follow Debreu (1959). All contracting is done at $t = 0$ via competitive markets. After $t = 0$ the economy just carries out the contracts agreed to at $t = 0$.
- Sequential markets follow Arrow. Each period has spot markets for goods and a financial market for trade in one-period ahead Arrow securities. Agents know/forecast future spot and security prices.

Language of Debreu

Debreu (1959) provided a modeling language for exchange and production economies. We employ his language with slight modification.

An economy is $E = (X^i, U^i, e^i)_{i \in \{1,\ldots,I\}}$.

- X^i is a consumption set.
- $U^i : X^i \to R$ is a utility function
- $e^i \in X^i$ is an endowment
- I is the number of agents

Implicit in these notions is a commodity space L and the concept of a commodity. Keep in mind that $X^i \subset L$. The simplest commodity space is $L = R^n$ where there are n commodities. In macro, we will often have infinite dimensional commodity spaces due to the abstaction that time is infinite.

Language of Debreu

$E = (X^i, U^i, e^i)_{i \in \{1,...,I\}}$. Consider applications with $L = R^n$.

Definition:

- A feasible allocation is $(x^1, ..., x^I)$ such that (i) $x^i \in X^i, \forall i$ and (ii) $\sum_{i=1}^I x^i = \sum_{i=1}^I e^i$.
- A feasible allocation $(x^1, ..., x^I)$ is Pareto efficient provided there is no feasible allocation $(\bar{x}^1, ..., \bar{x}^I)$ such that $U^i(\bar{x}^i) \geq U^i(x^i), \forall i$ and $U^i(\bar{x}^i) > U^i(x^i)$ for some i.
- A competitive equilibrium is $((x^1, ..., x^I), p)$ such that
  - $\forall i = 1, .., I, x^i \in argmax\ U^i(x^i)\ s.t.\ x^i \in X^i, px^i \leq pe^i$.
  - $\sum_{i=1}^I x^i = \sum_{i=1}^I e^i$.

Language of Debreu

Example 1: We will put A LOT of structure on our examples

- $I = 2$
- $U^i(x^i) = \sum_{t=0}^{T} \sum_{s^t \in S^t} \beta^t u(x^i(s^t))P(s^t)$
- $u(c) = c^{1-\sigma}/(1-\sigma)$ - CRRA utility
- $S^t = \{s^t = (s_0, ..., s_t) : s_0 = 1, and\ s_j \in S, \forall j = 1, .., t\}$
- $S = \{1, 2\}$
- $e^i = (e^i(s_0), ..., e^i(s^T))$, where $e^1(s^t) = s_t$ and $e^2(s^t) = 2s_t$

Roughly: two person exchange economy with one physical good per period, $T + 1$ periods and endowment risk.

Language of Debreu

Example 1: What is a competitive equilibrium for Example 1?

Guess: $p = (p(s^0), ..., p(s^T))$, where

$$p(s^t) = \frac{\beta^t u'(s_t) P(s^t)}{u'(s_0) P(s^0)} = \beta^t (\frac{s_0}{s_t})^\sigma P(s^t | s^0)$$

Guess: $x^1(s^t) = s_t$ and $x^2(s^t) = 2s_t, \forall t, \forall s^t$

Roughly: The guess is that each agent eats his endowment. Prices are ratios of marginal utilities, where the price of the time zero good is normalized to 1.

Language of Debreu

Example 1: Verify that the guess satisfies the two requirements of a comp. equil.

Requirement 1: Resource feasibility ... $\sum_i x^i = \sum_i e^i$. This holds for the guess as $x^i = e^i$.

Requirement 2: Best choice ...

$$L = \sum_{t=0}^{T} \sum_{s^t \in S^t} \beta^t u(x^i(s^t))P(s^t) + \lambda^i [\sum_{t=0}^{T} \sum_{s^t \in S^t} p(s^t)(e^i(s^t) - x^i(s^t))]$$

$$(1)\ \beta^t u'(x^i(s^t))P(s^t) = \lambda^i p(s^t), \forall t, \forall s^t$$

$$(2)\ \sum_{t=0}^{T} \sum_{s^t \in S^t} p(s^t)(e^i(s^t) - x^i(s^t)) = 0$$

Language of Debreu

Example 1: It remains to verify that condition (1) from previous slide holds for $i = 1, 2$.

$$(1)\ \beta^t u'(x^i(s^t))P(s^t) = \lambda^i p(s^t), \forall t, \forall s^t$$

$$(1)\ \beta^t u'(x^i(s^t))P(s^t) = \lambda^i \frac{\beta^t u'(s_t)P(s^t)}{u'(s_0)P(s^0)}$$

Holds with $\lambda^1 = u'(s_0)P(s^0)$ for agent $i = 1$.
Holds with $\lambda^2 = 2^{-\sigma}u'(s_0)P(s^0)$ for $i = 2$.

$$(1)\ \beta^t u'(x^2(s^t))P(s^t) = \beta^t 2^{-\sigma} u'(s_t)P(s^t) = \lambda^2 \frac{\beta^t u'(s_t)P(s^t)}{u'(s_0)P(s^0)}$$

Background:

One reason for using the language of Debreu is that the welfare theorems are already established within this language. However, the nature of markets is far removed from the structure of markets we see operating in modern economies. We see lots of spot trade in modern economies and trade in some financial assets by households and lots of trade in financial assets by various financial intermediaries.

A step towards the structure observed in modern economies is to allow spot trade at all time periods plus trade in some special financial assets. One reason why this is useful is that one can prove in different contexts that with spot markets and Arrow securities one achieves the same allocations as achieved by the time zero market structure. We will not prove this but we will set up language for this different equilibrium notion.

Consider the exchange economy in Example 1.

Definition: A competitive equilibrium with sequential markets is $\{x^i(s^t), a^i(s^{t+1})\}_{t=0}^{T}$ for $i = 1, 2$ and $\{q(s_{t+1}|s^t)\}_{t=0}^{T-1}$ such that

- (1) $\{x^i(s^t), a^i(s^{t+1})\}_{t=0}^{T}$ solves P1 for $i = 1, 2$.
- (2) $\sum_{i=1}^{2} x^i(s^t) = \sum_{i=1}^{2} e^i(s^t), \forall t, \forall s^t$.
- (3) $\sum_{i=1}^{2} a^i(s^t, s_{t+1}) = 0, \forall t, \forall s^t, \forall s_{t+1}$.

$$(P1) \quad \max \sum_{t=0}^{T} \sum_{s^t \in S^t} \beta^t u(x^i(s^t)) P(s^t) \ s.t.$$

$$x^i(s^t) + \sum_{s_{t+1}} q(s_{t+1}|s^t) a^i(s^t, s_{t+1}) \leq e^i(s^t) + a^i(s^t), \forall t < T, \forall s^t$$

$$x^i(s^T) \leq e^i(s^T) + a^i(s^T), \forall s^T$$

Example 1: Calculate an equilibrium with sequential markets.

Guess: $x^i(s^t) = e^i(s^t), \forall t, \forall s^t, \forall i$
Guess: $a^i(s^t, s_{t+1}) = 0, \forall t, \forall s^t, \forall s_{t+1}, \forall i$
Guess: $q(s^t, s_{t+1}) = \frac{\beta u'(s_{t+1}) P(s^{t+1}|s^t)}{u'(s_t)}$

Verify: equilibrium conditions 2-3 hold by construction. Thus, it remains to show that consumption and financial asset decisions are best decisions. To do this verify that the K-T conditions on the next page hold.

$$L = \sum_{t=0}^{T} \sum_{s^t \in S^t} \beta^t u(x^i(s^t))P(s^t) +$$

$$\sum_{t} \sum_{s^t \in S^t} \lambda^i(s^t)P(s^t)[e^i(s^t) + a^i(s^t) - x^i(s^t) - \sum_{s_{t+1}} q(s^t, s_{t+1})a^i(s^t, s_{t+1}))]$$

1. $\beta^t u'(x^i(s^t))P(s^t) - \lambda^i(s^t)P(s^t) = 0$
2. $\lambda^i(s^t)P(s^t) - \lambda^i(s^{t-1})P(s^{t-1})q(s^{t-1}, s_t) = 0$
3. all budget constraints

1. Holds with $\lambda^1(s^t) = \beta^t u'(s_t)$ and $\lambda^2(s^t) = \beta^t u'(2s_t)$
2. holds by substituting $\lambda^i(s^t)$ and q into condition 2.
3. holds by construction

[Note: Lagrange multipliers are "parameterized" in a convenient way to simplify algebra]

Efficient Allocations

A standard comparison is between the allocations produced by a market mechanism and Pareto efficient allocations. The welfare theorems make this comparison. We will develop some methods for producing efficient allocations via a planning problem (P1) in order to work out properties of efficient allocations.

$$(P1) \ \max \sum_{i=1}^{I} \gamma^i [\sum_{t=0}^{T} \sum_{s^t \in S^t} \beta^t u_i(x^i(s^t)) P(s^t)] \ s.t.$$

$$\sum_i x^i(s^t) = \sum_i e^i(s^t), \forall t, \forall s^t$$

Efficient Allocations

Claim: If $(x^1, ..., x^I)$ solves P1 for some $\gamma^1, ..., \gamma^I > 0$, then

1. $(x^1, ..., x^I)$ is Pareto efficient
2. the allocation $x^i(s^t)$ in any time period t is an increasing function of $\sum_i e^i(s^t)$
3. conditional on $\sum_i e^i(s^t)$, $x^i(s^t)$ does not vary with $e^i(s^t)$.

Proof:

1. If not, then there is an alternative feasible allocation that Pareto dominates. However, this alternative allocation must strictly increase the objective in P1. Contradiction.

Efficient Allocations

Proof of 2.:

1. $\gamma^i \beta^t u_i'(x^i(s^t))P(s^t) = \lambda(s^t), \forall i, \forall t, \forall s^t$
2. $x^i(s^t) = (u_i')^{-1}(\frac{\lambda(s^t)}{\gamma^i \beta^t P(s^t)}) \equiv f_i(\frac{\hat{\lambda}(s^t)}{\gamma^i})$
   $\Rightarrow x^i(s^t)\ is\ a\ st.\ decreasing\ function\ of\ \hat{\lambda}(s^t)$
3. $\sum_i x^i(s^t) = \sum_i f_i(\frac{\hat{\lambda}(s^t)}{\gamma^i}) = \sum_i e^i(s^t) \Rightarrow \hat{\lambda}(s^t) = g(\sum_i e^i(s^t))$
4. $x^i(s^t) = f_i(\frac{\hat{\lambda}(s^t)}{\gamma^i}) = f_i(\frac{g(\sum_i e^i(s^t))}{\gamma^i})$
5. Thus, $x^i(s^t)$ is an increasing function of the aggregate endowment in any period t as f_i and g are both decreasing functions.

Cochrane (1991, JPE)

$$(P1)\ \max \sum_{i=1}^{I} \gamma^i [\sum_{t=0}^{T} \sum_{s^t \in S^t} \beta_i^t u(x^i(s^t), \delta_i(s^t)) P(s^t)]\ s.t.$$

$$\sum_i x^i(s^t) = \sum_i e^i(s^t), \forall t, \forall s^t$$

$$(\delta_i, \beta_i, \gamma^i)\ preference\ and\ planning\ weight\ heterogeneity$$

Question: Can we reject the hypothesis that US household consumption data solves P1 (is Pareto efficient)?

Method: Use FOC to P1 to determine what elements determine consumption growth. Posit that we observe X_t^i that is independent of the elements determining consumption growth. See if can reject that consumption is independent of X_t^i.

Cochrane (1991, JPE)

$$L = \sum_{i=1}^{I} \gamma^i [\sum_{t=0}^{T} \sum_{s^t \in S^t} \beta_i^t u(x^i(s^t), \delta^i(s^t)) P(s^t)] +$$

$$\sum_t \sum_{s^t} P(s^t)\lambda(s^t)[\sum_i e^i(s^t) - x^i(s^t)]$$

1. $(FOC) \quad \gamma^i \beta_i^t u_1(x^i(s^t), \delta^i(s^t)) = \lambda(s^t)$
2. $\frac{\beta_i u_1(x^i(s^{t+1}), \delta^i(s^{t+1}))}{u_1(x^i(s^t), \delta^i(s^t))} = \frac{\lambda(s^{t+1})}{\lambda(s^t)}$
3. Let $u(x_t^i, \delta_t^i) = b_t^i \ (x_t^i)^{1+\sigma^i}/(1+\sigma^i)$, where $\delta_t^i \equiv (b_t^i, \sigma^i)$
4. $\log(\frac{x_{t+1}^i}{x_t^i}) = \frac{1}{\sigma^i}[\log \frac{\lambda(s^{t+1})}{\lambda(s^t)} - \log \frac{b_{t+1}^i}{b_t^i} - \log \beta_i]$
5. $\log(\frac{\hat{x}_{t+1}^i}{\hat{x}_t^i}) = \frac{1}{\sigma^i}[\log \frac{\lambda(s^{t+1})}{\lambda(s^t)} - \log \frac{b_{t+1}^i}{b_t^i} - \log \beta_i] + \epsilon_{t+1}^i$
6. Measurement Error: $\log(\frac{\hat{x}_{t+1}^i}{\hat{x}_t^i}) = \log(\frac{x_{t+1}^i}{x_t^i} \exp(\epsilon_{t+1}^i))$

$$\log(\frac{\hat{x}^i_{t+1}}{\hat{x}^i_t}) = \frac{1}{\sigma^i}[\log\frac{\lambda(s^{t+1})}{\lambda(s^t)} - \log\frac{b^i_{t+1}}{b^i_t} - \log\beta_i] + \epsilon^i_{t+1}$$

Assume: X^i_{t+1} is cross sectionally indep of $(\frac{b^i_{t+1}}{b^i_t}, \sigma^i, \beta_i, \epsilon^i_{t+1})$.

Regression: $\log(\frac{\hat{x}^i_{t+1}}{\hat{x}^i_t}) = \alpha + \beta X^i_{t+1} + \hat{\epsilon}^i_{t+1}$

Theory: If allocation solves P1 and independence holds, then

$$\beta = 0$$

$$\alpha = E[\frac{1}{\sigma^i}[\log\frac{\lambda(s^{t+1})}{\lambda(s^t)} - \log\frac{b^i_{t+1}}{b^i_t} - \log\beta_i] + \epsilon^i_{t+1}]$$

$$\hat{\epsilon}^i_{t+1} = [\frac{1}{\sigma^i}[\log\frac{\lambda(s^{t+1})}{\lambda(s^t)} - \log\frac{b^i_{t+1}}{b^i_t} - \log\beta_i] + \epsilon^i_{t+1}] - E[[\cdot]]$$

Data:

1. $\log(\frac{\hat{x}^i_{t+1}}{\hat{x}^i_t})$ is PSID log (food) consumption growth from 1980 to 1983.
2. X^i_{t+1} is whether or not a household experienced an involuntary job loss over 1980-83. The variable is 1 if this occured and 0 otherwise.
3. Two Samples
   Sample 1: 3373 observations and 291 w/ job loss
   Sample 2 (w/o Composition Change): 1173 observations and 76 w/ job loss

Results:

Sample 1: $\hat{\beta} = -26.74$
Sample 2: $\hat{\beta} = -24.03$

Both samples have more than a 24 percent difference in mean consumption growth between the involuntary job loss group and the no job loss group.

Author's summary statement: *"Thus the evidence does not contradict full insurance for illness of less than 100 days, spells of unemployment following an involuntary job loss, loss of work due to strike, and an involuntary move. On the other hand, the loss of more than 100 days of work due to illness and the involuntary loss of a job are important right-hand variables, whose associations with consumption growth are both economically and statistically significant."*

Commentary:

Some economists interpret this type of evidence as suggesting that some risks are not fully insured from the perspective of theory where all information is publically observed. This has motivated several lines of work.

1. Some work simply assumes that some risks cannot be insured in the market. Incomplete markets models fall into this group. Such models produce a lower consumption growth rate for the job loss group.
2. Some work assumes that some information is only privately observed. Feasible allocations then must be (i) resource feasible and (ii) incentive compatible. See the work on the Revelation Principle. Efficient allocations within this class of models may feature lower consumption growth for the job loss group.