Business Cycles: I

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Business Cycles: A Very Short History

- **Theory: Shocks and Propagation Mechanisms**

  Wicksell analogy: a club and a rocking horse

  Candidate Shocks: technology shocks, animal spirits, govt spending or tax shocks, erratic monetary policy, news, uncertainty shocks, financial shocks

  Propagation Mechanisms: physical capital, balance sheet, input-output links
- **Facts:** Burns and Mitchell (1946) vs Hodrick and Prescott (1980) vs other methods
- **BC Theory: Post Kydland and Prescott (1982)**
  - BC Theory- rooted in growth models
  - GE models central
  - Models judged on quantitative grounds
  - policy analysis done w/in model - GE analysis

Business Cycles: Agenda for these Slides

- Focus on three things
- BC Facts: apply HP filter
- Candidate Shock: Solow residual
- BC Theory: Employ aggregate prod fn w/ Solow residual as the only shock. Evaluate model in the style of KP (1982)

Business Cycles: Facts

- Follow Hodrick and Prescott (1980)
- Trend-cycle decomp: $y_t = \bar{y}_t + y_t^c$ where y_t is data, $\bar{y}_t$ is a trend and y_t^c is a cycle component
- Define HP trend: use $\lambda = 1600$ quarterly data and $\lambda = 1600/4^4 = 6.25$ yearly - Ravn and Uhlig (2002)

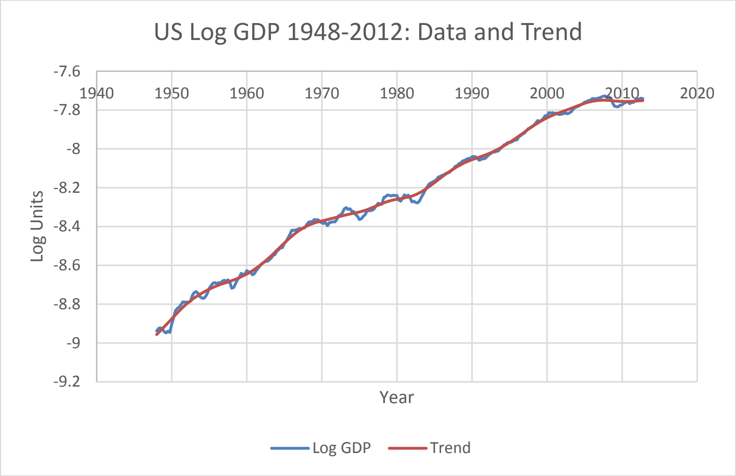
$$\{\bar{y}_t\}_{t=1}^{T} \in \ argmin \sum_{t=1}^{T}(y_t - \bar{y}_t)^2 + \lambda \sum_{t=2}^{T-1}[(\bar{y}_{t+1} - \bar{y}_t) - (\bar{y}_t - \bar{y}_{t-1})]^2$$

- BC Facts: 2nd moments of y_t^c for many series (e.g. log GDP, log Consumption, log Invest, log Hours,..)

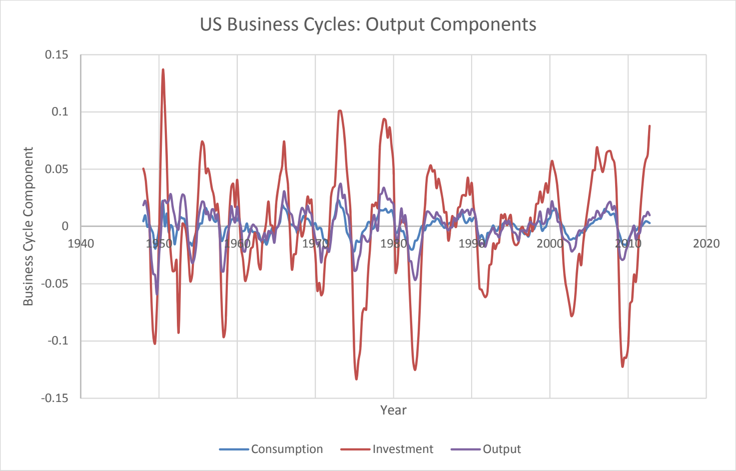
US Business-Cycle Facts

Variable	Amplitude (Std. Deviation)	Correlation w/ GDP
GDP	.017	1.0
Consumption Nondurables	.009	0.79
Consumption Durables	.051	0.65
Investment	.050	0.84
Government Spending	.032	0.05
Labor Hours	.015	0.87
Employment	.013	0.81
Labor Productivity	.008	0.36

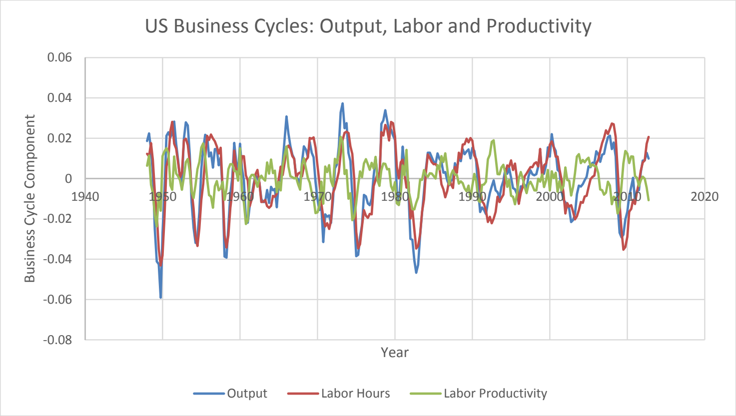
Source: US Bureau of Economic Analysis 1948- 2012 quarterly data


US Log GDP 1948-2012: Data and Trend
-7.6
-7.8
-8
-8.2
-8.4
-8.6
-8.8
-9
-9.2
1940
1950
1960
1970
1980
1990
2000
2010
2020
Log Units
Year
Log GDP
Trend

Business Cycles



Business Cycles



Facts:

Read Kydland-Prescott (1990) for a broader set of facts based on the same procedure and some perspective on the history of thought.

Candidate Shocks: Measure using Solow (1957)

$$Y_t = A_t F(x_t)\ C.R.S.$$

$$\dot{Y}_t = \dot{A}_t F(x_t) + \sum_{i=1}^{n} A_t F_i(x_t)\dot{x}_t^i$$

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{A}_t F(x_t)}{Y_t} + \sum_i \frac{A_t F_i(x_t)\dot{x}_t^i}{Y_t}$$

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{A}}{A_t} + \sum_i \left(\frac{A_t F_i(x_t) x_t^i}{Y_t}\right)\ \frac{\dot{x}_t^i}{x_t^i}$$

Solow Growth Accounting Equation w/ two Inputs:

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{A}_t}{A_t} + \beta_t \frac{\dot{K}_t}{K_t} + (1 - \beta_t) \frac{\dot{L}_t}{L_t}$$

An Approximation (also follows from C-D Prod Fn):

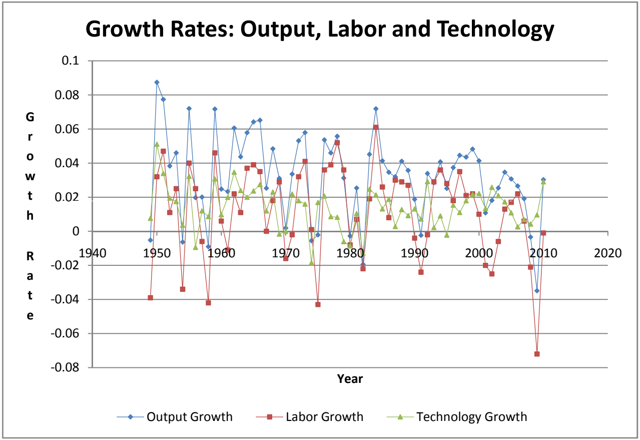
$$\frac{\Delta Y_t}{Y_t} = \frac{\Delta A_t}{A_t} + \beta \frac{\Delta K_t}{K_t} + (1 - \beta) \frac{\Delta L_t}{L_t}$$

Two Points: (1) an accounting decomp of output growth into factor input growth components and a technology component, (2) view technology component (under the theory) as true realized shock plus measurement error in inputs and output.

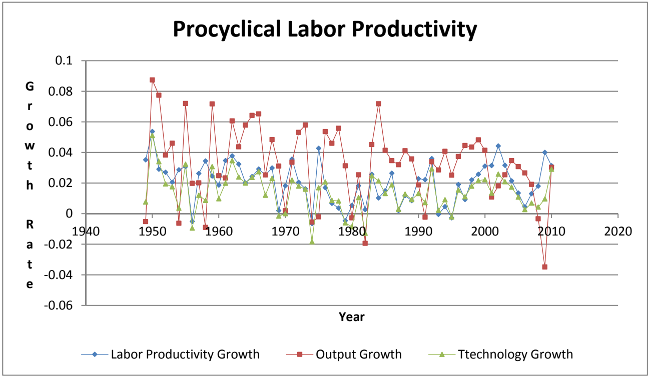
Solow (1957) Findings: US 1909- 1949

1. Output per unit of labor input grows by about 100 percent.
2. The capital-labor ratio grows by about 30 percent over the period. So there is "capital deepening".
3. Technology grows by about 80 percent. Thus, about 80 percent of the growth in output per unit of labor input over the period is accounted for by growth in the technology and the remaining 20 by increases in the capital-labor ratio.
4. The measure of the technology level falls in a number of recession and depression years and tends to increase in expansions. Thus, measured technology growth rates are "pro cyclical".

Repeat Using US 1948 -2010 data



Repeat Using US 1948 -2010 data



Theory: Based on an Aggregate Prod. Fn.

$$(*)\ Y_t = A_t F(K_t, L_t)\ CRS$$

Claim (1): If A_t is constant and K_t is constant, then all theories based on $(*)$ w/ rational or irrational sources of fluctuations in L_t will be counterfactual. Reason: Y/L is procyclical in US data.

Claim (2): Measurement of K_t in US data does not substantially change this conclusion. K_t doesn't move proportionally as much as L_t at BC frequencies and is not highly correlated w/ Y_t and L_t.

Claim (3): Since measured growth in A_t is procyclical, then theories based on $(*)$ can be consistent w/ procyclical labor productivity when labor and technology growth move together.

Theory: Based on an Aggregate Prod. Fn.

Preferences: $E[\sum_{t=0}^{\infty} \beta^t u(c_t, 1-l_t)]$
Technology: $y_t = \exp(z_t)F(k_t, l_t)$ and $c_t + x_t = y_t$

$$k_{t+1} = k_t(1-\delta) + x_t$$

$$z_t = \rho z_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2)$$

Endowments: $k_0 > 0$ and 1 unit of labor time

Theory: Based on an Aggregate Prod. Fn.

Def: A recursive competitive equilibrium is $(c(k,K,z), l(k,K,z), x(k,K,z))$ and $(w(K,z), R(K,z))$ and $g(K,z)$ s.t.

1. $(c(k,K,z), l(k,K,z), x(k,K,z))$ solve BE
2. $w(K,z) = \exp(z)F_2(K, l(K,K,z))$ and $R(K,z) = \exp(z)F_1(K, l(K,K,z))$
3. $c(K,K,z) + x(K,K,z) = \exp(z)F(K, l(K,K,z))$
4. $g(K,z) = K(1-\delta) + x(K,K,z)$

$$v(k,K,z) = \max_{(c,l,x)} u(c, 1-l) + \beta E[v(k', g(K,z), z')|z]$$

$$c + x = w(K,z)l + R(K,z)k \textit{ and } k' = k(1-\delta) + x$$

Evaluating the Model

Functional forms:

$$u(c, 1-l) = (1-\phi)\log c + \phi \log(1-l)$$

$$F(k, l) = exp(z)k^{\alpha}l^{1-\alpha}$$

$$k' = k(1-\delta) + x$$

$$z' = \rho z + \epsilon \ and \ \epsilon \sim N(0, \sigma_{\epsilon}^2)$$

Theory: Based on an Aggregate Prod. Fn.

A Procedure to Evaluate the Model: KP(1982)

1. Set many model parameters s.t absent shocks model matches averages (e.g. I/Y, labors share, ave. return to capital, fraction of time devoted to work) in US economy.
2. Set model shock parameters $(\rho, \sigma_\epsilon^2)$ based on growth accounting and aggregate inputs and outputs in US data.
3. Evaluation: Solve model using DP methods. Simulate model time series of same length as data length. Detrend model data using HP filter. Compute model second moments based on "cyclical component" just as in data. Repeat to get means across simulations.

Evaluating the Model

Set Model Parameters to Match Averages in Steady State:

1. $u_1(c, 1-l) = \beta u_1(c, 1-l)(1+r)$

$$1.04 = (1+r)^4 = (1/\beta)^4 \ implies \ \beta = (1/1.04)^{1/4}$$

2. $0.64 = labor\ share = \frac{wl}{y} = 1 - \alpha$
3. $0.15 = \frac{invest}{output} = \frac{\delta k}{y}\ and\ \frac{k}{y} \doteq 8\ imply\ \delta = 0.15/8 \doteq .02$
4. $\frac{-u_2(c,1-l)}{u_1(c,1-l)} = w$ implies $\frac{\phi}{1-\phi}\frac{c}{1-l} = (1-\alpha)\frac{y}{l}$
   $\frac{\phi}{1-\phi} = \frac{1-l}{l}(1-\alpha)\frac{y}{y-\delta k} = 2 \times 0.64 \times \frac{1}{1-\delta\frac{k}{y}}$

   Micro data implies $l = 1/3$. Back out ϕ from other values.

Evaluating the Model

How to Solve the Model, Given All Parameters?

MANY METHODS:

1. Solve Planning Problem using Finite DP methods
2. Solve Planning Problem using linear approx to policy function about deterministic steady. Use first-order Taylor Series approx in (k, z)
3. Write FOC and resource constraint to Planning Problem or Comp. Equil. and then linearize or log-linearize all equations. Get linear system in deviations from steady state quantities.
4. Nowadays, estimation, solution and simulation of (standard) BC models are done w/in packages such as DYNARE.

Model Implications: Cooley and Prescott (1995)

Variable	SD Data	SD Model	Corr w/ GDP Data	Corr w/ GDP Model
GDP	.017	.014	1.0	1.0
C Nondurables	.009	.003	0.79	0.84
C Durables	.051	-	0.65	-
Investment	.050	.06	0.84	0.99
Govt Spend	.032	-	0.05	-
Labor Hours	.015	.008	0.87	0.99
Employment	.013	-	0.81	-
Labor Prod	.008	.006	0.36	0.98

Data Source: US Bureau of Economic Analysis 1948- 2012

Model Implications:

1. Taking $\rho = 0.95$ and $\sigma_\epsilon^2 = 0.007^2$ for technology shocks, Cooley and Prescott (1995) arrive at the implications stated in the previous slide.
2. Taking $\rho = 0.9$ and $\sigma_\epsilon^2 = 0.00763^2$ for technology shocks, Prescott (1986, p. 16) "Theory Ahead of Business-Cycle Measurement" states:

   "With the standard deviation of the technology shock equal to 0.763, theory implies that the standard deviation of output will be 1.48 percent. In fact, it is 1.76 percent for the post-Korean War American economy."
3. Upshot: When these models are matched to data in this manner, the model economies produce a large fraction of the observed business-cycle movements in US output.

Discussion

1. So far these notes have described an approach to document business-cycle fluctuations and to build a quantitative model based on technology shocks as the only shocks.
2. There is substantial lore developed around trying to understand what is important in such a model for producing the SD and Corr structure of US aggregates.
3. Disagreements over theory and measurement: (1) Interpret Solow residuals as exog. shocks or as aggregation issues of other shocks across production units?, (2) How does aggregate hours variability at business-cycle frequencies arise and how is it tied to properties of utility functions at micro level? We address (2) next.

Preference Parameters: Frisch Elasticity

1. Prescott (1986) and Cooley and Prescott (1995) employ:

$$u(c,l) = (1-\phi)\log c + \phi \log(1-l)$$

2. They choose ϕ to get $1/3$ of available time is work time.
3. Some standard classes of preferences

$$u(c,l) = \frac{c^{1-\sigma}}{1-\sigma} + \phi \frac{(1-l)^{1-\frac{1}{\nu}}}{1-\frac{1}{\nu}}$$

$$u(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - \phi \frac{l^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}$$

4. Why is log utility from point 1. empirically relevant?

Preference Parameters: Frisch Elasticity

1. Following MaCurdy (1982), labor economists have tried to measure different notions of labor hours elasticity. One potentially useful elasticity is the Frisch elasticity because estimation of this elasticity relates in a fairly direct way to preference parameters.
2. Typical View: Frisch labor hours elasticity for prime-age males (age 25-55) is well below one.
3. Casual argument: Average (log) hours and wage rates are roughly hump-shaped over working lifetime but wages vary in percentage terms more than hours.

Preference Parameters: Frisch Elasticity

$\max \sum_{j=1}^{J} \beta^{j-1} u(c_j, l_j)$ s.t.
$c_j + a_{j+1} \leq w_j l_j + a_j(1+r)$ and $l_j \in [0,1]$ and $a_{J+1} \geq 0$

FONC for interior solution

$$(1) \quad u_1(c_j, l_j) = \lambda_j$$

$$(2) \quad -u_2(c_j, l_j) = \lambda_j w_j$$

$$(3) \quad \lambda_j = \lambda_{j+1} \beta (1+r)$$

Preference Parameters: Frisch Elasticity

$$(*) \quad u(c,l) = u(c) - \phi \frac{l^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}$$

Use (*) and FONC (2)-(3) for interior solution:

$$-u_2(c_j, l_j) = \lambda_j w_j \Rightarrow \phi l_j^{1/\nu} = \lambda_j w_j$$

$$\log l_j = \nu[\log \lambda_j + \log w_j - \log \phi]$$

$$\Delta \log l_j = \nu \Delta \log \lambda_j + \nu \Delta \log w_j$$

$$(**) \quad \Delta \log l_j = -\nu \log(\beta(1+r)) + \nu \Delta \log w_j$$

Preference Parameters: Frisch Elasticity

Equation (**) suggests that a linear regression of log differenced labor hours on a constant and log differenced wage rates will pick up the preference parameter ν as the slope:

$$(**) \quad \Delta \log l_j = -\nu \log(\beta(1+r)) + \nu \Delta \log w_j$$

Theory: based on perfect foresight and, hence, anticipated wage movements. Theory can be generalized to allow for wage rate risk. Get a modified version of $(**)$ - see Keane (2012, JEL) - under some assumptions. Estimation of this relationship using US males age 25-55 in PSID data produce estimated $\nu \leq 1/2$.

Equation $(**)$ is quite nice. Estimate a utility function parameter without knowing other model parameters or even a full model.

Preference Parameters: Frisch Elasticity

Some standard classes of preferences. First function is constant Frisch elasticity of leisure, Second function is hte constant Frisch elasticity of labor.

$$u(c,l) = u(c) + \phi \frac{(1-l)^{1-\frac{1}{\nu}}}{1-\frac{1}{\nu}}$$

$$u(c,l) = u(c) - \phi \frac{l^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}$$

We will now figure out theory-based Frisch elasticities for these two classes.

Preference Parameters: Frisch Elasticity

$\max \sum_{j=1}^{J} \sum_{s^j \in S^j} \beta^{j-1} u(c_j(s^j), l_j(s^j)) P(s^j)$ s.t.
$c_j(s^j) + a_{j+1}(s^j) \leq w_j(s^j) l_j(s^j) + a_j(s^{j-1})(1+r)$ and
$l_j(s^j) \in [0,1]$ and $a_{J+1} \geq 0$

FONC for interior solution

$$(1) \quad u_1(c_j(s^j), l_j(s^j)) = \lambda_j(s^j)$$

$$(2) \quad -u_2(c_j(s^j), l_j(s^j)) = \lambda_j(s^j) w_j(s^j)$$

$$(3) \quad \lambda_j(s^j) = E[\lambda_{j+1}(s^{j+1}) | s^j] \beta(1+r)$$

Define Frisch demand functions $(c(\lambda, w), l(\lambda, w), n(\lambda, w))$ as solutions to system below:

$$(1) \quad u_1(c(\lambda, w), l(\lambda, w)) = \lambda$$

$$(2) \quad -u_2(c(\lambda, w), l(\lambda, w)) = \lambda w$$

$$(3) \quad l(\lambda, w) + n(\lambda, w) = 1$$

Define labor ϵ^l and leisure ϵ^n elasticities as follows:

$$\epsilon^l(\lambda, w) \equiv \frac{\partial l(\lambda, w)}{\partial w} \frac{w}{l(\lambda, w)} \; and \; \epsilon^n(\lambda, w) \equiv \frac{\partial n(\lambda, w)}{\partial w} \frac{w}{n(\lambda, w)}$$

$$\epsilon^l(\lambda, w) = \frac{-\partial n(\lambda, w)}{\partial w} \frac{w}{1 - n(\lambda, w)} = -\epsilon^n(\lambda, w) \frac{n(\lambda, w)}{1 - n(\lambda, w)}$$

Example 1: $u(c,l) = u(c) - \phi\frac{l^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}$

$$(1) \quad u_1(c(\lambda,w),l(\lambda,w)) = \lambda$$

$$(2) \quad -u_2(c(\lambda,w),l(\lambda,w)) = \lambda w$$

$$(3) \quad l(\lambda,w) + n(\lambda,w) = 1$$

$$l(\lambda,w) = [\frac{\lambda w}{\phi}]^{\nu} \Rightarrow \epsilon^{l}(\lambda,w) = \nu$$

$$\epsilon^{n}(\lambda,w) = -\epsilon^{l}(\lambda,w)(\frac{1-n(\lambda,w)}{n(\lambda,w)}) = -\nu(\frac{1-n(\lambda,w)}{n(\lambda,w)})$$

Example 2: $u(c,l) = u(c) + \phi \frac{(1-l)^{1-\frac{1}{\nu}}}{1-\frac{1}{\nu}}$

$$(1) \quad u_1(c(\lambda,w), l(\lambda,w)) = \lambda$$

$$(2) \quad -u_2(c(\lambda,w), l(\lambda,w)) = \lambda w$$

$$(3) \quad l(\lambda,w) + n(\lambda,w) = 1$$

$$n(\lambda,w) = [\frac{\lambda w}{\phi}]^{-\nu} \Rightarrow \epsilon^n(\lambda,w) = -\nu$$

$$\epsilon^l(\lambda,w) = -\epsilon^n(\lambda,w)\frac{n(\lambda,w)}{1-n(\lambda,w)} = \nu\frac{n(\lambda,w)}{1-n(\lambda,w)}$$

Example 3: $u(c, l) = u(c) + \phi \log(1 - l)$

A limiting case of Example 2

$$n(\lambda, w) = [\frac{\lambda w}{\phi}]^{-1} \Rightarrow \epsilon^n(\lambda, w) = -1$$

$$\epsilon^l(\lambda, w) = -\epsilon^n(\lambda, w)\frac{n(\lambda, w)}{1 - n(\lambda, w)} = 1\frac{n(\lambda, w)}{1 - n(\lambda, w)}$$

$$Prescott : \epsilon^l(\lambda, w) = 1(\frac{n(\lambda, w)}{1 - n(\lambda, w)}) = 1 \times 2 = 2$$

Upshot: Frisch labor Elasticity of total work hours of 2 is several times the estimated Frisch elasticity for prime-age males w/ substantial attachment to the labor force.

More Discussion of Example 3: $u(c,l) = u(c) + \phi \log(1-l)$

1. The discussion of Example 3 up to this point is meant to get you up to speed in interpreting evidence on a key preference parameter from one theoretical perspective.
2. See "Micro and Macro Labor Supply Elasticities: A Reassessment of Conventional Wisdom." - Keane and Rogerson (2012, JEL) - for a recent review.
3. KR(2012): BC agg. hours fluctuations involve a large movement in and out of employment (extensive margin) and smaller agg. movement from varying hours worked per worker (intensive margin). Thus, BC agg. hours fluctuations need theoretical models where agg. hours variation involves extensive margin decisions (e.g. models w/ search, home production, multi-member households, retirement,...).