

Macro I

Homework 9 - A Business Cycle Model

Consider an economy with the primitives specified below. There is a single exogenous source of randomness - iid draws of the random variable z_t . A model period corresponds to a quarter of a year.

1. Preferences: $E[\sum_{t=0}^{\infty} \beta^t u(c_t, l_t)]$
2. Technology: $c_t + x_t = y_t = \exp(z_t)F(k_t, l_t)$ and $k_{t+1} = k_t(1 - \delta) + x_t$ and $z_t \in Z \equiv \{z_1, z_2, z_3\}$ and $P(z_1) = P(z_2) = P(z_3) = 1/3$
3. Endowments: $k_0 > 0$ and there is one unit of time each period.

$$F(k, l) = k^\alpha l^{1-\alpha}$$

$$u(c, l) = \log c + \phi \frac{(1-l)^{1-\frac{1}{\nu}}}{1-\frac{1}{\nu}}$$

$$Z \equiv \{z_1, z_2, z_3\} = \{-0.007, 0.0, 0.007\}$$

We consider two closely related models (Model 1 and Model 2) that differ in the values of the model parameters (ϕ, ν) . These two models are calibrated so that absent risk and setting the technology level at $\bar{z} = 0$, they both produce (i) an annual interest rate of $1 + r = 1.04$, (ii) an investment output ratio of $i/y = .16$, (iii) capital's share of income of 0.36, (iv) agents work one third of available time ($\bar{l} = 1/3$).

Parameter Values: Model 1 and 2

	β	ϕ	ν	α	δ
Model 1	$(1/1.04)^{1/4}$	0.4515	0.25	.36	0.00788
Model 2	$(1/1.04)^{1/4}$	1.4916	0.95	.36	0.00788

Assume that the economy starts out with $(k_0, z_0) = (36, 0)$. Construct one random draw $\{z_t\}_{t=0}^{100}$. Compute the realized path for output and labor input in a competitive equilibrium for this shock history in both Model 1 and Model 2. Plot the paths for output in Models 1-2 on the same graph. Do the same for labor input. Include your code and graphs with your homework. Offer an interpretation for the differences in the graphs for Models 1-2.

Suggested Methods:

1. I suggest that you approximate solutions to the model by solving the optimal planning problem. Use finite dynamic programming methods from homework 3. Put a uniform grid on k over the interval $[33, 38]$ with 401 points. When you solve Bellman's equation, restrict next periods capital to be on your capital

grid and restrict the choice of labor input to lie on a uniform grid on $[0.3, 0.35]$ with 101 points.

2. Use a random number generator to construct one random draw $\{z_t\}_{t=0}^{100}$. Use this fixed draw in all your analysis.

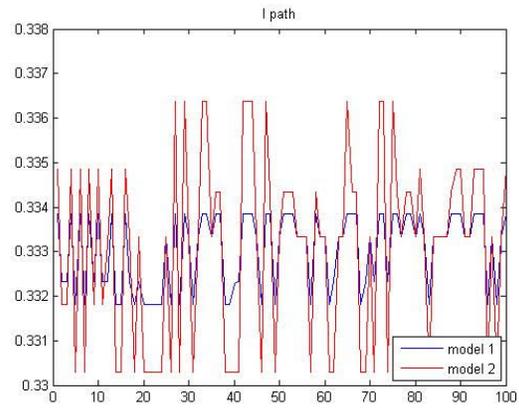
3. Use the computed optimal decision rules to simulate realizations $\{y_t, l_t\}_{t=0}^{100}$, starting from state $(k_0, z_0) = (36, 0)$ using the draw $\{z_t\}_{t=0}^{100}$.

Commentary: Labor input and output paths in Model 1 and Model 2 should look roughly like the graphs below. The graphs display “weak propagation” in that the output and labor paths are roughly scaled versions of the path for shocks themselves. So whatever is the persistence of the shocks is the same as the persistence of output. If this property were to hold more broadly for this type of model, then it could be successful for describing output fluctuations only if an estimated shocks process has similar persistence properties to estimated output persistence properties.

Labor Path and Output Path

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l path.jpg



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y path.jpg

