Macro I

Homework 6- Asset Pricing

1. (Lucas Tree Economy)

(a) Compute the stock price function and the price function of a risk-free asset paying 1 unit of goods one period in advance in a one-tree version of the Lucas asset pricing model. Assume the dividends are iid and equal 1 and 2 with equal probability. Period utility is $u(c) = c^{1-\rho}/(1-\rho)$, where $\rho = 2.0$, and the discount factor is $\beta = .96$.

(b) Define the gross realized return on stocks and bonds as $R^s(y, y')$ and $R^b(y, y')$, where y is the current state and y' is tomorrow's realized state. This is just the value of the asset tomorrow plus dividend divided by the value of the asset today.

Using this definition, calculate the conditional expected gross return $R^i(y) = E[R^s(y, y')|y]$ in each state for $i = s, b$. Also calculate the unconditional gross returns $E[R^i(y)]$. To calculate unconditional returns use the unconditional (steady state) probabilities of the Markov chain on y. Be sure to state these probabilities.

(c) Offer an explanation for why the conditional or unconditional returns to stocks are higher/lower than bonds.

[Hint: use the defintion of covariance.]

2. (Lucas Tree Economy with an Information Variable)

Imagine that we add to the Lucas asset pricing model a variable i_t that offers information that may be relevant to future dividends in the sense that it may help predict future dividends. You can think of the information variable as news. The agent knows the conditional joint probability distribution F on future dividends and information. Here we assume that dividends and information follow a Markov process specified by F. Assume that $y \in Y$ and $i \in I$, where both Y and I are finite sets.

$$F(y', i'|y, i) = Prob(y_{t+1} \leq y', i_{t+1} \leq i'|y_t = y, i_t = i)$$

(i) Provide a definition of equilibruim for this economy. Be sure to describe the maximization problem that the agent solves.

(ii) Under what circumstances does the pricing function from this model coincide with the pricing function for the model without the information variable?

(iii) Is there a unique price function for stock in this model? Explain carefully.

3. (Hansen-Jaganathan Bounds)

(a) Suppose that we observe 200 realizations of the stock price in the economy described in problem 1. In these 200 observations 100 were the high price and

100 were the low price. Using this data calculate the Hansen-Jaganathan bounds on the mean and the standard deviation of stochastic discount factors consistent with the data observations. Graph the resulting H-J bounds.

[Hint: Do this by concentrating on discrete points (i.e. .90, .91, ..., 1.09, 1.10) on the mean of the stochastic discount factor. To calculate the bounds for this particular example it is not important in what order the data on prices were observed.]

(b) On the same graph clearly plot the unconditional mean and standard deviation of the theoretical stochastic discount factor from problem 1.