Macro I

Homework 5- Exchange Economies

1. Consider an exchange economy with two agents $i = 1, 2$. The endowments of the two types are stochastic and are given by $e^1(s^t) = e^2(s^t) = s_t$. Thus, at date t in history $s^t = (s_0, ..., s_t)$ each agent has an endowment of s_t units of a consumption good. Let $P(s^t)$ denote the probability of history $s^t \in S^t$, where S^t denotes the set of possible t-period histories. It is understood that in any period t the realization s_t lies in a finite set $S = \{1, 2\}$ and that the stochastic process is Markovian with time invariant transition probability $\pi(s'|s)$, where s denotes current state and s' next periods state.

Preferences: $U^1(c) = U^2(c) = \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t u(c_t(s^t))P(s^t)$

Endowments: $e^1(s^t) = e^2(s^t) = s_t, \forall t, \forall s^t$

Calculate (i.e. write down) a competitive equlibrium with time-0 markets for this economy when $s_0 = 1$ and when u is a strictly increasing, strictly concave, differentiable period utility function. Offer some justification for why what you write down is an equilibrium.

2. Consider an exchange economy with the same two agents, the same preferences and the same endowments. The only difference with respect to question 1 is the market structure. Instead of a complete set of time-0 markets for each commodity, the economy has in each time period t two financial assets (Arrow securities). One asset pays off 1 unit of the time $t+1$ consumption good provided $s_{t+1} = 1$ and zero otherwise. The other asset pays 1 unit of the time $t + 1$ consumption good provided $s_{t+1} = 2$ and zero otherwise. Let $Q(s_{t+1}|s^t)$ denote the price in history s^t of the Arrow security that pays off when state s_{t+1} is realized.

Calculate (i.e. write down) a competitive equlibrium with sequential markets for this economy when $s_0 = 1$ and when u is a strictly increasing, strictly concave, differentiable period utility function. Offer some justification for why what you write down is an equilibrium.

3. Consider an overlapping generations economy in which there is one good in each period and there are infinitely many periods $t = 1, 2, ...$ There is one agent born in each period $t = 1, 2, ...$ and an initial old agent "born" in period $t = 0$. The utility of an agent born in period $t = 1, 2, ...$ is $U^t(c^t) = u(c_t^t) + u(c_{t+1}^t) = \log c_t^t + \log c_{t+1}^t$ and the utility of the agent born in period $t = 0$ is $U^0(c^0) = \log c_1^0$. The endowments of an agent born in period t are $(w_t^t, w_{t+1}^t) = (4, 2)$ for $t \geq 1$ so that they have 4 units of the consumption good when young in period t and 2 units when old in period $t + 1$. The agent born in period 0 is endowed with 2 units of the period 1 good (i.e. $w_1^0 = 2$).

(a) Define an Arrow-Debreu (i.e. "competitive equilibrium with time 1 markets") equilibrium for this economy. Calculate an Arrow-Debreu equilibrium for this economy.

(b) Define an equilibrium with Arrow securities (i.e. competitive equilibrium with sequential markets and Arrow securities) for this economy. Calculate an equilibrium with Arrow securities for this economy.

(c) Are the equilibrium allocations that you calculate in part a and/or part b Pareto efficient? Explain.