Macro I

Homework 1- Formulating Dynamic Programming Problems

Instructions:

Each problem describes a maximization problem specified in terms of agents choosing sequences. Reformulate each problem as a dynamic programming problem. Thus, for each problem specify (i) state x and control y variables, (ii) feasible sets ($Y(x)$ or $Y_j(x)$), (iii) return functions ($u(x,y)$ or $u_j(x,y)$), (iv) the law of motion for the state ($x' = g(x,y)$ or $x' = g_j(x,y)$) and (v) Bellman's equation.

1. [Consumption and Saving]

An agent lives forever and maximizes the following objective subject to the constraints indicated below. In this problem e is a constant labor endowment, r is a constant real interest rate, c_t is consumption and k_{t+1} is the amount of assets brought from period t to period $t+1$. On the preference side $\beta \in [0,1)$ is a discount factor and u is a period utility function. The initial assets k_0 are fixed.

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$c_t + k_{t+1} \leq k_t(1+r) + e$$

$$c_t, k_{t+1} \geq 0$$

2. [Optimal Growth]

This problem is the same as problem 1 but the agent has a production technology in place of a market for selling labor and holding financial assets. Here δ is a depreciation rate on physical capital. This is the standard problem studied in the optimal growth literature.

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$c_t + k_{t+1} \leq f(k_t) + k_t(1-\delta)$$

$$c_t, k_{t+1} \geq 0$$

3. [Firm's Problem]

A firm maximizes profit, the first expression below, by choosing the capital stock over time. Here $p_t = (1+r)^{-(t-1)}$ is the time 1 price of the time t good, where $r > 0$. The constraints are also provided below where the initial capital stock k_1 is taken as given.

$$\max \sum_{t=1}^{\infty} p_t[F(k_t) - i_t]$$

$$k_{t+1} = k_t(1 - \delta) + i_t$$

$$k_{t+1} \geq 0$$

4. [Consumption Decisions with Habit Formation]

A consumer maximizes utility $\sum_{t=1}^{T} u_t(c_{t-1}, c_t)$ by choosing a consumption plan. The constraints are that (i) c_0 is a given initial habit and (ii) $\sum_{t=1}^{T} p_t c_t \leq 100$, (iii) $c_t \geq 0$.

5. [Human Capital Problem]

An agent lives for T periods. Each period the agent makes a consumption-saving decision as well as a decision of what fraction l of the working day to spend working and what fraction $(1 - l)$ to spend in human capital accumulation. The objective is to maximize the utility function below. The constraints are listed below. Here (r, w, δ) are the real interest rate, the rental rate of human capital and the depreciation rate on human capital. The variables (h, k, c) denote human capital, asset holding and consumption. Initial human and physical capital stocks are known.

$$\max \sum_{t=1}^{T} u_t(c_t)$$

$$c_t + k_{t+1} \leq k_t(1 + r) + h_t w_t l_t$$

$$h_{t+1} = h_t(1 - \delta) + f(h_t, l_t)$$

$$c_t, k_{t+1} \geq 0, l_t \in [0, 1]$$

6. [Search]

An agent maximizes expected utility $E[\sum_{t=0}^{\infty} \beta^t u(c_t)]$. The agent receives a wage offer w at $t = 0$. If the agent accepts this offer, the agent works forever at wage w receiving utility $\sum_{t=0}^{\infty} \beta^t u(w)$. If the agent rejects this offer, the agent receives periods utility $u(0) = 0$ and gets an offer next period drawn from a distribution F. The decision problem repeats each period as long as the agent rejects the offer.