Business Cycles: Facts and Theory

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One of the most controversial questions in macroeconomics is what explains business-cycle fluctuations?

Economists largely agree on what are the key facts describing business-cycle fluctuations. Thus, it is fairly clear what business-cycle theorists are trying to explain. However, a unifying explanation for these facts is still hotly debated. Business-cycle policy is also hotly debated for the same reason.
According to Burns and Mitchell (1946, p. 3), business cycles are

“... a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own.”
Economists nowadays characterize business-cycle facts quite differently from Burns and Mitchell. A common approach is to divide a time series $\left(y_1, y_2, \ldots, y_n\right)$ into trend and cycle components. Business-cycle facts are then properties of the cycle component $y_t^{cycle}$:

$$y_t = y_t^{trend} + y_t^{cycle}$$
Modern view: What are the key facts?
Business-cycle facts describe the statistical regularities of the cycle component.

Key properties
1. amplitude of fluctuations
2. comovements
3. lead and lag patterns
Business Cycles

US Log GDP 1948-2012: Data and Trend
Business-Cycle Facts

How to define the smooth trend line?

We will follow the literature and use the Hodrick-Prescott filter to define the smooth trend line. Intuitively, this is a way to extract the long-run growth portion of a series. It can also be viewed as a way to take away “low frequency” components from the data.
How to define the smooth trend line?

Given data \((y_1, \ldots, y_T)\), Hodrick and Prescott defined the trend \((y_1^{trend}, \ldots, y_T^{trend})\) as the values for the trend that minimize the objective below when \(\lambda = 1600\):

\[
\sum_{t=1}^{T} (y_t - y_t^{trend})^2 + \lambda \sum_{t=2}^{T-1} [(y_{t+1}^{trend} - y_t^{trend}) - (y_t^{trend} - y_{t-1}^{trend})]^2
\]

This is a mechanical procedure that passes a trend through data.
The next two figures plot the cyclical components of output, consumption and investment in the US as well as the cyclical components of output, labor hours and labor productivity.

We will then be especially interested in the magnitude of the cyclical fluctuations and how correlated the cyclical fluctuations in various aggregate series are with output.
Business Cycles

US Business Cycles: Output Components

Year
Business Cycle Component

Consumption Investment Output
US Business Cycles: Output, Labor and Productivity

Year

Output Labor Hours Labor Productivity
Next we consider a study by Kydland and Prescott that documented the magnitude of cyclical fluctuations in US data and their correlation with the cyclical component of output. Their data runs from 1954-1989. They apply the same definition of trend as the series plotted earlier.
### Table 1
Cyclical Behavior of U.S. Production Inputs

<table>
<thead>
<tr>
<th>Variable x</th>
<th>Volatility (% Std. Dev.)</th>
<th>$x(t-5)$</th>
<th>$x(t-4)$</th>
<th>$x(t-3)$</th>
<th>$x(t-2)$</th>
<th>$x(t-1)$</th>
<th>$x(t)$</th>
<th>$x(t+1)$</th>
<th>$x(t+2)$</th>
<th>$x(t+3)$</th>
<th>$x(t+4)$</th>
<th>$x(t+5)$</th>
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<tbody>
<tr>
<td>Real Gross National Product</td>
<td>1.71</td>
<td>-0.03</td>
<td>0.15</td>
<td>0.38</td>
<td>0.63</td>
<td>0.85</td>
<td>1.00</td>
<td>0.85</td>
<td>0.63</td>
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<td></td>
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<tr>
<td>Hours (Household Survey)</td>
<td>1.47</td>
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<td>0.05</td>
<td>0.23</td>
<td>0.44</td>
<td>0.69</td>
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<td>0.82</td>
<td>0.82</td>
<td>0.67</td>
<td>0.47</td>
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<tr>
<td>Hours per Worker</td>
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<td>0.49</td>
<td>0.66</td>
<td>0.71</td>
<td>0.59</td>
<td>0.43</td>
<td>0.29</td>
<td>0.11</td>
<td>-0.02</td>
</tr>
<tr>
<td>Hours (Establishment Survey)</td>
<td>1.65</td>
<td>-0.23</td>
<td>-0.07</td>
<td>0.14</td>
<td>0.39</td>
<td>0.66</td>
<td>0.88</td>
<td>0.92</td>
<td>0.81</td>
<td>0.64</td>
<td>0.42</td>
<td>0.21</td>
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<tr>
<td>GNP/Hours (Household Survey)</td>
<td>0.88</td>
<td>0.11</td>
<td>0.21</td>
<td>0.34</td>
<td>0.48</td>
<td>0.60</td>
<td>0.71</td>
<td>0.59</td>
<td>0.43</td>
<td>0.29</td>
<td>0.11</td>
<td>-0.02</td>
</tr>
<tr>
<td>GNP/Hours (Establishment Survey)</td>
<td>0.83</td>
<td>0.40</td>
<td>0.46</td>
<td>0.49</td>
<td>0.53</td>
<td>0.43</td>
<td>0.31</td>
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<td>-0.31</td>
<td>-0.49</td>
<td>-0.52</td>
<td>-0.50</td>
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<td>Average Hourly Real Compensation (Business Sector)</td>
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<td>0.42</td>
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<tr>
<td>Nonresidential Capital Stock*</td>
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<td>-0.48</td>
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<td>-0.08</td>
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<td>-0.55</td>
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<td>0.09</td>
<td>0.25</td>
<td>0.38</td>
<td>0.45</td>
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<tr>
<td>Producers' Durable Equipment</td>
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<td>-0.58</td>
<td>-0.53</td>
<td>-0.41</td>
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<td>0.47</td>
<td>0.62</td>
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<td>0.71</td>
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<tr>
<td>Inventory Stock (Nonfarm)</td>
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<td>0.72</td>
<td>0.83</td>
<td>0.81</td>
<td>0.71</td>
<td>0.53</td>
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</table>

Source of basic data: Citicorp's Citibase data bank
## Business Cycles

### Table 2

**Cyclical Behavior of U.S. Output and Income Components**

**Deviations From Trend of Product and Income Variables**

Quarterly, 1954–1989

<table>
<thead>
<tr>
<th>Variable</th>
<th>Volatility (% Std. Dev.)</th>
<th>(x(t-5))</th>
<th>(x(t-4))</th>
<th>(x(t-3))</th>
<th>(x(t-2))</th>
<th>(x(t-1))</th>
<th>(x(t))</th>
<th>(x(t+1))</th>
<th>(x(t+2))</th>
<th>(x(t+3))</th>
<th>(x(t+4))</th>
<th>(x(t+5))</th>
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</thead>
<tbody>
<tr>
<td>Real Gross National Product</td>
<td>1.71</td>
<td>-0.03</td>
<td>0.15</td>
<td>0.38</td>
<td>0.63</td>
<td>0.85</td>
<td>1.00</td>
<td>0.85</td>
<td>0.63</td>
<td>0.38</td>
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<td>-0.03</td>
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<td>Consumption Expenditures</td>
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<td>0.41</td>
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<td>0.81</td>
<td>0.82</td>
<td>0.66</td>
<td>0.45</td>
<td>0.21</td>
<td>-0.02</td>
<td>-0.21</td>
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<td>Nondurables &amp; Services</td>
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<td>0.76</td>
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<td>0.69</td>
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<td>0.90</td>
<td>0.81</td>
<td>0.60</td>
<td>0.35</td>
<td>0.08</td>
<td>-0.14</td>
</tr>
<tr>
<td>Nonresidential</td>
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<td>-0.13</td>
<td>0.05</td>
<td>0.31</td>
<td>0.57</td>
<td>0.80</td>
<td>0.88</td>
<td>0.83</td>
<td>0.68</td>
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<td>-0.17</td>
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<td>0.73</td>
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<td>-0.34</td>
<td>-0.45</td>
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<td>Government Purchases</td>
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<td>-0.03</td>
<td>-0.01</td>
<td>-0.01</td>
<td>0.09</td>
<td>0.12</td>
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<td>0.06</td>
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<td>0.10</td>
<td>0.17</td>
<td>0.25</td>
<td>0.26</td>
<td>0.25</td>
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<td>0.16</td>
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<td>0.11</td>
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<td>0.71</td>
<td>0.51</td>
<td>0.28</td>
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<td>Real Net National Income</td>
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<tr>
<td>Labor Income*</td>
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<td>-0.02</td>
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<td>0.80</td>
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<td>0.60</td>
<td>0.30</td>
<td>0.02</td>
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<td>-0.29</td>
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<td>Proprietors' Income &amp; Misc.</td>
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<td>0.62</td>
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<td>0.29</td>
<td>0.11</td>
<td>0.02</td>
<td>-0.10</td>
</tr>
</tbody>
</table>

*Employee compensation is deflated by the implicit GNP price deflator.

**This variable includes corporate profits with inventory valuation and capital consumption adjustments, plus rental income of persons with capital consumption adjustment, all deflated by the implicit GNP price deflator.
Table 4
Cyclical Behavior of U.S. Monetary Aggregates and the Price Level
Devations From Trend of Money Stock, Velocity, and Price Level
Quarterly, 1954–1989

<table>
<thead>
<tr>
<th>Variable</th>
<th>Volatility (% Std. Dev.)</th>
<th>$x(t-5)$</th>
<th>$x(t-4)$</th>
<th>$x(t-3)$</th>
<th>$x(t-2)$</th>
<th>$x(t-1)$</th>
<th>$x(t)$</th>
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<tr>
<td>Monetary Base</td>
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<td>-0.12</td>
<td>0.02</td>
<td>0.14</td>
<td>0.25</td>
<td>0.36</td>
<td>0.41</td>
<td>0.40</td>
<td>0.37</td>
<td>0.32</td>
<td>0.28</td>
<td>0.26</td>
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<td>M1</td>
<td>1.68</td>
<td>0.01</td>
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<td>0.35</td>
<td>0.31</td>
<td>0.22</td>
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<td>M2</td>
<td>1.51</td>
<td>0.48</td>
<td>0.60</td>
<td>0.67</td>
<td>...0.68</td>
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<td>0.46</td>
<td>0.26</td>
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<td>0.67</td>
<td>0.65</td>
<td>0.56</td>
<td>0.40</td>
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<td>-0.01</td>
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<tr>
<td>Monetary Base</td>
<td>1.33</td>
<td>-0.26</td>
<td>-0.15</td>
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<td>0.22</td>
<td>0.40</td>
<td>0.59</td>
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<td>-0.08</td>
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<tr>
<td>M1</td>
<td>2.02</td>
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<td>-0.19</td>
<td>-0.12</td>
<td>-0.01</td>
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<td>M2</td>
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<td>0.40</td>
<td>0.43</td>
<td>0.44</td>
<td>0.43</td>
</tr>
<tr>
<td>Price Level</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Implicit GNP Deflator</td>
<td>0.89</td>
<td>-0.50</td>
<td>-0.61</td>
<td>-0.68</td>
<td>-0.69</td>
<td>-0.64</td>
<td>-0.55</td>
<td>-0.43</td>
<td>-0.31</td>
<td>-0.17</td>
<td>-0.04</td>
<td>0.09</td>
</tr>
<tr>
<td>Consumer Price Index</td>
<td>1.41</td>
<td>-0.52</td>
<td>-0.63</td>
<td>-0.70</td>
<td>-0.72</td>
<td>-0.68</td>
<td>-0.57</td>
<td>-0.41</td>
<td>-0.24</td>
<td>-0.05</td>
<td>0.14</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Source of basic data: Citicorp's Citibase data bank.
Key US Facts:

1. Labor hours are about as variable as output and are strongly procyclical. The standard deviation of output is 1.71, whereas labor is 1.47. Thus, a typical deviation from trend for output is 1.71 percent.
2. Labor productivity is procyclical.
3. Consumption is less variable than output but investment is much more variable than output.
4. Consumption and investment are procyclical. Government spending is acyclical.
5. Measures of money are procyclical but prices are countercyclical.
Business-Cycle Facts

Other Data Issues:
1. Have business-cycle fluctuations changed in magnitude or nature over time in US data?

There is a literature that debates whether business-cycle fluctuations in the US were larger before the Great Depression than after WW II. There is also a literature that documents smaller business-cycle fluctuations in the US after 1984 but before the Great Recession.

2. Is the systematic movement in quarterly GDP (the seasonal cycle) smaller or larger than business-cycle fluctuations?
A good place to begin discussing business-cycle theory is by discussing the procyclical productivity puzzle.

**Procyclical Productivity Puzzle:**

The data tell us that at business-cycle frequencies labor hours move a lot (SD=1.5 percent) and are procyclical but that capital input moves little and is acyclical. If we adopt a theory that features a neoclassical aggregate production function, then what are some different possibilities to produce procyclical labor productivity \((Y/L)\)?
A Problem:

Any theory with an unchanging, CRS production function \( Y_t = F(K_t, L_t) \) with diminishing marginal products will be problematic. We will see that this holds regardless of what are the fundamental shocks (e.g. animal spirits, government spending shocks, ...) driving business cycles other than technology shocks and regardless of whether or not consumer behavior in the face of such shocks is rational or irrational.

Qualifying Proviso: capital input varies little in percentage terms.
Business Cycles

\[ y = F(\bar{K}, L) \]

Diagram showing relationship between output (y) and inputs (K, L) with points y_1, y_2, L_1, and L_2.
Two Possible Solutions:

1. \( Y_t = A_t F(K_t, L_t) \) and \( A_t \) varies.

2. \( Y_t = F(K_t, L_t) \) w/ increasing returns to labor

The next slide plots the same two data points on two graphs. The data points are consistent with procyclical productivity. The possible “solutions” proposed involve the aggregate production function passing through the data points, holding capital input unchanged.
A large class of business-cycle theories are based on “impulses” or shocks on the one hand and “propagation mechanisms” on the other hand. This follows Knut Wicksell who reportedly said:

“If you hit a wooden rocking horse with a club, the movement of the horse will be very different to that of the club.”
Business-Cycle Theory

**Impulses:** technology shocks, government spending-tax-default shocks, animal spirits, erratic monetary policy, uncertainty shocks, news about the probability of various shocks, sunspots, financial shocks.

Impulses are viewed as exogenous

**Propagation Mechanisms:** physical capital, human capital, future consumption is a normal good, balance sheets of a firm, inventories, input-output linkages across industries
While there are many different business-cycle theories to consider, we will consider just two. They will be vastly different in theoretical structure and in policy implications.

* Theory 1: Technology Shock Theory

* Theory 2: Keynesian Animal Spirits Theory
Theory 1: Life-Cycle Model w/ Technology Shocks

Main Ingredients:
- rational choice: Yes ... consumers optimize
- technology: $Y_t = A_t F(K_t, L_t) = A_t K_t^\beta L_t^{1-\beta}$
- impulses: $A_t$ - technology shocks
- shock propagation: capital accumulation + future consumption is a normal good
Theory 1: A Permanent Increase in Technology Assume:

1. The Life-Cycle Model is the way the economy works.
2. Economy is at steady state at $t = 1$
3. Assume $A_1 = 1, A_2 = 2, A_3 = 2, A_4 = 2, ...$

To figure out what happens ... graph the law of motion.
\[ k_{t+1} = (1 - \alpha)(1 - \beta) A^B_2 k_t^B \]

\[ k_{t+1} = (1 - \alpha)(1 - \beta) A_1 k_t^B \]
Business Cycles

Permanent Technology Shock: Life-Cycle Model

- Output
- Capital
- Wage
- Investment
- Technology
Theory 1: Basic Story

The growth rate of technology $A_t$ (as measured by the Solow residual) is variable in data. This theory implies high $A$ is associated with high output $Y$, high wage $W$ and high $Y/L$. This pattern is supported in data.

The life-cycle model abstracts from a labor-leisure choice. Thus, it does not get labor hours to vary. Procyclical labor hours can be achieved by adding such a choice to the model. However, much of the cyclical variation in labor hours in the data comes via changes in employment - unemployment. Modeling unemployment is way beyond what we can do in this course.
Theory 1: Policy

If the Life-Cycle model with shocks is the theory of business cycles, then attempts to smooth out business cycle fluctuations will not (with positive real interest rates) lead to Pareto improvements. Recall the Proposition established in the chapter on the life-cycle model.
Theory 2: Old-Time Keynesian Story

Main Ingredients:

- rational choice: None ... Keynes abandons microeconomics
- technology: no production function
- impulses: animal spirits of investors are exogenous impulses
- shock propagation: unclear as the model is static and does not model time periods
Rational Choice: None

“The fundamental psychological law, upon which we are entitled to depend with great confidence both a priori and from our knowledge of human nature and from the detailed facts of experience, is that men are disposed, as a rule and on the average, to increase their consumption as their income increases, but not by as much as the increase in their income.” - Keynes (1936, Chapter 8, p. 96.).
Source of Shocks: Animal Spirits

“Even apart from the instability due to speculation, there is the instability due to the characteristic of human nature that a large proportion of our positive activities depend on spontaneous optimism rather than on a mathematical expectation, whether moral or hedonistic or economic. Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as a result of animal spirits - of a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities.” - Keynes (1936, Chapter 12, p. 161)
Stylized Keynesian Model

\[ C + I + G = Y \] - NIPA identity
\[ C = a + b(Y - T) \] - consumption \((a > 0\) and \(0 < b < 1\))
\[ I \] - determined by animal spirits
\[ T = G \] - assumption of a balanced budget

Solve for Output \(Y\):
\[ a + b(Y - G) + I + G = Y \] or \[ Y = \frac{a + I + (1 - b)G}{1 - b} \]
Theory 2: Basic Story

The economy is buffeted by animal spirits of investors. Thus, investment moves exogenously. Times of low investment are times of low output as the model implies (under the assumptions above): \[ Y = \frac{a + I + (1-b)G}{1-b}. \]

Times of low output are taken to be indicative of a poorly functioning economy. A formal welfare analysis is not undertaken (as it would in other areas of economics) using the Pareto criteria and the utility functions of the consumers.
Theory 2: Policy

Typical advice: increase government spending (or reduce taxes) in a recession.

Balanced Budget Multiplier: \[ \frac{\Delta Y}{\Delta G} = \frac{1-b}{1-b} = 1 \]
\[ Y = \frac{a+I+(1-b)G}{1-b} \Rightarrow \Delta Y = \frac{(1-b)\Delta G}{1-b} \]

Unbalanced Budget Multiplier: \[ \frac{\Delta Y}{\Delta G} = \frac{1}{1-b} > 1 \]
\[ Y = \frac{a-bT+I+G}{1-b} \Rightarrow \Delta Y = \frac{\Delta G}{1-b} \]
Government Spending Multipliers:

**Keynesian Model:** An increase in government spending (whether balanced budget or deficit financed) increases current output as both multipliers are POSITIVE. If the multiplier is positive but less than 1 then \((G, Y)\) go up but \(C\) falls. Model does not have dynamic multipliers as the model is static.
Government Spending Multipliers:

Life-Cycle Model: A multiplier is defined as the change in output (at some horizon) due to the increased spending divided by the change in government spending. To determine multipliers, the output path under two different government spending plans must be calculated. Basic Conclusion: current multiplier is ZERO but the multiplier is NEGATIVE in all future periods as the taxes/borrowing financing extra spending depresses the capital stock.

NOTE: If a labor-leisure decision is added to the life-cycle model, then the output multiplier could be POSITIVE. This could occur if the negative income effect (from the increased taxes funding government spending) leads to an increase in labor that is sufficiently strong to offset the fall in savings from the increase in taxes.
Multipliers: Empirical Work

It is not surprising that different theories have completely different multipliers or that one theory could produce either positive or negative multipliers under different assumptions.

Romer and Romer (2010) argue, using US data, that the contemporaneous tax multiplier is roughly zero but is negative at longer horizons. Thus, an exogenous increase in taxes leads to lower future output. We will discuss their work in chapter 7.
Gains to Business-Cycle Smoothing

Are Gains to Business-cycle Smoothing Large?

Although the source(s) of business-cycle fluctuations are controversial, could we figure out whether the maximum potential gain to (somehow) eliminating these fluctuations is large?

Robert Lucas’ back-of-the-envelope calculation was that the maximum potential gain was worth about $8.50 per person per year! We will review the logic behind his calculation.
Gains to Business-Cycle Smoothing

Digression: Expected Utility Theory

\( \Omega = \{1, 2, \ldots, N\} \) - \( N \) possible distinct states of nature.

\( (x_1, x_2, \ldots, x_N) \) - A gamble that pays off \( x_i \) units of output if state \( i = 1, \ldots, N \) occurs.

\( (p_1, p_2, \ldots, p_N) \) - \( p_i \) is the probability that state \( i \) occurs.

\( E[x] = \sum_{i=1}^{N} x_i p_i \) - the expected value of a gamble

\( u(x) \) - the utility obtained for a sure payoff of \( x \) units

\( E[u(x)] = \sum_{i=1}^{N} u(x_i) p_i \) - the expected utility of a gamble
Example 1: [Coin Toss]

\[ \Omega = \{ H, T \} \]
\[ (x_H, x_T) = (90, 110) \]
\[ (p_H, p_T) = (0.5, 0.5) \]
\[ u(x) = \log(x) \]

Expected utility:
\[ u(x_H)p_H + u(x_T)p_T = \log(90) \times 0.5 + \log(110) \times 0.5 \]
Example 1: [Coin Toss]

If the utility function $u$ displays diminishing marginal utility, then getting the expected value (or mean) of a gamble as a sure payout is strictly preferred to taking the gamble. Utility functions with this property display risk aversion. How valuable is getting the mean for sure?

$$
\log(100) = \log(90(1 + \lambda)) \times 0.5 + \log(110(1 + \lambda)) \times 0.5
$$

$$
\log(1 + \lambda) = \log(100) - \log(90) \times 0.5 + \log(110) \times 0.5 = 0.00218
$$

$\lambda = 0.005$ implies getting the mean is worth 1/2 percent of consumption
A Simplified Version of Lucas’ Argument:

1. Assume the government engineers the smooth trend consumption rather than a risky consumption process with the same period-by-period mean.
2. Assume agents are risk averse.
3. Quantify the gain achieved by such business-cycle smoothing.
4. Need to measure consumption variation. A one standard deviation movement of the cyclical component of aggregate consumption from trend is about a 1.25 percent movement over 1954-89.
5. Need knowledge of \( u \).
Specific Assumptions:

\[ E[u(c)] = u(c_{\text{low}})P(\text{low}) + u(c_{\text{high}})P(\text{high}) \]

Probability: \( P(\text{low}) = P(\text{high}) = 1/2 \)

Utility: \( u(c) = \frac{c^{1-\gamma}}{1-\gamma} \), where \( \gamma > 1 \)

Consumption: \( c_{\text{low}} = 98 \) and \( c_{\text{high}} = 102 \)
Gain ($\lambda$) to getting $c = 100$ for sure?

$$E[U((1 + \lambda)c)] = U(100)$$

$$\frac{(102(1+\lambda))^{1-\gamma}}{1-\gamma}(1/2) + \frac{(98(1+\lambda))^{1-\gamma}}{1-\gamma}(1/2) = \frac{(100)^{1-\gamma}}{1-\gamma}$$

$$\Rightarrow (1 + \lambda)^{1-\gamma} = \frac{100^{1-\gamma}}{102^{1-\gamma}(1/2) + 98^{1-\gamma}(1/2)}$$

$$\Rightarrow \lambda = \left[\frac{100^{1-\gamma}}{102^{1-\gamma}(1/2) + 98^{1-\gamma}(1/2)}\right]^{1/(1-\gamma)} - 1$$
Table: Gain \( \lambda \) to Eliminating Aggregate Fluctuations:

<table>
<thead>
<tr>
<th>Coefficient of Risk Aversion (( \gamma ))</th>
<th>Compensation (( \lambda ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma = 2 )</td>
<td>( \lambda = .00040 )</td>
</tr>
<tr>
<td>( \gamma = 4 )</td>
<td>( \lambda = .00080 )</td>
</tr>
<tr>
<td>( \gamma = 10 )</td>
<td>( \lambda = .00199 )</td>
</tr>
</tbody>
</table>

Lucas: $8.50 is a tenth of a percent of consumption/person in US in 1983.
Table above says that if risk aversion \( \gamma \leq 4 \), then the gain is less than a tenth of a percent of consumption.
How Risk Averse Are You?
Pick the row best matching your risk aversion. Each row describes the maximum percentage of total wealth that a consumer with utility function $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$ is willing to give up to avoid an even-odds gamble of gaining or losing a fraction $\alpha$ of total wealth.

Percentage of Wealth Given Up to Avoid a Proportional Wealth Gamble of Size $\alpha$

<table>
<thead>
<tr>
<th>Coefficient of Risk Aversion ($\gamma$)</th>
<th>$\alpha = 10%$</th>
<th>$\alpha = 30%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma = 1$</td>
<td>0.5%</td>
<td>4.6%</td>
</tr>
<tr>
<td>$\gamma = 4$</td>
<td>2.0%</td>
<td>16.0%</td>
</tr>
<tr>
<td>$\gamma = 10$</td>
<td>4.4%</td>
<td>24.4%</td>
</tr>
<tr>
<td>$\gamma = 40$</td>
<td>8.4%</td>
<td>28.7%</td>
</tr>
</tbody>
</table>
Potential Reasons Why Lucas’ number is so small:

1. Aggregate consumption movements over the business cycle are small after WWII.
2. Individual households experience much larger consumption fluctuations than those in aggregate data.
3. Eliminating or reducing aggregate fluctuations may reduce (but not eliminate) individual fluctuations. If so, should start calculations with the magnitude of individual fluctuations not aggregate fluctuations.