Homework 6- Business Cycle Fluctuations

1. (Impulse and Propagation)
Calculate time paths for the capital-labor ratio (k), output per person (y), investment (i) and the wage rate (w) in the life-cycle model economy when it is hit with a temporary increase in the technology level. Calculate these four variables for time periods \( t = 0, 1, 2, 3, 4 \) and graph them.

Assumptions:
\( \alpha = .5 \) - preference parameter
\( (\beta, \delta) = (.5,.5) \) - capital’s share of output and the depreciation rate
\( A_0 = 10, A_1 = 11, A_2 = 10, A_3 = 10, A_4 = 10 \) - technology level over time
\( k_0 = 6.25 \) - the capital-labor ratio at time \( t = 0 \)

Hint: First calculate \( k_t \) using the law of motion: 
\[
k_{t+1} = (1 - \alpha)(1 - \beta)A_t k_t^\beta.
\]
Then use the fact that all the other variables are simple functions of the capital-labor ratio: 
\[
y_t = A_t k_t^\beta, \quad w_t = (1 - \beta)A_t k_t^\beta \quad \text{and} \quad i_t = k_{t+1} - k_t (1 - \delta).
\]

2. Using the data from Tables 1 and 2 in the paper "Business Cycles: Real Facts and a Monetary Myth" in the column labeled \( x(t) \) to state whether each of the following variables are pro-cyclical, counter-cyclical or acyclical:
(i) investment
(ii) wage rate (i.e. average hourly real compensation)
(iii) labor hours
(iv) capital input

Hint: A series is pro-cyclical if its correlation with GDP is positive and is counter-cyclical if its correlation with GDP is negative. A series is acyclical if its correlation with GDP is near zero.

Does the model from question 1 seem like it can generate any of these qualitative features of the data? Explain.

3. Calculate whether or not hours worked \( L \), capital input \( K \) and average wages \( w \) are procyclical or countercyclical in US data. Answer the question by calculating the correlation of the growth rates of each of these three series with the output growth rate. Use the data on (i) output \( Y \), (ii) hours worked \( L \) and (iii) capital \( K \) from Homework 3 to calculate growth rates. T

Step 1: You have data on all of these variables except average wages. Calculate average wages \( w_t \) using \( w_t = (1 - \alpha)Y_t/L_t \), where \( \alpha = .36 \).

Step 2: Calculate annual growth rates for (i) output, (ii) hours, (iii) capital and (iv) wages.

Step 3: Calculate the correlation of each of these three series with output growth. Use the correlation function in Excel (i.e. CORREL(array1, array2)) to calculate each of these correlations.
4. (Keynesian Economics)

Consider the simple Keynesian model below. Equation 1 is an identity, Equation 2 is a behavioral equation for consumption. Equation 3 is the government budget constraint which requires budget balance. The government chooses government spending $G$ and taxes $T$ each model period.

Exogenous model parameters: (i) parameters governing consumption are $a = 10$ and $b = 0.8$ and (ii) investment $I_t$ at time $t$ is $(I_1, I_2, I_3, I_4, I_5) = (5, 5, 2, 5, 5)$ over model periods 1 to 5.

1. $C + I + G = Y$
2. $C = a + b(Y - T)$
3. $G = T$

(a) If the government wants to engineer a constant level $Y$ of GDP equal to 100 in each model periods $t = 1, ..., 5$ taking investment as exogenous, then find the path of government spending and taxes in these five periods that makes $Y_t = 100$ in each model period.

(b) In the model calculate the balanced budget government spending multiplier.