Tipping Points and Business-as-Usual in a Global Carbon Commons

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Abstract

This paper analyzes a dynamic strategic model of carbon consumption among nations. Each period, countries extract carbon from the global ecosystem. A country's output depends both on its carbon usage and on"stored carbon" in the ecosystem. The desired mix of extracted versus stored carbon varies across countries and evolve stochastically over time.

A *Business-as-usual (BAU) equilibrium* characterizes countries' strategic incentives in the absence of an effective international agreement. Under non-concave carbon dynamics, depletion of the carbon stock in a BAU equilibrium may reach a *tipping point* below which the global commons spirals downward toward a steady state of marginal sustainability. These *tipping points* emerge endogenously. We show that if elasticities with respect to carbon usage become large enough, a tipping point will be breached. We find that countries will, in fact, *accelerate* their rates of carbon usage the closer they are to tipping. Even so, there remains a time span (a "negotiation window") in which a collapse may be averted if the countries agree to implement the socially efficient profile of carbon usage.

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Key Words and Phrases: carbon consumption, global carbon commons, tipping points, safe operating space for humanity, international carbon agreements.

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1 Introduction

Human consumption is based on carbon usage. Alarmed by increases in anthropogenic GHGs, many scientists and policy experts focus on finding an effective international response to limit carbon emissions.[1]

This paper formulates a dynamic model of global carbon consumption in the absence of such a response. Our objective is to understand the strategic incentives of nations in a "business-as-usual" (or "BAU" from here on) scenario. What are the long run implications of BAU? How does it compare with socially efficient usage? Are outcomes under BAU sustainable or is economic collapse inevitable? What determines the transition, if any, from sustainability to collapse?

To make sense of the last few questions in particular, our model integrates a strategic model of emissions into a nonlinear dynamic model of carbon. A key feature of this model is that consumption and economic output may collapse and shrink if a key state variable falls below some critical threshold — a *tipping point* — determined endogenously in equilbrium.

Tipping points are commonly discussed and modeled in the earth science literature. See, for instance, Lenton, et al. (2008), Kerr (2008), Rockstrom, et al. (2009), Anderies, et al. (2013), and Steffen, et al. (2015). Most of these posit nonlinear dynamical systems that describe a *safe operating space (SOS) for humanity*, i.e., a region in a multi-dimensional space consisting of levels of methane and CO_2 concentrations, degrees of biodiversity, and so on,... that sustains human innovation, growth, and development. Tipping points are defined as the *planetary boundaries* of these regions (see Rockstrom et al. (2009)).

The earth science models contain a detailed account of the different forms of carbon mass and their movements throughout the carbon cycle. Human incentives are not usually modeled explicitly. The present paper complements these by modeling the dynamic incentives of nations to extract carbon in an, albeit rudimentary, model of the carbon cycle.

Economic incentives appear in the integrated assessment models of Nordhaus (2006, 2007, 2008), Lemoine and Traeger (2014), Hope (2006), Stern (2006), and Cai, Judd, and Lontzek (2012) all of whom integrate tipping dynamics into (atomistic) GE market economies.[2] They also appear in the shallow lake models of Carpenter, et al. (1999) and Maler et al (2003) who analyze open loop Nash equilibria of a game with non-linear accumulation of pollutants.

[1] e.g., IPCC Fourth Assessment Report: Climate Change 2007.

[2] See also Krusell and Smith (2009), Acemoglu et al. (2012), and Golosov et al. (2014) for useful quantitive assessments of carbon taxation and cap and trade policies.

Our focus on real time strategic incentives of state actors is, to us, a sensible addition to the IAM and shallow lakes literatures since the most critical policy choices are made by large, powerful nations with divergent interests.

We therefore posit a dynamic stochastic game in which each country produces a composite consumption good for its citizens. Production depends both on a carbon-based input and on the carbon-based ecosystem. The ecosystem is an open access source of stored carbon from which countries can freely extract. The model builds on the common pool framework initiated by Levhari and Mirman (LM) (1980).[3] The potential for over-depletion here echoes the "tragedy of the commons" theme running through models in this framework.

While each country's carbon extraction is essential for its own production, the global depletion of the carbon-based ecosystem also affects output. Some preservation of the ecosystem and its repository of stored carbon is, therefore, beneficial for purely economic reasons (Section 2.1 further elaborates). The productive inputs therefore include both emitted and "stored" (non-atmospheric) carbon. The latter is summarized by a carbon resource stock ω_t at each date t representing all usable sources of non-atmospheric carbon in the global ecosystem. The stock ω_t may be thought of as known reserves of "stored" or "preserved" carbon in biomass, soil, or fossils.

Each country's desired mix of stored and extracted carbon is determined by its relative output elasticities. Countries with relatively high output elasticities of extracted carbon prefer to extract more than countries with low elasticities. The elasticities are assumed to evolve stochastically, and the country-specific shocks to these may be serially correlated. This assumption captures a common feature in studies of greenhouse gas emissions: both environmental costs and factor composition vary over time, are difficult to forecast, and often vary widely across countries. Heterogeneity reflects variation in geographic, demographic, and politico-economic influences.[4] The profile of country-specific carbon elasticities, together with the carbon stock constitute the state variables of the system.

The driving force of the model is a low-end instability in the law of motion for carbon. When ω_t is large enough, dynamic forces of release and recapture produce a

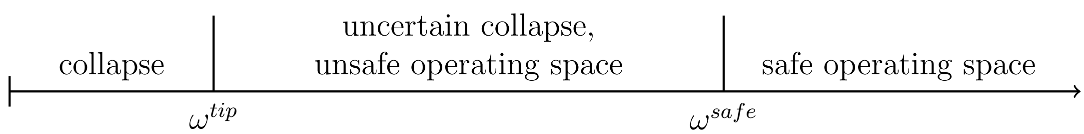
[3]See also Cave (1987), Mirman and Fisher (1992), Dutta and Sundaram (1993), Sorger (1996), Mirman and Fisher (1996), Finus (2001), Barrett (2003), Dutta and Radner (2004, 2006, 2009), and Battaglini and Harstad (2012), all of whom examine strategic incentives in dynamic games with a commons or with climate externalities. Our emphasis on tipping point dynamics presents a distinct set of challenges for the common pool problem.

[4]Burke, et. al. (2011) find, for example, widely varying estimates of the effect of climate change on US agriculture when climate model uncertainty is taken into account. Desmet and Rossi-Hansberg (2014) document substantial cross country variation in a calibrated model of spatial differences in welfare losses across countries due to global warming.

stable carbon cycle. A low enough stock, however can destabilizes the cycle leading to a collapse — a tipping problem. The tipping point at which this occurs is endogenously determined in a *business-as-usual (BAU) equilibrium*, a smooth Markov Perfect equilibrium profile of carbon usage across countries. The main results characterize BAU equilibria, endogenous tipping points, and the incentives of countries in the face of a tipping threat.

Sakamoto (2014) also studies BAU equilibria in a tipping model. In his model, a given resource threshold (a tipping point) triggers a change in the likelihood of a regime switch, i.e., a switch from a high growth state to a low growth one. Strategic play determines whether the tipping point is breached.

In the present model the growth rates and tipping points are determined *endogenously* from strategic play. In turn, strategic choices depend on technical change. Thus, the "deep" parameters that determine tipping here are technological: the factor elasticities determining the mix of extracted and stored carbon. The tipping points separate regions of ω_t in which collapse will never occur (the *safe operating space (SOS)*) from regions of possible collapse. Regions of possible collapse can be distinguished from regions of inevitable collapse. Consequently, the mechanism produces a schematic consistent with the planetary boundaries framework of Rockstrom et al. (2009) and Steffan, et al. (2015) and is displayed below.[5]



In this schematic, the tipping points are the boundaries of the support of an equilibrium distribution. This distribution describes the probability of collapse starting from any given stock. In the case where the equilibrium distribution is degenerate (i.e., if all sources of stochastic variation are removed), then $\omega^{tip} = \omega^{safe}$. In that case, there is a single tipping point that divides the space into tipped and non-tipped regions, somewhat analogous to Skiba points in non-convex optimal control problems.[6] Even in that case, the point is endogenously determined in a multi-player game and, in particular, no one nation can unilaterally ensure that tipping will not occur. With stochastic variation in extraction elasticities, the equilibrium induces a distribution on such points.

[5]The schematic display in Steffen, et al. (2015 and reprinted in the Washington Post, January 15, 2015) uses emissions rather than stocks and so the direction of risk is rightward rather than leftward.

[6]Skiba (1978).

We find parametric configurations to support both SOS and collapse. Our results show the following. First, a global economy in a BAU equilibrium can remain in SOS if elasticities of carbon usage remain low. If, however, the carbon factor elasticities become large enough and remain so over a long enough span of time, the BAU equilibrium will eventually breach a tipping point, precipitating a collapse.

Second, countries actually accelerate their rates of carbon usage the closer the carbon commons comes to tipping. By contrast, countries are more cautious the *further* they are from tipping. The intuition is vaguely reminiscent of the Green Paradox (Sinn (2013)) which posits that firms increase their extraction of fossil fuels if more stringent regulations are expected in the future.

In the Green Paradox, the future regime change is exogenous from the firms' perspective. Here, regime change is endogenously determined by the strategic actors themselves. In our model, each actor expects his marginal continuation value of preserving the carbon stock to vanish when the likelihood of tipping is high due to the strategic play of others. Hence the actor accelerates his extraction *which, in turn, feeds back to the tipping likelihood itself.*

This result is accentuated by heterogeneity in elasticities across countries. Countries with either very high or very low carbon factor elasticities have larger output than those with intermediate elasticities. This means that even as the global commons reaches a tipping point, the leading carbon emitters are not the first to suffer consequences for output of an accelerated decline.

Third, the BAU equilibrium generates lower aggregate output and higher carbon use each period than the extraction plan chosen by a social planner. This is a standard result in non-tipping models of the commons. We show that it remains true with tipping. Moreover, the relative difference between the BAU and socially optimal paths of aggregate carbon stock grow over time. Unlike most commons models, some countries might actually use *less* carbon under BAU.

Fourth, we show that collapse may be avoided if the international community moves away from business-as-usual and toward a socially optimal extraction regime. Indeed, because the tipping points under BAU arrives sooner than in the social optimum, even if the BAU equilibrium tipping point is breached there is still some time (a "negotiation window") in which a collapse may be averted if the countries agree to implement the optimal plan. The upshot is that an effective international agreement provides an additional buffer against large shocks to the carbon stock. Thus the scenario is more optimistic if countries can achieve such an agreement.[7]

[7] See, for instance, Finus (2001), Barrett (2003), Calcott and Petkov (2012), and Harrison and Lagunoff (2016).

2 A Tipping Model of Carbon Usage

2.1 An Overview

This section lays out a rudimentary model of carbon usage. The model consists of an infinite horizon global economy with n countries. Each country makes use of an essential resource — carbon — each period. Carbon is extracted from a carbon-based commons which we refer to as the "global ecosystem." Extraction includes, but is not limited to, fossil fuels. It also includes depletion of any form of biomass (e.g., deforestation) and soil nutrients.

The framework is reminiscent of the classic common pool model of Levhari and Mirman (LM) (1980). In the LM model, identical users choose how much of a depletable, open access resource to consume each period. Examples include fisheries or forestry. There are no direct costs or externalities from usage. More importantly there is no tangible constraint on consumption/production *until* the resource stock literally hits zero. Conservation is thus valued in LM only for instrumental reasons: preserving the stock allows one to smooth consumption.

By contrast, we analyze a production technology where the carbon-based ecosystem enters as an input. This means that, unlike a pure commons, there are tangible impediments to production even when the stock is not fully depleted. As a result, there are incentives to conserve even if there is no threat whatsoever of full depletion. We further modify the model by (1) introducing heterogeneous shocks that affect each country's desired mix of productive inputs, and (2) positing a carbon dynamic with a low-end non-concavity capable of tipping the system.

Some features of the setup should be clarified at the outset. First, one could object to lumping all forms of carbon stock, including terrestrial, marine, and geological sources, into a single state variable. One might argue that the accumulation of geological carbon, for instance, is a long term process and should therefore be considered separate from the ecosystem. Our inclusion of fossil fuels is based on evidence that extraction of fossil fuels can deplete the ecosystem (Rockstrom, et al. (2009)). Fracking, strip-mining, oil drilling all involve potential depletion of biomass or limits on its growth. Stored carbon stock represents a "flip side" of carbon emissions in our model, and so avoiding emissions is equivalent to preserving the stock — including geological carbon. By separating out the various stocks, the game theoretic aspects of use versus preservation are obscured.

Second, one might observe that fossil fuels are not open access; its distribution around the world is non-uniform. Yet a full accounting for *all* forms of emittable carbon makes open access a defensible approximation. Countries like Brazil and

Tanzania have large rain forests and agricultural production. Other countries like Russia and Saudi Arabia extract fossil fuels. Along these lines, asymmetries in access are incorporated indirectly by assuming heterogeneous technologies across countries.

2.2 Output and Carbon Extraction

Countries make inter-temporal strategic decisions regarding how much carbon to extract and use. Country i's ($i = 1, \ldots, n$) carbon extraction in date t is denoted by c_{it}. Let $C_t = \sum_i c_{it}$ represent the level of global carbon consumption at t. Consumption of C_t units of carbon produces C_t units of emissions, and so the two terms are sometimes used interchangeably. The global consumption C_t is consumed from a stock ω_t that represents all usable sources of non-atmospheric carbon in the global ecosystem. It is worth noting that C_t includes usage of all forms of carbon, not just fossil fuels. The stock ω_t may be thought of as known reserves of "stored" or "preserved" carbon in soil, biomass, or fossils. Extracted carbon is emitted into the atmosphere. The simple distinction between stored and released carbon forms the basis for all dynamic changes in the model.[8]

A composite good y_{it} for each country represents the output consumed by the representative citizen from country i at date t. The long run payoff to the representative citizen from country i for consuming y_{it} at each date t is

$$\sum_t \delta^t \; u(y_{it}) \tag{1}$$

with u strictly increasing, differentiably concave, and $u' \to \infty$ as $y_{it} \to 0$. The main equilibrium characterization results will assume $u(y_{it}) = \log(y_{it})$. All countries discount the future according to δ.

The production of y_{it} depends on both extracted carbon and the carbon-based global ecosystem according to the production technology

$$y_{it} = c_{it}^{\theta_{it}} (\omega_t - C_t)^{1-\theta_{it}}. \tag{2}$$

In (2), $\theta_{it} \in [0, 1]$ is the output elasticity of extracted carbon, while $1 - \theta_{it}$ is the output elasticity of the global ecosystem net of aggregate consumption. Countries with larger θ_{it} in date t will typically extract and emit more carbon, other things equal.[9]

[8]The stock variable includes carbon netted out of the emissions process via photosynthesis and marine absorption.

[9]The assumption of linearly homogeneous production is purely for tractability. None of the results depend on it. One can work with general coefficients α_{it} and β_{it} and derive the same qualitative conclusions.

The formulation accounts for the fact that all countries' economies have carbon requirements, but production also requires that countries draw upon a viable ecosystem. Recall that the stored carbon stock represents a "flip side" of carbon emissions, and so the assumption that the stored stock is a productive input is equivalent to modeling carbon emissions as a GDP-reducing cost. By design, the production function in (2) excludes the traditional inputs capital and labor on the grounds that these are largely derived from elemental, carbon-based inputs in (2) and, hence, would be double-counted.[10]

The model, moreover, builds in correlated shocks to the relative elasticities. The elasticities are assumed to vary both over time and between countries, the latter reflecting the fact that both benefits and costs of extraction differ across countries. Warmer average temperatures resulting from GHG emissions are viewed differently in Greenland than in Sub-saharan Africa. Time variation comes from the fact that countries may be hit with serially correlated shocks. The shocks capture the unpredictability of technological change and the persistence of climatic change within each country.

A *type profile* in date t is a vector

$$\theta_t = (\theta_{1t}, \theta_{2t}, \ldots, \theta_{it}, \ldots \theta_{nt}),$$

and is publicly observed at the beginning of each period t. Let $\theta^t = \{\theta_0, \theta_1, \ldots, \theta_t\}$ be the history of realized type profiles up to and including date t, and let

$$\theta^\infty \;=\; \{\theta_0, \theta_1, \ldots, \theta_t, \ldots\}$$

the infinite time path of elasticity profiles.

Fixing the initial profile θ_0, the profile θ_t is assumed to evolve according to a stationary, though not necessarily ergodic, Markov process $\pi(\theta_t|\theta_{t-1})$. In what follows, "almost everywhere" will refer to the paths θ^∞ in the probability space $(\Theta^\infty, \mathcal{F}, P)$ such that π is the Markov density associated with a filtration $\{\mathcal{F}_t\}$ on the space $(\Theta^\infty, \mathcal{F}, P)$. We allow for π to exhibit both persistence across time and correlation of carbon elasticities across countries.

2.3 Carbon Stock Dynamics

We introduce a law of motion which, in the absence of human consumption (i.e., $C_t = 0$), will balance the dynamic forces of release and recapture of carbon through

[10] Labor, for instance, is derived from carbon usage (caloric intake, respiration, etc). Many if not most types of capital embody carbon, including metal alloys (steel), plastics, and organic inputs (rubber, cotton, wool, wood, livestock, etc.).These would be included in the stock ω_t.

sequestration to produce a stable carbon cycle if the stock ω_t is not too low. The law of motion, however, also contains a non-concavity that can destabilizes the cycle if the stock is low enough. Expressed formally, the ecosystem evolves according to:

$$\omega_{t+1} = \begin{cases} A(\omega_t - C_t - b)^\gamma & if \quad A(\omega_t - C_t - b)^\gamma \geq F \\ F & \text{otherwise} \end{cases} \tag{3}$$

with the initial stock fixed at some level ω_0. By assumption, $\gamma < 1$ which allows for depreciation (e.g., plant respiration), while $A > 1$ which allows for accumulation due to natural reabsorption (e.g., plant photosynthesis). For now the parameter A is assumed to be constant but the model can be generalized to allow for a time-varying ergodic process $\{A_t\}$, in which case there is a stable carbon cycle if the stock is large enough.[11] We assume $\delta\gamma < 1$.

The parameter b describes the exogenous "off-take", the subtraction of carbon from the stock that is independent of human decisions. It can be interpreted as a lower bound below which natural recapture or sequestration cannot occur. When $b > 0$ the carbon-based ecosystem can collapse and shrink if the stock falls below some critical carbon threshold — a *tipping point* — a concept explicitly defined and characterized in Section 3.2. Figure 1 illustrates the dynamic in (3) (for illustrative purposes, C_t is held constant in the Figure). There are three fixed points, one of which, ω°, is unstable (the analogue of a Skiba point in optimal control problems). In a non-stochastic world with fixed exogenous human behavior, ω° would correspond to a "tipping point." In this case, F represents an "environmental poverty trap" since a stock that reaches the carbon floor F remains stuck there forever.[12]

In the present model, however, the law of motion in Figure 1 is one realization from a possible continuum of laws of motion, each corresponding to the realization of θ from the stochastic process π. The resulting stochastic law of motion is itself endogenously determined by equilibrium behavior. Hence, from the point of view of the participants, tipping is a stochastically and endogenously determined phenomenon.

When $F > 0$ carbon stocks are never fully depleted. This assumption is largely a matter of convenience.[13] The exogenous off-take $b > 0$ is critical because there is no tipping problem if $b = 0$ (a fact established later in Section 3.1). For this reason, the

[11] An even richer model would allow A to depend on the existing stock. For tractability, however, we assume it is fixed and exogenous.

[12] The figure is canonical under a parametric restriction. Specifically, all fixed points of $\omega_{t+1} = A(\omega_t - C_t - b)^\gamma$, if any exist, must lie above F. A sufficient condition for this is to require b sufficiently large such that any stock ω satisfying to $\omega = A(\omega - b)^\gamma$ (fixing $C_t = 0$) must lie above F.

[13] Specifically, because $u = -\infty$ at zero, a floor $F > 0$ rules out full depletion, thus avoiding the limit at $\omega_t = 0$.

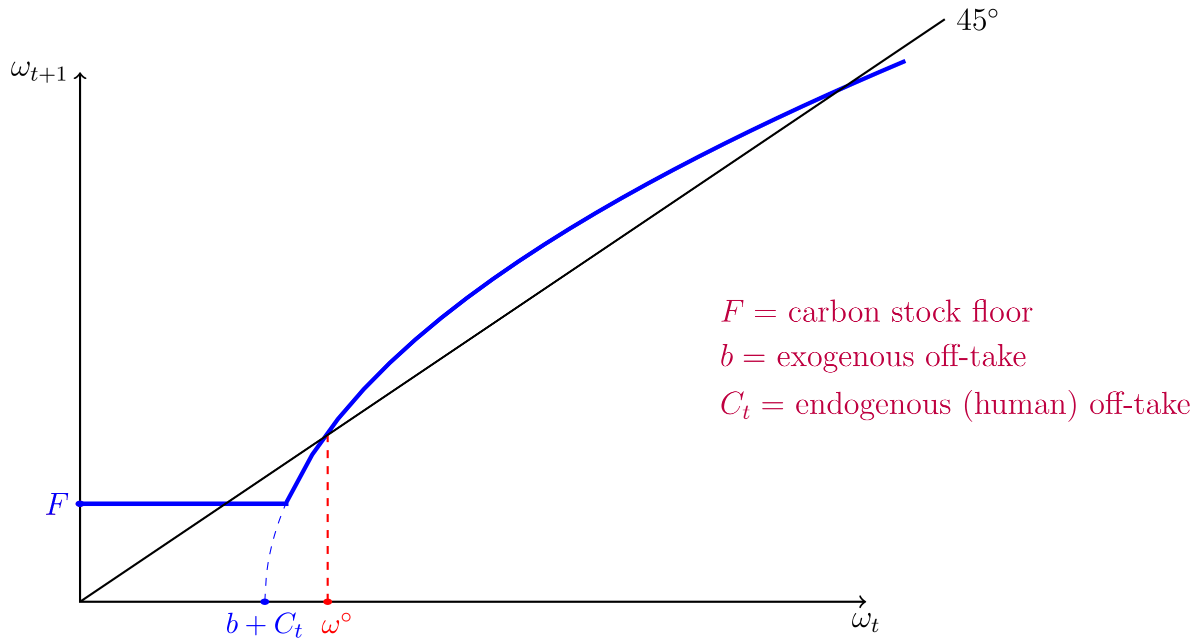


Figure 1: Carbon dynamics

model with $b > 0$ and $F > 0$ will be referred to as the "tipping model" — as distinct from the benchmark no-tipping case of $b = 0$ and $F = 0$.[14]

The dynamic in (3) is not intended to be a literal description of an earth system. Rather, we view it as a tractable heuristic that incorporates a local instability at the low-end of the carbon stock. Specifically, the carbon dynamic allows for growth, depreciation, and/or sudden collapse to the stock, depending on parameters. We later show that a number of modifications intended to make the model more suitable to standard models of the commons, for instance allowing F to vary stochastically, will not change the main results.

Let $\mathbf{c}_t = (c_{1t} \ldots, c_{nt})$ denote the date t profile of resource consumption (and emissions). The entire dynamic path profile of resource consumption is then given by

$$\mathbf{c} \;=\; \{\mathbf{c}_t\}_{t=0}^{\infty}$$

A consumption path $\mathbf{c}$ is *feasible* if it is consistent with the dynamic constraint (3) and $C_t \leq \omega_t - b$ at each date t.

Overall, the model presents a simplification of the geophysical dynamics of carbon. It nevertheless captures what Cai, et al. (2012, p.2) argue are two critical

[14]The cases $b > 0, F = 0$ and $b = 0, F > 0$ are not formally considered since they are tangential to the main focus of the paper.

features that should be included in a reasonable representation of tipping. Namely, "(i) a fully stochastic formulation of abrupt changes, and (ii) a representation of the irreversibility" of the collapse. Regarding (ii), the law of motion in Equation (3) converges to a low but finite steady state F whenever the carbon stock falls below a critical "tipping" point. The fact that the low steady state is independent of human activity is roughly consistent with simulations by Hansen et. al (2013), demonstrating a "soft" or "low-end" runaway greenhouse effect. Their simulations "indicate that no plausible human-made GHG forcing can cause an instability and runaway greenhouse effect" in which extreme, amplified feedbacks fully dissipate the stored carbon stock and evaporate all planetary surface water — as believed to have happened on Venus.

3 The Business-As-Usual Equilibrium

In any period, the state of the global carbon economy is summarized by the pair (ω_t, θ_t) consisting of the ecosystem and the elasticity profile. A *Markov-contingent plan* is a state-contingent profile

$$c^*(\omega_t, \theta_t) = (c_1^*(\omega_t, \theta_t), \ldots, c_n^*(\omega_t, \theta_t))$$

that specifies each country's usage $c_i^*(\omega_t, \theta_t)$ as a function of the state (ω_t, θ_t). The corresponding aggregate consumption is $C^*(\omega_t, \theta_t) = \sum_i c_i^*(\omega_t, \theta_t)$.

The long run payoff of a Markov-contingent plan $\mathbf{c}^*$ to the representative citizen in country i may be expressed as

$$U_i(\omega_t, \mathbf{c}^*, \theta_{it}) \;\equiv\; E\left[\sum_{\tau=t}^{\infty} \delta^{\tau-t} u\left(\; (c_{i\tau}^*(\omega_\tau, \theta_\tau))^{\theta_{i\tau}} (\omega_\tau - C_\tau^*(\omega_\tau, \theta_\tau)\;)^{1-\theta_{i\tau}}\;\right) \middle|\; \omega_t, \theta_t\right] \quad (4)$$

A *Markov Perfect equilibrium (MPE)* is a Subgame Perfect equilibrium in which each country's strategy is a Markov-contingent plan.[15] The MPE is often interpreted as a " business-as-usual" benchmark since it represents a scenario that prevails in the *absence* of any agreement or coordination among the participants. The MPE requires no special coordination, no monitoring beyond the initial quota, and no explicit sanctions.[16] The definition is fairly standard in the dynamic common pool literature (e.g., Dutta and Radner (2009)).

[15]In any MPE each country's Markov-contingent plan c_i^* maximizes $U_i(\mathbf{c}^*, \omega_t, \theta_{it})$ given c_{-i}^* in any state (ω_t, θ_{it}) over the set of *full history-contingent* consumption plans. For brevity, we omit the specification of full history contingent strategies. Payoffs corresponding to infeasible paths must be formally defined as well. For our purposes, the simplest approach is to define the payoff on the extended real line, setting flow payoffs equal to $-\infty$ whenever $C_t \geq \omega_t + b$.

[16]Without the Markov restriction, a version of a Folk Theorem can be applied (see, for instance, Dutta (1995) for a general statement), and efficient plans can be implemented by international coordination on the appropriate punishments.

We further restrict attention to *smooth* MPE, that is, Markov-contingent plans that are both Subgame Perfect and smooth functions of the state (smooth everywhere except possibly at the floor F). This restriction rules out certain MPE that use discontinuities in the state to create triggers on which participants can tacitly coordinate.

Consequently, we refer to any such MPE as a *Business-as-usual (BAU) equilibrium*. In any BAU equilibrium, country i's Markov-contingent plan $c_i^*(\omega_t, \theta_t)$ must maximize its long run payoff from date t, given the carbon dynamic in Equation (3) and production technology (2), and given any past history of consumption and elasticity profiles.

Let $\omega_{t+1}^*(\omega_t, \theta_t)$ denote the BAU equilibrium law of motion when (3) is evaluated at $c^*(\omega_t, \theta_t)$. Iterating forward, $\omega^{*\,t+s}(\omega_t, \theta^{t+s-1}),\ s = 0, 1, 2 \ldots$ denotes the equilibrium path from ω_t.[17] Depending on trends in elasticities over time, both growth or contraction in output and carbon stock can occur in equilibrium.

3.1 Euler equation

Using the parameterization $u(y_{it}) = \log(y_{it})$ in (4), the BAU equilibrium consumption $c_i^*(\omega_t, \theta_t)$ may be found as a solution to the Bellman equation

$$U_i(\omega_t, \mathbf{c}^*, \theta_{it}) = \max_{c_{it}} \left\{ \theta_{it} \log c_{it} + (1 - \theta_{it}) \log(\omega_t - C_t) \ + \ \delta\ E \left[U_i(\omega_{t+1}, \mathbf{c}^*, \theta_{i\,t+1}) \middle|\ \omega_t, \theta_{it} \right] \right\} \tag{5}$$

subject to (3) after for every state (ω_t, θ_t).

To calculate the BAU it is simpler to work with extraction *rates* rather than levels. The extraction rate e_{it} is defined implicitly by $c_{it} = e_{it}\omega_t$. Denote the global extraction rate by $\mathcal{E}_t = \sum_i e_{it}$.

With this transformation, the Bellman equation can be expressed in terms of a Markov-contingent extraction *rate*, $\mathbf{e}^*$. The payoff (5) is thus expressed as

$$\widetilde{U}_i(\omega_t, \mathbf{e}^*, \theta_{it}) = \max_{e_{it}} \left\{ \theta_{it} \log \omega_t e_{it} + (1 - \theta_{it}) \log(\omega_t(1 - \mathcal{E}_t)) \ + \ \delta\ E \left[\widetilde{U}_i(\omega_{t+1}, \mathbf{e}^*, \theta_{i\,t+1}) \middle|\ \omega_t, \theta_{it} \right] \right\} \tag{6}$$

[17] This path of carbon stock is defined inductively by

$$\omega^{*\,t+1}(\omega_t, \theta^t) \ = \ \omega_{t+1}^*(\omega_t, \theta_t), \quad \omega^{*\,t+2}(\omega_t, \theta^{t+1}) \ = \ \omega_{t+2}^*(\omega^{*\,t+1}(\omega_t, \theta^t), \theta_{t+1}), \ \cdots$$

$$\cdots \ \omega^{*\,t+s}(\omega_t, \theta^{t+s-1}) \ = \ \omega_{t+s}^*(\omega^{*\,t+s-1}(\omega_t, \theta^{t+s-2}), \theta_{t+s-1}\ ), \quad \cdots\cdots$$

By construction, $\widetilde{U}_i(\omega_t, \mathbf{e}^*, \theta_{it}) = U_i(\omega_t, \mathbf{c}^*, \theta_{it})$.

In the subsequent analysis, we also employ the following notation. Let $1^*_{\{\omega_t, \mathcal{E}_t\}}$ be an indicator function taking value "1" whenever $A(\omega_t(1-\mathcal{E}_t)-b)^\gamma > F$, and taking value zero otherwise. The indicator registers a value of "1" whenever the carbon floor is not reached next period.

Our first result shows that the BAU equilibrium solves a system of Euler equations in extraction rates. Each Euler equation is derived by applying the usual Envelope theorems to the first order conditions associated with (5). The Euler equations are given by

$$\underbrace{\frac{\theta_{it}}{e_{it}} - \frac{1-\theta_{it}}{1-\mathcal{E}_t}}_{\text{net marginal benefit of extraction to country } i \text{ in per. } t}$$

$$=$$

$$\underbrace{\frac{\delta\gamma\ \omega_t}{\omega_t(1-\mathcal{E}_t)-b}\left\{1 + E\left[\left(\frac{\theta_{i\,t+1}}{e_{i\,t+1}} - \frac{1-\theta_{i\,t+1}}{1-\mathcal{E}_{t+1}} - \frac{\theta_{i\,t+1}}{e_{i\,t+1}}\xi^{-i}_{t+1}\right)(1-\mathcal{E}_{t+1})\Big|\ \omega_t, \theta_{i\,t}\right]\right\}\ 1^*_{\{\omega_t,\mathcal{E}_t\}}}_{\text{marginal cost of extraction}} \tag{7}$$

where

$$\xi^{-i}_{t+1} \equiv -\frac{\omega_{t+1}}{1-\mathcal{E}_{t+1}}\sum_{j\neq i}\frac{\partial e_{j\,t+1}}{\partial \omega_{t+1}} = \frac{\text{\% reduction other countries' extraction rates}}{\text{\% increase in stock}}$$

In other words, ξ^{-i}_t is the overall elasticity of reduced extraction in other countries due to an increase in the carbon stock. As with all elasticities, it measures the responsiveness of countries to a change in the stock brought about by i's current rate of extraction. This elasticity will typically, but not always, be negative: an increase in the current stock decreases the future incentives of countries to preserve the stock (or increases their incentives to extract more). One of the striking results shown later on is that the reverse is true when countries are closer to tipping.

A complete derivation of (7) is contained in the Appendix. Equation (7) equates country i's marginal extraction benefit (MEB), as measured by its net present benefit in terms of period t flow payoff, with its marginal extraction cost (MEC) as measured by the loss of stored carbon used for maintaining the ecosystem and for future extraction opportunities. Generally, the equation system in (7) yields no closed form solution and is necessary but not sufficient to characterize BAU equilibria.

Nevertheless, any BAU equilibrium must generate a solution to (7). To summarize,

Proposition 1 *Let* $\mathbf{c}^*$ *be a business-as-usual (BAU) equilibrium. Then* $c_i^*(\omega_t, \theta_t) = e_i^*(\omega_t, \theta_t)\,\omega_t$ *for each* i *and* t*, where* $e_i^*(\theta_t, \omega_t)$*, country* i*'s equilibrium extraction rate, is an implicit solution of the system of equations described in Equation (7) for each country* i*.*

In a BAU equilibrium, each country i's consumption/emissions is decreasing in the effective discount factor $\delta\gamma$, increasing in its own output elasticity θ_{it} with respect to carbon extraction, and increasing in the sum ξ_{t+1}^{-i} of other countries' extraction elasticities in $t+1$.

The last comparative static can be understood as follows. The larger are the extraction elasticities of other countries, the more responsive are these countries to lower carbon stocks. This, in turn, gives added incentive by country i to extract more today.

To simplify notation, we evaluate the extraction rates of all countries except i at their equilibrium values and then express the marginal extraction cost (the right-hand side of (7)) as $G_i^*(e_{it}, \omega_t, \theta_t)$.[18] This cost is comprised of future marginal extraction benefits net of other countries' responses, multiplied by the ratio of post-consumption stock each period to the post-consumption stock net of b.

While we have outlined some of the more obvious comparative statics results, the overall response of G_i^* to changes in i's extraction rate e_{it} is subtle. Holding fixed all other countries' rates at the equilibrium level, the MEC G^* is not generally increasing in e_{it}. Such a function is expressed in Figure 2. When the current stock ω_t is sufficiently large and/or aggregate extraction $\mathcal{E}_t$ is sufficiently low, then the constraint implied by $1^*_{\{(\omega_t, \mathcal{E}_t)\}}$ does not bind. In this case G_i^* is increasing in e_i as one might expect.

Yet, if e_i is large enough, then one or more of the constraints are very likely to

[18] Formally,

$$G_i^*(e_{it}, \omega_t, \theta_t) \;=\; \frac{\delta\gamma\;\omega_t}{\omega_t(1 - \mathcal{E}^*_{-i\,t}(\omega_t, \theta_t) - e_{it}) - b} \times$$

$$\Bigg\{1 + E\left[\left(\frac{\theta_{i\,t+1}}{e^*_{i\,t+1}(\omega_{t+1}, \theta_{t+1})} - \right.\right.$$

$$\left.\left.\frac{1-\theta_{i\,t+1}}{1 - \mathcal{E}^*_{t+1}(\omega_{t+1}, \theta_{t+1})} \;-\; \frac{\theta_{i\,t+1}}{e^*_{i\,t+1}(\omega_{t+1}, \theta_{t+1})}\xi^{*\,-i}_{t+1}(\omega_{t+1}, \theta_{t+1})\right)(1 - \mathcal{E}^*_{t+1}(\omega_{t+1}, \theta_{t+1}))\,\Big|\,\omega_t, \theta_{i\,t}\right]\Bigg\}\; 1^*_{\{\omega_t, e_{it} + \mathcal{E}^*_{-i\,t}(\omega_{t+1}, \theta_{t+1})\}}$$

bind in the near future in which case the marginal cost G^* may, in fact, *decrease* in e_i. The logic is as follows. If e_i is large, then next period's carbon stock ω_{t+1} is likely to hit the carbon floor F. When countries are certain that it will, then they anticipate that their date t decisions can have no effect on future stocks. But if, by cutting back extraction in t, country i can lower this likelihood of hitting the floor in $t+1$, then doing so will extend the expected time horizon over which its date t extraction affects future stocks. These incentives in the presence of tipping are explored in Section 3.3.[19]

The Proposition characterizes Euler equations for a given BAU equilibrium, but does not establish existence of such equilibria. In an external appendix, an existence result is obtained in a robust class of parameters.[20] Below we construct an explicit BAU equilibrium in closed form whenever $b = 0$ and $F = 0$ (the case of no-tipping).

The no-tipping special case. When $b = 0$ and $F = 0$, the BAU forward solution to Equation (7) yields a right-hand side of $G_i^*(e_{it}, \omega_t, \theta_t) = \frac{\delta\gamma}{(1-\delta\gamma)(1-\mathcal{E}_t)}$. Thus the no-tipping model admits a simple closed form solution for the BAU equilibrium:

$$c_i^*(\omega_t, \theta_t) \;=\; \bar{e}_i(\theta)\,\omega_t \tag{8}$$

where $\bar{e}_i(\theta)$, i's extraction rate, is independent of the stock and is given by

$$\bar{e}_i(\theta) \;=\; \frac{\frac{\theta_{it}(1-\gamma\delta)}{1-\theta_{it}(1-\gamma\delta)}}{1 + \sum_{j=1}^{n} \frac{\theta_{jt}(1-\gamma\delta)}{1-\theta_{jt}(1-\gamma\delta)}} \tag{9}$$

The derivation of c_{it} in Equation (8) can be obtained directly from the Euler equation in the Proposition when $b = 0$. Equation (9) reveals that, without a threshold b, the BAU extraction rate is stationary.

Observe that if $\theta_{it} = 1$ for all i and t then the equilibrium coincides with the Levhari-Mirman (LM) (1980) fish war model as a special case. In their model, $\theta_{it} = 1$ and $b = F = 0$ for all i and t. In other words, if there is no tipping problem, no direct value from preserving the ecosystem, and no heterogeneity, in that value, then the BAU equilibrium coincides with the one calculated in LM.[21]

[19] This logic does not change if the floor F is either not known or varies stochastically. The latter case complicates the extraction decision in a post-tipping world, but does not change the qualitative trade offs outlined here. In the Euler equation (7), F enters implicitly in the indicator function $1^*_{\{\cdot\}}$. Since the threshold at which the indicator $1^*_{\{\omega_t,\mathcal{E}_t\}}$ tips to zero is already stochastic given π, the addition of a stochastic F alters the distribution in the Euler equation, but does not change the basic structure of e.

[20] See faculty.georgetown.edu/lagunofr/BAU4-External-Appendix.pdf.

[21] Namely, for all i and t,

$$c_{it}^*(\theta = \mathbf{1}) = \frac{\left(\frac{(1-\gamma\delta)}{1-(1-\gamma\delta)}\right)}{1 + \frac{(1-\gamma\delta)}{1-(1-\gamma\delta)}n}\omega_t = \frac{(1-\gamma\delta)}{n(1-\gamma\delta)+\gamma\delta}\,\omega_t.$$

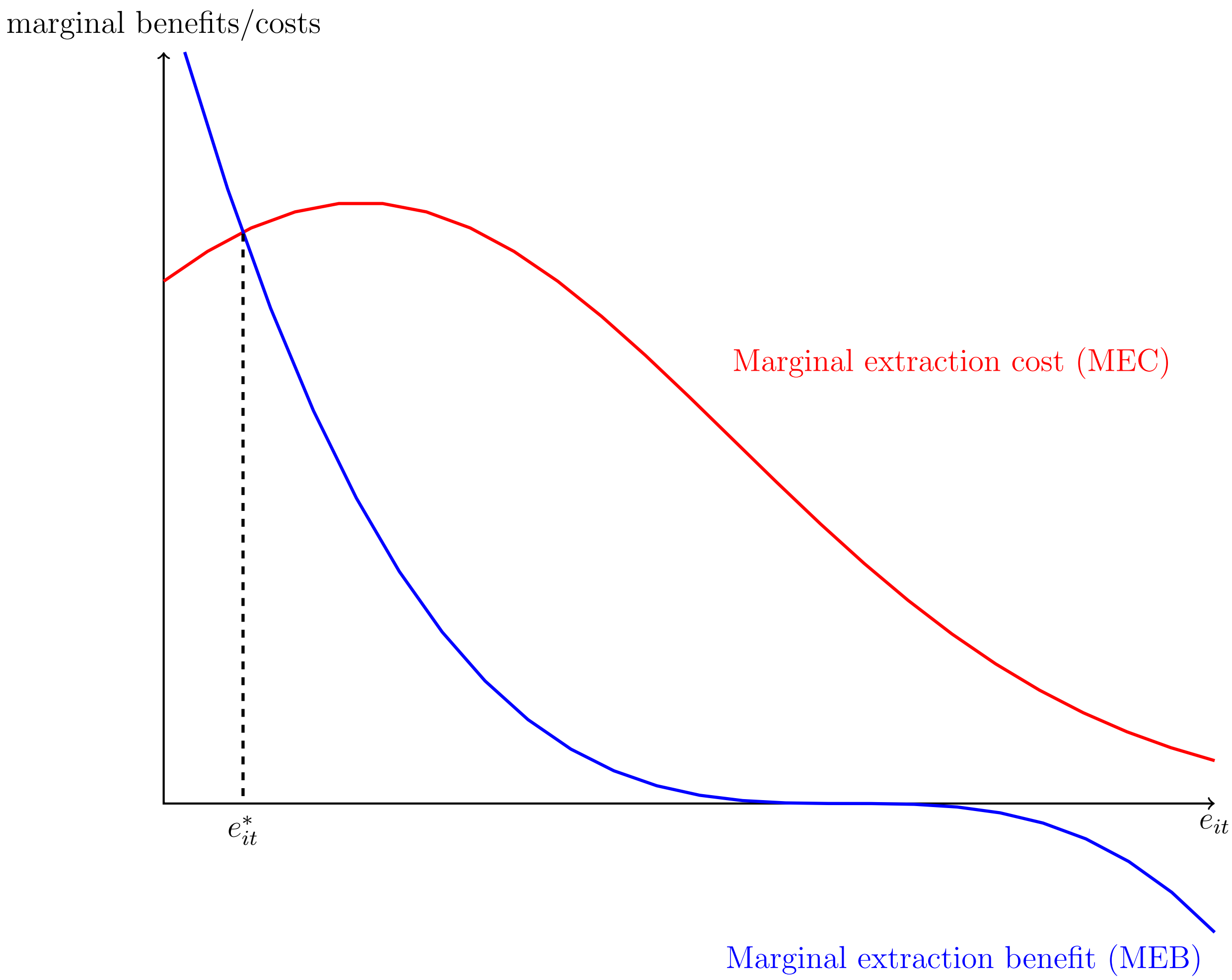


Figure 2: Euler equation

Even without tipping, the carbon externality has both aggregate and distributional effects that are not present in the standard common pool problem. Extraction rates exhibit cross-sectional dispersion in which countries with either very high or very low resource elasticities have larger consumption than those with intermediate elasticities. This is due to the fact that a country's output $y_i^*(\omega_t, \theta)$ is U-shaped in θ_{it}, ceteris parabus. The U-shape also helps explain why reversing course is problematic: starting from a high elasticity θ_{it}, as a country's carbon footprint recedes, output must initially fall before growth is possible again.

3.2 Tipping Points and Collapse

Notice from Figure 1 that when $b > 0$ there is a possibility that the stock can depreciate down to the floor F. More precisely, the global commons in a BAU equilibrium will be said to *collapse under BAU at stock* ω_0 if the equilibrium path $\{\omega^{*t}\}$ of carbon stock converges to F for almost every path θ^∞ of elasticity profiles. That is, the commons *collapses* at ω_0 if

$$\lim_{t\to\infty} \omega^{*t}(\omega_0, \theta^{t-1}) \;=\; F \quad a.e. \;\; \theta^\infty$$

More generally, let

$$\mu(\omega_0) \;=\; P\left(\left\{\theta^\infty : \lim_{t\to\infty} \omega^{*t}(\omega_0, \theta^{t-1}) \;=\; F\right\}\right)$$

denoting the probability of collapse. In the no-tipping model, $\mu(\omega_0) = 0$ if $\omega_0 > 0$. At the other extreme, if the commons collapses the stock spirals downward toward threshold F. A tipping point is therefore the largest stock from which the collapse must occur. Specifically, a *tipping point* is a carbon stock ω^{tip} satisfying

$$\begin{aligned}\omega^{tip} &= \sup\{\omega_0 : \text{ the commons under BAU collapses at } \omega_0\} \\ &= \sup\{\omega_0 : \; \mu(\omega_0) = 1\}.\end{aligned}$$

By these definitions, if the global commons collapses at every initial stock, then the tipping point is infinite.

The critical feature of a tipping point is that it is *endogenously determined* in equilibrium. Hence, different types of equilibria give rise to different tipping points, a fact we elaborate on later when making the comparison to "optimal" tipping points.

The tipping point can be distinguished from a carbon threshold above which exists a *safe operating space for humanity*, in the sense of Rockstrom et. al. (2009). In the present model, the global commons under BAU is in a *safe operating space* at ω_0 if

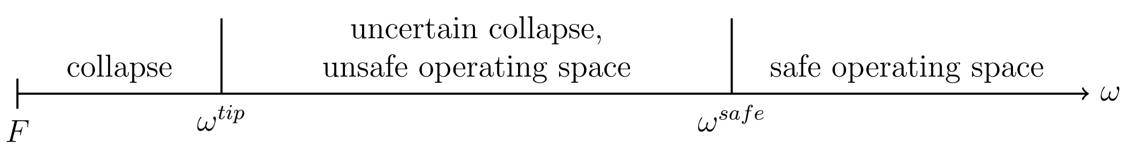
$$\lim_{t \to \infty} \omega^{*t}(\omega_0, \theta^{t-1}) \; > \; \omega^{tip} \quad a.e. \;\; \theta^{\infty}$$

This leads naturally to a notion of a *safe operating bound*, defined as a carbon stock ω^{safe} satisfying

$$\begin{aligned} \omega^{safe} &= \inf\{\omega_0 : \text{ the commons under BAU is in a safe operating space at } \omega_0\}. \\ &= \inf\{\omega_0 : \; \mu(\omega_0) = 0\}. \end{aligned}$$

Proposition 2 *Suppose that $\omega_0 > 0$. Then $\mu(\omega_0)$ is weakly decreasing in ω_0 and is strictly decreasing on any interval (ω', ω'') for which $\mu(\omega_0) \in (0,1) \quad \forall \, \omega_0 \in (\omega', \omega'')$.*

The proof is in the Appendix. By the Proposition, it follows that $\omega^{safe} \geq \omega^{tip}$ with strict inequality if ω^{safe} is finite and π is non-degenerate. In particular, if there is no variation in θ_t, i.e., if θ_t is constant over all t, then $\omega^{safe} = \omega^{tip}$. If the interval $(\omega^{tip}, \omega^{safe})$ is nonempty, then it consists of stocks that are neither safe nor collapsing. In this interval tipping is stochastically determined by the evolution of factor elasticities. The various regions are delineated below.



Consider, as an example a stationary Markov process on the two profiles $\{\theta', \theta''\}$ with $\theta' < \theta''$. Suppose that either profile can be reached from the other each period with positive probability bounded away from 0. There are two possibilities. Either the carbon dynamic has a finite tipping point or it does not. The case of a finite tipping point is displayed in Figure 3. Since the equilibrium carbon dynamic for both stocks has fixed points, the tipping point ω^{tip} corresponds to the lowest unstable fixed point. From any stock strictly larger than ω^{tip}, the process can avoid collapse with positive probability. In particular, if it reaches stock ω^{safe}, then the commons is guaranteed to avoid collapse, thus defining the *safe operating space (SOS)* described in the planetary boundaries literature of Rockstrom et. al (2009), Anderies et. al. (2013), and others.

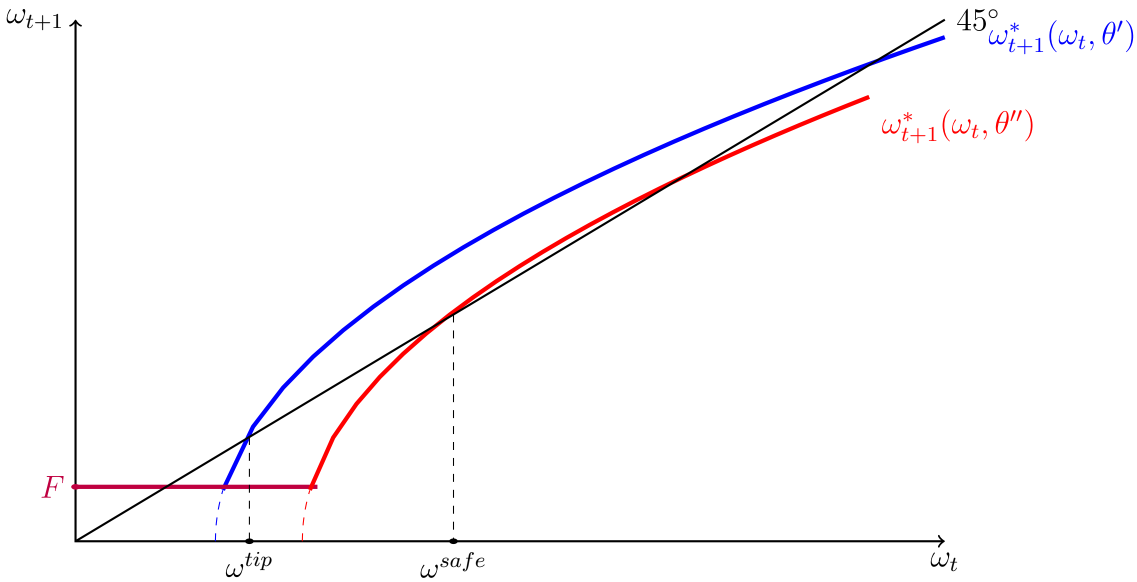


Figure 3: Carbon dynamics with tipping point is ω^{tip}

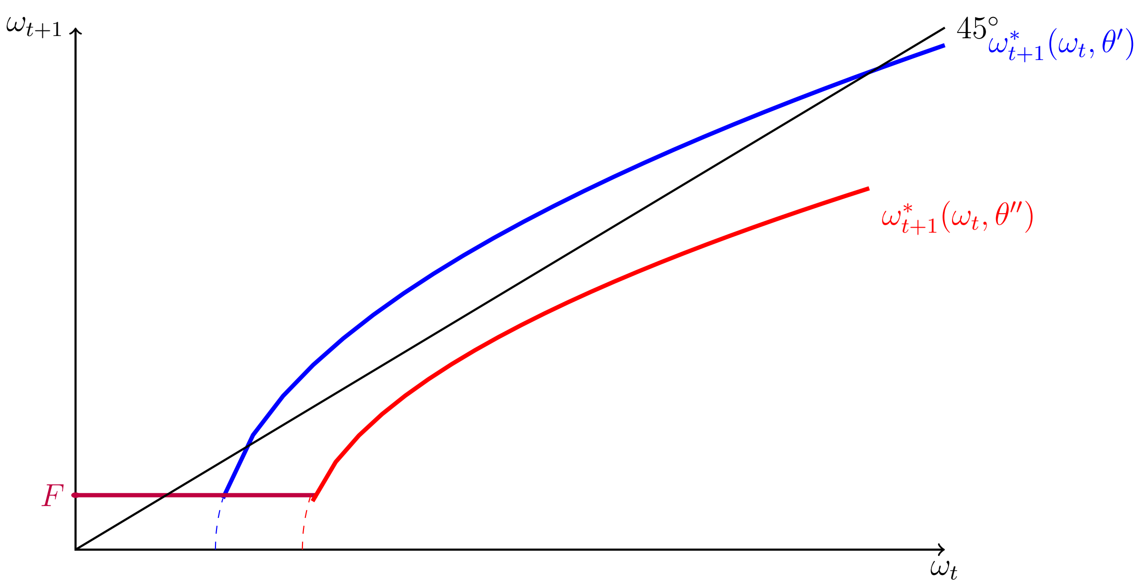


Figure 4: Carbon dynamic with certain collapse.

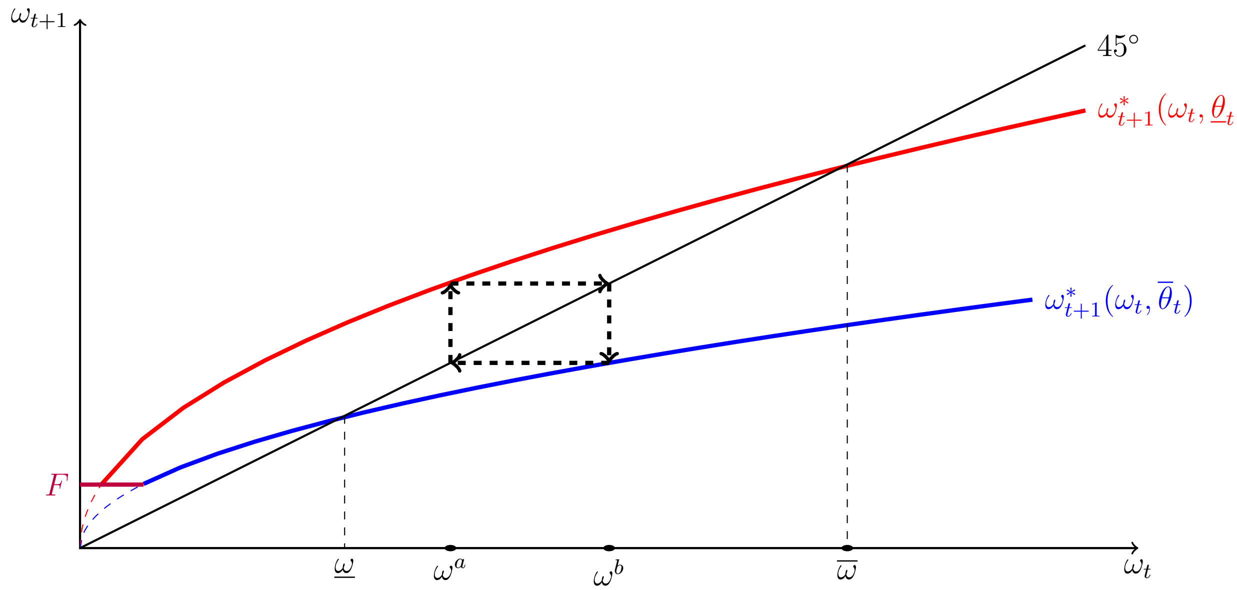


Figure 5: When $b = 0$, a stable carbon cycle alternates between ω^a and ω^b.

The case where the tipping "point" is infinite is displayed in Figure 4. In this case the parameters generate certain collapse. Specifically, from any stock ω and any initial profile, for any time length T, there is a date t at which the process will remain "stuck" at θ'' for T periods starting from t. Since this occurs at infinitely many t, then for T sufficiently long the commons will eventually collapse from ω with probability one. Consequently, $\omega^{tip} = \infty$.

Finally, when $b = 0$ then $\mu(\omega_0) = 0$ for any $\omega_0 > 0$. In other words, $\omega^{tip} = 0$ and so tipping never occurs.[22] In this case, the BAU equilibrium law of motion $\omega^*_{t+1}(\omega_t, \theta_t)$, derived from (8), converges to a stationary distribution on the carbon stock (a stable carbon cycle) if the underlying process on θ^∞ is ergodic. This is illustrated in a particularly simple case. Consider a two-state stationary, irreducible Markov process on the two bounds $\underline{\theta}$ and $\overline{\theta}$ with p denoting the switching probability between the two. Figure 5 displays a literal cycle when $p = 1$, that is, when the process alternates deterministically between $\underline{\theta}$ and $\overline{\theta}$. The equilibrium dynamic then cycles between carbon stocks, ω^a and ω^b.

[22]For this to hold F cannot be too large. See Footnote 12 for an upper bound on F.

3.3 Equilibrium Incentives with Tipping Points

To understand how the equilibrium incentives are affected by tipping, we offer a heuristical outline of two polar cases.

First, suppose that the initial stock ω_0 is sufficiently large so $\mu(\omega_0) = 0$, i.e., the economy remains in the safe operating space. Formally, this means $1^*_{\{\omega_t, \mathcal{E}_t\}} = 1$ with probability one in all periods t. The carbon dynamic then reduces to $\omega_{t+1} = A(\omega_t - C_t - b)^\gamma$. Moreover, because the initial stock is large, $\frac{\omega_t(1-\mathcal{E}_t)}{(\omega_t(1-\mathcal{E}_t)-b)}$ is approximately one in which case future extraction elasticities are approximately zero, i.e., $\xi^{-i}_{t+1} \approx 0$. A small increase in the bound b therefore increases the marginal cost of extraction, and so $e^*_i(\omega_t, \theta_t) < \bar{e}_i(\theta_t)$. That is, the BAU equilibrium extraction rate in the tipping model ($b > 0$) is less than that in the non-tipping model.

Next, suppose that ω_0 is small enough so that $\mu(\omega_0) = 1$, i.e., collapse is certain. This occurs, for instance, if $A(\omega_t(1 - \mathcal{E}^*_t) - b)^\gamma \leq F$ so that $1^*_{\{\omega_t, \mathcal{E}_t\}} = 0$ in the current period t. But this means $\dfrac{\partial \omega_{t+1}}{\partial e_{it}} = 0$, in other words, the country's marginal cost of extraction is zero. Since current extraction rates do not affect future stocks, each country therefore solves a one period static problem. Each country solves its static first order condition

$$\frac{\theta_{it}}{e_{it}} - \frac{1-\theta_{it}}{1-\mathcal{E}_t} = 0.$$

corresponding to the case where the marginal extraction cost is zero. Country i's BAU equilibrium extraction then coincides with its one-shot or static equilibrium extraction rate is $e^{static}_i(\theta_t) = \frac{\theta_{it}}{1-\theta_{it}} \left(1 + \sum_j \frac{\theta_{jt}}{1-\theta_{jt}}\right)^{-1}$. In the static equilibrium, countries extract carbon as if $\delta = 0$.

It is not difficult to verify that $e^{static}_i(\theta_t) > \bar{e}_i(\theta_t)$ where the latter is the closed form calculated in the no-tipping special case.

Intuitively, if the constraint $1^*_{\{\omega_t, \mathcal{E}_t\}} = 0$ holds or will hold with high probability in the near future, countries have little to lose by extracting as much as possible for the present. In this case, other countries' future responses to i are negligible, and thus have a negligible effect on the current extraction of country i. Thus i extracts as if tipping is a *fait accompli*, something that occurs independently of its own decision. It follows that when ω_t is small enough that the economy is close to tipping, $e^*_i(\omega_t, \theta_t) > \bar{e}_i(\theta_t)$.

Taken together, this intuition can be summarized as follows: *the off-take parameter b reduces the incentives to extract carbon when the threat of tipping is low, but increases the incentive to extract when the tipping threat is large.* Clearly, these are

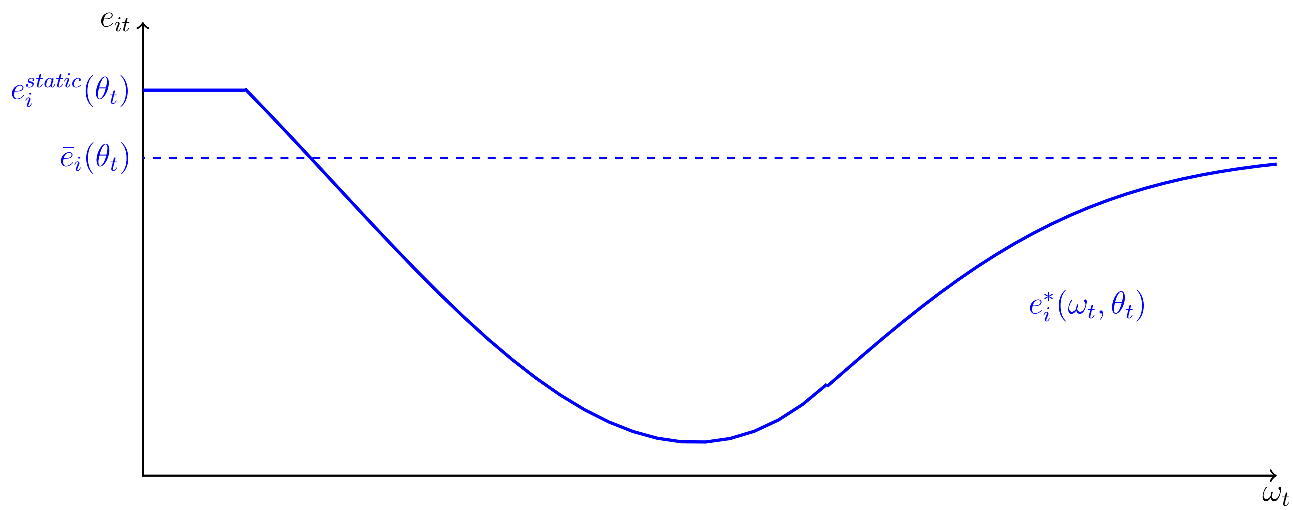


Figure 6: Non-monotonicity in extraction rates

heuristical rather than precise arguments. The intuition, however, is formalized in the following result.

Theorem 1 *If $b > 0$, then for each country i, there exists ω^1 and ω^2 with $\omega^1 \leq \omega^2$ such that*

1. *if $\omega_t \geq \omega^2$ then $e_i^*(\omega_t, \theta_t) < \bar{e}_i(\theta_t)$ and $e_i^*(\omega_t, \theta_t)$ is non-decreasing in ω_t, and and*
2. *if $\omega_t \leq \omega^1$ then $e_i^*(\omega_t, \theta_t) = e_i^{static}(\theta_t) > \bar{e}_i(\theta_t)$.*

Part 1 asserts that countries exhibit greater caution when there is tipping threat if ω_t is large, i.e., *when the threat of tipping is relatively low*. Part 2 asserts that countries' extraction rates are higher when there is a tipping threat if ω_t is low, i.e., *when the threat of tipping is relatively high*. Of course, the likelihood of reaching the threshold is endogenous. Later on, we show that there are realized values of the elasticity path profile in which the threshold will be reached, and that this incentives accelerate one's extraction rate intensify with the level of strategic competition.

The proof is in the Appendix. Figure 6 demonstrate the non-monotonicity of a country's equilibrium extraction as the current stock varies. For low enough stock, the extraction resembles a static solution $\mathbf{e}^{static}$ where current extraction has no effect on future payoffs. For large enough stock, the extraction resembles the BAU equilibrium in the no-tipping model ($b = 0$ and $F = 0$). Because the equilibrium extraction rates approach each constant rate from below, the lowest extraction rate in equilibrium occurs for intermediate stocks, as shown in the Figure.

The Figure indicates that proximity to tipping leads countries to *accelerate their rate of extraction*. The result is reminiscent of the "Green Paradox" (Sinn (2008)), whereby the extraction increases when more stringent emissions regulations are anticipated in the future. In that case, the tipping event — the policy change — is taken as exogenous. Here, the tipping event is endogenous. Sakamoto (2014) obtains a related result in a model where an exogenous tipping point determines a stochastic shift from a good environmental state to a bad one.

In our case, both the tipping point and the acceleration are jointly derived in equilibrium.

3.4 Reaching a Tipping Point

With tipping there are parameter configurations under which an economy will collapse, and alternative configurations under which the commons remains in the safe operating space.

Theorem 2 *Suppose that* $[\underline{\theta}, \overline{\theta}] = [0, 1]$.

1. *Let* π *satisfy: there exists* $\epsilon > 0$ *such that for any integer* T *and a.e.* θ^∞,
$$\int_{\theta_{t+s} \in (1-\epsilon, 1]^n} \pi(\theta_{t+s}|\theta_{t+s-1}) d\theta_{t+s} \;\geq\; \epsilon \tag{10}$$
for all $s = 1, \ldots, T$ *and infinitely many* t. *Then there exists* n' *such that* $n \geq n'$ *implies that the global commons under BAU collapses at every* ω_0 *(i.e.,* $\omega^{tip} = \omega^{safe} = \infty$*).*

2. *There exists* $\epsilon > 0$ *such that if, for almost every* θ^∞,
$$\theta_t \in [0, \epsilon]^n \;\; \forall \; t \tag{11}$$
then the global commons has a finite tipping point ω^{tip}.

Part 1 asserts that if there are sufficiently many countries and if for almost every process the carbon elasticities will become and remain high for T periods for any T, the commons will collapse in the BAU equilibrium. The result can be generated by many types of stochastic processes on θ consistent with the historical pattern of increased reliance on fossil fuels.

Part 2 asserts that finite tipping points exist if for almost every process the carbon elasticities remain low. In that case, collapse is avoided if the stock starts out large enough. The proof is in the Appendix. Notice that the sufficient conditions for Part 1 are, in a sense, less restrictive than those of Part 2. To guarantee at least the possibility of reaching a safe operating space, the process cannot stay very long at *any* point in time in profiles with high elasticities of extraction.

In comparing the assumptions underlying Parts 1 and 2, notice that far less is required to guarantee a collapse. The process needs to hit the high range of elasticities at *some* point in time, and remain there for a while. This will be generally true of ergodic processes with full support. It also holds for a wide class of super martingales.

The Theorem makes clear that while the proximate cause of tipping is the depletion of the carbon stocks, the "deeper" parameters that drive the tipping and collapse are technological: the factor elasticities that determine the mix of extracted and stored carbon.

The Theorem's logic is straightforward. Let $\mathcal{E}^*(\omega_t, \theta_t, b, n)$ denote the BAU equilibrium aggregate extraction rate, expressed as a function of the relevant parameters. First, as $\theta_t \to \mathbf{0} \equiv (0, \ldots, 0)$, it happens that $\mathcal{E}^*(\omega_t, \theta_t, b, n) \to 0$. This is intuitive since in the limit elasticities are uniformly zero and so countries do not care at all about individual carbon extraction. In that case, the equilibrium law of motion equals the law of motion without human consumption — the latter always has a finite tipping point which will not be approached if the initial stock is large enough.

By contrast, observe that it is *not* true that $\mathcal{E}^*(\omega_t, \theta_t, b, n) \to 1$ as $\theta_t \to \mathbf{1} \equiv (1, \ldots, 1)$. That is, even when the ecosystem is not valued at all, countries will not fully extract the stock in equilibrium. This simply because their desire smooth intertemporal consumption leads to some degree of temporal rationing. This was, in fact, first observed by Levhari and Mirman who analyzed precisely the case $\theta_t = \mathbf{1}$ (without the bound b). Nevertheless, it is easy to show that $\mathcal{E}^*(\omega_t, \theta_t, b, n) \to 1$ as both $\theta_t \to \mathbf{1}$ *and* $n \to \infty$. In other words, full extraction does occur in any commons problem when the number of participants is large enough. Thus, when the stochastic process on elasticities moves the global economy to this limiting case for a large enough period of time, a global collapse occurs.

One limitation of Theorem 1 is that it only deals with tail events, i.e., tipping properties are only stated for elasticities close to 1 or close to 0. Whether the tail events occur depends on the likelihood of collapse. Part 1, for instance, can be shown to hold if the Markov process is stationary, ergodic, and has full support. Generally, tipping is more likely in distributions that place increasing weight on higher elasticity profiles.

To see this, start with the original Markov kernel π, then let $\tilde{\pi}$ be another Markov kernel associated with probability $\tilde{P}$ on the same measurable space $(\Theta^\infty, \mathcal{F})$. The density π will be said to *dominate* $\tilde{\pi}$ (we write $\pi \succ_D \tilde{\pi}$) if for all t and all θ_t and all nondecreasing functions $w(\theta)$,

$$\int_{\theta_{1\,t+1}=\underline{\theta}}^{\bar{\theta}} \cdots \int_{\theta_{n\,t+1}=\underline{\theta}}^{\bar{\theta}} w(\theta_{t+1})\, \pi(\theta_{t+1}|\theta_t) d\theta_{t+1} \;\geq\; \int_{\theta_{1\,t+1}=\underline{\theta}}^{\bar{\theta}} \cdots \int_{\theta_{n\,t+1}=\underline{\theta}}^{\bar{\theta}} w(\theta_{t+1})\, \tilde{\pi}(\theta_{t+1}|\theta_t) d\theta_{t+1}$$

The definition above is a standard one for multivariate stochastic dominance, although there are others.[23] We use it to show that the likelihood of collapse is stochastically increasing in carbon usage elasticities.

Theorem 3 *Suppose that* $\omega_0 > F$ *and* $\pi \succ_D \tilde{\pi}$. *Then* $\mu(\omega_0) \geq \tilde{\mu}(\omega_0)$ *and* $\omega^{tip} \geq \tilde{\omega}^{tip}$, *and these inequalities are strict if* $\tilde{\omega}^{tip} < \infty$.

4 Optimal Extraction and Optimal Tipping Points

The BAU equilibrium can also be compared to the socially efficient rate carbon usage. The latter is defined as the solution to a utilitarian social planner's problem. From this planner's perspective, an Markov-contingent plan, denoted by $c^\circ(\omega_t, \theta_t) = (c_1^\circ(\omega_t, \theta_t), \ldots, c_n^\circ(\omega_t, \theta_t))$, is *optimal* if it solves

$$\max_{\mathbf{c}^\circ} \; E\left[\sum_{i=1}^{n} \sum_{t=0}^{\infty} \; \delta^t u(y_{it}) \;\middle|\; \omega_0, \theta_0 \right] \quad \text{subject to (2) and (3).} \tag{12}$$

As with the BAU equilibrium, combining log utility with the production technology (2) in the planner's objective, an optimal plan $\mathbf{c}^\circ$ solves the Bellman's equation

$$V(\omega_t, \mathbf{c}^\circ, \theta_t) \;=\; \max_{\mathbf{c}_\mathbf{t}^\circ} \; \left\{ \sum_{i=1}^{n} \; \theta_{it} \log c_{it} + (1-\theta_{it}) \log(\omega_t - C_t) \;+\; \delta\; E\left[V(\omega_{t+1}, \mathbf{c}^\circ, \theta_{t+1}) \middle| \; \omega_t, \theta_t \right] \right\} \tag{13}$$

One can interpret the planner's problem as an "ideal benchmark" against which BAU equilibrium may be compared. Alternatively, the planner's problem can be viewed as the result of an international agreement. The planner's solution can then be interpreted is the prescribed carbon usage in an alternative Subgame Perfect equilibrium — albeit one that requires agreed-upon triggers to punish deviations.[24]

[23] See Zoli (2009) or Maasoumi and Yalonetzky (2013).

[24] The construction of such triggers is non-trivial in this heterogeneous environment. Barrett (2013)

4.1 Equilibrium Over-extraction

The planner's Euler equation in e_{it} satisfies

$$\frac{\theta_{it}}{e_{it}} - \frac{\sum_{j=1}^{n} (1-\theta_{jt})}{1-\mathcal{E}^{\circ}_{-i\,t}(\omega_t,\theta_t) - e_{it}} = nG^{\circ}(e_t,\omega_t,\theta_t) \tag{14}$$

for each country i, where G° has the same functional form as a country's equilibrium marginal extraction cost (MEC), except that G° is evaluated at the socially optimal profile $\mathbf{e}^{\circ}$. Because the planner internalizes the effect of country i's extraction on the global economy, the social MEC is nG°. The left-hand side then determines the social marginal benefit from increasing i's carbon extraction.

Theorem 4 *Let $\mathbf{c}^*$ be a BAU equilibrium and $\mathbf{c}^{\circ}$ an optimal plan. Then for any state (ω_t,θ_t), $C^*(\omega_t,\theta_t) > C^{\circ}(\omega_t,\theta_t)$, i.e., the BAU equilibrium is characterized by aggregate over-extraction.*

By itself, the Proposition is not surprising. It serves mainly as a useful background check, verifying that the tipping threshold b does not eliminate the free rider problem in the aggregate. The Proposition also implies that the BAU equilibrium transition $\omega^*_{t+1}(\omega_t,\theta_t)$ on the carbon stock is lower than its efficient counterpart $\omega^{\circ}_{t+1}(\omega_t,\theta_t)$ for every realized state (ω_t,θ_t).

Unlike many common pool problems, it is not generally true here that all *individual* countries over-extract. The next section considers the benchmark model without the tipping threshold (i.e., the case where $b=0$). In this special case, some countries will under-extract relative to the planner's optimum.

Social optimum in the No-tipping Model. When $b=0$, the planner's optimal extraction plan has, in fact, a closed form solution

$$c^{\circ}_i(\omega_t,\theta_{it}) = \frac{\phi_{it}}{n}\omega_t \quad \forall\ i \tag{15}$$

where $\phi_{it} \equiv \theta_{it}(1-\gamma\delta)$. Not surprisingly, each country's carbon emission is increasing in its resource elasticity, and decreasing in the effective discount factor $\delta\gamma$. The aggregate under-extraction result of Proposition 4 obviously applies to the special case of $b=0$. However, more can be said here about the comparison between BAU equilibrium and the socially optimal plan.

explores the problems with international coordination when the location of a tipping threshold is uncertain. In a prior paper, Harrison and Lagunoff (2016), we show that the planner's solution cannot necessarily be implemented by simple reversion to Markov Perfect (BAU) equilibrium in the event of a deviation.

Proposition 3 *Let $b = 0$ (the no tipping model), and let $\mathbf{c}^*$ and $\mathbf{c}^\circ$ represent a BAU equilibrium and the socially optimal plan, resp. Then for any state (ω_t, θ_t),*

1. *For each country i, and each profile θ_{-i} of others' elasticities, there exists a cutoff carbon elasticity $\tilde{\theta}_i \in [\underline{\theta}, \overline{\theta}]$ such that for any stock ω_t, and in any date t,*

$$c_i^*(\omega_t, \theta_{it}, \theta_{-i}) \; \geq (>) \; c_i^\circ(\omega_t, \theta_{it}) \quad if \quad \theta_{it} \; \geq (>) \; \tilde{\theta}_i, \quad and$$

$$c_i^*(\omega_t, \theta_{it}, \theta_{-i}) \; \leq (<) \; c_i^\circ(\omega_t, \theta_{it}) \quad if \quad \theta_{it} \; \leq (<) \; \tilde{\theta}_i, \quad and$$

2. *along any path of realized carbon elasticity profiles θ^t, the relative differences between efficient and equilibrium output $\frac{y_i^{\circ\, t}(\omega_0, \theta^t)}{y_i^{*t}(\omega_0, \theta^t)}$, carbon consumption $\frac{c_i^{\circ\, t}(\omega_0, \theta^t)}{c_i^{*t}(\omega_0, \theta^t)}$, and carbon stock $\frac{\omega^{\circ\, t}(\omega_0, \theta^t)}{\omega^{*t}(\omega_0, \theta^t)}$ all increase in t.*

The proof is in the Appendix. Notice that the planner exercise more caution not because of concern that the carbon stock will be fully depleted. Even without any such threat, the planner internalizes the effects of depreciation of the ecosystem on aggregate output.

Significantly, the Proposition demonstrates that while all BAU equilibria are characterized by aggregate over-extraction, individual countries may over- *or* under extract depending on their resource elasticity. High intensity carbon users over-extract in the BAU while low intensity users may actually extract less than in the efficient plan. While this is shown only for the special case of a degenerate threshold ($b = 0$) the strict inequalities suggest that it should hold for small but positive thresholds as well.

The possibility of under-extraction in a Markov equilibrium is unusual but not unheard of. Dutta and Sundaram (1993) show this possibility in a LM resource model where the state variable can trigger a punishment. In our model, smoothness of the Markov strategy rules out Markov "trigger" strategies. Instead, heterogeneity is the key. Under-extraction by low intensity carbon users occurs as a compensating response to massive over-extraction by the high intensity users. Low intensity users never fully compensate, however, since over-extraction always occurs in the aggregate.

4.2 Optimal Tipping Points and International Agreements

It may be possible to avoid imminent collapse by having countries agree to implement the globally optimal emissions plan. At this point, another sort of tipping point becomes relevant: the threshold stock above which an optimal emissions plan can forestall collapse. The main result of this section establishes that as long as the initial carbon stock is not too low, it is always possible to construct an agreement to forestall collapse.

The *optimal* tipping point, denoted by ω^{otip}, is the threshold below which economy collapses under the welfare maximizing planner's solution:

$$\omega^{otip} = \sup\{\omega_0 : \text{ the planner's commons collapses at } \omega_0\}.$$

Tipping point ω^{otip} a point at which it is too late for the countries to avoid collapse even if they sign on to an international agreement that implements the optimal extraction plan. Not surprisingly, one can verify that $\omega^{otip} \leq \omega^{tip}$. The inequality, moreover, is strict, if the policy tipping point is finite and if $\underline{\theta} > 0$. This follows directly from the fact that $\mathcal{E}^*(\omega, \theta_t; b) > \mathcal{E}^\circ(\omega, \theta_t; b)$ in any state pair (ω_t, θ_t). The policy tipping point thus provides the economy more breathing space to avoid collapse. This is especially important in the case of Part 1 in the Theorem — the parameters under which collapse is certain in the business-as-usual equilibrium. In that case, a coordinated international agreement is necessary.

Below, we prove that policy tipping points are always finite, and so it is always possible to avoid collapse if the initial carbon stock is not too low.

Theorem 5 *In any global commons, the optimal tipping point ω^{otip} is finite.*

As before, the proof is in the Appendix. The policy tipping point thus leads to a more nuanced delineation of the state space as follows

Notice, moreover, that for any stock ω_t with $\omega^{otip} < \omega_t < \omega^{tip}$, the international community has a stochastic but finite period of time to implement an optimal international agreement in order to avert a collapse.

5 Conclusion

This paper formulates a model of global carbon consumption that integrates strategic incentives of countries into a dynamic model of nonlinear carbon emissions. Our focus is specifically on the strategic interaction among the largest players — the countries themselves. The objective is to understand the strategic incentives to extract carbon in a business-as-usual equilibrium when tipping is possible.

The papers models a world in which a country's GDP depends on both its carbon usage and on the preservation of the global ecosystem. Each country therefore faces a trade off between, on the one hand, extracting and emitting carbon, and on the other, maintaining a stock of stored or "unextracted" carbon to preserve a healthy ecosystem. Countries naturally differ in how they evaluate this trade off, and even the same country can make different trade offs at different points in time, depending on economic shocks.

The results describe scenarios in which consumption and economic output may collapse and shrink if the carbon stock sustaining the ecosystem falls below some critical threshold — a tipping point. The results delineate between stocks the guarantee a safe operating space for humanity from carbon stocks in which tipping *can* occur. In turn, stocks in which tipping can occur are delineated from those in which tipping *must* occur. These distinctions are roughly consistent with certain *planetary boundaries* as defined by Rockstrom et. al. (2009).

In an unsettling result, we show that if the there are sufficiently many participants in the BAU and if output elasticity of extracted carbon is high enough for a long enough time period, a tipping point will certainly be breached. The silver lining is that even in this case, there remains a small window in which tipping may be averted if the countries can depart from BAU and sign on to an effective international treaty to limit emissions.

Together, the results underscore the idea that while the proximate cause of tipping is the ongoing depletion of the carbon stocks (or, equivalently, accretion of atmospheric carbon), the "deeper" parameters that drive the tipping and collapse are technological. Future research may be directed toward understanding of the sources of change for these technologies.

6 Appendix

Proof of Proposition 1.

Starting from the Euler equation,

$$\frac{\theta_{it}}{e_{it}} - \frac{(1-\theta_{it})}{(1-\mathcal{E}_t)} + \delta[\frac{\partial E[U_i(\omega_{t+1},\mathbf{e},\theta_{i\,t+1})\Big|\,\theta_{it}]}{\partial\omega_{t+1}}\frac{\partial\omega_{t+1}}{\partial e_{it}}]1^*_{\{\omega_t,\mathcal{E}_t\}} \;=\; 0$$

with $\frac{\partial\omega_{t+1}}{\partial e_{it}} = -\frac{A\gamma\omega_t}{(\omega_t(1-\mathcal{E}_t)-b)^{1-\gamma}}$. Then the Euler equation is

$$\frac{\theta_{it}}{e_{it}} - \frac{(1-\theta_{it})}{(1-\mathcal{E}_t)} \;=\; A\delta\gamma\frac{\partial E[U_i(\omega_{t+1},\mathbf{e},\theta_{i\,t+1})\Big|\,\theta_{it}]}{\partial\omega_{t+1}}\frac{\omega_t}{(\omega_t(1-\mathcal{E}_t)-b)^{1-\gamma}}1^*_{\{\omega_t,\mathcal{E}_t\}}. \qquad (16)$$

Differentiating the value function $U_i(\omega_{t+1},\mathbf{e},\theta_{i\,t+1})$ with respect to ω_{t+1}

$$\begin{aligned}\frac{\partial E_t[U_i(\omega_{t+1},\mathbf{e},\theta_{i\,t+1})\Big|\,\theta_{i\,t}]}{\partial\omega_{t+1}} \;&=\; E_t\left[\frac{1}{\omega_{t+1}} - \frac{(1-\theta_{i\,t+1})}{1-\mathcal{E}_{t+1}}\sum_{j\neq i}\frac{\partial e_{j\,t+1}}{\partial\omega_{t+1}} + \right.\\ &A\delta\gamma\left(\frac{\partial E_{t+1}[U_i(\omega_{t+2},\mathbf{e},\theta_{i\,t+2})\Big|\,\omega_{t+1},\theta_{i\,t+1}]}{\partial\omega_{t+2}}\right)\times\\ &\left.\left(\frac{1-\mathcal{E}_{t+1}}{(\omega_{t+1}(1-\mathcal{E}_{t+1})-b)^{1-\gamma}} - \frac{\omega_{t+1}}{(\omega_{t+1}(1-\mathcal{E}_{t+1})-b)^{1-\gamma}}\sum_{j\neq i}\frac{\partial e_{j\,t+1}}{\partial\omega_{t+1}}\right)\;1^*_{\{\omega_{t+1},\mathcal{E}_{t+1}\}}\Big|\,\omega_t,\theta_{i\,t}\right]\end{aligned}$$

From i's first order condition in $t+1$ we obtain,

$$\begin{aligned}&\frac{\partial E_{t+1}[U_i(\omega_{t+2},\mathbf{e},\theta_{i\,t+2})\Big|\,\omega_{t+1},\theta_{i\,t+1}]}{\partial\omega_{t+2}} \;=\\ &\frac{1}{A\delta\gamma}\left(\frac{\theta_{i\,t+1}}{e_{i\,t+1}} - \frac{(1-\theta_{i\,t+1})}{(1-\mathcal{E}_{t+1})}\right)\left(\frac{\omega_{t+1}}{(\omega_{t+1}(1-\mathcal{E}_{t+1})-b)^{1-\gamma}}\right)^{-1}\;1^*_{\{\omega_{t+1},\mathcal{E}_{t+1}\}}\end{aligned}$$

Hence substituting this in to the marginal value function above, we obtain

$$\begin{aligned}\frac{\partial E_t[U_i(\omega_{t+1},\mathbf{e},\theta_{i\,t+1})\Big|\,\theta_{i\,t}]}{\partial\omega_{t+1}} \;&=\; E_t\left[\frac{1}{\omega_{t+1}} - \frac{(1-\theta_{i\,t+1})}{1-\mathcal{E}_{t+1}}\sum_{j\neq i}\frac{\partial e_{j\,t+1}}{\partial\omega_{t+1}} + \right.\\ &\left.\left(\frac{\theta_{i\,t+1}}{e_{i\,t+1}} - \frac{(1-\theta_{i\,t+1})}{(1-\mathcal{E}_{t+1})}\right)\left(\frac{1-\mathcal{E}_{t+1}}{\omega_{t+1}} - \sum_{j\neq i}\frac{\partial e_{j\,t+1}}{\partial\omega_{t+1}}\right)\;1^*_{\{\omega_{t+1},\mathcal{E}_{t+1}\}}\Big|\,\omega_t,\theta_{i\,t}\right]\end{aligned}$$

Substituting this into the first order condition (16), we then obtain

$$
\begin{aligned}
& \frac{\theta_{it}}{e_{it}} - \frac{(1-\theta_{it})}{(1-\mathcal{E}_t)} \\
= \;& A\delta\gamma \left\{ \frac{\partial E_t[U_i(\omega_{t+1}, \mathbf{e}, \theta_{i\,t+1}) \big| \, \theta_{i\,t}]}{\partial \omega_{t+1}} \left(\frac{\omega_t}{(\omega_t(1-\mathcal{E}_t)-b)^{1-\gamma}} \right) \right\} 1^*_{\{\omega_t, \mathcal{E}_t\}} \\
= \;& \delta\gamma \left\{ E_t \left[\frac{1}{\omega_{t+1}} \; - \; \frac{(1-\theta_{i\,t+1})}{1-\mathcal{E}_{t+1}} \sum_{j\neq i} \frac{\partial e_{j\,t+1}}{\partial \omega_{t+1}} \; + \right.\right. \\
& \left.\left(\frac{\theta_{i\,t+1}}{e_{i\,t+1}} - \frac{(1-\theta_{i\,t+1})}{(1-\mathcal{E}_{t+1})} \right) \left(\frac{1-\mathcal{E}_{t+1}}{\omega_{t+1}} \; - \; \sum_{j\neq i} \frac{\partial e_{j\,t+1}}{\partial \omega_{t+1}} \right) \; 1^*_{\{\omega_{t+1}, \mathcal{E}_{t+1}\}} \Big| \; \omega_t, \theta_{i\,t} \right] \times \\
& \left. \frac{\omega_t}{(\omega_t(1-\mathcal{E}_t)-b)} \omega_{t+1} \right\} 1^*_{\{\omega_t, \mathcal{E}_t\}} \\
= \;& \delta\gamma \left\{ E_t \left[1 \; - \; \frac{\omega_{t+1}(1-\theta_{i\,t+1})}{1-\mathcal{E}_{t+1}} \sum_{j\neq i} \frac{\partial e_{j\,t+1}}{\partial \omega_{t+1}} \; + \right.\right. \\
& \left.\left(\frac{\theta_{i\,t+1}(1-\mathcal{E}_{t+1})}{e_{i\,t+1}} - (1-\theta_{i\,t+1}) \right) \left(1 \; - \; \frac{\omega_{t+1}}{1-\mathcal{E}_{t+1}} \sum_{j\neq i} \frac{\partial e_{j\,t+1}}{\partial \omega_{t+1}} \right) \; 1^*_{\{\omega_{t+1}, \mathcal{E}_{t+1}\}} \Big| \; \omega_t, \theta_{i\,t} \right] \times \\
& \left. \frac{\omega_t}{(\omega_t(1-\mathcal{E}_t)-b)} \right\} 1^*_{\{\omega_t, \mathcal{E}_t\}}
\end{aligned}
$$

Reorganizing and canceling appropriate terms, we obtain the BAU Euler equation

$$
\begin{aligned}
\theta_{it}\left(1 + \frac{(1-\mathcal{E}_t)}{e_{it}}\right) \; = \; & 1 \; + \; \delta\gamma \left\{ \frac{\omega_t(1-\mathcal{E}_t)}{(\omega_t(1-\mathcal{E}_t)-b)} \times \right. \\
E_{t+1}\Bigg[\Bigg(\theta_{i\,t+1}\left(1 + \frac{(1-\mathcal{E}_{t+1})}{e_{i\,t+1}}\right) \; - \; & \theta_{i\,t+1}\frac{(1-\mathcal{E}_{t+1})}{e_{i\,t+1}} \xi^{-i}_{t+1} \Bigg) \; 1^*_{\{\omega_{t+1}, \mathcal{E}_{t+1}\}} \; \Big| \; \omega_t, \theta_{i\,t} \Bigg] \Bigg\} 1^*_{\{\omega_t, \mathcal{E}_t\}}.
\end{aligned} \tag{17}
$$

where, recall, ξ^{-i}_{t+1} is the overall extraction elasticity of others with respect to carbon stock. After some minor algebra, this is equivalent to Equation (7).

Finally, by setting $b = 0$ as required for the no-tipping model, we obtain the closed for solution for a BAU equilibrium in Equation (8). ∎

Proof of Proposition 2 . Let $\omega_0 > \bar{\omega}_0 > F$. It suffices to show

$$P\left(\left\{\theta^\infty : \lim_{t\to\infty} \omega^{*\,t}(\omega_0, \theta^t) \;\leq\; \omega^{tip}\right\}\right) \;<\; P\left(\left\{\theta^\infty : \lim_{t\to\infty} \omega^{*\,t}(\overline{\omega}_0, \theta^t) \;\leq\; \omega^{tip}\right\}\right).$$

In turn, this holds if

$$\omega^{*\,t}(\omega_0, \theta^t) \;>\; \omega^{*\,t}(\bar{\omega}_0, \theta^t) \quad \forall\, \theta^t \;\; \forall\, t.$$

We proceed by induction. Observe that $\omega^{*\,t}(\omega_0, \theta^t) = A(\omega^{*\,t-1}(\omega_0, \theta^{t-1}) - C^*(\omega_0, \theta^{t-1}) - b)^\gamma$ and so we proceed by induction. Suppose, by contradiction, that for $t = 1$,

$$\omega_1^*(\omega_0, \theta_0) = A(\omega_0 - C^*(\omega_0, \theta^0) - b)^\gamma \;<\; A(\bar{\omega}_0 - C^*(\bar{\omega}_0, \theta^0) - b)^\gamma = \omega_1^*(\bar{\omega}_0, \theta_0).$$

In particular, this implies

$$\omega_0 - C^*(\omega_0, \theta^0) \;<\; \bar{\omega}_0 - C^*(\bar{\omega}_0, \theta^0). \tag{18}$$

Notice, first, that it is not possible for $\omega_0 - C^*_{-i}(\omega_0, \theta^0) < \bar{\omega}_0 - C^*_{-i}(\bar{\omega}_0, \theta^0)$ for all i. If that were true, then $c_i^*(\omega_0, \theta^0) < c_i^*(\bar{\omega}_0, \theta^0)$ for all i, a contradiction of the fact that $C^*(\omega_0, \theta^0) \;>\; C^*(\bar{\omega}_0, \theta^0)$ given $\omega_0 > \bar{\omega}_0$.

Hence, there is some country j for whom

$$\omega_0 - C^*_{-j}(\omega_0, \theta^0) > \bar{\omega}_0 - C^*_{-j}(\bar{\omega}_0, \theta^0) \quad \text{and} \quad c_j^*(\omega_0, \theta^0) > c_j^*(\bar{\omega}_0, \theta^0).$$

For this country j, payoffs in an arbitrary state ω_0 and for an arbitrary choice c_j can be expressed as

$$\theta_{it} \log c_j + (1 - \theta_{it}) \log(\omega_0 - C^*_{-j}(\omega_0, \theta^0) - c_j) + \delta E[V(\omega_0 - C^*_{-j}(\omega_0, \theta^0) - c_j)].$$

By strict concavity of this objective as a function of c_j, $c_j^*(\omega_0, \theta^0)$ is optimal in state ω_0 only if

$$c_j^*(\omega_0, \theta^0) - c_j^*(\bar{\omega}_0, \theta^0) \;\leq\; (\omega_0 - C^*_{-j}(\omega_0, \theta^0)) - (\bar{\omega}_0 - C^*_{-j}(\bar{\omega}_0, \theta^0)),$$

contradicting (18).

Proceeding by induction, it can be established that for all t,

$$\omega^{*\,t}(\omega_0, \theta^t) \;>\; \omega^{*\,t}(\bar{\omega}_0, \theta^t) \quad \forall\, \theta^t \;\; \forall\, t.$$

We thus conclude the proof ∎

Proof of Theorem 1.

Recall that any BAU equilibrium is characterized by the fundamental Euler equation (7). This equation relates current extraction rates e_t^* to next period's e_{t+1}^*. Observe that equation is bounded due to the fact that BAU extraction rates are bounded above by the one-shot or myopically optimal rate $e_i^{static}(\theta)$ for each i, and bounded below by the planner's optimal extraction rate, e_i° assigned to each i.

To prove the theorem we return to the fundamental Euler equation (7), notice that if we let

$$Z_{it} \;=\; \frac{\theta_{it}(1-\mathcal{E}_t)}{e_{it}} - (1-\theta_{it})$$

and

$$\Gamma(\omega_t,\theta_t) \;=\; \left(\frac{\delta\gamma\;\omega_t(1-\mathcal{E}^*(\omega_t,\theta_t))}{\omega_t(1-\mathcal{E}^*(\omega_t,\theta_t)\;)-b}\right) 1^*_{\{\omega_t,\mathcal{E}^*(\omega_t,\theta_t)\}},$$

then the Euler equation (7) can be expressed as

$$Z_{it}(\omega_t,\theta_t) \;=\; \Gamma(\omega_t,\theta_t)\left(1+E\left[Z_{i\,t+1} \;-\; \xi_{t+1}^{-i}\,(Z_{i\,t+1}+(1-\theta_{i\,t+1}))\;\Big|\;\omega_t,\theta_{i\,t}\right]\right) \tag{19}$$

In the case of $b=0$ and $F=0$, this reduces to

$$Z_{it}^0 \;=\; \delta\gamma\left(1+E\left[Z_{i\,t+1}^0\Big|\;\omega_t,\theta_{i\,t}\right]\right) \tag{20}$$

which is solved by $\bar{e}_i(\theta)$.

Step 1. First show that there exists ω^1 satisfying $e_i^*(\omega_t,\theta) > \bar{e}_i(\theta)$ whenever $\omega_t < \omega^1$.

Observe that $\Gamma(\omega_t,\theta_t) > \delta\gamma$ whenever $1^*_{\{\omega_t,\mathcal{E}^*(\omega_t,\theta_t)\}} = 1$ and $\Gamma(\omega_t,\theta_t)=0$ otherwise. Hence, for ω^1 that triggers $1^*_{\{\omega^1,\mathcal{E}^*(\omega^1,\theta)\}} = 0$ we obtain $\Gamma(\omega,\theta)=0$ for all $\omega \le \omega^1$ in which case

$$e_i^*(\omega,\theta) > \bar{e}_i \;\text{ if }\; \omega < \omega^1 \tag{21}$$

Note that in this case $e_i^*(\omega_t,\theta) = e_i^{static}(\theta)$.

Step 2. Show that there exists ω^2 satisfying $e_i^*(\omega_t,\theta) < \bar{e}_i(\theta)$ whenever $\omega_t > \omega^2$.

Observe from the bounds on e^* that $e_j^*(\omega,\theta) \to \bar{e}_j(\theta)$ as $\omega \to \infty$ for all θ. It follows by the Intermediate Value Theorem that there is some $\omega^*(\theta)$ such that

$$E[\frac{\partial e_{j\,t+1}^*(\omega,\theta)}{\partial\omega}|\;\omega^*(\theta_t),\theta_t] = 0,$$

Using this $\omega^*(\theta)$ and the definition of Z_{it} we obtain

$$Z^*_{it}(\theta_t) \equiv Z_{it}(\omega^*(\theta_t),\theta_t) \;=\; \Gamma(\omega^*(\theta_t),\theta_t)\left(1+E\left[Z^*_{i\,t+1}(\theta_{t+1}) \;\middle|\; \omega^*(\theta_t),\theta_{i\,t}\right]\right)$$

Thus we have $Z^*_{it}(\theta_t) > Z^0_i(\theta_t)$ whenever $\omega > \omega^*(\theta)$. Taking $\omega^2_t(\theta) \equiv \omega^*(\theta)$ we have

$$e^*_i(\omega,\theta) < \bar{e}_i \;\text{ if }\; \omega > \omega^2_t(\theta) \tag{22}$$

Finally since e^* is stationary, these inequalities must hold at all dates and in all states.

We conclude the proof. ∎

Proof of Theorem 2. Let $\mathcal{E}^*(\omega_t,\theta_t,b,n)$ denote the equilibrium aggregate extraction rate, expressed as a function of the relevant parameters. Let

$$G^*(\omega_t,\theta_t,b) \;=\; \left(\frac{\delta\gamma\;\omega_t(1-\mathcal{E}^*(\omega_t,\theta_t))}{\omega_t(1-\mathcal{E}^*(\omega_t,\theta_t)\;)-b}\right)1^*_{\{\omega_t,\mathcal{E}^*(\omega_t,\theta_t)\}}\times$$
$$\left(1+E\left[\frac{\theta_{i\,t+1}(1-\mathcal{E}_{t+1})}{e_{i\,t+1}}-(1-\theta_{i\,t+1})\;-\;\xi^{-i}_{t+1}\left(\frac{\theta_{i\,t+1}(1-\mathcal{E}_{t+1})}{e_{i\,t+1}}\right)\;\middle|\;\omega_t,\theta_{i\,t}\right]\right)$$

representing the marginal future cost of extraction, multiplied by $1-\mathcal{E}_t$.

Part 1. We first show that there is a stationary lower bound on the aggregate extraction rate, namely, an extraction rate $\underline{\mathcal{E}}(\theta_t,b,n)$ with $\underline{\mathcal{E}}(\theta_t,b,n) \le \mathcal{E}^*(\omega_t,\theta_t,b,n)\;\;\forall\,\omega_t$ such that if $\theta_t \in (1-\epsilon,1]^n$ where ϵ is small enough and n is large enough, we have

$$\omega_t \;>\; \omega^*_{t+1}(\omega_t,\theta_t;b)\quad \forall\,\omega_t \ge F\;.$$

In other words, the only fixed point of ω^* is the floor F.

We first find a lower bound $\underline{\mathcal{E}}(\theta_t,b,n)$ that is stationary in the stock ω. Observe that $1^*_{\{\omega_t,\mathcal{E}_t\}} = 0$ for all ω_t such that $A(\omega_t(1-\mathcal{E}_t)-b)^\gamma \le F$ or equivalently, $\omega_t \le \frac{1}{1-\mathcal{E}_t}\left(b+(\frac{F}{A})^{1/\gamma}\right) \equiv K$. Moreover, K is the upper bound on stocks for which $1^*_{\{\omega_t,\mathcal{E}_t\}} = 0$. Hence, fixing θ_t the marginal future cost of extraction (the right-hand side of (7)) is bounded above by its stationary limit when ω approaches K from the right, so that $1^*_{\{\omega_t,\mathcal{E}_t\}} = 1$. Stated precisely:

$$\forall\,\omega_t,\qquad G^*(\omega_t,\theta_t,b) \le \lim_{\omega\searrow K} G^*(\omega,\theta_t,b).$$

This is obviously true if $\omega \le K$ since $G^*(\omega_t,\theta_t,b) = 0$ when the carbon floor is reached. It is also true if $\omega > K$ since marginal cost is declining in stock.

Let $\mathcal{E}^*(K, \theta_t, b, n) \equiv \underline{\mathcal{E}}(\theta_t, b, n)$ so, by definition, $\underline{\mathcal{E}}(\theta_t, b, n)$ is the solution to (7) when $\omega = K$. we evaluate the Euler equation in the limit as $\theta_t \to \mathbf{1}$. In that case follows that $e^*_{it} = e^*_{i\,t+1} = \mathcal{E}^*/n$ and $\xi_t^{-i} = \xi_{t+1}^{-i} = (n-1)\xi < 0$. This last inequality follows by the acceleration of extraction in a neighborhood of K. The Euler equation becomes

$$\frac{n(1-\mathcal{E})}{\mathcal{E}} \;=\; \delta\gamma \frac{K(1-\mathcal{E})}{K(1-\mathcal{E})-b}\left(1+\frac{n(1-\mathcal{E})}{\mathcal{E}}(1-(n-1)\xi)\right)$$

which can be expressed as

$$n\frac{1-\mathcal{E}}{\mathcal{E}}(1-(n-1)\xi) \;=\; \frac{\delta\gamma K(1-\mathcal{E})}{(1-\delta\gamma)K(1-\mathcal{E})-b}.$$

Using the fact that $K \equiv \frac{1}{1-\mathcal{E}}\left(b+(\frac{F}{A})^{1/\gamma}\right)$, the Euler equation becomes

$$n\frac{1-\mathcal{E}}{\mathcal{E}}(1-(n-1)\xi) \;=\; \frac{\delta\gamma\left(b+(\frac{F}{A})^{1/\gamma}\right)}{(1-\delta\gamma)\left(b+(\frac{F}{A})^{1/\gamma}\right)-b}. \tag{23}$$

Since, by construction $\underline{\mathcal{E}}(\mathbf{1}, b, n)$ satisfies (23) and $\xi < 0$, it is easy to see that

$$\lim_{n\to\infty} \underline{\mathcal{E}}(\mathbf{1}, b, n) = 1.$$

Next we show that for ϵ small enough and n large enough,

$$\omega_t \;>\; \omega^*_{t+1}(\omega_t, \theta_t; b) \quad \forall\, \omega_t \quad \forall\, \theta_t \in (1-\epsilon, 1]^n. \tag{24}$$

The inequality (24) may be rewritten as

$$\left(\frac{1}{(1-\mathcal{E}^*(\omega_t, \theta_t, b, n))}\left[(\frac{\omega_t}{A})^{1/\gamma}+b\right]-\omega_t\right) \;>\; 0 \quad \forall\, \omega_t. \tag{25}$$

To verify that (25) holds, we show that

$$\underline{P} \;\equiv\; \min_{\omega}\left(\frac{1}{(1-\underline{\mathcal{E}}(\theta_t; b, n))}\left[(\frac{\omega}{A})^{1/\gamma}+b\right]-\omega\right) \quad > \quad 0 \tag{26}$$

where $\underline{\mathcal{E}}_t(\theta_t; b, n)$ is, recall, a stationary lower bound of $\mathcal{E}^*(\omega_t, \theta_t, b, n)$.

The first order condition for $\underline{P}$ is

$$(\gamma(1-\underline{\mathcal{E}}(\theta_t; b, n))A^{1/\gamma})^{-1}\omega^{\frac{1-\gamma}{\gamma}} - 1 = 0.$$

Solving for ω, we obtain $\omega^m \equiv (\gamma(1-\underline{\mathcal{E}}(\theta_t; b, n)))^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}}$. Substituting ω^m back into the problem we obtain,

$$\begin{aligned}
\underline{P} &= \left(\frac{1}{(1-\underline{\mathcal{E}}(\theta_t; b, n))} \left[(\frac{\omega^m}{A})^{1/\gamma} + b \right] - \omega^m \right) \\
&= \left(\frac{1}{(1-\underline{\mathcal{E}}(\theta_t; b, n))} \left[(\frac{(\gamma(1-\underline{\mathcal{E}}(\theta_t; b, n)))^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}}}{A})^{1/\gamma} + b \right] - (\gamma(1-\underline{\mathcal{E}}(\theta_t; b, n)))^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}} \right) \\
&= \frac{b}{1-\underline{\mathcal{E}}(\theta_t; b, n)} \; - \; (1-\underline{\mathcal{E}}(\theta_t; b, n))^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}} \left(\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}} \right).
\end{aligned}$$

Hence, (25) holds if

$$\underline{P} \; = \; \frac{b}{1-\underline{\mathcal{E}}(\theta_t; b, n)} \; - \; (1-\underline{\mathcal{E}}(\theta_t; b, n))^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}} \left(\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}} \right) \; > \; 0 \tag{27}$$

holds. But (27) clearly holds in the limit as $\theta_t \to \mathbf{1} \equiv (1, \ldots, 1)$ and $n \to \infty$ since $\underline{\mathcal{E}}(\theta_t; b, n) \to 1$ in that case.

Since the argument is strict, it holds for sufficiently large n and θ_t sufficiently close to one. Thus, for any fixed profile if θ_t, there is a finite time length $T(\theta_t)$ such that $\omega^{*t}(\omega_0, \theta^t) \to F$ in at most $T(\theta_t)$ iterations. Let

$$T = \max_{\theta_t \in [1-\epsilon, 1]} T(\theta_t).$$

Thus T is a time length (dependent on ω_0) such that if (24) holds for all $\theta_t \in (1-\epsilon, 1]^n$ then $\omega^{*t}(\omega_0, \theta^t) \to F$ in at most T iterations.

Observe that (10) implies for any finite $T > 0$, that for a.e. θ_t,

$$\begin{aligned}
& \Pr \left(\theta_{t+s} \in (1-\epsilon, 1]^n, \; s = 1, \ldots, T \;\middle|\; \theta_t \right) \\
= \; & \int_{\theta_{t+1} \in (1-\epsilon,1]^n} \cdots \int_{\theta_{t+T} \in (1-\epsilon,1]^n} \prod_{s=1}^{T} dF(\theta_{t+s} | \theta_{t+s-1}) \qquad (28) \\
\geq \; & \epsilon^T.
\end{aligned}$$

It follows that for almost every process $\{\theta_t\}$, there is a date t (infinitely many dates actually) such that (24) holds for realized values $\theta_t, \theta_{t+1}, \ldots, \theta_{t+T}$, in which case $\omega^{*\,t+T}(\omega_t, \theta^{t+T}) = F$. Consequently, the economy collapses at ω_t, concluding the proof of Part 1.

Part 2. The proof here largely reverse engineers some of the logic of part 1. In particular, we now find a stationary *upper* bound $\overline{\mathcal{E}}(\theta_t, b, n) \geq \mathcal{E}^*(\omega_t, \theta_t, b, n) \;\; \forall \; \omega_t$ with the property that for $\theta_t \in (0, \epsilon]^n$ and ϵ small enough, we have

$$\omega_t \; < \; \omega^*_{t+1}(\omega_t, \theta_t; b) \quad \text{on a nonnull set of stocks } \omega_t. \tag{29}$$

Notice that (29) is just the negation of (25).

The simplest upper bound is the extraction rate when each country its static, one shot optimal rate. Namely, we have as our upper bound,

$$\overline{\mathcal{E}}(\theta_t, b, n) \; = \; \sum_i \frac{\theta_{it}}{1-\theta_{it}} \left(1 + \sum_i \frac{\theta_{it}}{1-\theta_{it}}\right)^{-1}$$

Now using an analogous argument to that of the steps from Equations (26) to (27), the Inequality in (29) above holds if

$$\overline{P} \; \equiv \; \min_\omega \; \left(\frac{1}{(1-\overline{\mathcal{E}}(\theta_t; b, n))} \left[(\frac{\omega}{A})^{1/\gamma} + b \right] - \omega \right) \;\; < \;\; 0 \tag{30}$$

The first order condition for $\overline{P}$ is

$$(\gamma(1-\overline{\mathcal{E}}(\theta_t; b, n))A^{1/\gamma})^{-1}\omega^{\frac{1-\gamma}{\gamma}} - 1 = 0.$$

Solving for ω, we obtain $\omega^0 \equiv (\gamma(1-\overline{\mathcal{E}}(\theta_t; b, n)))^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}}$. Substituting ω^0 back into the problem we obtain,

$$\overline{P} \; = \; \frac{b}{1-\overline{\mathcal{E}}(\theta_t; b, n)} \; - \; (1-\overline{\mathcal{E}}(\theta_t; b, n))^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}} \left(\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}} \right).$$

Hence, (29) holds if

$$\overline{P} \; = \; \frac{b}{1-\overline{\mathcal{E}}(\theta_t; b, n)} \; - \; (1-\overline{\mathcal{E}}(\theta_t; b, n))^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}} \left(\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}} \right) \;\; < \;\; 0 \tag{31}$$

But (31) is easily observed to hold in the limit as $\theta_t \to \mathbf{0} \equiv (0, \ldots, 0)$ since

$$\overline{\mathcal{E}}(\theta_t, b, n) \; \equiv \; \sum_i \frac{\theta_{it}}{1-\theta_{it}} \left(1 + \sum_i \frac{\theta_{it}}{1-\theta_{it}}\right)^{-1} \; \to \; 0 \;\; \text{as} \;\; \theta_t \to \mathbf{0}$$

in the limit. Since the inequality is strict, (31) holds for $\theta_t \in [0, \epsilon]^n$ if ϵ is nonzero but sufficiently small. This concludes the proof of Part 2. ∎

Proof of Theorem 3 . Let $\theta_t \geq \tilde{\theta}_t$. Then by the definition of dominance, it suffices to show

$$\omega^{*\,t}(\omega_0, \theta^t) \;<\; \omega^{*\,t}(\omega_0, \tilde{\theta}^t) \quad \forall\, \omega_0 \tag{32}$$

We now proceed to verify (32). We first show $C^*(\omega_t, \theta_t) > C^*(\omega_t, \tilde{\theta}_t)$, in which case (32) holds by an induction argument.

Using the derivation of the Euler equation, in terms of c_i^* it can be expressed as

$$\begin{aligned}\frac{\theta_{it}(\omega_0 - C_t)}{c_{it}} - (1-\theta_{it}) \;&=\; \delta\gamma \left\{ \frac{\omega_t - C_t}{(\omega_t - C_t - b)} \times \right.\\ &\left. \left[1 + E\left[\left(\frac{\theta_{i\,t+1}(\omega_0 - C_{t+1})}{c_{i\,t+1}} - (1-\theta_{i\,t+1})\right) \;-\; \xi_{t+1}^{-i}\left(\frac{\theta_{i\,t+1}(\omega_t - C_{t+1})}{c_{i\,t+1}}\right) \;\middle|\; \omega_t, \theta_{i\,t}\right]\right] 1^*_{\{\omega_t, C_t\}} \right\}\end{aligned} \tag{33}$$

Since $\xi_{t+1}^{-i} < 1$, it is clear from (33) that c_{it}^* is increasing in θ_{it}, and since $\theta_{-i\,t}$ enters c_{it}^* only through its effect on $C_{-i\,t}^*$, it follows from the Envelope Theorem that $C_t^*(\theta_t)$ is increasing in θ_t.

The ordering of tipping points follows from the fact that

$$P\left(\left\{\theta^\infty : \lim_{t\to\infty} \omega^{*\,t}(\omega_0, \theta^t) \;\to\; F\right\}\right) \;=\; P\left(\left\{\theta^\infty : \lim_{t\to\infty} \omega^{*\,t}(\omega_0, \theta^t) \;\leq\; \omega^{tip}\right\}\right) \quad \forall\, \omega_0$$

and from Proposition 5.

$$P\left(\left\{\theta^\infty : \lim_{t\to\infty} \omega^{*\,t}(\omega_0, \theta^t) \;\leq\; \omega^{tip}\right\}\right) \;<\; P\left(\left\{\theta^\infty : \lim_{t\to\infty} \omega^{*\,t}(\overline{\omega}_0, \theta^t) \;\leq\; \omega^{tip}\right\}\right)$$

∎

Proof of Theorem 4. The Euler equation in the BAU equilibrium (in (7)) can be expressed as

$$\begin{aligned}&\left(\theta_{it} - \frac{(1-\theta_{it})e_{it}}{1-\mathcal{E}_t}\right)(\omega_t(1-\mathcal{E}_t) - b) \quad - \\ &A\delta\gamma\; e_{it}\omega_t \left\{1 + E\left[\left(\frac{\theta_{i\,t+1}}{e_{i\,t+1}} - \frac{1-\theta_{i\,t+1}}{1-\mathcal{E}_{t+1}} \;-\; \xi_{t+1}^{-i}\left(\frac{\theta_{i\,t+1}}{e_{i\,t+1}}\right)\right)(1-\mathcal{E}_{t+1}) \middle|\, \omega_t, \theta_{i\,t}\right]\right\}\; 1^*_{\{\omega_t, \mathcal{E}_t\}} \;=\; 0\end{aligned} \tag{34}$$

Similarly, the Euler equation for the planner's problem can be expressed as

$$\begin{aligned}
&\frac{1}{n}\left(\theta_{it} - \frac{e_{it}\sum_j(1-\theta_{jt})}{1-\mathcal{E}_t}\right)(\omega_t(1-\mathcal{E}_t)-b) \quad - \\
&A\delta\gamma\ e_{it}\omega_t\left\{1+E\left[\left(\frac{\theta_{i\,t+1}}{e_{i\,t+1}} \ - \ \frac{1-\theta_{i\,t+1}}{1-\mathcal{E}_{t+1}}\right)(1-\mathcal{E}_{t+1})\middle|\ \omega_t,\theta_{i\,t}\right]\right\}\ 1^*_{\{\omega_t,\mathcal{E}_t\}} \ = \ 0
\end{aligned} \tag{35}$$

The left-hand sides of Equations (34) and (35) are the marginal values to country i and the planner, respectively, from i's extraction of carbon. Summing both equations over all countries, we obtain,

$$\begin{aligned}
&H_t^*(\mathcal{E}_t,\mathbf{e}_{t+1}) \ \equiv \ \left(\sum_i \theta_{it} - \sum_i \frac{(1-\theta_{it})e_{it}}{1-\mathcal{E}_t}\right)(\omega_t(1-\mathcal{E}_t)-b) \quad - \\
&A\delta\gamma\ \mathcal{E}_t\omega_t\left\{1+E\sum_i\left[\left(\frac{\theta_{i\,t+1}}{e_{i\,t+1}} - \frac{1-\theta_{i\,t+1}}{1-\mathcal{E}_{t+1}} \ - \ \xi^{-i}_{t+1}\left(\frac{\theta_{i\,t+1}}{e_{i\,t+1}}\right)\ \right)(1-\mathcal{E}_{t+1})\middle|\ \omega_t,\theta_{i\,t}\right]\right\}\ 1^*_{\{\omega_t,\mathcal{E}_t\}} \ = \ 0
\end{aligned} \tag{36}$$

and

$$\begin{aligned}
&H_t^\circ(\mathcal{E}_t,\mathbf{e}_{t+1}) \ \equiv \ \frac{1}{n}\left(\sum_i \theta_{it} - \frac{\mathcal{E}_t\sum_j(1-\theta_{jt})}{1-\mathcal{E}_t}\right)(\omega_t(1-\mathcal{E}_t)-b) \quad - \\
&A\delta\gamma\ \mathcal{E}_t\omega_t\left\{1+E\left[\left(\frac{\theta_{i\,t+1}}{e_{i\,t+1}} \ - \ \frac{1-\theta_{i\,t+1}}{1-\mathcal{E}_{t+1}}\right)(1-\mathcal{E}_{t+1})\middle|\ \omega_t,\theta_{i\,t}\right]\right\}\ 1^*_{\{\omega_t,\mathcal{E}_t\}} \ = \ 0
\end{aligned} \tag{37}$$

We now compare the marginal values $H_t^*(\mathcal{E}_t,\mathbf{e}_{t+1})$ and $H_t^\circ(\mathcal{E}_t,\mathbf{e}_{t+1})$. In the following Lemmata, we refer to the top lines of H^* and of H° as the first term, and the bottom line of each as the second term.

Lemma 1 *The second term of $H_t^*(\mathcal{E}_t,\mathbf{e}_{t+1})$ is smaller than the second term of $H_t^\circ(\mathcal{E}_t,\mathbf{e}_{t+1})$*

Proof of Lemma 1. Clear by inspection.

Lemma 2 *Regarding the first terms of H^* and H°, for any $\mathcal{E}$ satisfying $\mathcal{E} < \frac{n}{n+1}$,*

$$\sum_i \theta_{it} - \sum_i \frac{(1-\theta_{it})e_{it}}{1-\mathcal{E}_t} \ > \ \frac{1}{n}\left(\sum_i \theta_{it} - \frac{\mathcal{E}_t\sum_j(1-\theta_{jt})}{1-\mathcal{E}_t}\right).$$

Proof of Lemma 2. For simplicity let $\Theta_t = \sum_i \theta_{it}$. Then we seek to show

$$\Theta_t(1-\mathcal{E}_t) - \mathcal{E}_t + \sum_i \theta_{it} e_{it} \overset{?}{>} \frac{1}{n}\left(\Theta_t(1-\mathcal{E}_t) - n\mathcal{E}_t + \Theta_t\mathcal{E}_t\right)$$

or

$$\Theta_t(1-\mathcal{E}_t) + \sum_i \theta_{it} e_{it} \overset{?}{>} \frac{1}{n}\left(\Theta_t(1-\mathcal{E}_t) + \Theta_t\mathcal{E}_t\right)$$

or

$$\frac{n-1}{n}\Theta_t(1-\mathcal{E}_t) + \sum_i \theta_{it} e_{it} \overset{?}{>} \frac{1}{n}\Theta_t\mathcal{E}_t$$

or

$$\frac{n-1}{n}\Theta_t - \Theta_t\mathcal{E}_t + \sum_i \theta_{it} e_{it} \overset{?}{>} 0$$

which clearly holds if $\frac{n}{n+1} > \mathcal{E}_t$.

Combining Lemma 1 with Lemma 2, it follows that for all $\mathcal{E}_t$ satisfying $\mathcal{E}_t < \frac{n}{n+1}$, and all $\mathbf{e}_{t+1}$,

$$H_t^*(\mathcal{E}_t, \mathbf{e}_{t+1}) > H_t^\circ(\mathcal{E}_t, \mathbf{e}_{t+1}).$$

Since $\mathcal{E}_t^\circ < \frac{n}{n+1}$,

$$H_t^*(\mathcal{E}_t^\circ, \mathbf{e}_{t+1}) > H_t^\circ(\mathcal{E}_t^\circ, \mathbf{e}_{t+1}) \;=\; 0,$$

for all $\mathbf{e}_{t+1}$. This yields $\mathcal{E}_t^* > \mathcal{E}_t^\circ$. ∎

Proof of Proposition 3

Part 1. Over- and Under-extraction by Individual Countries. To evaluate whether a country over or under extracts in the BAU equilibrium, one need only compare e_{it}° to e_{it}^*. Country over (under) extracts if $e_{it}^* > (<) e_{it}^\circ$. We therefore compare:

$$e_{it}^\circ(\theta_t) = \frac{\theta_{it}(1-A\gamma\delta)}{n} = \frac{\phi_{it}}{n} \overset{?}{>} e_{it}^*(\theta_t) = \frac{\left(\frac{\theta_{it}(1-A\gamma\delta)}{1-\theta_{it}(1-A\gamma\delta)}\right)}{1+\left(\sum_{j=1}^n \frac{\theta_{jt}(1-A\gamma\delta)}{1-\theta_{jt}(1-A\gamma\delta)}\right)} = \frac{\left(\frac{\phi_{it}}{1-\phi_{it}}\right)}{1+\left(\sum_{j=1}^n \frac{\phi_{jt}}{1-\phi_{jt}}\right)}$$

with, recall, $\phi_{it} = \theta_{it}(1-A\gamma\delta)$. Since $\phi_{it} > 0$, country i over-extracts if

$$\frac{\left(\frac{1}{1-\phi_{it}}\right)}{1+\left(\sum_{j=1}^n \frac{\phi_{jt}}{1-\phi_{jt}}\right)} > \frac{1}{n},$$

and solving for ϕ_{it}, country i will over (under) extract if

$$\phi_{it} \;>\; (<)\; 1 - \frac{n-1}{\sum_{j\neq i} \frac{\phi_{jt}}{1-\phi_{jt}}} \tag{38}$$

By choosing $\tilde{\theta}$ such that $\tilde{\theta}(1 - A\delta\gamma)$ equals the right hand side of (38), we have found out threshold.

Notice, moreover, that the larger the profile of her opponents the (weakly) smaller is the set of types for which is optimal to her over extract.[25]

Part 2. Output paths. The final part must prove that relative output, carbon consumption and carbon stock shrinks in the BAU relative to that of the efficient plan.

We first compute the socially optimal extraction rate and the optimal carbon path when $b = 0$ and setting $\phi_{it} = \theta_{it}(1 - A\delta\gamma)$. The extraction rate is: $e_{it} = \frac{\phi_{it}}{n}$ and the time path of the carbon stock in the planner's optimum is

$$\begin{aligned} \omega^{*\,t}(\omega_0, \theta^t) &= \omega_0^{\gamma^t} A^{\frac{1-\gamma^t}{1-\gamma}} \prod_{\tau=1}^{t} (1 - \mathcal{E}^{\circ}_{t-\tau}(\theta_{t-\tau}))^{\gamma^\tau} \\ &= \omega_0^{\gamma^t} A^{\frac{1-\gamma^t}{1-\gamma}} \prod_{\tau=1}^{t} \left(1 - \frac{\sum_j \phi_{j\,t-\tau}}{n}\right)^{\gamma^\tau}. \end{aligned} \tag{39}$$

[25] Example: suppose a symmetric profile ϕ_{-i}, i.e. $\phi_j = \phi_k = \phi$ for all $k, j \neq i$.

$$\phi_i > 1 - \frac{n-1}{\sum_{j\neq i} \frac{\phi_j}{1-\phi_j}} = 1 - \frac{n-1}{(n-1)\frac{\phi}{1-\phi}} = \frac{2\phi - 1}{\phi}. \qquad (**)$$

Note that the extreme (highest) profile player i can be facing is a profile of opponents with the highest type, *i.e.* $\theta_j = \overline{\theta} < 1$ for all $j \neq i$. Then from equation $(**)$ above,

$$\theta_i > \frac{2\overline{\theta}(1-\gamma\delta) - 1}{\overline{\theta}(1-\gamma\delta)^2}$$

or

$$\phi_i > \frac{2\overline{\phi} - 1}{\overline{\phi}}.$$

So if we require all θ_i over extract, the condition is:

$$\underline{\theta} > \frac{2\overline{\theta}(1-\gamma\delta) - 1}{\overline{\theta}(1-\gamma\delta)^2}.$$

This implies the following sufficient condition: if $\delta\gamma \geq \frac{1}{2}$ all types θ_i over extract.

A country's output path in the social planner's problem is given by

$$y_i^{*t} \;=\; \left(\frac{\phi_{i\,t}}{n}\right)^{\theta_{it}} \left(1-\frac{\sum_j \phi_{j\,t}}{n}\right)^{(1-\theta_{it})} \omega_0^{\gamma^t} A^{\frac{1-\gamma^t}{1-\gamma}} \prod_{\tau=1}^{t}\left(1-\frac{\sum_j \phi_{j\,t-\tau}}{n}\right)^{\gamma^\tau}. \tag{40}$$

A particularly useful illustration of (39) is the case without shocks. In that case $\theta_t = \theta_{t'} = \theta$ and so (39) reduces to

$$\omega^{*t}(\omega_0,\theta^t) = \omega_0^{\gamma^t}\left(1-\frac{\sum_j \phi_j}{n}\right)^{\frac{\gamma(1-\gamma^t)}{1-\gamma}} A^{\frac{1-\gamma^t}{1-\gamma}} \tag{41}$$

in which case the output path simplifies to

$$y_i^{*t} \;=\; \left(\frac{\phi_i}{n}\right)^{\theta_i}\left(1-\frac{\sum_j \psi_j}{n}\right)^{(1-\theta_i)} \omega_0^{\gamma^t}\left(1-\frac{\sum_j \phi_j}{n}\right)^{\frac{\gamma(1-\gamma^t)}{1-\gamma}} A^{\frac{1-\gamma^t}{1-\gamma}}. \tag{42}$$

These paths may be compared to the BAU equilibrium. Iterating on the equilibrium law of motion, one derives the time path of the carbon stock as

$$\begin{aligned}\omega^{*t}(\omega_0,\theta^t) \;&=\; \omega_0^{\gamma^t} A^{\frac{1-\gamma^t}{1-\gamma}} \prod_{\tau=1}^{t}(1-\mathcal{E}^*_{t-\tau}(\theta_{t-\tau}))^{\gamma^\tau}\\ &=\; \omega_0^{\gamma^t} A^{\frac{1-\gamma^t}{1-\gamma}} \prod_{\tau=1}^{t}\left(1-\frac{\sum_j(\frac{\phi_{j\,t-\tau}}{1-\phi_{j\,t-\tau}})}{1+(\sum_j \frac{\phi_{j\,t-\tau}}{1-\phi_{j\,t-\tau}})}\right)^{\gamma^\tau}.\end{aligned} \tag{43}$$

A country's output path in the BAU equilibrium is given by

$$\begin{aligned}y_i^{*t}(\omega_0,\theta^t) \;&=\; \left(\frac{(\frac{\phi_{it}}{1-\phi_{it}})}{1+(\sum_{j=1}\frac{\phi_{jt}}{1-\phi_{jt}})}\right)^{\theta_{it}}\left(1-\frac{\sum_j(\frac{\phi_{jt}}{1-\phi_{jt}})}{1+(\sum_j\frac{\phi_{jt}}{1-\phi_{jt}})}\right)^{(1-\theta_{it})}\omega^{*t}(\omega_0,\theta^t)\\ &=\; \left(\frac{(\frac{\phi_{it}}{1-\phi_{it}})}{1+(\sum_{j=1}\frac{\phi_{jt}}{1-\phi_{jt}})}\right)^{\theta_{it}}\left(1-\frac{\sum_j(\frac{\phi_{jt}}{1-\phi_{jt}})}{1+(\sum_j\frac{\phi_{jt}}{1-\phi_{jt}})}\right)^{(1-\theta_{it})}\times\\ &\qquad \omega_0^{\gamma^t} A^{\frac{1-\gamma^t}{1-\gamma}}\prod_{\tau=1}^{t}\left(1-\frac{\sum_j(\frac{\phi_{j\,t-\tau}}{1-\phi_{j\,t-\tau}})}{1+(\sum_j\frac{\phi_{j\,t-\tau}}{1-\phi_{j\,t-\tau}})}\right)^{\gamma^\tau}.\end{aligned} \tag{44}$$

Comparing the BAU in (44) with the optimal output in (40). We see that $y^*_{it} < y^\circ_{it}$ iff

$$\left(\frac{(\frac{\phi_{it}}{1-\phi_{it}})}{1+(\sum_{j=1}\frac{\phi_{jt}}{1-\phi_{jt}})}\right)^{\theta_{it}}\left(1-\frac{\sum_j(\frac{\phi_{jt}}{1-\phi_{jt}})}{1+(\sum_j\frac{\phi_{jt}}{1-\phi_{jt}})}\right)^{(1-\theta_{it})}\prod_{\tau=1}^{t}\left(1-\frac{\sum_j(\frac{\phi_{j\,t-\tau}}{1-\phi_{j\,t-\tau}})}{1+(\sum_j\frac{\phi_{j\,t-\tau}}{1-\phi_{j\,t-\tau}})}\right)^{\gamma^\tau}$$
$$< \left(\frac{\phi_{it}}{n}\right)^{\theta_{it}}\left(\frac{1-\sum_j\phi_{jt}}{n}\right)^{(1-\theta_{it})}\prod_{\tau=1}^{t}\left(1-\frac{\sum_j\phi_{j\,t-\tau}}{n}\right)^{\gamma^\tau}.$$

In order to evaluate the relative growth in output paths, we compare:

$$\prod_{\tau=1}^{t}\left(1-\frac{\sum_j(\frac{\phi_{j\,t-\tau}}{1-\phi_{j\,t-\tau}})}{1+(\sum_j\frac{\phi_{j\,t-\tau}}{1-\phi_{j\,t-\tau}})}\right)^{\gamma^\tau} < \prod_{\tau=1}^{t}\left(1-\frac{\sum_j\phi_{j\,t-\tau}}{n}\right)^{\gamma^\tau}$$

which holds due to the fact that the aggregate extraction rate is larger (hence conservation rate is smaller) in the MPE. Moreover the relative difference

$$\prod_{\tau=1}^{t}\left(1-\frac{\sum_j\phi_{j\,t-\tau}}{n}\right)^{\gamma^\tau} / \prod_{\tau=1}^{t}\left(1-\frac{\sum_j(\frac{\phi_{j\,t-\tau}}{1-\phi_{j\,t-\tau}})}{1+(\sum_j\frac{\phi_{j\,t-\tau}}{1-\phi_{j\,t-\tau}})}\right)^{\gamma^\tau}$$

is increasing as time passes. Hence, both the expected ratio $E[\frac{y^\circ_{it}}{y^*_{it}}]$ and the expected difference $E[y^\circ_{it} - y^*_{it}]$ are increasing in t. ∎

Proof of Theorem 5. It suffices to show that the planner's solution admits a finite tipping in the worst case: $\theta_t = \mathbf{1}$. In an economy restricted to $\theta_t = \mathbf{1}$, the policy tipping point is defined by the unstable fixed point solution to $\omega_t = \omega^\circ(\omega_t, \mathbf{1}; b)$ or

$$\left(\frac{1}{(1-\mathcal{E}^\circ(\omega_t,\mathbf{1};b))}\left[(\frac{\omega_t}{A})^{1/\gamma}+b\right]-\omega_t\right) = 0 \tag{45}$$

In fact, a finite tipping point exists if we can show that (45) has any solution, stable or unstable. From the planner's solution, it follows that

$$\mathcal{E}^\circ(\omega_t,\mathbf{1};b) = \frac{1}{1+G^\circ(\omega_t,\mathbf{1};b)}.$$

Notice that this expression does not vary in n. Equation (45) then becomes

$$H(\omega_t,b) \equiv \left(\frac{1+G^\circ(\omega_t,\mathbf{1};b)}{G^\circ(\omega_t,\mathbf{1};b)}\left[(\frac{\omega_t}{A})^{1/\gamma}+b\right]-\omega_t\right) = 0 \tag{46}$$

To verify that this equation has a solution, observe that $G^{\circ}(\omega_t, \mathbf{1}; b) \to 0$ as $\omega_t \to 0$ while $G^{\circ}(\omega_t, \mathbf{1}; b) \to G^{\circ}(\omega_t, \mathbf{1}; 0) = \frac{A\delta\gamma}{1-A\delta\gamma}$ as $\omega_t \to \infty$. These limits imply $H(\omega_t, b) \to \infty$ as $\omega \to 0$ and $H(\omega_t, b) \to H(\omega_t, 0) < \omega_t$ as $\omega_t \to \infty$. The Intermediate Value Theorem immediately implies the existence of a solution to (46). Consequently, a finite policy tipping point exists, concluding the proof of this theorem. ∎

References

[1] Acemoglu, D., P. Aghion, L. Bursztyn, and D. Hemous (2012), "The Environment and Directed Technical Change," *American Economic Review*, 102: 131-166.

[2] Anderies, J.M., S.R. Carpenter, W. Steffan, and J. Rockstrom (2013), "The Topology of Nonlinear Global Carbon Dynamics: from the tipping points to planetary boundaries." *Environmental Research Letters*, 8: 1-13.

[3] Barrett, S. (2003), *Environment and Statecraft: The Strategy of Environmental Treaty- Making*, Oxford University Press, Oxford.

[4] Barrett, S. (2013), "Climate Treaties and Approaching Catastrophes," *Journal of Environmental Economics and Management*, 66: 235-50.

[5] Battaglini, M. and Bard Harstad (2012), "Participation and Duration of Environmental Agreements," NBER working paper, #18585.

[6] Bodansky, D. (2004), "International Climate Efforts Beyond 2012: A Survey of Approaches," *PEW Center on Global Climate Change.*

[7] Cai, Y., K. Judd, and T. Lontzek (2012), "Tipping Points in a Dynamic Stochastic IAM," mimeo, January.

[8] Calcott, P. and V. Petkov (2012), "International Environmental Agreements: Design of Optimal Transfers Under Heterogeneity," *Environmental Modeling and Assessment*, 17: 209-220.

[9] Carpenter, S.R., D. Ludwig, and W. A. Brock (1999), "Management of Eutrophication for Lakes Subject to Potentially Irreversible Change," *Ecological Applications*, 9: 751-771.

[10] Cave, J. (1987), "Long-Term Competition in a Dynamic Game: The Cold Fish War," *Rand Journal*, 18: 596-610.

[11] Desmet, K. and E. Rossi-Hansberg (2014), "On the Spatial Economic Impact of Global Warming," mimeo, Princeton University, July.

[12] Dutta, P. (1995), "A Folk Theorem for Stochastic Games," *Journal of Economic Theory*, 66: 1-32.

[13] Dutta, P. and R. Radner (2009), "A Strategic Analysis of Global Warming: theory and some numbers," *Journal of Economic Behavior and Organization*, 71: 187-2009.

[14] Dutta, P. and R. Radner (2006), "A Game-Theoretic Approach to Global Warming," *Advances in Mathematical Economics*, 8: 135-53.

[15] Dutta, P. and R. Radner (2004), "Self-enforcing climate-change treaties," *Proceedings of the National Academy of Sciences.*

[16] Dutta, P. and R. Sundaram (1993), "Tragedy of the Commons?" *Economic Theory*, 3: 413-26.

[17] Finus, M. (2001), *Game Theory and International Environmental Cooperation*, Cheltenham, Edward Elgar.

[18] Fisher, R. and L. Mirman (1992), "Strategic Dynamic Interactions: Fish Wars," *Journal of Economic Dynamics and Control*, 16: 267-87.

[19] Fisher, R. and L. Mirman (1996), "The Compleat Fish Wars: Biological and Dynamic Interaction," *Journal of Environmental Economics and Management*, 30: 34-42.

[20] Golosov, M., J. Hassler, P. Krusell, and A. Tsyvinski (2014), "Optimal Taxes on Fossil Fuel in General Equilibrium," *Econometrica*, 82: 41-88.

[21] Hansen, J., M. Sato, G. Russell, and P. Kharecha (2013), "Climate Sensitivity, Sea Level and Atmospheric Carbon Dioxide," *Royal Society Publishing: Philosophical Transactions A*, 371 (2001): 1-42.

[22] Harrison, R. and R. Lagunoff (2016), "Dynamic Mechanism Design for a Global Commons," *International Economic Review*, forthcoming.

[23] Haurie, A. (2008), "A Dynamic Game Model of Strategic R&D Cooperation and GHG Emission Mitigation," mimeo.

[24] Hope C (2006), "The Marginal Impact of CO2 from PAGE2002: An integrated assessment model incorporating the IPCC's five reasons for concern," *Integrated Assessment*, 6: 19-56.

[25] Krusell, P. and A. Smith (2009), "Macroeconomics and Global Climate Change: Transition for a Many-Region Economy," working paper.

[26] Kerr, R. (2008), "Climate Tipping Points Come in from the Cold," *Science* 319: 153.

[27] Lemoine, D. and C. Traeger (2014), "Watch Your Step: Optimal Policy in a Tipping Climate," *American Economic Journal: Economic Policy*, 6: forthcoming.

[28] Lenton, T.M., H. Held, E. Kriegler, J.W. Hall, and W. Lucht (2008), "Tipping Elements in the Earth's Climate System," *Proceedings of the National Academy of Sciences*, 105: 1786.

[29] Levhari, D. and L. Mirman (1980), "The Great Fish War: An Example Using a Dynamic Cournot-Nash Solution," *Bell Journal of Economics*, 14: 322-34.

[30] Maasoumi, E. and G. Yalonetzky (2013), "Introduction to Robustness in Multidimensional Wellbeing Analysis," *Econometric Reviews*, 32: 1-6.

[31] Maler, K-G., A. Xepapadeas, and A.J. Zeeuw (2003), "The economics of shallow lakes," *Environmental and Resource Economics* , 26: 603-624.

[32] N-V Long (2011), "Dynamic Games in the Economics of Natural Resources: A Survey," *Dynamic Games and Applications* , 1: 115-148.

[33] Nordhaus, W. (2006), "After Kyoto: Alternative Mechanisms to Control Global Warming," *American Economic Review*, 96: 31-34.

[34] Nordhaus, W. (2007), "To Tax or Not to Tax: The Case for a Carbon Tax," *Review of Environmental Economics and Policy.*

[35] Ostrom, E. (1990), *Governing the Commons: The Evolution of Institutions for Collective Action,* Cambridge University Press, New York.

[36] Ostrom, E. (2002), "Common Pool Resource and Institutions: Towards a Revised Theory," *Handbook of Agricultural Economics, Volume 2.*

[37] Rockstrom, J., W. Steffen, K. Noone, A. Persson, F. Chapin, E. Lambin, T. Lenton, M. Scheffer, C. Folke, H. Schellnhuber, B. Nykvist, C. de Wit, T. Hughes, S. van der Leeuw, H. Rodhe, S. Sorlin, P. Snyder, R. Costanza, U. Svedin, M. Falkenmark, L. Karlberg, R. Corell, V. Fabry, J. Hansen, B. Walker, D. Liverman, K. Richardson, P. Crutzen, and J. Foley (2009), " A Safe Operating Space for Humanity," *Nature*, 461: 472-75, DOI: 10.1038/461472a

[38] Rouillon, S. (2010), "Optimal decentralized management of a natural resource," mimeo, Universite Bordeaux.

[39] Sakamoto, H. (2014), "Dynamic Resource Management under the Risk of Regime Shifts," *Journal of Environmental Economics and Management*, 68: 1-19.

[40] Sinn, H.W. (2008), "Public Policies against Global Warming," *International Tax and Public Finance*, 15: 360-394.

[41] Sorger, G. (1996), "Markov-Perfect Nash Equilibria in a Class of Resource Games," CIRANO Working paper, April.

[42] Steffen, W., K. Richardson, J. Rockstrom, S. Cornell, I. Fetzer, E. Bennett, R. Biggs, S. Carpenter, W. de Vries, C. de Wit, C. Folke, D. Gerten, J. Heinke, G. Mace, L. Persson, V. Ramanathan, B. Reyers, S. Sorlin (2015), "Planetary Boundaries: Guiding Human Development on a Changing Planet," *Science*, 347: DOI: 10.1126/science.1259855.

[43] Skiba, A. (1978), "Optimal Growth with a Convex-Concave Production Function," *Econometrica*, 46: 527539.

[44] Stern, N. (2006), *The Stern Review Report on the Economics of Climate Change.*

[45] United Nations Inter-Governmental Panel on Climate Change (2007), *Fourth Assessment Report: Climate Change.*

[46] Zoli, C. (2009), "From Unidimensional to Multidimensional Inequality, Welfare and Poverty Measurement," mimeo, University of Verona.