

Tipping Points and Business-as-Usual in a Global Carbon Commons

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ABSTRACT

This paper analyzes a dynamic strategic model of carbon consumption among nations. Each period, countries extract carbon from the global ecosystem. A country's output depends both on its carbon usage and on the stock of carbon biomass (stored carbon) in the ecosystem. The output elasticities of extracted and stored carbon vary across countries and evolve stochastically over time.

A *Business-as-usual (BAU) equilibrium* characterizes each country's carbon footprint in the absence of an effective international agreement. Under non-concave carbon dynamics, depletion of the carbon stock in a BAU equilibrium may reach a *tipping point* below which the global commons spirals downward toward a steady state of marginal sustainability. These tipping points emerge endogenously. We show that if the number of carbon extractors is large enough, the commons always tips, regardless of the initial stock. We find that countries will *accelerate* their rates of carbon usage the closer they are to reaching the low-end steady state. By contrast, in the socially efficient plan the commons never tips if the initial carbon stock is large.

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Key Words and Phrases: Carbon consumption, global carbon commons, tipping points, safe operating space for humanity, Business-as-usual equilibrium.

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1 Introduction

Human consumption is based on carbon usage. Alarmed by increases in anthropogenic greenhouse gas emissions, many scientists and policy experts focus on finding an effective international response to limit carbon emissions.¹

This paper formulates a dynamic model of carbon consumption in the absence of such a response. Our objective is to understand the strategic incentives of nations in a “business-as-usual” (or “BAU” from here on) scenario. Our approach integrates a strategic model of emissions into a non-concave dynamic model of carbon. A key feature of this model is that consumption and economic output may shrink if a key state variable falls below some critical threshold, a *tipping point*, determined endogenously in equilibrium.

We construct a dynamic stochastic game with heterogeneous countries. Each country produces a composite consumption good for its citizens. Production depends on two carbon-based inputs. One input comprises the country’s extracted carbon from fossil fuels, forestry, and agricultural practices. The other input, which we refer to as the *carbon-based ecosystem*, consists of the stock of stored or preserved carbon in biomass, soil, or below-ground sources. This carbon stock is a productive input as it prevents soil erosion, pollution, and climate change.

Thus, for purely economic reasons a country seeks to balance its need to use carbon against its desire to preserve the ecosystem. A country’s optimal mix of extracted and preserved carbon is determined by its relative output elasticities with respect to each input. These elasticities differ across countries and are assumed to evolve stochastically. At any given date a country with high output elasticity with respect to extracted carbon (i.e. a “pro-extractive” country) prefers to extract more than a country with lower elasticity (a “pro-conservation” country). The assumptions on elasticities capture a common feature of greenhouse gas emissions: both environmental costs and factor composition vary over time, are difficult to forecast, and often vary widely across countries. Heterogeneity reflects variation in geographic, demographic, and politico-economic influences.²

The driving force of the model is a non-concavity in the transition law for carbon. The non-concavity introduces a bifurcation in the stock dynamics. Given any level of global carbon extraction, when the stock of preserved carbon, denoted here by ω_t , is large enough the natural process of sequestration and respiration produce a

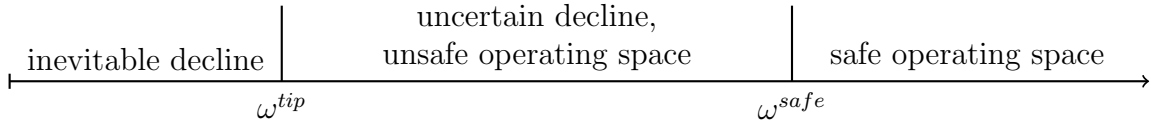
¹e.g., IPCC Fourth Assessment Report: Climate Change 2007.

²Burke, et. al. (2011) find, for example, widely varying estimates of the effect of climate change on US agriculture when climate model uncertainty is taken into account. Desmet and Rossi-Hansberg (2014) document substantial cross country variation in a calibrated model of spatial differences in welfare losses across countries due to global warming.

stable carbon cycle. However, a low enough stock can destabilize the cycle, leading to a precipitous decline toward a *carbon floor* — a low end steady state of marginal sustainability. The threshold at which this decline begins is referred to as a *tipping point*.

Because the global extraction of carbon is determined endogenously by the countries’ strategic decisions, the tipping point will be as well. We study a *business-as-usual (BAU) equilibrium*, defined as a smooth Markov Perfect equilibrium profile of carbon usage across countries.

We derive the boundary points of the support of a distribution that describes the likelihood of reaching the carbon floor starting from an initial given stock. The tipping point, denoted here by ω^{tip} , is lower bound of the support. The upper bound of the support, denoted by ω^{safe} , is the threshold at which the low end is never reached. The region above ω^{safe} is thus often referred to as the *safe operating space (SOS)* in the planetary boundaries literature (Rockstrom et al. (2009) and Steffan, et al. (2015)). In this region, the stock converges to a high end, “sustainable” stationary distribution of carbon stocks. The regions are displayed below.³



In the knife-edged case where the equilibrium distribution is degenerate (i.e., if all sources of stochastic variation are removed), then $\omega^{tip} = \omega^{safe}$. In that case a single tipping point divides the space into tipped and non-tipped regions, analogous to Skiba points in non-convex optimal control problems.⁴

We compare extraction rates and tipping properties of any BAU equilibrium to those coming from a concave (“no-tipping”) transition model. We further compare the BAU equilibrium to the solution of a standard utilitarian planner’s problem.

We first show that, relative to the concave model, countries in a BAU equilibrium actually accelerate their rates of carbon usage the closer/sooner the commons comes to reaching the carbon floor. This is done unevenly, as persistently pro-extractive countries accelerate sooner than pro-conservation ones. Relative to the concave model, countries extract more slowly the further/later they are to reaching the floor.

³The schematic display in Steffan, et al. (2015 and reprinted in the Washington Post, January 15, 2015) uses emissions rather than stocks and so the direction of risk is rightward rather than leftward.

⁴Skiba (1978).

The intuition is reminiscent of the Green Paradox (Sinn (2013)) which posits that firms increase their extraction of fossil fuels if more stringent regulations are expected in the future. In the Green Paradox, the future regime change is exogenous from the firms' perspectives. In our model, regime change is endogenously determined by the strategic actors themselves. Each actor expects his marginal continuation value of preserving the carbon stock to vanish when the likelihood of tipping is high due to the strategic play of others. Hence the actor accelerates his extraction which, in turn, feeds back to the tipping likelihood itself.

Second, we show that if the number of countries in the BAU equilibrium is at or above some threshold number, the tipping point ω^{tip} is infinite. This means that there is no safe operating space, and the global commons reaches the low end state with certainty. This happens even if countries are pro-conservation most of the time.

Third, we show that the tipping properties of the Planner's problem are fundamentally different. Not surprisingly, prescribed carbon is usage lower under the Planner's solution, and the relative difference grows over time. More strikingly, under the Planner's solution, both the tipping point and the safe operating bound are finite and invariant to the number of countries. This means that in the Planner's solution, there is a threshold level of carbon stock above which the countries remain in safe operating space. This last result highlights the potential value of an international climate agreement that limits global carbon usage.

The paper may be compared to a number of related literatures on climate tipping. Tipping is discussed and modeled in the earth science literature. For instance, Lenton, et al. (2008), Kerr (2008), Rockstrom, et al. (2009), Anderies, et al. (2013), and Steffen, et al. (2015) posit nonlinear dynamical systems that describe the *safe operating space (SOS) for humanity*, i.e., a region in a multi-dimensional space consisting of levels of methane and CO_2 concentrations, degrees of biodiversity, and so on,... that sustains human innovation, growth, and development. Tipping points are the *planetary boundaries* of these regions (see Rockstrom et al. (2009)).

The earth science models contain a detailed account of the different forms of carbon mass and their movements throughout the carbon cycle. Human incentives are not usually modeled explicitly. The present paper complements these by modeling the dynamic incentives of nations to extract carbon in an, albeit rudimentary, model of the carbon cycle.

Economic incentives appear in the integrated assessment models of Nordhaus (2006, 2007, 2008), Lemoine and Traeger (2014), Hope (2006), Stern (2006), and Cai, Judd, and Lontzek (2012), etc., all of whom integrate tipping dynamics into atomistic GE market economies.⁵ They also appear in the shallow lake models of

⁵See also Krusell and Smith (2009), Acemoglu et al. (2012), and Golosov et al. (2014) for useful

Carpenter, et al. (1999) and Maler et al (2003) who analyze open loop Nash equilibria of a symmetric game with non-linear accumulation of pollutants, and Kossioris, et. al. (2008) and Sakamoto (2014) who examine symmetric closed loop equilibria in related models.

Our focus on strategic incentives of heterogeneous state actors is, to us, a sensible addition to the literature since the most critical policy choices are made by large, powerful nations with divergent interests.

In the rest of the paper, Section 2 introduces the model. Section 3 analyzes the strategic incentives of countries in a BAU equilibrium. Section 4 examines its tipping properties. Section 5 introduces the Planner’s solution and compares it to BAU. Section 6 concludes with a discussion of international agreements. Section 7 is an Appendix with proofs and detailed calculations.

2 A Tipping Model of Carbon Usage

This Section lays out a rudimentary model of carbon usage and follows it up with a discussion of the key assumptions of the model.

2.1 Output and Carbon Extraction

An infinite horizon global economy consists of n countries. At each date t , a composite good y_{it} is produced and consumed by the citizens of country i ($i = 1, \dots, n$). The long run payoff to the representative citizen of country i from consuming y_{it} at each date t is

$$\sum_{t=0}^{\infty} \delta^t u(y_{it}) \tag{1}$$

where u is strictly increasing, differentially concave, and $u' \rightarrow \infty$ as $y_{it} \rightarrow 0$. The main equilibrium results will assume $u(y_{it}) = \log(y_{it})$. All countries discount the future according to δ .

Country i ’s production of y_{it} requires two inputs, both made of carbon and both derived from a carbon-based “global ecosystem.” The first input comprises extracted carbon denoted by c_{it} . Extracted carbon c_{it} produces c_{it} units of emissions, and so the terms “extraction,” “consumption,” and “emissions” of carbon are used interchangeably.

quantitative assessments of carbon taxation and cap and trade policies.

Let $C_t = \sum_i c_{it}$ represent the level of global carbon consumed/emitted at t . Emissions come from various sources, including fossil fuels, forest biomass, and agricultural practices. All of these are taken from a stock ω_t representing the carbon-based ecosystem. This ecosystem consists of known reserves of stored or “preserved” carbon in soil, biomass, and fossils. It represents all usable sources of non-atmospheric carbon. Thus, carbon that is not emitted into the atmosphere is preserved in this ecosystem.⁶

The second productive input is the ecosystem itself. Preservation of the ecosystem enhances production by maintaining soil and plant health and reducing the scope for damage from runoff, erosion, pollution, and climate change. A more detailed discussion of the ecosystem’s role is contained in the next subsection.

The production technology for y_{it} is

$$y_{it} = c_{it}^{\theta_{it}} (\omega_t - C_t)^{1-\theta_{it}}. \quad (2)$$

In (2), $\theta_{it} \in (0, 1]$ is the output elasticity of extracted carbon, while $1 - \theta_{it}$ is the output elasticity of the global ecosystem net of aggregate consumption. Cross-country differences in the θ_{it} reflect differences in geography and demography. We refer to countries with larger θ_{it} as “pro-extractive” in date t since they will typically extract and emit more carbon, other things equal.⁷ Countries with smaller θ_{it} are referred to as “pro-conservation.”

The elasticities are assumed to vary both over time and between countries. Thus, a pro-extractive country at one point in time may become pro-conservation in the future.

A *type profile* in date t is a vector

$$\theta_t = (\theta_{1t}, \theta_{2t}, \dots, \theta_{it}, \dots, \theta_{nt}),$$

and is publicly observed at the beginning of each period t . The profile θ_t of country-specific elasticities, together with the carbon stock ω_t constitute the state of the system.

Let $\theta^t = \{\theta_0, \theta_1, \dots, \theta_t\}$ be the history of realized type profiles up to and including date t , and let

$$\theta^\infty = \{\theta_0, \theta_1, \dots, \theta_t, \dots\}$$

the infinite time path of elasticity profiles.

⁶This definition excludes marine carbon which plays no role in the model.

⁷The assumption of linearly homogeneous production is purely for tractability. None of the results depend on it. One can work with general coefficients α_{it} and β_{it} and derive the same qualitative conclusions.

Fixing the initial profile θ_0 , the profile θ_t is assumed to evolve according to a stationary Markov process $\pi(\theta_t|\theta_{t-1})$. In what follows, “almost everywhere” will refer to the paths θ^∞ in the probability space $(\Theta^\infty, \mathcal{F}, P)$ such that π is the Markov density associated with a filtration $\{\mathcal{F}_t\}$ on the space $(\Theta^\infty, \mathcal{F}, P)$. We allow for π to exhibit both persistence across time and correlation of carbon elasticities across countries.

We assume that there is some $\epsilon > 0$ such that for all T , π places mass of at least ϵ on the n -dimensional cube $(1 - \epsilon, 1]^n$ for T consecutive periods. Formally, assume that π satisfies: there exists $\epsilon > 0$ such that for any integer T and a.e. θ^∞ ,

$$\int_{\theta_{t+s} \in (1-\epsilon, 1]^n} \pi(\theta_{t+s}|\theta_{t+s-1}) d\theta_{t+s} \geq \epsilon \quad (3)$$

for all $s = 1, \dots, T$ and infinitely many t .

The inequality in (3) is a natural ergodic property. It requires that the process will eventually hit the upper ϵ -interval of elasticities and remain there for T periods or more. This will hold, for example, for any number of processes that have full support uniformly bounded away from zero. The assumption also holds for a wide class of supermartingales where the θ 's trend upward, reflecting the historical pattern of increased reliance on fossil fuels.

2.2 Discussion of the Production Technology

In the classic common pool model of Levhari and Mirman (LM) (1980) identical users choose how much of a depletable, open access resource to consume each period. Examples include fisheries or forestry. There are no direct costs or externalities from usage. More importantly there is no tangible constraint on consumption/production *until* the resource stock literally hits zero. Conservation is thus valued in LM only for instrumental reasons: preserving the stock allows one to smooth consumption.

Here, we propose a production technology where the carbon-based ecosystem enters as an input. This means that, unlike a pure commons, there are incentives to conserve even if there is no threat of full depletion. The formulation accounts for the fact that all countries' economies have carbon requirements, but production also requires that countries draw upon a viable ecosystem. Stored carbon stock represents a “flip side” of carbon emissions, and so the assumption that the stored stock is a productive input is equivalent to modeling carbon emissions as a GDP-reducing cost.

By design, the production function in (2) excludes the traditional inputs of capital and labor on the grounds that carbon would be double-counted.⁸ The simple dis-

⁸Labor, for instance, is derived from carbon usage (caloric intake, respiration, etc). Many if not

inction between stored and released carbon forms the basis for all dynamic changes introduced later in the model.

Two features of the model warrant further discussion. First, the model lumps all forms of carbon stock into a single state variable. One might argue that the accumulation of geological carbon, for instance, is a long term process and should therefore be considered separate from the ecosystem. Our inclusion of fossil fuels is based on evidence that extraction of fossil fuels can deplete the ecosystem (Rockstrom, et al. (2009)). Fracking, strip-mining, oil drilling all involve potential depletion of biomass or limits on its growth. Avoiding emissions is thus equivalent to preserving the stock — including fossilized carbon. By separating out the various stocks, the game theoretic aspects of extraction versus preservation are obscured.

Second, one could argue that fossil fuels are not open access resources; their distribution around the world is non-uniform. Yet a full accounting for all forms of emittable carbon makes open access a defensible approximation. Countries like Brazil and Tanzania have large rain forests and agricultural production. Other countries like Russia and Saudi Arabia extract fossil fuels. Along these lines, asymmetries in access are incorporated indirectly by assuming heterogeneous technologies across countries. Warmer average temperatures resulting from GHG emissions are viewed differently in Greenland than in Sub-saharan Africa.

The next subsection, we posit a carbon dynamic with a low-end non-concavity capable of tipping the system.

2.3 Carbon Stock Dynamics

In the absence of human consumption (i.e., $C_t = 0$), the stock dynamics will balance the dynamic forces of release and recapture of carbon to produce a stable carbon cycle if the stock ω_t is not too low. However, the law of motion contains a non-concavity so that high levels of human consumption can destabilize the cycle.

Expressed formally, the ecosystem evolves according to:

$$\omega_{t+1} = \begin{cases} A(\omega_t - C_t - b)^\gamma & \text{if } A(\omega_t - C_t - b)^\gamma > F \\ F & \text{otherwise} \end{cases} \quad (4)$$

with the initial stock ω_0 . By assumption, $\gamma < 1$ which allows for depreciation (e.g., most types of capital embody carbon, including metal alloys (steel), plastics, and organic inputs (rubber, cotton, wool, wood, livestock, etc.)). These would be included in the stock ω_t .

plant respiration), while $A > 1$ which allows for accumulation due to natural reabsorption (e.g., plant photosynthesis).⁹

The parameter $b \geq 0$ describes the exogenous “off-take”, the subtraction of carbon from the stock that is independent of human decisions. If $b > 0$ the carbon-based ecosystem can dramatically shrink if the stock falls below some critical carbon threshold — a *tipping point* — a concept explicitly defined and characterized in Section 3.

The parameter F is the carbon floor — a lower bound on the stock below which stock depletion cannot occur. Payoffs that might otherwise hit $-\infty$ as the stock is depleted are bounded below when $F > 0$. When $F > 0$ carbon stocks are never fully depleted.¹⁰ We assume that there is some stock $\tilde{\omega} > F$ for which $A(\tilde{\omega} - b)^\gamma = \tilde{\omega}$. That is, in the absence of human behavior, there is at least one steady state carbon stock above the floor F . The assumption means that without human consumption, a stable carbon stock or carbon cycle (if noise is added) exists.

Finally, we assume that F is small, specifically $F \leq b$, so that F sustains very low output.¹¹ From here on, the model with $b \geq F > 0$ will be referred to as the “non-concave model” — as distinct from the benchmark concave model of $b = F = 0$.

Figure 1 illustrates the dynamic in (4). For illustrative purposes, C_t is exogenous and constant in the Figure but is endogenous in the model. There are three fixed points, one of which is unstable. In a non-stochastic world with fixed exogenous human behavior, the unstable fixed point (ω° in the Figure) would correspond to a “tipping point.” The carbon floor F represents an “environmental poverty trap” since a stock that reaches F remains stuck there forever.

Let $\mathbf{c}_t = (c_{1t} \dots, c_{nt})$ denote the date t profile of carbon consumption. The entire dynamic path profile of resource consumption is then given by

$$\mathbf{c} = \{\mathbf{c}_t\}_{t=0}^\infty$$

A consumption path \mathbf{c} is *feasible* if it is consistent with the dynamic constraint (4) and $C_t \leq \omega_t - b$ at each date t .

⁹For tractability, A is assumed to be exogenous and constant. However, the model can be generalized to allow for a time-varying ergodic process $\{A_t\}$, in which case there is a stable carbon cycle if the stock is large enough. An even richer model would allow A to depend on the existing stock.

¹⁰Specifically, because $\log(y_{it}) = -\infty$ when $\omega_t = 0$, a floor $F > 0$ rules out full depletion, thus avoiding the limit at $\omega_t = 0$.

¹¹The condition $F \leq b$ also ensures that the non-concavity model has non-trivial implications for tipping. Specifically, $F \leq b$ ensures that there is an unstable steady state above F . If $F > b$ then it could be the case that the unstable steady state of $\omega_{t+1} = A(\omega_t - C_t - b)^\gamma$ would lie below F , in which case the floor would never be reached from any stock above F .

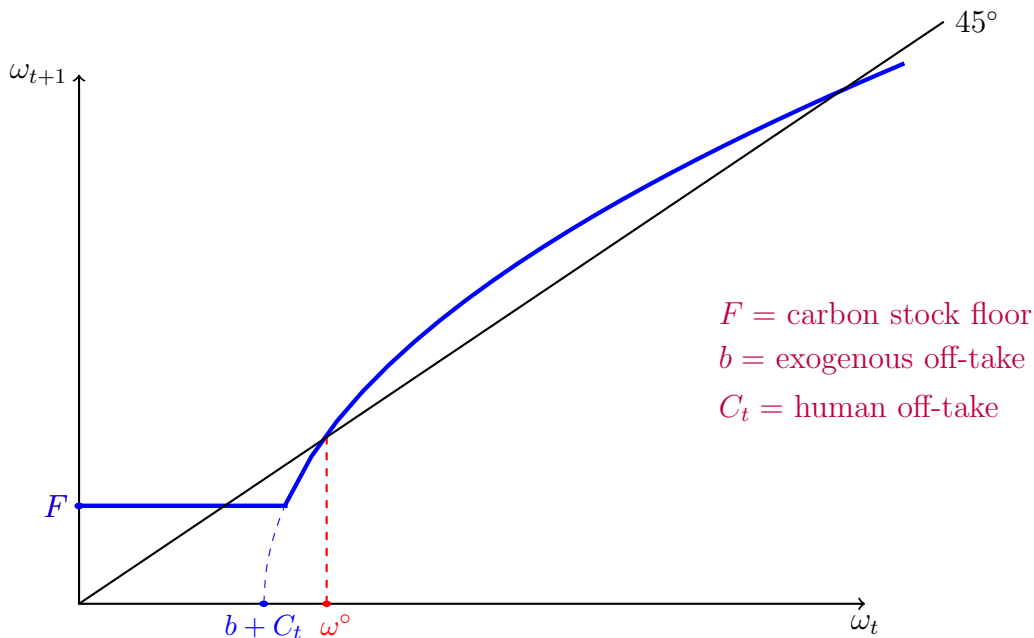


Figure 1: Carbon dynamics with Fixed, Exogenous Consumption

2.4 Discussion of the Carbon Stock Dynamics

The dynamic in (4) is not intended to be a literal description of an earth system. Rather, we view it as a tractable heuristic that incorporates a local instability at the low-end of the carbon stock. Specifically, the carbon dynamic allows for growth, depreciation, and/or sudden decline in the stock, depending on parameters.

Overall, the model presents a simplification of the geophysical dynamics of carbon. It nevertheless captures what Cai, et al. (2012, p.2) argue are two critical features that should be included in a reasonable representation of tipping. Namely, “(i) a fully stochastic formulation of abrupt changes, and (ii) a representation of the irreversibility” of the decline. Regarding (ii), the law of motion in Equation (4) converges to a low but finite steady state F whenever the carbon stock falls below a critical point. The fact that the low steady state is independent of human activity is roughly consistent with simulations by Hansen et. al (2013), demonstrating a “soft” or “low-end” greenhouse damage.¹²

¹²Their simulations “indicate that no plausible human-made GHG forcing can cause an instability and runaway greenhouse effect” in which extreme, amplified feedbacks fully dissipate the stored carbon stock and evaporate all planetary surface water — as believed to have happened on Venus.

3 Business-As-Usual Equilibria

In any period, the state of the global carbon economy is summarized by the pair (ω_t, θ_t) consisting of the ecosystem and the elasticity profile. A *Markov-contingent plan* is a state-contingent profile

$$c^*(\omega_t, \theta_t) = (c_1^*(\omega_t, \theta_t), \dots, c_n^*(\omega_t, \theta_t))$$

that specifies each country's usage $c_i^*(\omega_t, \theta_t)$ as a function of the state (ω_t, θ_t) . The corresponding aggregate consumption is $C^*(\omega_t, \theta_t) = \sum_i c_i^*(\omega_t, \theta_t)$.

The long run payoff of a Markov-contingent plan \mathbf{c}^* to the representative citizen of country i is expressed as

$$U_i(\omega_t, \mathbf{c}^*, \theta_{it}) \equiv \mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \delta^{\tau-t} u \left((c_{i\tau}^*(\omega_\tau, \theta_\tau))^{\theta_{i\tau}} (\omega_\tau - C_\tau^*(\omega_\tau, \theta_\tau))^{1-\theta_{i\tau}} \right) \middle| \omega_t, \theta_t \right] \quad (5)$$

where the expectation \mathbb{E}_t is taken with respect to the process π , conditional on θ_t and the transition equation (4).

A *Markov Perfect equilibrium (MPE)* is a Subgame Perfect equilibrium in which each country's strategy is a Markov-contingent plan.¹³ The MPE is often interpreted as a “business-as-usual” benchmark since it represents a scenario that prevails in the *absence* of any agreement or coordination among the participants. The MPE requires no special coordination, no monitoring beyond the initial quota, and no explicit sanctions.¹⁴ The definition is fairly standard in the dynamic common pool literature (e.g., Dutta and Radner (2009)).

We further restrict attention to *smooth* MPE, that is, Markov-contingent plans that are both Subgame Perfect and smooth functions of the state (smooth everywhere except possibly at the floor F). This restriction rules out certain MPE that use discontinuities in the state to create triggers on which participants can tacitly coordinate. Consequently, we refer to any such MPE as a *Business-as-usual (BAU) equilibrium*.

¹³In any MPE each country's Markov-contingent plan c_i^* maximizes $U_i(\mathbf{c}^*, \omega_t, \theta_{it})$ given c_{-i}^* in any state (ω_t, θ_{it}) over the set of full history-contingent consumption plans. For brevity, we omit the specification of full history contingent strategies. Payoffs corresponding to infeasible paths must be formally defined as well. For our purposes, the simplest approach is to define the payoff on the extended real line, setting flow payoffs equal to $-\infty$ whenever $C_t \geq \omega_t + b$.

¹⁴Without the Markov restriction, a version of a Folk Theorem can be applied (see, for instance, Dutta (1995) for a general statement), and efficient plans can be implemented by international coordination on the appropriate punishments.

3.1 Euler equations

Using the parameterization $u(y_{it}) = \log(y_{it})$ in (5), the BAU equilibrium consumption $c_i^*(\omega_t, \theta_t)$ for country i is a solution to the Bellman equation

$$U_i(\omega_t, \mathbf{c}^*, \theta_{it}) = \max_{c_{it}} \left\{ \theta_{it} \log c_{it} + (1 - \theta_{it}) \log(\omega_t - C_t) + \delta \mathbb{E} \left[U_i(\omega_{t+1}, \mathbf{c}^*, \theta_{it+1}) \mid \omega_t, \theta_{it} \right] \right\} \quad (6)$$

subject to (4) after for every state (ω_t, θ_t) .

To calculate the BAU it is simpler to work with extraction *rates* rather than levels. Extraction rate e_{it} is defined implicitly by $c_{it} = e_{it}\omega_t$. The global extraction rate is denoted by $\mathcal{E}_t = \sum_i e_{it}$. With this transformation, the Bellman equation can be expressed in terms of a Markov-contingent extraction *rate*, \mathbf{e}^* . The payoff in (6) is rewritten as

$$\tilde{U}_i(\omega_t, \mathbf{e}^*, \theta_{it}) = \max_{e_{it}} \left\{ \theta_{it} \log \omega_t e_{it} + (1 - \theta_{it}) \log(\omega_t(1 - \mathcal{E}_t)) + \delta \mathbb{E} \left[\tilde{U}_i(\omega_{t+1}, \mathbf{e}^*, \theta_{it+1}) \mid \omega_t, \theta_{it} \right] \right\} \quad (7)$$

subject to (4). By construction, $\tilde{U}_i(\omega_t, \mathbf{e}^*, \theta_{it}) = U_i(\omega_t, \mathbf{c}^*, \theta_{it})$.

In the subsequent analysis, we also employ the following notation. Let $X(\omega_t) = \{\mathcal{E}_t : A(\omega_t(1 - \mathcal{E}_t) - b)^\gamma > F\}$ denoting the aggregate extraction rates that do not force the stock down to the floor given stock ω_t . Then let $1_{\{\mathcal{E}_t \in X(\omega_t)\}}$ be an indicator function taking value “1” whenever $\mathcal{E}_t \in X(\omega_t)$ and taking value zero otherwise.

Our first result, Proposition 1 stated below, shows that the BAU equilibrium solves a system of Euler equations in extraction rates. As usual, the Euler equations are derived from each country’s first order conditions for (7):

$$\frac{\theta_{it}}{e_{it}} - \frac{1 - \theta_{it}}{1 - \mathcal{E}_t} - A\delta\gamma\omega_t((1 - \mathcal{E}_t)\omega_t - b)^{\gamma-1} \mathbb{E}_t \left[\frac{\partial \tilde{U}_{t+1}}{\partial \omega_{t+1}} \right] 1_{\{\mathcal{E}_t \in X(\omega_t)\}} = 0$$

The first two terms correspond to the country i ’s marginal flow payoff in period t from its extraction of carbon in that period. The third term represents the country’s marginal value (gained or lost) in future periods from its period t extraction. After some calculations and the standard application of the Envelope Theorem to future decisions, the first order condition can be expressed conveniently as an Euler equation described in the following Proposition.

Proposition 1 *Let \mathbf{c}^* be a business-as-usual (BAU) equilibrium. Then $c_i^*(\omega_t, \theta_t) = e_i^*(\omega_t, \theta_t)\omega_t$ for each i , where $e^*(\theta_t, \omega_t)$ is a profile of extraction rates that implicitly solve a system of n equations. Each equation is given by*

$$\frac{\theta_{it}}{e_{it}}(1 - \mathcal{E}_t^{-i}) - 1 = \frac{\delta\gamma(1 - \mathcal{E}_t)\omega_t}{(\omega_t(1 - \mathcal{E}_t) - b)} \mathbb{E}_t \left[\frac{\theta_{it+1}}{e_{it+1}}(1 - \mathcal{E}_{t+1}^{-i})(1 + \sigma_{t+1}^{-i}) \right] 1_{\{\mathcal{E}_t \in X(\omega_t)\}} \quad (8)$$

where $\mathcal{E}_t^{-i} = \sum_{j \neq i} e_{jt}$ and

$$\sigma_{t+1}^{-i} \equiv \frac{\omega_{t+1}}{1 - \mathcal{E}_{t+1}} \frac{\partial(1 - \mathcal{E}_{t+1}^{-i})}{\partial\omega_{t+1}} = \frac{\% \text{ reduction other countries' extraction rates}}{\% \text{ increase in stock}}$$

is the overall elasticity of other countries' extraction rates due to an increase in carbon stock.

Generally, the equation system in (8) yields no closed form solution and is necessary but not sufficient to characterize BAU equilibria. The full derivation of (8) is contained in the Appendix. In an external appendix, an existence result is obtained in a robust class of parameters.¹⁵

The Euler equation in (8) displays the marginal rate of substitution between the current and future payoff from current extraction. Not surprisingly, each country i 's consumption/emissions is decreasing in the effective discount factor $\delta\gamma$, increasing in its own current output elasticity θ_{it} .

The Euler equation (8) takes a standard form in the concave model ($b = F = 0$). In the concave model $1_{\{\mathcal{E}_t \in X(\omega_t)\}} = 1$ and the Euler equation is then solved by guessing and verifying that $\sigma_{t+1}^{-i} = 0$. In this case, the marginal rate of substitution equals the effective discount factor $\delta\gamma$ (the “golden rule”).

To understand how the non-concave model departs from the golden rule, observe that if $1_{\{\mathcal{E}_t \in X(\omega_t)\}} = 1$, then $b > 0$ results in the overweighting of future payoffs, ceteris paribus. The overweighting will result in lower extraction rates in period t than in the concave model. The magnitude of this “excess caution” depends on σ_{t+1}^{-i} which measures the responsiveness of countries to a change in the stock brought about by i 's current rate of extraction. If the countries are not in imminent danger of hitting the floor F , then σ_{t+1}^{-i} will be negative: an increase in the current stock decreases the future incentives of countries to preserve the stock (or increases their incentives to extract more). A large $|\sigma_{t+1}^{-i}|$ dampens the incentives for conservation even to the point at which underweighting future payoffs can occur.

The extreme case of underweighting the future occurs when $1_{\{\mathcal{E}_t \in X(\omega_t)\}} = 0$, i.e., the countries will hit the carbon floor in period $t + 1$. Then, the right-hand side of (8) is zero and so future payoffs are underweighted in the extreme. The country then

¹⁵See faculty.georgetown.edu/lagunofr/BAU4-External-Appendix.pdf.

treats the extraction decision as a static problem, and so extraction in period t will be higher than in the concave model.

Since countries will revert to static optimization whenever the floor is reached, each country's relative weighting of present versus future depends on the timing and likelihood of $1_{\{\mathcal{E}_{t+s} \in X(\omega_{t+s})\}} = 0$ at each future date $t + s$, $s = 1, 2, \dots$. This intuition forms the basis for a later comparison between the concave and non-concave models.

4 The Tipping Problem

In the concave model, the solution to Equation (8) admits a simple closed form solution for the BAU equilibrium:

$$c_i^*(\omega_t, \theta_t) = \bar{e}_i(\theta) \omega_t \quad (9)$$

where

$$\bar{e}_i(\theta) = \frac{\frac{\theta_{it}(1-\gamma\delta)}{1-\theta_{it}(1-\gamma\delta)}}{1 + \sum_{j=1}^n \frac{\theta_{jt}(1-\gamma\delta)}{1-\theta_{jt}(1-\gamma\delta)}} \quad (10)$$

This derivation can be obtained directly from the Euler equation in the Proposition when $b = F = 0$.¹⁶

Observe that if $\theta_{it} = 1$ for all i and t then the equilibrium coincides with the Levhari-Mirman (LM) (1980) fish war model as a special case. In the LM model, $\theta_{it} = 1$ for all i and t , and the transition is concave.¹⁷ In other words, there is no tipping problem, no direct value from preserving the ecosystem, and no heterogeneity.

Even without the non-concavity, the BAU equilibrium calculated in (10) reveals both aggregate and distributional effects that are not present in the standard common pool problem. Extraction rates exhibit cross-sectional dispersion in which countries with either very high or very low resource elasticities have larger consumption than those with intermediate elasticities. This is due to the fact that a country's output $y_i^*(\omega_t, \theta)$ is U -shaped in θ_{it} , ceteris parabis. The U -shape also helps explain why

¹⁶In general the Euler equations cannot rule out multiple solutions even in this case. The set of solutions are reduced by our restriction to continuously differentiable linear MPE. The restriction rules out nonlinear and/or discontinuous solutions or implicit trigger in which countries implement punishments by using the stock itself as a trigger.

¹⁷Namely, for all i and t ,

$$c_{it}^*(\theta = \mathbf{1}) = \frac{\left(\frac{1-\gamma\delta}{1-(1-\gamma\delta)}\right)}{1 + \frac{(1-\gamma\delta)}{1-(1-\gamma\delta)}n} \omega_t = \frac{(1-\gamma\delta)}{n(1-\gamma\delta) + \gamma\delta} \omega_t.$$

reversing course is problematic: starting from a high elasticity θ_{it} , the output of a pro-extraction country initially falls as it makes the transition to a pro-conservation technology.

When $b \geq F > 0$ the non-concavity gives rise to potentially multiple steady states, including one at the carbon floor F . The Theorem below describes both how equilibrium incentives of each country are altered by the non-concavity. We later show how the non-concave model leads to a tipping problem.

Theorem 1 *Let \mathbf{e}^* be a BAU equilibrium in the non-concave model and $\bar{\mathbf{e}}$ the BAU equilibrium in the concave model given in Equation (10). Then for each country i , there exists ω^1 and ω^2 with $\omega^1 \leq \omega^2$ such that for all θ_t ,*

1. *if $\omega_t \geq \omega^2$ then $e_i^*(\omega_t, \theta_t) < \bar{e}_i(\theta_t)$ and $e_i^*(\omega_t, \theta_t)$ is non-decreasing in ω_t , and*
2. *if $\omega_t \leq \omega^1$ then $e_i^*(\omega_t, \theta_t) > \bar{e}_i(\theta_t)$.*

Part 1 asserts that, relative the concave (“no-tipping”) model, countries’ extraction rates in the non-concave model are lower when ω_t is large, i.e., *when the stock is far from the floor F* . Part 2 asserts that countries’ extraction rates are higher than in the concave model when ω_t is low, i.e., *when the stock is close to F* . The likelihood of reaching the floor in any given interval of time is endogenous.

The proof is in the Appendix. The logic, however, is intuitive and mirrors our earlier discussion of the Euler equation. Specifically, suppose that the initial stock ω_0 is sufficiently large so that $1_{\{\mathcal{E} \in X(\omega_t)\}} = 1$ with probability one for all $t = 0, \dots, T$ where T is large. For the dates in this interval of time, the carbon dynamic reduces to $\omega_{t+1} = A(\omega_t - C_t - b)^\gamma$. Moreover, because the initial stock is large, $\frac{\omega_t(1-\mathcal{E}_t)}{(\omega_t(1-\mathcal{E}_t)-b)}$ is approximately one in which case extraction elasticities in the time interval are approximately zero, i.e., $\sigma_{t+1}^{-i} \approx 0$. A small increase in the bound b therefore increases the marginal cost of extraction, and so $e_i^*(\omega_t, \theta_t) < \bar{e}_i(\theta_t)$. That is, countries exhibit greater caution in the non-concave model when ω_t is high.

Next, suppose that ω_t is small enough so that $1_{\{\mathcal{E} \in X(\omega_t)\}} = 0$. But this means $\frac{\partial \omega_{t+1}}{\partial e_{it}} = 0$. In other words, the global commons has reached the floor F in which case each country’s marginal extraction cost is zero. Since current extraction rates do not affect future stocks, each country solves the one period first order condition

$$\frac{\theta_{it}}{e_{it}} - \frac{1 - \theta_{it}}{1 - \mathcal{E}_t} = 0.$$

Country i 's BAU equilibrium extraction then is $e_i^{static}(\theta_t) = \frac{\theta_{it}}{1-\theta_{it}} \left(1 + \sum_j \frac{\theta_{jt}}{1-\theta_{jt}}\right)^{-1}$. In the static equilibrium, countries extract carbon as if the future is irrelevant.

It is not difficult to verify that $e_i^{static}(\theta_t) > \bar{e}_i(\theta_t)$ where the latter is the closed form expression (10) in the concave model.

Intuitively, if the constraint $1_{\{\omega_t, \mathcal{E}_t\}}^* = 0$ holds or will hold with high probability in the near future, countries have little to lose by extracting as much as possible for the present. In this case, other countries' future responses to i are negligible, and thus have a negligible effect on the current extraction of country i . Thus i extracts as if tipping is a *fait accompli*, something that occurs independently of its own decision. It follows that when ω_t is small enough that the low end steady state F will be shortly reached, $e_i^*(\omega_t, \theta_t) > \bar{e}_i(\theta_t)$.

To summarize, the off-take parameter b reduces the incentives to extract carbon when the stock ω is large, but increases the incentive to extract when ω is small.

Figure 2 demonstrates this non-monotonicity of a country's equilibrium extraction as the current stock varies. For low enough stock, the extraction resembles a static solution e^{static} where current extraction has no effect on future payoffs. For large enough stock, the extraction resembles the BAU equilibrium in the concave model ($b = 0$ and $F = 0$). Because the equilibrium extraction rates approach each constant rate from below, the lowest extraction rate in equilibrium occurs for some intermediate stocks, as shown in the Figure.¹⁸

Figure 2 indicates that proximity to F leads countries to *accelerate their rate of extraction*. The result is reminiscent of the ‘‘Green Paradox’’ (Sinn (2008)), whereby the extraction increases when more stringent emissions regulations are anticipated in the future. In that case, the tipping event — the policy change — is taken as exogenous. Sakamoto (2014) obtains a related result in a model where an exogenous threshold determines a stochastic shift from a good environmental state to a bad one.

4.1 Tipping Points and Safe Operating Bounds

The previous section compared BAU equilibria in the non-concave commons to the concave benchmark. Incentives hinged on whether and/or how soon it is that the commons reaches the floor F . This Section examines that likelihood in more detail.

Let $\omega_{t+1}^*(\omega_t, \theta_t)$ denote the realized BAU equilibrium law of motion when the stock

¹⁸The hole in the middle is due to the fact that we cannot determine the precise shape of e^* for intermediate values.

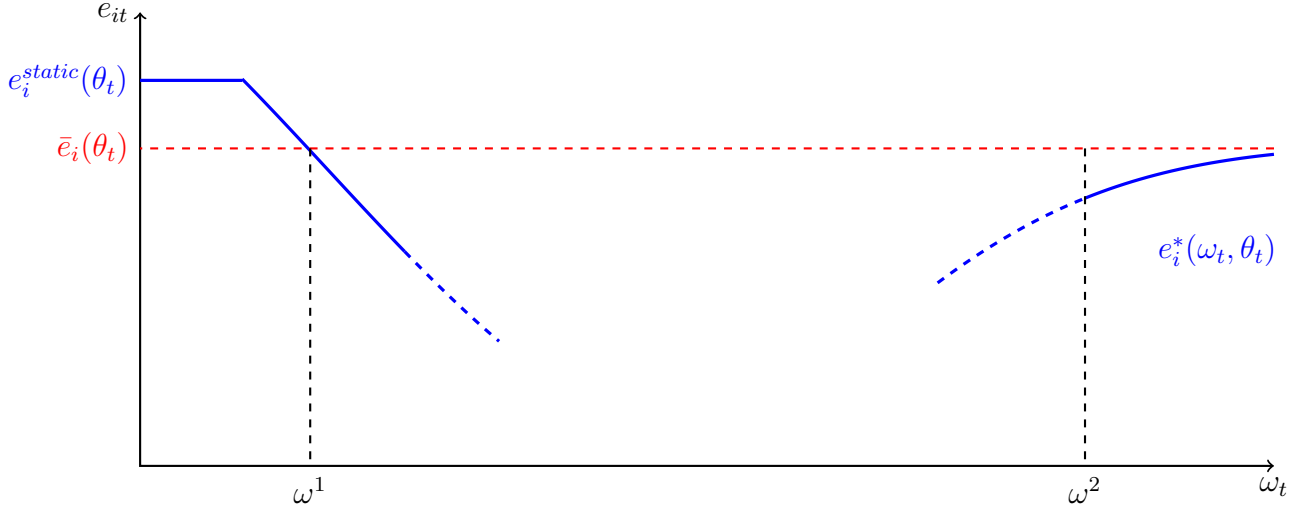


Figure 2: Non-monotonicity in extraction rates

transition rule in (4) is evaluated at $c^*(\omega_t, \theta_t)$. Transition law $\omega_{t+1}^*(\omega_t, \cdot)$ is a random variable endogenously determined by equilibrium behavior. Hence, from the point of view of the participants, tipping is a stochastically and endogenously determined phenomenon, formally defined as follows.

First, ω^* can be iterated forward, generating realizations $\omega^{*t+s}(\omega_t, \theta^{t+s-1})$, $s = 0, 1, 2, \dots$. These determine a realized equilibrium path starting from ω_t .¹⁹ Depending on trends in elasticities over time, both growth or contraction in output and carbon stock can occur in equilibrium.

Recall from Figure 1 that when $b > 0$ there is a possibility that the stock can depreciate down to the floor F . More precisely, we are interested in the probability

$$\mu(\omega_0) = P\left(\left\{\theta^\infty : \lim_{t \rightarrow \infty} \omega^{*t}(\omega_0, \theta^{t-1}) = F\right\}\right)$$

This defines the probability that the stock reaches the floor F . A tipping point is therefore the largest stock from which the floor is reached with probability one.

¹⁹This path of carbon stock is defined inductively by

$$\begin{aligned} \omega^{*t+1}(\omega_t, \theta^t) &= \omega_{t+1}^*(\omega_t, \theta_t), & \omega^{*t+2}(\omega_t, \theta^{t+1}) &= \omega_{t+2}^*(\omega^{*t+1}(\omega_t, \theta^t), \theta_{t+1}), \dots \\ \dots \omega^{*t+s}(\omega_t, \theta^{t+s-1}) &= \omega_{t+s}^*(\omega^{*t+s-1}(\omega_t, \theta^{t+s-2}), \theta_{t+s-1}), \dots \end{aligned}$$

Specifically, a *tipping point* is a carbon stock ω^{tip} satisfying

$$\omega^{tip} = \sup\{\omega_0 : \mu(\omega_0) = 1\}. \quad (11)$$

If the global commons reaches F from every initial stock, then the tipping point is infinite. Different types of equilibria give rise to different tipping points, a fact we elaborate on later when making the comparison to “optimal” tipping points.

The tipping point can be distinguished from a carbon threshold above which exists a *safe operating space for humanity*, in the sense of Rockstrom et. al. (2009). In the present model, the global commons under BAU is in a safe operating space at ω_0 if the commons reaches a high-end steady state or stationary distribution above the floor F . This leads naturally to a notion of a *safe operating bound*, defined as a carbon stock ω^{safe} satisfying

$$\omega^{safe} = \inf\{\omega_0 : \mu(\omega_0) = 0\}. \quad (12)$$

By construction, $\omega^{safe} \geq \omega^{tip}$. Consider, as an example a stationary Markov process on the two profiles $\{\theta', \theta''\}$ with $\theta' < \theta''$. Suppose that either profile can be reached from the other each period with positive probability bounded away from 0. There are two possibilities. Either the carbon dynamic has a finite tipping point or it does not. The case of a finite tipping point is displayed in Figure 3. Since the equilibrium carbon dynamic for both stocks has fixed points, the tipping point ω^{tip} corresponds to the lowest unstable fixed point. From any stock strictly larger than ω^{tip} , the process can avoid the low end steady state with positive probability. In particular, if it reaches stock ω^{safe} , then the commons is guaranteed to avoid the low steady state, thus defining for our purposes the *safe operating space (SOS)* described in the planetary boundaries literature of Rockstrom et. al (2009), Anderies et. al. (2013), and others.

The case where the tipping “point” is infinite is displayed in Figure 4. In this case the parameters generate a transition toward the floor F . Specifically, from any stock ω and any initial profile, for any time length T , there is a date t at which the process will remain “stuck” at θ'' for T periods starting from t . Since this occurs at infinitely many t , then for T sufficiently long the commons will eventually reach F from ω with probability one. Consequently, $\omega^{tip} = \infty$.

Finally, when $b = 0$ then $\mu(\omega_0) = 0$ for any $\omega_0 > 0$. In other words, $\omega^{tip} = 0$ and so tipping never occurs. In this case, the BAU equilibrium law of motion $\omega_{t+1}^*(\omega_t, \theta_t)$, derived from (9), converges to a stationary distribution on the carbon stock (a stable carbon cycle) if the underlying process on θ^∞ is ergodic. This is illustrated in a particularly simple case. Consider a two-state stationary, irreducible Markov process on two stocks, $\underline{\theta}$ and $\bar{\theta}$ with p denoting the switching probability between the two.

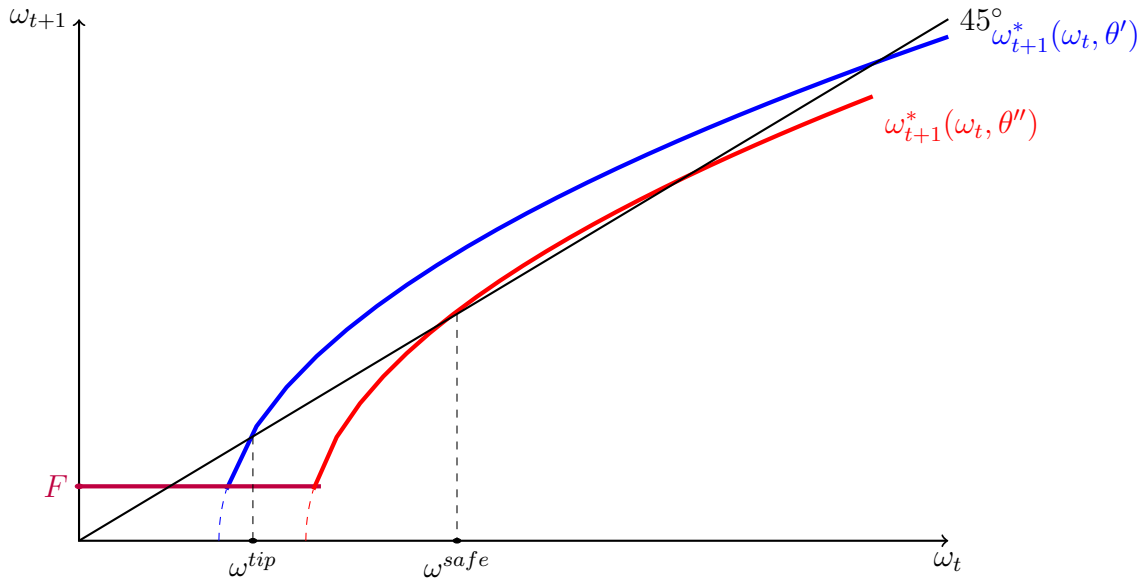


Figure 3: Carbon dynamics with finite tipping point ω^{tip}

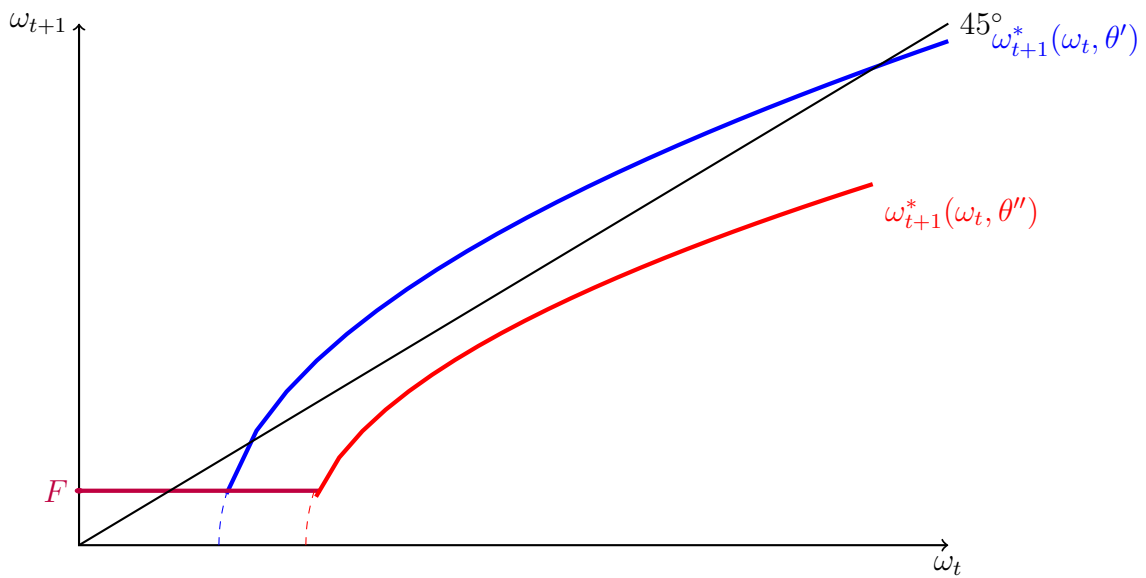


Figure 4: Carbon dynamic with infinite tipping point, inevitably reaching F .

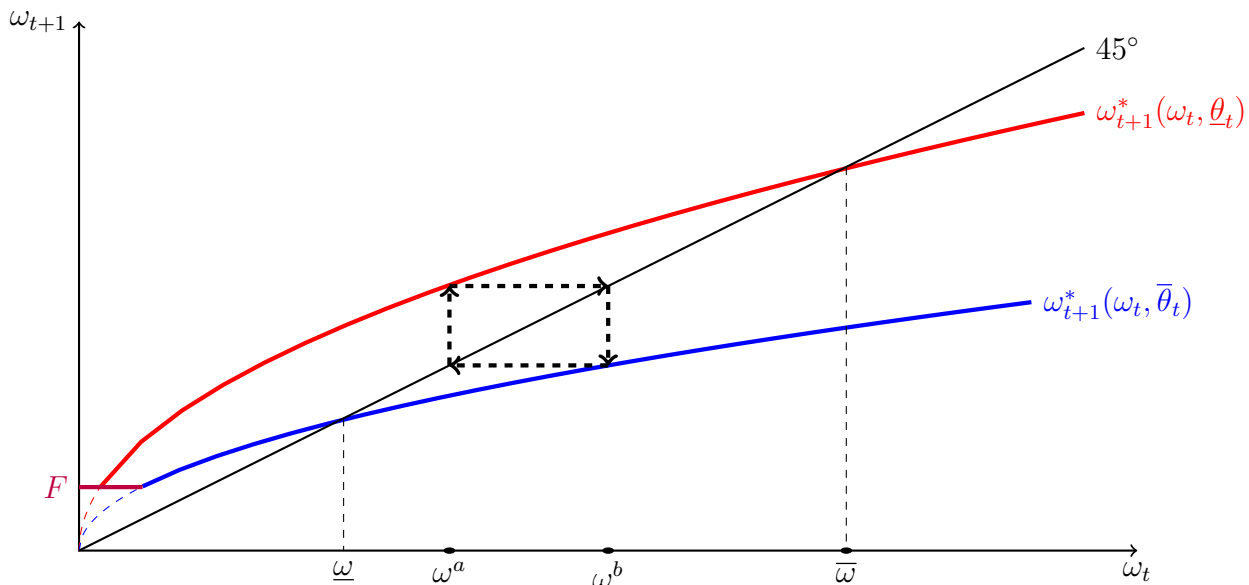


Figure 5: When $b = 0$, a stable carbon cycle alternates between ω^a and ω^b .

Figure 5 displays a literal cycle when $p = 1$, that is, when the process alternates deterministically between $\underline{\theta}$ and $\bar{\theta}$. The equilibrium dynamic then cycles between carbon stocks, ω^a and ω^b .

Theorem 2 Consider any Business-as-Usual equilibrium with n countries in the non-concave ($b \geq F > 0$) model. Then there is an integer $n' > 0$ such that if $n > n'$, $\omega^{tip} = \omega^{safe} = \infty$, i.e., the global commons reaches the carbon floor F .

The precise n' at which tipping becomes inevitable depends on parameters, and the paper has no quantitative prediction about it. Still, the large n case is not a remote possibility; the ongoing devolution (i.e., the partitioning large nations into ever smaller ones) has substantially increased the number of nations since World War II, each nation making separate carbon decisions.

In contrast with the Theorem, when $b = 0$, then $\omega^{tip} = \omega^{safe} = 0$ for any n as one would expect in the concave model.

The discontinuity between the $b = 0$ and $b > 0$ cases appears stark though understandable. Even in the concave model, full extraction occurs in the limit as $n \rightarrow \infty$ when countries are pro-extractive ($\theta = \mathbf{1}$). The difference is that when $b = F = 0$, the extraction rate hits one and stocks are fully depleted *only in the limit* $n = \infty$. In other words, in the concave model with n countries, the only steady state above the

floor is stable and so for any finite n , extraction is less than one. Consequently, from any initial stock the process will converge to the stable steady state above the floor. By contrast, when $b \geq F > 0$ and n is large enough, the Theorem asserts that the BAU equilibrium necessarily hits the floor, even as the extraction rate is less than one.

The proof in the Appendix is long, but the intuition is straightforward. We first show that full extraction occurs as $n \rightarrow \infty$ when $\theta = \mathbf{1}$, just as in the concave model. In this case, however, the non-concave dynamics drives the stock down to the floor when extraction is close, but not equal, to one. The result then makes use of the ergodicity-like assumption on π . Under this assumption, the process eventually moves the output elasticities into a neighborhood of $\theta_t = \mathbf{1}$ for a long enough period of time for the stock to decline toward the floor.

It is important to note that the dynamics are not strictly monotone since θ is bouncing up and down according to π . In periods where θ is low (pro-conservation) the stock may recover if it hasn't fallen too far. Eventually, however, it will reach the floor as pro-extractive types will emerge and remain in place for while.

Hence, while the proximate cause of tipping is the depletion of the carbon stocks, the “deeper” parameters that drive the tipping and decline are technological: the evolution of factor elasticities that determine the mix of extracted and stored carbon.

5 The Planner's Solution

The BAU equilibrium can be compared to the socially efficient carbon profile. We define the latter as the solution to a Utilitarian Social Planner's problem. From this Planner's perspective, a Markov-contingent plan, denoted by $c^\circ(\omega_t, \theta_t) = (c_1^\circ(\omega_t, \theta_t), \dots, c_n^\circ(\omega_t, \theta_t))$, is *optimal* if it solves

$$\max_{\mathbf{c}^\circ} E \left[\sum_{i=1}^n \sum_{t=0}^{\infty} \delta^t u(y_{it}) \mid \omega_0, \theta_0 \right] \quad \text{subject to (2) and (4)}. \quad (13)$$

As with the BAU equilibrium, combining log utility with the production technology (2) in the Planner's objective, an optimal plan \mathbf{c}° solves the Bellman's equation

$$V(\omega_t, \mathbf{c}^\circ, \theta_t) = \max_{\mathbf{c}_t^\circ} \left\{ \sum_{i=1}^n \theta_{it} \log c_{it} + (1 - \theta_{it}) \log(\omega_t - C_t) + \delta E \left[V(\omega_{t+1}, \mathbf{c}^\circ, \theta_{t+1}) \mid \omega_t, \theta_t \right] \right\} \quad (14)$$

The Planner’s problem treats nations equally. We use it as a benchmark against which BAU equilibrium may be compared. Clearly, there are other solutions if the Planner were to use different welfare weights. The equally weighted utilitarian solution can be viewed as a result of symmetric Nash Bargaining in an international agreement. The Planner’s solution can then be implemented by a Subgame Perfect equilibrium in which agreed-upon triggers are used to punish deviations.²⁰

The Planner’s Euler equation in extraction rate e_{it} satisfies

$$\frac{\theta_{it}}{e_{it}}(1 - \mathcal{E}_t^{-i}) + \sum_{j \neq i} \theta_{jt-n} = \left[\frac{\delta \gamma \omega_t (1 - \mathcal{E}_t)}{\omega_t (1 - \mathcal{E}_t) - b} \right] \mathbb{E} \left[\frac{\theta_{it+1}}{e_{it+1}} (1 - \mathcal{E}_{t+1}^{-i}) + \sum_{j \neq i} \theta_{jt+1} \right] 1_{\{\mathcal{E}_t \in X(\omega_t)\}} \quad (15)$$

The derivation of Equation (15) is in the Appendix (Section 7.4). The equation relates the Planner’s marginal rate of substitution between present and future payoffs. The Planner naturally internalizes the effect of country i ’s extraction on the global economy.

As with the BAU equilibrium, the Planner’s optimal extraction plan in the concave model has a closed form solution:

$$c_i^\circ(\omega_t, \theta_{it}) = \frac{\phi_{it}}{n} \omega_t \quad \forall i \quad (16)$$

where $\phi_{it} \equiv \theta_{it}(1 - \gamma\delta)$.²¹ Not surprisingly, each country’s carbon emission is increasing in its elasticity θ_{it} and decreasing in the effective discount factor $\delta\gamma$.

5.1 Some Comparisons

Relative to the Planner’s solution, the BAU equilibrium is characterized by aggregate over-extraction:

Proposition 2 *Let \mathbf{c}^* be a BAU equilibrium and \mathbf{c}° an optimal plan in either the concave or non-concave model. Then for any state (ω_t, θ_t) , $C^*(\omega_t, \theta_t) > C^\circ(\omega_t, \theta_t)$.*

²⁰The construction of triggers is non-trivial in this heterogeneous environment. Barrett (2013) explores the problems with international coordination when the location of an exogenous tipping threshold is uncertain. In a prior paper, Harrison and Lagunoff (2016), we show that the planner’s solution cannot necessarily be implemented by simple reversion to Markov Perfect (BAU) equilibrium in the event of a deviation.

²¹See the Appendix.

The over-extraction result echoes a classic “tragedy of the commons” theme running through common pool resource games.²² This literature also examines strategic incentives in dynamic games with a commons or with climate externalities. The over-extraction result, while not surprising, is more subtle when tipping is possible. Theorem 1 showed that countries exercise increased caution at high values of ω_t . The effects of this excess caution is not internalized by the individual countries and so it is conceivable that over-extraction would not occur at high stock values.

In fact, over-extraction does not uniformly hold for all countries even in the absence of tipping. This is shown in a result below in the concave model.

Proposition 3 *Let \mathbf{c}^* and \mathbf{c}° represent a BAU equilibrium and the socially optimal plan, resp., in the concave model. Then for any state (ω_t, θ_t) ,*

1. *For each country i , and each profile θ_{-i} of others’ elasticities, there exists a cutoff carbon elasticity $\tilde{\theta}_i \in [\underline{\theta}, \bar{\theta}]$ such that for any stock ω_t , and in any date t ,*

$$c_i^*(\omega_t, \theta_{it}, \theta_{-i}) \geq (>) c_i^\circ(\omega_t, \theta_{it}) \text{ if } \theta_{it} \geq (>) \tilde{\theta}_i, \text{ and}$$

$$c_i^*(\omega_t, \theta_{it}, \theta_{-i}) \leq (<) c_i^\circ(\omega_t, \theta_{it}) \text{ if } \theta_{it} \leq (<) \tilde{\theta}_i, \text{ and}$$

2. *along any path of realized carbon elasticity profiles θ^t , the relative differences between efficient and equilibrium output $\frac{y_i^{\circ t}(\omega_0, \theta^t)}{y_i^{*t}(\omega_0, \theta^t)}$, carbon consumption $\frac{c_i^{\circ t}(\omega_0, \theta^t)}{c_i^{*t}(\omega_0, \theta^t)}$, and carbon stock $\frac{\omega^{\circ t}(\omega_0, \theta^t)}{\omega^{*t}(\omega_0, \theta^t)}$ all increase in t .*

The proof is in the Appendix. Significantly, the Proposition demonstrates that while all BAU equilibria are characterized by aggregate over-extraction, individual countries may over- or under-extract depending on their resource elasticities. Pro-extraction countries over-extract in the BAU while pro-conservation countries may actually extract less than in the efficient plan. Under-extraction occurs as a compensating response to massive over-extraction by the pro-extractors. Pro-conservators never fully compensate, and so over-extraction always occurs in the aggregate.

While the Proposition applies to the concave model, the strict inequalities suggest that it should hold for small but positive b values as well.²³

²²See Levhari and Mirman (LM) (1980), Cave (1987), Mirman and Fisher (1992), Dutta and Sundaram (1993), Sorger (1996), Mirman and Fisher (1996), Finus (2001), Barrett (2003), Dutta and Radner (2004, 2006, 2009), and Battaglini and Harstad (2012), and many others.

²³The possibility of under-extraction in a Markov equilibrium is unusual but not unheard of. Dutta and Sundaram (1993) show this possibility in a LM resource model where the state variable can trigger a punishment. In our model, smoothness of the Markov strategy rules out Markov “trigger” strategies. Instead, heterogeneity is the key.

5.2 Optimal Tipping

The tipping and safe operating bounds are also defined in the Planner's problem. Let ω^{otip} and ω^{osafe} denote the tipping point and safe operating bound for the Planner's problem, applying the definitions in (11) and (12) to the Planner's solution.

The tipping properties of the BAU and the Planner's solution can be compared as follows.

Theorem 3 *There is a carbon floor \bar{F} such that for any $F \leq \bar{F}$, the planner's optimal tipping point ω^{otip} and its safe operating bound ω^{osafe} are finite and independent of the number of countries.*

The result provides a sharp contrast between the BAU and the Planner's solution. Recall that in the BAU model, the global commons tips with certainty, regardless of the value of F and regardless of the initial stock if there are many countries. In the planner's problem, for a low enough floor the commons tips only below some finite threshold. Above some finite threshold, it does not tip at all regardless of the number of countries. These differences are displayed in Figure 6.

The intuition for the Theorem is not complicated. The flow payoff to any country goes to $-\infty$ as F goes to zero. The Planner obviously wishes to avoid this and has the capacity to do so. Because there are natural steady states (steady states without human intervention) above F , there is an unstable one. This means that at stocks slightly above the unstable state, the Planner can drive up the stock to a stable steady state by temporarily reducing aggregate extraction. Even at this reduced level of extraction, the Planner's payoff is higher than the low payoff from reaching the carbon floor F .

Why is this option not available to countries in the BAU equilibrium? With large n , each country's extraction decision has a small effect on the stock dynamics. Consequently, unlike the Planner, there is little an individual country can do except to make the best of a bad situation. Since it views the downward spiral toward F as inevitable, the country will attempt to grab as much of the carbon stock as it can when its θ_{it} is high. This behavior, in turn, fulfills the expectation that hitting the floor F is inevitable.

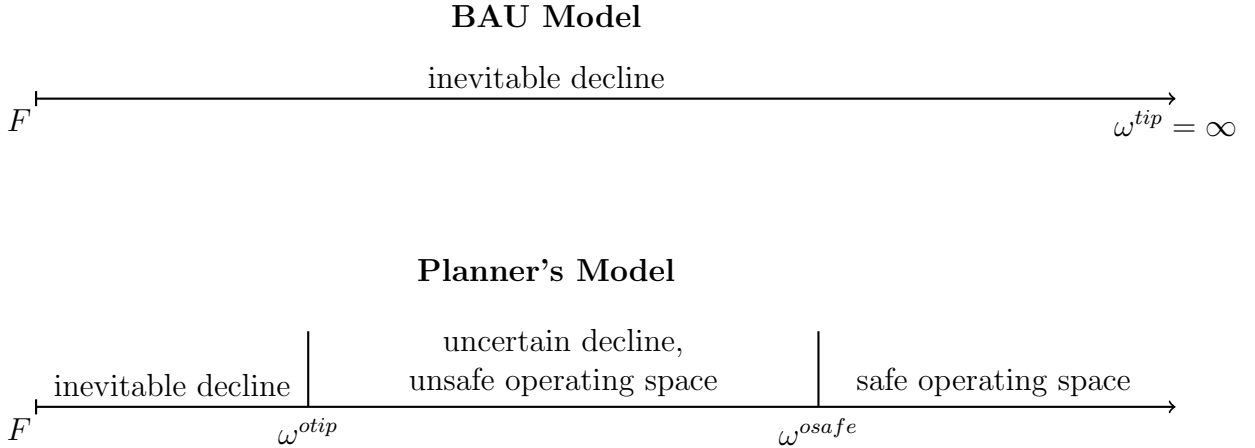


Figure 6: Comparison of Tipping Points in the Different Models

6 International Agreements and Other Considerations

The present paper introduces a non-concave transition where tipping toward a low-end floor is possible. BAU equilibria in the non-concave model differ sharply from those in the concave model. They also differ sharply from the Planner’s solution. The shocks play a critical role since the tipping problem intensifies as technologies become more pro-extractive. The heterogeneity introduces a further consideration, as the most pro-extractive countries will maintain the largest output as the stock declines toward the floor.

The comparison between the BAU and the Planner’s problem highlights the importance of an international agreement to control carbon emissions. An agreement that implements the Planner’s solution could avert tipping, provided that the stock is above the planner’s tipping point.

Such an agreement would require self-enforcing incentives to prevent cheating. In an earlier paper (Harrison and Lagunoff (2016)), we lay out the version of the concave model. We show that the planner’s solution can be implemented by a series of triggers that implement increasing levels of extraction to punish earlier deviations. The implementation, however, requires full information. When countries have private information about their technologies or environmental hazards, then the constrained optimal agreement pools across pro-extraction and pro-conservation types, revealing the limits to global cooperation.

An alternative, one that the present model does not explore, is that countries agree to invest in pro-conservation or mitigation technologies. Even in the BAU, tipping only becomes inevitable if countries become pro-extractive for a long enough stretch of time. If the technologies can cap θ , then safe operating spaces can exist even in under BAU extraction.

Finally, we observe that while model is parametric, the qualitative aspects are intuitive and we think it's unlikely that results will differ in similar models with non-concave transitions. Clearly, there is much work to do. This includes empirical explorations that integrate quantitative features of carbon cycle dynamics into these types of models.

7 Appendix

7.1 Proof of Proposition 1

The agent seeks to maximize the sum of discounted utilities on each period, by choosing the level of consumption (c_t). The maximization problem in Bellman form can be written as:

$$\tilde{U}(\omega_t; \mathbf{e}^*, \theta_{it}) = \max_{e_{it}} u_{it} + \delta \mathbb{E}_t \left[\tilde{U}(\omega_{t+1}; \mathbf{e}^*, \theta_{it+1}) \right] \quad (17)$$

$$\text{s.t. } \omega_{t+1} = \begin{cases} A((1 - \mathcal{E}_t)\omega_t - b)^\gamma & \text{if } A((1 - \mathcal{E}_t)\omega_t - b)^\gamma > F \\ F & \text{otherwise} \end{cases} \quad (18)$$

The chosen rates must be such as to maximize the value function. Taking derivatives respect to the control variables e_{it} , the first order conditions are:

$$\forall i \quad [e_{it}] : \quad \frac{\theta_{it}}{e_{it}\omega_t} \omega_t + \frac{1 - \theta_{it}}{(1 - \mathcal{E}_t)\omega_t} (-\omega_t) + \delta \mathbb{E}_t \left[\frac{\partial \tilde{U}}{\partial \omega_{t+1}} \frac{\partial \omega_{t+1}}{\partial e_{it}} \right] = 0 \quad (19)$$

The derivative of the stock movement equation respect to the consumption is the following:

$$\begin{aligned} \frac{\partial \omega_{t+1}}{\partial e_{it}} &= \begin{cases} A\gamma((1 - \mathcal{E}_t)\omega_t - b)^{\gamma-1} (-\omega_t) & \text{if } A((1 - \mathcal{E}_t)\omega_t - b)^\gamma > F \\ 0 & \text{otherwise} \end{cases} \\ &= -A\gamma\omega_t((1 - \mathcal{E}_t)\omega_t - b)^{\gamma-1} \cdot 1_{\mathcal{E}_t \in X(\omega_t)} \end{aligned}$$

where

$$X(\omega_t) \equiv \{\mathcal{E}_t : A((1 - \mathcal{E}_t)\omega_t - b)^\gamma > F\} \quad (20)$$

is the event that the commons has not reached the floor F .

We then rewrite 38 as:

$$[e_{it}] : \quad \frac{\theta_{it}}{e_{it}} - \frac{1 - \theta_{it}}{1 - \mathcal{E}_t} - A\delta\gamma\omega_t((1 - \mathcal{E}_t)\omega_t - b)^{\gamma-1} \mathbb{E}_t \left[\frac{\partial \tilde{U}_{t+1}}{\partial \omega_{t+1}} \right] 1_{\{\mathcal{E}_t \in X(\omega_t)\}} = 0 \quad (21)$$

By the Envelope Theorem, deriving the value function with respect to ω_t would yield:

$$\frac{\partial \tilde{U}(\omega_t, \mathbf{e})}{\partial \omega_t} = \frac{\partial \tilde{U}(\omega_t, \mathbf{e})}{\partial \omega_t} + \sum_{j \neq i} \frac{\partial \tilde{U}(\omega_t, \mathbf{e})}{\partial e_{jt}} \frac{\partial e_{jt}}{\partial \omega_t} \quad (22)$$

where:

$$\begin{aligned}
\frac{\partial \tilde{U}(\omega_t, \mathbf{e})}{\partial \omega_t} &= \frac{\theta_{it}}{e_{it}\omega_t} e_{it} + \frac{1 - \theta_{it}}{(1 - \mathcal{E}_t)\omega_t} (1 - \mathcal{E}_t) + \frac{A\gamma\delta(1 - \mathcal{E}_t)}{((1 - \mathcal{E}_t)\omega_t - b)^{1-\gamma}} \mathbb{E}_t \left[\frac{\partial \tilde{U}_{t+1}}{\partial \omega_{t+1}} \right] \\
&= \frac{\theta_{it}}{\omega_t} + \frac{1 - \theta_{it}}{\omega_t} + \frac{A\gamma\delta(1 - \mathcal{E}_t)}{((1 - \mathcal{E}_t)\omega_t - b)^{1-\gamma}} \mathbb{E}_t \left[\frac{\partial \tilde{U}_{t+1}}{\partial \omega_{t+1}} \right] \\
&= \frac{1}{\omega_t} + \frac{A\gamma\delta(1 - \mathcal{E}_t)}{((1 - \mathcal{E}_t)\omega_t - b)^{1-\gamma}} \mathbb{E}_t \left[\frac{\partial \tilde{U}_{t+1}}{\partial \omega_{t+1}} \right]
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial \tilde{U}(\omega_t, \mathbf{e})}{\partial e_{jt}} \frac{\partial e_{jt}}{\partial \omega_t} &= \left(-\omega_t \frac{1 - \theta_{it}}{\omega_t(1 - \mathcal{E}_t)} + \frac{A\gamma\delta\omega_t}{((1 - \mathcal{E}_t)\omega_t - b)^{1-\gamma}} \mathbb{E}_t \left[\frac{\partial \tilde{U}_{t+1}}{\partial \omega_{t+1}} \right] \right) \frac{\partial e_{jt}}{\partial \omega_t} \\
&= \left(-\frac{1 - \theta_{it}}{1 - \mathcal{E}_t} + \frac{A\gamma\delta\omega_t}{((1 - \mathcal{E}_t)\omega_t - b)^{1-\gamma}} \mathbb{E}_t \left[\frac{\partial \tilde{U}_{t+1}}{\partial \omega_{t+1}} \right] \right) \frac{\partial e_{jt}}{\partial \omega_t} \\
\sum_{j \neq i} \frac{\partial \tilde{U}(\omega_t, \mathbf{e})}{\partial e_{jt}} \frac{\partial e_{jt}}{\partial \omega_t} &= \left(-\frac{1 - \theta_{it}}{1 - \mathcal{E}_t} + \frac{A\gamma\delta\omega_t}{((1 - \mathcal{E}_t)\omega_t - b)^{1-\gamma}} \mathbb{E}_t \left[\frac{\partial \tilde{U}_{t+1}}{\partial \omega_{t+1}} \right] \right) \sum_{j \neq i} \frac{\partial e_{jt}}{\partial \omega_t}
\end{aligned}$$

In the optimum, $\frac{\partial \tilde{U}(\omega_t, \mathbf{e}_{it}^*)}{\partial e_{it}} = 0$ for being the function that maximizes u_{it} at time t .

Therefore, condition 44 can be written as:

$$\frac{\partial \tilde{U}}{\partial \omega_t} = \frac{1}{\omega_t} - \frac{1 - \theta_{it}}{1 - \mathcal{E}_t} \sum_{j \neq i} \frac{\partial e_{jt}}{\partial \omega_t} \tag{23}$$

$$+ A\delta\gamma((1 - \mathcal{E}_t)\omega_t - b)^{\gamma-1} \left(1 - \mathcal{E}_t - \omega_t \sum_{j \neq i} \frac{\partial e_{jt}}{\partial \omega_t} \right) \mathbb{E}_t \left[\frac{\partial \tilde{U}_{t+1}}{\partial \omega_{t+1}} \right] 1_{\{\mathcal{E}_t \in X(\omega_t)\}} \tag{24}$$

Solving for $\mathbb{E}_t \left[\frac{\partial \tilde{U}_{t+1}}{\partial \omega_{t+1}} \right] 1_{\{\mathcal{E}_t \in X(\omega_t)\}}$ from the first order condition yields:

$$\mathbb{E}_t \left[\frac{\partial \tilde{U}_{t+1}}{\partial \omega_{t+1}} \right] 1_{\{\mathcal{E}_t \in X(\omega_t)\}} = (A\delta\gamma\omega_t ((1 - \mathcal{E}_t)\omega_t - b)^{\gamma-1})^{-1} \left(\frac{\theta_{it}}{e_t} - \frac{1 - \theta_{it}}{1 - \mathcal{E}_t} \right) \tag{25}$$

Substituting (46) into (45):

$$\begin{aligned}
\frac{\partial \tilde{U}_t}{\partial \omega_t} &= \frac{\partial u(\omega_t, e_{it}^*)}{\partial \omega_t} + \delta \mathbb{E}_t \left[\frac{\partial \tilde{U}_{t+1}}{\partial \omega_{t+1}} \frac{\partial \omega_{t+1}}{\partial \omega_t} \right] \\
&= \frac{1}{\omega_t} - \frac{1 - \theta_{it}}{1 - \mathcal{E}_t} \sum_{j \neq i} \frac{\partial e_{jt}}{\partial \omega_t} + \left(1 - \mathcal{E}_t - \omega_t \sum_j \frac{\partial e_{jt}}{\partial \omega_t} \right) (\omega_t)^{-1} \left(\frac{\theta_{it}}{e_{it}} - \frac{1 - \theta_{it}}{1 - \mathcal{E}_t} \right) \\
&= \frac{1}{\omega_t} \left(1 + (1 - \mathcal{E}_t) \left(\frac{\theta_{it}}{e_{it}} - \frac{1 - \theta_{it}}{1 - \mathcal{E}_t} \right) \right) + \sum_{j \neq i} \frac{\partial e_{jt}}{\partial \omega_t} \left(\frac{1 - \theta_{it}}{1 - \mathcal{E}_t} - \left(\frac{\theta_{it}}{e_{it}} - \frac{1 - \theta_{it}}{1 - \mathcal{E}_t} \right) \right) \\
&= \frac{\theta_{it}}{\omega_t} \left(\frac{1 - \sum_{j \neq i} e_{jt}}{e_{it}} \right) - \frac{\theta_{it}}{e_{it}} \sum_{j \neq i} \frac{\partial e_{jt}}{\partial \omega_t} \\
&= \frac{\theta_{it}}{e_{it}} \left(\frac{1 - \mathcal{E}_t^{-i}}{\omega_t} + \sum_{j \neq i} \frac{\partial(1 - e_{jt})}{\partial \omega_t} \right) \\
&= \frac{\theta_{it}}{e_{it}} \frac{(1 - \mathcal{E}_t^{-i})}{\omega_t} \left(1 + \frac{\omega_t}{1 - \mathcal{E}_t^{-i}} \frac{\partial(1 - \mathcal{E}_t^{-i})}{\partial \omega_t} \right) \\
&= \frac{\theta_{it}}{\omega_t e_{it}} (1 - \mathcal{E}_t^{-i}) (1 + \sigma_t^{-i})
\end{aligned}$$

where $\sigma_t^{-i} = \frac{\omega_t}{1 - \mathcal{E}_t^{-i}} \frac{\partial(1 - \mathcal{E}_t^{-i})}{\partial \omega_t}$ is the overall elasticity of consumption of the rest of the countries with respect to the carbon stock at time t . Now, forwarding the last equation one period, we obtain:

$$\frac{\partial \tilde{U}_{t+1}}{\partial \omega_{t+1}} = \frac{\theta_{it+1}}{\omega_{t+1} e_{it+1}} (1 - \mathcal{E}_{t+1}^{-i}) (1 + \sigma_{t+1}^{-i}) \quad (26)$$

And replacing (50) on the first order condition yields the Euler equation for the system:

$$\begin{aligned}
\frac{\theta_{it}}{e_{it}} - \frac{1 - \theta_{it}}{1 - \mathcal{E}_t} - A \delta \gamma \omega_t ((1 - \mathcal{E}_t) \omega_t - b)^{\gamma-1} \mathbb{E}_t \left[\frac{\theta_{it+1}}{\omega_{t+1} e_{it+1}} (1 - \mathcal{E}_{t+1}^{-i}) (1 + \sigma_{t+1}^{-i}) \right] 1_{\{\mathcal{E}_t \in X(\omega_t)\}} &= 0 \\
\frac{\theta_{it}}{e_{it}} - \frac{1 - \theta_{it}}{1 - \mathcal{E}_t} - \delta \gamma \omega_t ((1 - \mathcal{E}_t) \omega_t - b)^{-1} \mathbb{E}_t \left[\frac{\theta_{it+1}}{e_{it+1}} (1 - \mathcal{E}_{t+1}^{-i}) (1 + \sigma_{t+1}^{-i}) \right] 1_{\{\mathcal{E}_t \in X(\omega_t)\}} &= 0
\end{aligned}$$

Therefore, the Euler equation can be rewritten as:

$$\frac{\theta_{it}}{e_{it}} - \frac{1 - \theta_{it}}{1 - \mathcal{E}_t} = \frac{\delta \gamma \omega_t}{((1 - \mathcal{E}_t) \omega_t - b)} \mathbb{E}_t \left[\frac{\theta_{it+1}}{e_{it+1}} (1 - \mathcal{E}_{t+1}^{-i}) (1 + \sigma_{t+1}^{-i}) \right] 1_{\{\mathcal{E}_t \in X(\omega_t)\}} \quad (27)$$

or, to simplify further,

$$\frac{\theta_{it}(1 - \mathcal{E}_t^{-i})}{e_{it}} = 1 + \frac{\delta\gamma(1 - \mathcal{E}_t)\omega_t}{((1 - \mathcal{E}_t)\omega_t - b)} \mathbb{E}_t \left[\frac{\theta_{it+1}}{e_{it+1}} (1 - \mathcal{E}_{t+1}^{-i}) (1 + \sigma_{t+1}^{-i}) \right] 1_{\{\mathcal{E}_t \in X(\omega_t)\}} \quad (28)$$

which coincides with the Euler equation in (8).

Finally, by setting $b = 0$ as required for the no-tipping model, we obtain the closed form solution for a BAU equilibrium in Equation (9). \blacksquare

7.2 Proof of Theorem 1

To prove the theorem we return to the BAU equilibrium Euler equation (8).

$$\frac{\theta_{it}(1 - \mathcal{E}_t^{-i})}{e_{it}} = 1 + \frac{\delta\gamma(1 - \mathcal{E}_t)\omega_t}{((1 - \mathcal{E}_t)\omega_t - b)} \mathbb{E}_t \left[\frac{\theta_{it+1}}{e_{it+1}} (1 - \mathcal{E}_{t+1}^{-i}) (1 + \sigma_{t+1}^{-i}) \right] 1_{\{\mathcal{E}_t \in X(\omega_t)\}} \quad (29)$$

where $X(\omega_t)$ is defined in (20).

Notice that when $b = F = 0$ (the concave, “no-tipping” model), the Euler equation reduces to

$$\frac{\theta_{it}(1 - \mathcal{E}_t^{-i})}{e_{it}} = 1 + \delta\gamma \mathbb{E}_t \left[\frac{\theta_{it+1}}{e_{it+1}} (1 - \mathcal{E}_{t+1}^{-i}) (1 + \sigma_{t+1}^{-i}) \right] \quad (30)$$

which is solved by the ω -stationary extraction plan

$$\bar{e}_{it}(\theta_t) = \frac{\frac{\theta_{it}(1-\gamma\delta)}{1-\theta_{it}(1-\gamma\delta)}}{1 + \sum_{j=1}^n \frac{\theta_{jt}(1-\gamma\delta)}{1-\theta_{jt}(1-\gamma\delta)}}.$$

In this stationary solution, $\sigma_{t+1}^{-i} = 0$ for all i, t , and θ .

First, we show that there exists ω^1 satisfying $e_{it}^*(\omega_t, \theta_t) > \bar{e}_{it}(\theta_t)$ for all θ_t whenever $\omega_t < \omega^1$. Observe that whenever $1_{\{\mathcal{E}_t \in X(\omega_t)\}} = 0$, then the right-hand side of the Euler equation (29) becomes zero. Given \mathcal{E} , the triggering event is independent of θ . Let ω^1 satisfy

$$\omega^1 \in \arg \inf \{ \omega : 1_{\{\mathcal{E}^*(\omega, \theta) \in X(\omega)\}} = 1 \ \forall \theta \}$$

A non-zero infimum exists since $1_{\{F \in X(\omega)\}} = 0$. By construction, for all $\omega_t \leq \omega^1$ and for all θ_t , $e_{it}^*(\omega_t, \theta_t) = e_i^{static}(\theta_t) > \bar{e}_{it}(\theta_t)$.

For the second part, we show that there is some ω^2 such that for all θ_t , $e_{it}^*(\omega_t, \theta_t) < \bar{e}_{it}(\theta_t)$ when $\omega_t \geq \omega^2$. Observe that the Euler equation can be rewritten as:

$$\frac{\theta_{it}(1 - \mathcal{E}_t^{-i})}{e_{it}} = 1 + \frac{\delta\gamma(1 - \mathcal{E}_t)}{(1 - \mathcal{E}_t) - b/\omega_t} \mathbb{E}_t \left[\frac{\theta_{it+1}}{e_{it+1}} (1 - \mathcal{E}_{t+1}^{-i}) (1 + \sigma_{t+1}^{-i}) \right] 1_{\{\mathcal{E}_t \in X(\omega_t)\}}$$

As $\omega \rightarrow \infty$, $b/\omega_t \rightarrow 0$ in which case the solution to the Euler equation is approximated by an stationary extraction plan \mathbf{e} . The stationary extraction plan, in turn, is the solution $\bar{\mathbf{e}}$ to the concave model. Hence, $e_j^*(\omega, \theta) \rightarrow \bar{e}_j(\theta)$ for all θ , as $\omega \rightarrow \infty$. Since the marginal extraction cost (the right-hand side of (29)) is larger than the marginal extraction cost in the concave model, the limit is reached from below. Consequently, $e_j^*(\omega, \theta) \nearrow \bar{e}_j(\theta)$. Fixing some $\epsilon > 0$, we choose ω^2 such that $|e_j^*(\omega^2, \theta) - \bar{e}_j(\theta)| < \epsilon$. Then we obtain $e_j^*(\omega^2, \theta) < \bar{e}_j(\theta)$ for all $\omega \geq \omega^2$.

We conclude the proof. ■

7.3 Proof of Theorem 2

Abusing notation, let $\mathcal{E}^*(\omega_t, \theta_t, b, n)$ denote the BAU equilibrium aggregate extraction rate, expressed as a function of both the state and the relevant parameters b and n . Let

$$G^*(\omega_t, \theta_t, b) \equiv \frac{\delta\gamma\omega_t(1 - \mathcal{E}^*(\omega_t, \theta_t, b, n))}{\omega_t(1 - \mathcal{E}^*(\omega_t, \theta_t, b, n)) - b} \times$$

$$\mathbb{E}_t \left[\frac{\theta_{it+1}}{e_i^*(\omega_{t+1}, \theta_{t+1}, b, n)} (1 - \mathcal{E}^{*-i}(\omega_{t+1}, \theta_{t+1}, b, n)) (1 + \sigma_{t+1}^{*-i}) \right] 1_{\{\mathcal{E}^*(\omega_t, \theta_t, b, n) \in X(\omega_t)\}}$$

So G^* is the marginal extraction cost evaluated at the BAU equilibrium.

We first show that there is a ω -stationary lower bound on the aggregate extraction rate, namely, an extraction rate $\underline{\mathcal{E}}(\theta_t, b, n)$ with $\underline{\mathcal{E}}(\theta_t, b, n) \leq \mathcal{E}^*(\omega_t, \theta_t, b, n) \forall \omega_t$, there exists ϵ sufficiently small and n sufficiently large, so that if $\theta_t \in (1 - \epsilon, 1]^n$ then

$$\omega_t > \omega_{t+1}^*(\omega_t, \theta_t; b) \quad \forall \omega_t > F.$$

In other words, the only fixed point of ω^* is the floor F .

We first find this lower bound $\underline{\mathcal{E}}(\theta_t, b, n)$. Observe that $1_{\{\mathcal{E}_t \in X(\omega_t)\}} = 0$ for all ω_t and \mathcal{E}_t such that $A(\omega_t(1 - \mathcal{E}_t) - b)^\gamma \leq F$ or equivalently, $\omega_t \leq \frac{1}{1 - \mathcal{E}_t} (b + (\frac{F}{A})^{1/\gamma}) \equiv K$. Moreover, K is the upper bound on stocks for which $1_{\{\mathcal{E}_t \in X(\omega_t)\}} = 0$. Hence, fixing θ_t the marginal future cost of extraction (the right-hand side of (8)) is bounded above by its stationary limit when ω approaches K from the right, so that $1_{\{\mathcal{E}_t \in X(\omega_t)\}} = 1$. Stated precisely:

$$\forall \omega_t, \quad G^*(\omega_t, \theta_t, b) \leq \lim_{\omega \rightarrow K^+} G^*(\omega, \theta_t, b).$$

This is obviously true if $\omega \leq K$ since $G^*(\omega_t, \theta_t, b) = 0$ when the carbon floor is reached. It is also true if $\omega > K$ since marginal extraction cost is declining in stock.

Hence, set $\underline{\mathcal{E}}(\theta_t, b, n) \equiv \mathcal{E}^*(K, \theta_t, b, n)$. By definition, $\underline{\mathcal{E}}(\theta_t, b, n)$ is the solution to (8) at the threshold value $\omega = K^+$. We evaluate the Euler equation at $\mathcal{E} = \underline{\mathcal{E}}(\theta_t, b, n)$ when $\theta_t = \mathbf{1}$. In that case, extraction rates are stationary and symmetric, i.e., $\mathcal{E} = ne$ and $\sigma_{-it} = (n-1)\sigma$.

The Euler equation becomes

$$\frac{1 - \mathcal{E}^{-i}}{\mathcal{E}/n} = 1 + \delta\gamma \frac{K(1 - \mathcal{E})}{K(1 - \mathcal{E}) - b} \left(\frac{1 - \mathcal{E}^{-i}}{\mathcal{E}/n} (1 + (n-1)\sigma) \right)$$

Considering $\mathcal{E}^{-i} = \frac{(n-1)}{n}\mathcal{E}$, the condition can be expressed as

$$\begin{aligned} n \frac{1 - \mathcal{E}}{\mathcal{E}} &= \delta\gamma \frac{K(1 - \mathcal{E})}{K(1 - \mathcal{E}) - b} \left(\left(n \frac{1 - \mathcal{E}}{\mathcal{E}} + 1 \right) (1 + (n-1)\sigma) \right) \\ n \frac{1 - \mathcal{E}}{\mathcal{E}} \left(1 - \delta\gamma \frac{K(1 - \mathcal{E})}{K(1 - \mathcal{E}) - b} (1 + (n-1)\sigma) \right) &= \delta\gamma \frac{K(1 - \mathcal{E})}{K(1 - \mathcal{E}) - b} (1 + (n-1)\sigma) \\ n \frac{1 - \mathcal{E}}{\mathcal{E}} &= \frac{\delta\gamma \frac{K(1 - \mathcal{E})}{K(1 - \mathcal{E}) - b} (1 + (n-1)\sigma)}{1 - \delta\gamma \frac{K(1 - \mathcal{E})}{K(1 - \mathcal{E}) - b} (1 + (n-1)\sigma)} \\ n \frac{1 - \mathcal{E}}{\mathcal{E}} &= \frac{\delta\gamma K(1 - \mathcal{E}) (1 + (n-1)\sigma)}{K(1 - \mathcal{E}) (1 - \delta\gamma (1 + (n-1)\sigma)) - b} \end{aligned}$$

Using the fact that $K \equiv \frac{1}{1-\mathcal{E}} \left(b + \left(\frac{F}{A} \right)^{1/\gamma} \right)$, the Euler equation becomes

$$n \frac{1 - \mathcal{E}}{\mathcal{E}} = \frac{\delta\gamma (1 + (n-1)\sigma) \left(b + \left(\frac{F}{A} \right)^{1/\gamma} \right)}{(1 - \delta\gamma (1 + (n-1)\sigma)) \left(b + \left(\frac{F}{A} \right)^{1/\gamma} \right) - b}. \quad (31)$$

Since, by construction $\underline{\mathcal{E}}(\mathbf{1}, b, n)$ satisfies (31) and $\sigma < 0$ in an interval $[K, K + \epsilon)$, it follows that

$$\lim_{n \rightarrow \infty} \underline{\mathcal{E}}(\mathbf{1}, b, n) = 1.$$

Next we show that for ϵ small enough and n large enough,

$$\omega_t > \omega_{t+1}^*(\omega_t, \theta_t; b) \quad \forall \omega_t \quad \forall \theta_t \in (1 - \epsilon, 1]^n. \quad (32)$$

The inequality (32) may be rewritten as

$$\frac{1}{(1 - \mathcal{E}^*(\omega_t, \theta_t, b, n))} \left[\left(\frac{\omega_t}{A} \right)^{1/\gamma} + b \right] - \omega_t > 0 \quad \forall \omega_t. \quad (33)$$

To verify that (33) holds, we show that

$$\underline{P} \equiv \min_{\omega} \left(\frac{1}{(1 - \underline{\mathcal{E}}_t(\theta_t; b, n))} \left[\left(\frac{\omega_t}{A} \right)^{1/\gamma} + b \right] - \omega \right) > 0 \quad (34)$$

where $\underline{\mathcal{E}}_t(\theta_t; b, n)$ is, recall, a stationary lower bound of $\mathcal{E}^*(\omega_t, \theta_t, b, n)$.

The first order condition for \underline{P} is

$$(\gamma(1 - \underline{\mathcal{E}}(\theta_t; b, n))A^{1/\gamma})^{-1}\omega^{\frac{1-\gamma}{\gamma}} - 1 = 0.$$

Solving for ω , we obtain $\omega^m \equiv (\gamma(1 - \underline{\mathcal{E}}(\theta_t; b, n)))^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}}$. Substituting ω^m back into the problem we obtain,

$$\begin{aligned} \underline{P} &= \left(\frac{1}{(1 - \underline{\mathcal{E}}(\theta_t; b, n))} \left[\left(\frac{\omega^m}{A} \right)^{1/\gamma} + b \right] - \omega^m \right) \\ &= \left(\frac{1}{(1 - \underline{\mathcal{E}}(\theta_t; b, n))} \left[\left(\frac{(\gamma(1 - \underline{\mathcal{E}}(\theta_t; b, n)))^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}}}{A} \right)^{1/\gamma} + b \right] - (\gamma(1 - \underline{\mathcal{E}}(\theta_t; b, n)))^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}} \right) \\ &= \frac{b}{1 - \underline{\mathcal{E}}(\theta_t; b, n)} - (1 - \underline{\mathcal{E}}(\theta_t; b, n))^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}} \left(\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}} \right). \end{aligned}$$

Hence, (33) holds if

$$\underline{P} = \frac{b}{1 - \underline{\mathcal{E}}(\theta_t; b, n)} - (1 - \underline{\mathcal{E}}(\theta_t; b, n))^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}} \left(\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}} \right) > 0 \quad (35)$$

holds. But (35) clearly holds in the limit as $\theta_t \rightarrow \mathbf{1} \equiv (1, \dots, 1)$ and $n \rightarrow \infty$ since $\underline{\mathcal{E}}(\theta_t; b, n) \rightarrow 1$ in that case.

Since the argument is strict, there exists n' sufficiently large, and θ_t sufficiently close to one such that there is a finite time length $T(\theta_t)$ such that $\omega^{*t}(\omega_0, \theta^t) \rightarrow F$ in at most $T(\theta_t)$ iterations. Let

$$T = \max_{\theta_t \in [1-\epsilon, 1]} T(\theta_t).$$

Thus T is a time length (dependent on ω_0) such that if (32) holds for all $\theta_t \in (1-\epsilon, 1]^n$ then $\omega^{*t}(\omega_0, \theta^t) \rightarrow F$ in at most T iterations.

Observe that (3) implies for any finite $T > 0$, that for a.e. θ_t ,

$$\begin{aligned} &\Pr \left(\theta_{t+s} \in (1 - \epsilon, 1]^n, s = 1, \dots, T \mid \theta_t \right) \\ &= \int_{\theta_{t+1} \in (1-\epsilon, 1]^n} \cdots \int_{\theta_{t+T} \in (1-\epsilon, 1]^n} \prod_{s=1}^T dF(\theta_{t+s} | \theta_{t+s-1}) \\ &\geq \epsilon^T. \end{aligned} \quad (36)$$

It follows that for almost every process $\{\theta_t\}$, there is a date t (infinitely many dates actually) such that (32) holds for realized values $\theta_t, \theta_{t+1}, \dots, \theta_{t+T}$, in which case $\omega^{*t+T}(\omega_t, \theta^{t+T}) = F$. Consequently, the commons reaches the floor F from ω_t , concluding the proof. \blacksquare

7.4 Derivation of the Planner's Euler Equation

We first derive the Planner's Euler equation before making comparisons to the BAU equilibrium.

Taking derivatives respect to the control variables e_{it} , the first order conditions are:

$$[e_{it}] : \quad \frac{\theta_{it}}{e_{it}\omega_t}\omega_t + \frac{\sum_{i=1}^n(1-\theta_{it})}{(1-\mathcal{E}_t)\omega_t}(-\omega_t) + \delta \mathbb{E} \left[\frac{\partial V}{\partial \omega_{t+1}} \frac{\partial \omega_{t+1}}{\partial e_{it}} \right] = 0 \quad (37)$$

$$\frac{\theta_{it}}{e_{it}} - \frac{\sum_{i=1}^n(1-\theta_{it})}{1-\mathcal{E}_t} + \delta \mathbb{E} \left[\frac{\partial V}{\partial \omega_{t+1}} \frac{\partial \omega_{t+1}}{\partial e_{it}} \right] = 0 \quad (38)$$

Considering that the derivative of the stock movement equation respect to the consumption is the following:

$$\frac{\partial \omega_{t+1}}{\partial e_{it}} = \begin{cases} A\gamma((1-\mathcal{E}_t)\omega_t - b)^{\gamma-1}(-\omega_t) & (1-\mathcal{E}_t)\omega_t - b \geq F \\ 0 & otherwise \end{cases} \quad (39)$$

$$= -A\gamma\omega_t((1-\mathcal{E}_t)\omega_t - b)^{\gamma-1} 1_{\mathcal{E}_t \in X(\omega_t)} \quad (40)$$

We can rewrite (38) as:

$$[e_{it}] : \quad \frac{\theta_{it}}{e_{it}} - \frac{\sum_{i=1}^n(1-\theta_{it})}{1-\mathcal{E}_t} = A\delta\gamma\omega_t((1-\mathcal{E}_t)\omega_t - b)^{\gamma-1} \mathbb{E} \left[\frac{\partial V_{t+1}}{\partial \omega_{t+1}} \right] 1_{\{\mathcal{E}_t \in X(\omega_t)\}} \quad (41)$$

Deriving the value function with respect to ω_t would yield:

$$\frac{\partial V(\omega_t)}{\partial \omega_t} = \sum_{i=1}^n \left(\frac{\theta_{it}e_{it}}{e_{it}\omega_t} + \frac{(1-\theta_{it})(1-\mathcal{E}_t)}{(1-\mathcal{E}_t)\omega_t} \right) + \delta \mathbb{E} \left[\frac{\partial V_{t+1}}{\partial \omega_{t+1}} \frac{\partial \omega_{t+1}}{\partial \omega_t} \right] \quad (42)$$

$$= \sum_{i=1}^n \left(\frac{\theta_{it}}{\omega_t} + \frac{1-\theta_{it}}{\omega_t} \right) + \delta \mathbb{E} \left[\frac{\partial V_{t+1}}{\partial \omega_{t+1}} \frac{\partial \omega_{t+1}}{\partial \omega_t} \right] \quad (43)$$

$$= \frac{n}{\omega_t} + \delta \mathbb{E} \left[\frac{\partial V_{t+1}}{\partial \omega_{t+1}} \frac{\partial \omega_{t+1}}{\partial \omega_t} \right] \quad (44)$$

where

$$\frac{\partial \omega_{t+1}}{\partial \omega_t} = A\gamma((1 - \mathcal{E}_t)\omega_t - b)^{\gamma-1} (1 - \mathcal{E}_t) 1_{\{\mathcal{E}_t \in X(\omega_t)\}}$$

The condition (44) can be written as:

$$\frac{\partial V_t}{\partial \omega_t} = \frac{n}{\omega_t} + A\delta\gamma((1 - \mathcal{E}_t)\omega_t - b)^{\gamma-1} (1 - \mathcal{E}_t) 1_{\{\mathcal{E}_t \in X(\omega_t)\}} \mathbb{E} \left[\frac{\partial V_{t+1}}{\partial \omega_{t+1}} \right] \quad (45)$$

From the FOC, solving for $\mathbb{E} \left[\frac{\partial V_{t+1}}{\partial \omega_{t+1}} \right]$ yields:

$$\mathbb{E} \left[\frac{\partial V_{t+1}}{\partial \omega_{t+1}} \right] = (A\delta\gamma\omega_t((1 - \mathcal{E}_t)\omega_t - b)^{\gamma-1})^{-1} \left(\frac{\theta_{it}}{e_{it}} - \frac{\sum_{i=1}^n (1 - \theta_{it})}{1 - \mathcal{E}_t} \right) \quad (46)$$

Replacing (46) on (45):

$$\frac{\partial V_t}{\partial \omega_t} = \frac{n}{\omega_t} + \frac{1 - \mathcal{E}_t}{\omega_t} \left(\frac{\theta_{it}}{e_{it}} - \frac{\sum_{i=1}^n (1 - \theta_{it})}{1 - \mathcal{E}_t} \right) \quad (47)$$

$$= \frac{n}{\omega_t} + \frac{(1 - \mathcal{E}_t)\theta_{it}}{e_{it}\omega_t} - \frac{\sum_{i=1}^n (1 - \theta_{it})}{\omega_t} \quad (48)$$

$$= \frac{(1 - \mathcal{E}_t)\theta_{it}}{e_{it}\omega_t} + \frac{\sum_{i=1}^n \theta_{it}}{\omega_t} \quad (49)$$

Now, forwarding (49) one period, we obtain:

$$\frac{\partial V_{t+1}}{\partial \omega_{t+1}} = \frac{(1 - \mathcal{E}_{t+1})\theta_{it+1}}{e_{it+1}\omega_{t+1}} + \frac{\sum_{i=1}^n \theta_{it+1}}{\omega_{t+1}} \quad (50)$$

And replacing (50) on the first order condition yields the Euler equation for the system:

$$\begin{aligned} \frac{\theta_{it}}{e_{it}} - \frac{\sum_{i=1}^n (1 - \theta_{it})}{1 - \mathcal{E}_t} &= A\delta\gamma\omega_t((1 - \mathcal{E}_t)\omega_t - b)^{\gamma-1} \mathbb{E} \left[\frac{(1 - \mathcal{E}_{t+1})\theta_{it+1}}{e_{it+1}\omega_{t+1}} + \frac{\sum_{i=1}^n \theta_{it+1}}{\omega_{t+1}} \right] 1_{\{\mathcal{E}_t \in X(\omega_t)\}} \\ &= \frac{A\delta\gamma\omega_t((1 - \mathcal{E}_t)\omega_t - b)^{\gamma-1}}{A((1 - \mathcal{E}_t)\omega_t - b)^\gamma} \mathbb{E} \left[\frac{(1 - \mathcal{E}_{t+1})\theta_{it+1}}{e_{it+1}} + \sum_{i=1}^n \theta_{it+1} \right] 1_{\{\mathcal{E}_t \in X(\omega_t)\}} \\ &= \frac{\delta\gamma\omega_t}{(1 - \mathcal{E}_t)\omega_t - b} \mathbb{E} \left[\frac{\theta_{it+1}}{e_{it+1}} - \frac{\theta_{it+1}}{e_{it+1}} \sum_{j \neq i} e_{jt+1} + \sum_{j \neq i} \theta_{jt+1} \right] 1_{\{\mathcal{E}_t \in X(\omega_t)\}} \end{aligned} \quad (51)$$

Therefore, the Planner's Euler equation for country i can be rewritten as:

$$\frac{\theta_{it}(1 - \mathcal{E}_t^{-i})}{e_{it}} - (n - \sum_{j \neq i} \theta_{jt}) = \left[\frac{\delta \gamma \omega_t (1 - \mathcal{E}_t)}{(1 - \mathcal{E}_t) \omega_t - b} \right] \mathbb{E} \left[\frac{\theta_{it+1}}{e_{it+1}} (1 - \mathcal{E}_{t+1}^{-i}) + \sum_{j \neq i} \theta_{jt+1} \right] 1_{\{\mathcal{E}_t \in X(\omega_t)\}} \quad (52)$$

7.5 Proof of Proposition 2

We first rewrite both the Planner's and the BAU Equation to obtain more convenient expressions.

Using the BAU Euler equation as expressed in Equation (27), we obtain

$$\begin{aligned} & \left(\theta_{it} - \frac{(1 - \theta_{it})e_{it}}{1 - \mathcal{E}_t} \right) (\omega_t (1 - \mathcal{E}_t) - b) - \\ & A \delta \gamma e_{it} \omega_t \left\{ 1 + E \left[\left(\frac{\theta_{it+1}}{e_{it+1}} - \frac{1 - \theta_{it+1}}{1 - \mathcal{E}_{t+1}} - \sigma_{t+1}^{-i} \left(\frac{\theta_{it+1}}{e_{it+1}} \right) \right) (1 - \mathcal{E}_{t+1}) \middle| \omega_t, \theta_{it} \right] \right\} 1_{\{\mathcal{E}_t \in X(\omega_t)\}} = 0 \end{aligned} \quad (53)$$

Similarly, Planner's Euler equation as expressed in Equation (51), we obtain

$$\begin{aligned} & \frac{1}{n} \left(\theta_{it} - \frac{e_{it} \sum_j (1 - \theta_{jt})}{1 - \mathcal{E}_t} \right) (\omega_t (1 - \mathcal{E}_t) - b) - \\ & A \delta \gamma e_{it} \omega_t \left\{ 1 + E \left[\left(\frac{\theta_{it+1}}{e_{it+1}} - \frac{1 - \theta_{it+1}}{1 - \mathcal{E}_{t+1}} \right) (1 - \mathcal{E}_{t+1}) \middle| \omega_t, \theta_{it} \right] \right\} 1_{\{\mathcal{E}_t \in X(\omega_t)\}} = 0 \end{aligned} \quad (54)$$

The left-hand sides of Equations (53) and (54) are the marginal values to country i and the Planner, respectively, from i 's extraction of carbon. Summing both equations over all countries, we obtain,

$$\begin{aligned} & H_t^*(\mathcal{E}_t, \mathbf{e}_{t+1}) \equiv \left(\sum_i \theta_{it} - \sum_i \frac{(1 - \theta_{it})e_{it}}{1 - \mathcal{E}_t} \right) (\omega_t (1 - \mathcal{E}_t) - b) - \\ & A \delta \gamma \mathcal{E}_t \omega_t \left\{ 1 + E \sum_i \left[\left(\frac{\theta_{it+1}}{e_{it+1}} - \frac{1 - \theta_{it+1}}{1 - \mathcal{E}_{t+1}} - \sigma_{t+1}^{-i} \left(\frac{\theta_{it+1}}{e_{it+1}} \right) \right) (1 - \mathcal{E}_{t+1}) \middle| \omega_t, \theta_{it} \right] \right\} 1_{\{\mathcal{E}_t \in X(\omega_t)\}} = 0 \end{aligned} \quad (55)$$

and

$$\begin{aligned}
H_t^\circ(\mathcal{E}_t, \mathbf{e}_{t+1}) &\equiv \frac{1}{n} \left(\sum_i \theta_{it} - \frac{\mathcal{E}_t \sum_j (1 - \theta_{jt})}{1 - \mathcal{E}_t} \right) (\omega_t(1 - \mathcal{E}_t) - b) - \\
A\delta\gamma \mathcal{E}_t \omega_t &\left\{ 1 + E \left[\left(\frac{\theta_{it+1}}{e_{it+1}} - \frac{1 - \theta_{it+1}}{1 - \mathcal{E}_{t+1}} \right) (1 - \mathcal{E}_{t+1}) \middle| \omega_t, \theta_{it} \right] \right\} 1_{\{\mathcal{E}_t \in X(\omega_t)\}} = 0
\end{aligned} \tag{56}$$

We now compare the marginal values $H_t^*(\mathcal{E}_t, \mathbf{e}_{t+1})$ and $H_t^\circ(\mathcal{E}_t, \mathbf{e}_{t+1})$. In the following Lemmata, we refer to the top lines of H^* and of H° as the first term, and the bottom line of each as the second term.

Lemma 1 *The second term of $H_t^*(\mathcal{E}_t, \mathbf{e}_{t+1})$ is smaller than the second term of $H_t^\circ(\mathcal{E}_t, \mathbf{e}_{t+1})$*

Proof of Lemma 1. Clear by inspection.

Lemma 2 *Regarding the first terms of H^* and H° , for any \mathcal{E} satisfying $\mathcal{E} < \frac{n}{n+1}$,*

$$\sum_i \theta_{it} - \sum_i \frac{(1 - \theta_{it})e_{it}}{1 - \mathcal{E}_t} > \frac{1}{n} \left(\sum_i \theta_{it} - \frac{\mathcal{E}_t \sum_j (1 - \theta_{jt})}{1 - \mathcal{E}_t} \right).$$

Proof of Lemma 2. For simplicity let $\Theta_t = \sum_i \theta_{it}$. Then we seek to show

$$\Theta_t(1 - \mathcal{E}_t) - \mathcal{E}_t + \sum_i \theta_{it} e_{it} \stackrel{?}{>} \frac{1}{n} (\Theta_t(1 - \mathcal{E}_t) - n\mathcal{E}_t + \Theta_t \mathcal{E}_t)$$

or

$$\Theta_t(1 - \mathcal{E}_t) + \sum_i \theta_{it} e_{it} \stackrel{?}{>} \frac{1}{n} (\Theta_t(1 - \mathcal{E}_t) + \Theta_t \mathcal{E}_t)$$

or

$$\frac{n-1}{n} \Theta_t(1 - \mathcal{E}_t) + \sum_i \theta_{it} e_{it} \stackrel{?}{>} \frac{1}{n} \Theta_t \mathcal{E}_t$$

or

$$\frac{n-1}{n} \Theta_t - \Theta_t \mathcal{E}_t + \sum_i \theta_{it} e_{it} \stackrel{?}{>} 0$$

which clearly holds if $\frac{n}{n+1} > \mathcal{E}_t$.

Combining Lemma 1 with Lemma 2, it follows that for all \mathcal{E}_t satisfying $\mathcal{E}_t < \frac{n}{n+1}$, and all \mathbf{e}_{t+1} ,

$$H_t^*(\mathcal{E}_t, \mathbf{e}_{t+1}) > H_t^\circ(\mathcal{E}_t, \mathbf{e}_{t+1}).$$

Since $\mathcal{E}_t^\circ < \frac{n}{n+1}$,

$$H_t^*(\mathcal{E}_t^\circ, \mathbf{e}_{t+1}) > H_t^\circ(\mathcal{E}_t^\circ, \mathbf{e}_{t+1}) = 0,$$

for all \mathbf{e}_{t+1} . This yields $\mathcal{E}_t^* > \mathcal{E}_t^\circ$. ■

7.6 Proof of Proposition 3

Part 1. Over- and Under-extraction by Individual Countries. To evaluate whether a country over or under extracts in the BAU equilibrium, one need only compare e_{it}° to e_{it}^* . Country over (under) extracts if $e_{it}^* > (<)e_{it}^\circ$. We therefore compare:

$$e_{it}^\circ(\theta_t) = \frac{\theta_{it}(1 - A\gamma\delta)}{n} = \frac{\phi_{it}}{n} \stackrel{?}{>} e_{it}^*(\theta_t) = \frac{\left(\frac{\theta_{it}(1-A\gamma\delta)}{1-\theta_{it}(1-A\gamma\delta)}\right)}{1 + \left(\sum_{j=1}^n \frac{\theta_{jt}(1-A\gamma\delta)}{1-\theta_{jt}(1-A\gamma\delta)}\right)} = \frac{\left(\frac{\phi_{it}}{1-\phi_{it}}\right)}{1 + \left(\sum_{j=1}^n \frac{\phi_{jt}}{1-\phi_{jt}}\right)}$$

with, recall, $\phi_{it} = \theta_{it}(1 - A\gamma\delta)$. Since $\phi_{it} > 0$, country i over-extracts if

$$\frac{\left(\frac{1}{1-\phi_{it}}\right)}{1 + \left(\sum_{j=1}^n \frac{\phi_{jt}}{1-\phi_{jt}}\right)} > \frac{1}{n},$$

and solving for ϕ_{it} , country i will over (under) extract if

$$\phi_{it} > (<) 1 - \frac{n-1}{\sum_{j \neq i} \frac{\phi_{jt}}{1-\phi_{jt}}} \tag{57}$$

By choosing $\tilde{\theta}$ such that $\tilde{\theta}(1 - A\delta\gamma)$ equals the right hand side of (57), we have found out threshold.

Notice, moreover, that the larger the profile of her opponents the (weakly) smaller is the set of types for which is optimal to her over extract.²⁴

²⁴Example: suppose a symmetric profile ϕ_{-i} , i.e. $\phi_j = \phi_k = \phi$ for all $k, j \neq i$.

$$\phi_i > 1 - \frac{n-1}{\sum_{j \neq i} \frac{\phi_j}{1-\phi_j}} = 1 - \frac{n-1}{(n-1)\frac{\phi}{1-\phi}} = \frac{2\phi-1}{\phi}. \quad (**)$$

Note that the extreme (highest) profile player i can be facing is a profile of opponents with the highest type, i.e. $\theta_j = \bar{\theta} < 1$ for all $j \neq i$. Then from equation (**) above,

$$\theta_i > \frac{2\bar{\theta}(1-\gamma\delta)-1}{\bar{\theta}(1-\gamma\delta)^2}$$

Part 2. Output paths. The final part must prove that relative output, carbon consumption and carbon stock shrinks in the BAU relative to that of the efficient plan.

We first compute the socially optimal extraction rate and the optimal carbon path when $b = 0$ and setting $\phi_{it} = \theta_{it}(1 - A\delta\gamma)$. The extraction rate is: $e_{it} = \frac{\phi_{it}}{n}$ and the time path of the carbon stock in the Planner's optimum is

$$\begin{aligned}\omega^{*t}(\omega_0, \theta^t) &= \omega_0^{\gamma^t} A^{\frac{1-\gamma^t}{1-\gamma}} \prod_{\tau=1}^t (1 - \mathcal{E}_{t-\tau}^\circ(\theta_{t-\tau}))^{\gamma^\tau} \\ &= \omega_0^{\gamma^t} A^{\frac{1-\gamma^t}{1-\gamma}} \prod_{\tau=1}^t \left(1 - \frac{\sum_j \phi_{jt-\tau}}{n}\right)^{\gamma^\tau}.\end{aligned}\quad (58)$$

A country's output path in the Planner's problem is given by

$$y_i^{*t} = \left(\frac{\phi_{it}}{n}\right)^{\theta_{it}} \left(1 - \frac{\sum_j \phi_{jt}}{n}\right)^{(1-\theta_{it})} \omega_0^{\gamma^t} A^{\frac{1-\gamma^t}{1-\gamma}} \prod_{\tau=1}^t \left(1 - \frac{\sum_j \phi_{jt-\tau}}{n}\right)^{\gamma^\tau}.\quad (59)$$

A particularly useful illustration of (58) is the case without shocks. In that case $\theta_t = \theta_{t'} = \theta$ and so (58) reduces to

$$\omega^{*t}(\omega_0, \theta^t) = \omega_0^{\gamma^t} \left(1 - \frac{\sum_j \phi_j}{n}\right)^{\frac{\gamma(1-\gamma^t)}{1-\gamma}} A^{\frac{1-\gamma^t}{1-\gamma}}\quad (60)$$

in which case the output path simplifies to

$$y_i^{*t} = \left(\frac{\phi_i}{n}\right)^{\theta_i} \left(1 - \frac{\sum_j \psi_j}{n}\right)^{(1-\theta_i)} \omega_0^{\gamma^t} \left(1 - \frac{\sum_j \phi_j}{n}\right)^{\frac{\gamma(1-\gamma^t)}{1-\gamma}} A^{\frac{1-\gamma^t}{1-\gamma}}.\quad (61)$$

or

$$\phi_i > \frac{2\bar{\phi} - 1}{\bar{\phi}}.$$

So if we require all θ_i over extract, the condition is:

$$\underline{\theta} > \frac{2\bar{\theta}(1 - \gamma\delta) - 1}{\bar{\theta}(1 - \gamma\delta)^2}.$$

This implies the following sufficient condition: if $\delta\gamma \geq \frac{1}{2}$ all types θ_i over extract.

These paths may be compared to the BAU equilibrium. Iterating on the equilibrium law of motion, one derives the time path of the carbon stock as

$$\begin{aligned}\omega^{*t}(\omega_0, \theta^t) &= \omega_0^{\gamma^t} A^{\frac{1-\gamma^t}{1-\gamma}} \prod_{\tau=1}^t (1 - \mathcal{E}_{t-\tau}^*(\theta_{t-\tau}))^{\gamma^\tau} \\ &= \omega_0^{\gamma^t} A^{\frac{1-\gamma^t}{1-\gamma}} \prod_{\tau=1}^t \left(1 - \frac{\sum_j (\frac{\phi_{jt-\tau}}{1-\phi_{jt-\tau}})}{1 + (\sum_j \frac{\phi_{jt-\tau}}{1-\phi_{jt-\tau}})} \right)^{\gamma^\tau}.\end{aligned}\quad (62)$$

A country's output path in the BAU equilibrium is given by

$$\begin{aligned}y_i^{*t}(\omega_0, \theta^t) &= \left(\frac{(\frac{\phi_{it}}{1-\phi_{it}})}{1 + (\sum_{j=1} \frac{\phi_{jt}}{1-\phi_{jt}})} \right)^{\theta_{it}} \left(1 - \frac{\sum_j (\frac{\phi_{jt}}{1-\phi_{jt}})}{1 + (\sum_j \frac{\phi_{jt}}{1-\phi_{jt}})} \right)^{(1-\theta_{it})} \omega^{*t}(\omega_0, \theta^t) \\ &= \left(\frac{(\frac{\phi_{it}}{1-\phi_{it}})}{1 + (\sum_{j=1} \frac{\phi_{jt}}{1-\phi_{jt}})} \right)^{\theta_{it}} \left(1 - \frac{\sum_j (\frac{\phi_{jt}}{1-\phi_{jt}})}{1 + (\sum_j \frac{\phi_{jt}}{1-\phi_{jt}})} \right)^{(1-\theta_{it})} \times \\ &\quad \omega_0^{\gamma^t} A^{\frac{1-\gamma^t}{1-\gamma}} \prod_{\tau=1}^t \left(1 - \frac{\sum_j (\frac{\phi_{jt-\tau}}{1-\phi_{jt-\tau}})}{1 + (\sum_j \frac{\phi_{jt-\tau}}{1-\phi_{jt-\tau}})} \right)^{\gamma^\tau}.\end{aligned}\quad (63)$$

Comparing the BAU in (63) with the optimal output in (59). We see that $y_{it}^* < y_{it}^\circ$ iff

$$\begin{aligned}&\left(\frac{(\frac{\phi_{it}}{1-\phi_{it}})}{1 + (\sum_{j=1} \frac{\phi_{jt}}{1-\phi_{jt}})} \right)^{\theta_{it}} \left(1 - \frac{\sum_j (\frac{\phi_{jt}}{1-\phi_{jt}})}{1 + (\sum_j \frac{\phi_{jt}}{1-\phi_{jt}})} \right)^{(1-\theta_{it})} \prod_{\tau=1}^t \left(1 - \frac{\sum_j (\frac{\phi_{jt-\tau}}{1-\phi_{jt-\tau}})}{1 + (\sum_j \frac{\phi_{jt-\tau}}{1-\phi_{jt-\tau}})} \right)^{\gamma^\tau} \\ &< \left(\frac{\phi_{it}}{n} \right)^{\theta_{it}} \left(\frac{1 - \sum_j \phi_{jt}}{n} \right)^{(1-\theta_{it})} \prod_{\tau=1}^t \left(1 - \frac{\sum_j \phi_{jt-\tau}}{n} \right)^{\gamma^\tau}.\end{aligned}$$

In order to evaluate the relative growth in output paths, we compare:

$$\prod_{\tau=1}^t \left(1 - \frac{\sum_j (\frac{\phi_{jt-\tau}}{1-\phi_{jt-\tau}})}{1 + (\sum_j \frac{\phi_{jt-\tau}}{1-\phi_{jt-\tau}})} \right)^{\gamma^\tau} < \prod_{\tau=1}^t \left(1 - \frac{\sum_j \phi_{jt-\tau}}{n} \right)^{\gamma^\tau}$$

which holds due to the fact that the aggregate extraction rate is larger (hence conservation rate is smaller) in the MPE. Moreover the relative difference

$$\prod_{\tau=1}^t \left(1 - \frac{\sum_j \phi_{jt-\tau}}{n} \right)^{\gamma^\tau} / \prod_{\tau=1}^t \left(1 - \frac{\sum_j (\frac{\phi_{jt-\tau}}{1-\phi_{jt-\tau}})}{1 + (\sum_j \frac{\phi_{jt-\tau}}{1-\phi_{jt-\tau}})} \right)^{\gamma^\tau}$$

is increasing as time passes. Hence, both the expected ratio $E\left[\frac{y_{it}^\circ}{y_{it}^*}\right]$ and the expected difference $E[y_{it}^\circ - y_{it}^*]$ are increasing in t . \blacksquare

7.7 Proof of Theorem 3

We show that the Planner's solution admits a finite tipping in the worst case: $\theta_t = \mathbf{1}$. It suffices to show that transition map $\omega_{t+1}^\circ(\omega_t, 1)$ admits a fixed point and this fixed point lies above F .

We first verify that when $\theta_t = \mathbf{1}$, the Planner's optimization problem for multiple countries yields aggregate extraction that is identical to the BAU aggregate extraction rate when the BAU model has only one country. In the former case, all countries have identical preferences, and so the Planner assigns each country an identical carbon quota. The Planner can therefore maximize the long run payoff of the representative agent.

We refer to the one country case as the monopolistic extractor. Side-by-side, the Planner's and monopolistic extractor's optimization problems are

$$\begin{aligned} \max_{\mathcal{E}_t} & \left\{ \log \omega_t \mathcal{E}_t / n + \delta E \left[\tilde{U}(\omega_{t+1}, \mathcal{E}^\circ, \mathbf{1}) \mid \omega_t, \right] \right\} \\ \text{s.t.} & \quad \omega_{t+1} = A(\omega_t(1 - \mathcal{E}_t) - b)^\gamma \quad \text{if } A(\omega_t(1 - \mathcal{E}_t) - b)^\gamma > F \text{ and } = F \text{ otherwise.} \end{aligned} \tag{64}$$

for the planner, and

$$\begin{aligned} \max_{e_t} & \left\{ \log \omega_t e_t + \delta E \left[\tilde{U}(\omega_{t+1}, \mathbf{e}^*, \mathbf{1}) \mid \omega_t, \right] \right\} \\ \text{s.t.} & \quad \omega_{t+1} = A(\omega_t(1 - e_t) - b)^\gamma \quad \text{if } A(\omega_t(1 - e_t) - b)^\gamma > F \text{ and } = F \text{ otherwise.} \end{aligned} \tag{65}$$

for the monopolistic extractor.

The two first order conditions are, respectively,

$$\frac{1}{e_t \omega_t} \omega_t + \delta \mathbb{E}_t \left[\frac{\partial \tilde{U}}{\partial \omega_{t+1}} \frac{\partial \omega_{t+1}}{\partial \mathbb{E}_t} \right] = 0$$

with

$$\frac{\partial \omega_{t+1}}{\partial c E_t} = \begin{cases} A\gamma((1 - \mathcal{E}_t)\omega_t - b)^{\gamma-1}(-\omega_t) & \text{if } A((1 - \mathcal{E}_t)\omega_t - b)^\gamma > F \\ 0 & \text{otherwise} \end{cases}$$

for the Planner, and

$$\frac{1}{e_t \omega_t} \omega_t + \delta \mathbb{E}_t \left[\frac{\partial \tilde{U}}{\partial \omega_{t+1}} \frac{\partial \omega_{t+1}}{\partial e_t} \right] = 0$$

with

$$\frac{\partial \omega_{t+1}}{\partial e_t} = \begin{cases} A\gamma((1 - e_t)\omega_t - b)^{\gamma-1}(-\omega_t) & \text{if } A((1 - e_t)\omega_t - b)^\gamma > F \\ 0 & \text{otherwise} \end{cases}$$

for the monopolist. The marginal extraction cost of increasing e is weakly higher for the Planner. Thus, if the commons has not reached the floor F , we have $\mathcal{E}^*(\omega_t, \mathbf{1}; b, n = 1) \equiv e^*(\omega_t, \mathbf{1}; b, n = 1) = \mathcal{E}^\circ(\omega_t, \mathbf{1}; b, n \geq 1)$.

It suffices to consider to show finite tipping point for the monopolistic country when $\theta_t = 1$. From the first Proposition, the Euler equation is

$$\frac{1}{e_t} = 1 + \frac{\delta \gamma (1 - e_t) \omega_t}{((1 - e_t)\omega_t - b)} \mathbb{E}_t \left[\frac{1}{e_{t+1}} \right] \mathbf{1}_{\{e_t \in X(\omega_t)\}}$$

It is easy to check that if the commons reaches floor F , then $\mathbf{1}_{\{e_t \in X(\omega_t)\}} = 0$ in which case $e_t = 1$ solves the Euler equation. The payoff to the monopolist if the floor is reached is

$$\frac{\log F}{1 - \delta} \tag{66}$$

We construct a bound on F by considering the payoff of the Planner near the natural unstable steady state stock. Specifically, recall the assumption from Section 2.3 that there exists a lowest possible stock $\tilde{\omega}$ satisfying

$$A(\tilde{\omega} - b)^\gamma = \tilde{\omega}, \tag{67}$$

i.e., $\tilde{\omega}$ is a fixed point of the map $A(\omega - b)^\gamma$, and is the minimum fixed point of the map. Earlier, $\tilde{\omega}$ was referred to as a natural steady state. By construction, it is the unstable steady state and so $A(\omega - b)^\gamma < \omega$ if $\omega < \tilde{\omega}$. It therefore follows that there exists $\eta > 0$ satisfying $A(\tilde{\omega} + \eta - b)^\gamma > \tilde{\omega} + \eta$. Now choose \hat{e} to satisfy

$$A((\hat{\omega} + \eta)(1 - \hat{e}) - b)^\gamma > \tilde{\omega} + \eta \tag{68}$$

If the Planner were to consume at this stock and extraction rate permanently, his payoff would be

$$\frac{\log((\tilde{\omega} + \eta)\hat{e})}{1 - \delta} \tag{69}$$

Yet, if the Planner were to maintain the extraction rate \hat{e} permanently, then since $\gamma < 1$, a large enough stock ω yields $A(\omega(1 - \hat{e}) - b)^\gamma < \omega$. Combining this inequality with (68), we invoke the Intermediate Value Theorem to establish a stock $\hat{\omega}$ satisfying

$$A(\hat{\omega}(1 - \hat{e}) - b)^\gamma = \hat{\omega}$$

Thus, $\hat{\omega}$ is a steady state under constant extraction rate \hat{e} . Moreover, this steady state is stable.

Since any tipping point must be weakly smaller than $\hat{\omega}$, the Planner has a finite tipping point if he were to maintain \hat{e} .

Let $\hat{\omega}^{ot}$ denote the t -period forward transition from any initial stock when \hat{e} is used each period. It follows from the definition of $\tilde{\omega}$ in (67) and by (68) that

$$\lim_{t \rightarrow \infty} \hat{\omega}^{ot}(\tilde{\omega} + \eta, \mathbf{1}) = \hat{\omega}$$

and so, starting from $\tilde{\omega} + \eta$ and maintaining \hat{e} , the transition dynamics will take the stock to the higher, stable steady state $\hat{\omega}$.

All along the path, the Planner's payoff will be larger than his payoff in (69) since he consumes the same fraction of an ever increasing stock. Hence, the payoff in (69) is a lower bound on the Planner's long run payoff starting from $\tilde{\omega} + \eta$.

We proceed to show that there is a bound on F at which the planner's payoff in (69) dominates the payoff in (66). We require only $\bar{F} < (\tilde{\omega} + \eta)\hat{e}$ so that $\log(\bar{F}) < \log((\tilde{\omega} + \eta)\hat{e})$. Thus, for $F < \bar{F}$, the Planner will never choose to reach F from $\tilde{\omega} + \eta$ and so some $\omega \leq \tilde{\omega} + \eta$ constitutes a tipping point. This completes the proof. ■

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