Electoral Uncertainty and Divisive Politics as the Foundation of Party Competition

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This paper studies the incentives of organized, policy-oriented groups to form coalitions to nominate candidates for office, and work toward their election; that is, the incentive to form electoral parties. We build on model of legislative parties as “long coalitions” of legislators (Schwartz 1989, Aldrich 1995.) The long coalitions logic does not, however, translate seamlessly to the electoral arena because of uncertainty about the behavior of unpredictable voters. When we retain the Schwartz-Aldrich model’s emphasis on distributive politics (policies with concentrated benefits and diffuse costs), this uncertainty creates a strong incentive to form a single, universalist electoral party, with no groups in opposition. When we consider more divisive policy demands, those with organized groups both in favor and opposed, however, two-party emerges in as equilibrium behavior in a sequential game of coalition formation.
One of the most compelling models for why parties form is that developed by (Schwartz, 1989) and (Aldrich, 1995), which demonstrates that legislators have an incentive to form “long coalitions” over many issues in the legislature’s agenda. Such coalitions are minimally winning but maximally long, amounting to permanent alliances, or parties.

The Schwartz-Aldrich model stands in contrast to the finding of (Weingast, 1979) that legislators will support a norm of universalism. Weingast argues that the advantage legislators might gain from distributing the spoils of victory to only a bare majority of the legislature are offset by the uncertainty of being in the majority. Sure, it would be better if we only gave pork to $\frac{n}{2} + 1$ legislators, but any one legislator’s probability of being one of the lucky ones is only $(\frac{n}{2} + 1)/n$. Better to be universalist.

But if that probability of being in the winning coalition is raised to 1, the coalition is more attractive. Thus, Schwartz and Aldrich conclude that legislators have an incentive to form parties.

However, we observe party development happening outside of legislators, primarily through elections and in particular through the nomination process. Groups seem to form parties so that they can nominate candidates who will give them what they want. They then throw their resources behind their party, helping it win. (Masket, 2009; Bawn et al., 2012; Cohen et al., 2008)

Moving the logic of long coalitions from the legislature to the electorate is not straightforward, however. Elections are not won with certainty. Economic conditions, scandals, personal charisma and its lack, even shark attacks\(^1\) can cause unexpected outcomes. In this paper, we explore the implications of extending the Schwartz-Aldrich model to the electoral arena. We find that parties can form, but only when groups have policy enemies.

1 Modeling Electoral Parties

*Players.* There are $n$ groups. For convenience, we will focus on the case in which $n$ is even. Each group $i$ is distinguished by a policy demand, which if implemented gives the group benefit $B_i$.

*Sequence, stages and strategies.* First, one group is chosen at random to propose a nominating coalition (electoral party.) A nominating coalition is a set of groups who support a single candidate. Then the groups included in the proposal simultaneously decide whether to agree to join the nominating coalition.

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1See, for example Achen and Bartels (2002)
coalition or not. We will refer to this set of decisions – one group proposes a coalition and those included accept or reject as a “stage.” but we emphasize that our model is not a repeated game, rather just a (potentially) long single interaction. Once a group that joins a nominating coalition, it is affiliated, and cannot be invited to join other coalitions.

If all groups are affiliated at the end of a stage, the game ends. If not, the game progresses to another stage with only the unaffiliated groups participating. Again, one of the unaffiliated groups is chosen to propose a nominating coalition, and each included group sequentially accepts or rejects inclusion. This process continues until all groups are affiliated.

Nominating coalitions can range in size from a single group to all \( n \). The group that is chosen at any stage must join its own coalition. This insures that the game has a clear ending (at most \( n \) nominating coalitions will be proposed) and means that we can find subgame perfect equilibria via backward induction.

**Pay-offs.** Pay-offs depend on the policy choices of the candidate who wins the election. Each candidate’s policies and her probability of winning depend on her nominating coalition. Specifically

- A candidate’s probability of winning the election is the fraction of the groups who are in her nominating coalition.

- If elected, the candidate implements the policy demands of all groups in her nominating coalition.

If, for example, there are 6 groups, and groups 1 and 2 support candidate A, A will win with probability 1/3. If A wins, the policy demands of groups 1 and 2 will be implemented and the benefits and costs of those policies will determine pay-offs. Note that candidates are not strategic players in this model; they mechanically implement the policy demands of their supporters.

These assumptions define the basic form of the models we consider below. What remains to be specified is the cost structure of policy demands. That is, how does one group’s policy demand affect other organized groups? As we will show, the cost structure of policy demand critically influences the nature of party competition.

## 2 The Distributive Model of Electoral Parties

We begin with the cost structure used by Schwartz (1989) and Aldrich (1995) to study the formation of parties in a legislative setting. Following Weingast
(1979), they assume that policy demands have concentrated benefits and diffuse costs. That is, policy demands are exclusively of the pork barrel form. This type of policy has been called “distributive” (Lowi 1964, see also Wilson 1980) and we will use that term here.

In the Distributive Model of Electoral Parties (DMEP), each group’s policy demand imposes diffuse costs on the rest of society. A large fraction of these diffuse costs will be borne by unorganized citizens. The unorganized (by definition) will not react systematically to costs imposed on them. Thus, the only relevant portion of the costs of policy demands are those that fall on organized groups. Let $c_i$ thus represent costs of policy demand $i$ born by organized groups, so that each group’s share of the costs is $\frac{c_i}{n}$. As is typical for distributive politics models, we assume for simplicity that benefits and costs are the same for all policy demands, so that $B_i = B$ and $c_i = c$ for all $i$.

We expect $c$ to be only a small fraction of the true costs of each policy demand, and that $c$ will be much less than concentrated benefits $B$.

With the distributive cost structure in place, the game is now fully-specified. Before solving for equilibrium strategies, we note some differences between our electoral model, and existing models of legislative parties. The most important difference is has to do with the impact of a majority coalition. In legislative models, a coalition containing a majority of legislators can implement its policy demands with certainty. This is not the case in the electoral arena, due to the behavior of unorganized voters. In the legislative arena, once $\frac{n+1}{2}$ legislators are in the coalition, there is no advantage to adding more. The minimal majority coalition wins with probability one. In the electoral arena, however, a coalition containing $50 + \epsilon$ percent of the groups has only a $50 + \epsilon$ percent chance of winning. In the legislative arena, a predictable minimum winning coalition offers its members higher expected pay-offs than a universal coalition. In the electoral arena, this is not the case. Expected pay-offs to members of an electoral party with $\frac{n+1}{2}$ groups range from

$$\frac{n+1}{2n} \left( B - \left( \frac{n+1}{2} \right) \frac{c}{n} \right) + \frac{n-1}{2n} \left( - \left( \frac{n-1}{2} \right) \frac{c}{n} \right) = \left( \frac{n+1}{2n} \right) B - \left( \frac{n^2 + 1}{2n^2} \right) c$$

when the remaining groups unite in a single competing party, to

$$\frac{n+1}{2n} \left( B - \left( \frac{n+1}{2} \right) \frac{c}{n} \right) + \frac{n-1}{2n} \left( - \frac{c}{n} \right) = \left( \frac{n+1}{2n} \right) B - \left( \frac{n^2 + 2n - 1}{2n^2} \right) c$$

(1)
when each remaining group supports its own candidate. Both of these are less than the pay-off received when all $n$ groups join a single party, $B - c$. We will refer to this latter case as a universal party. Note however that it is “universal” only in the sense of including all organized interests.

The following strategy, if played by all groups, will bring about a universal party as an outcome.

**Universal Strategy:** When chosen to propose a nominating coalition, include all groups not yet affiliated. When not chosen, only agree to join a proposal (1) if it includes at least a majority of groups not yet affiliated and (2) if the number of groups refusing the proposal and those not included do not together constitute a majority among groups not affiliated at a prior stage.

**Proposition 1.** The condition $B > 2c$ is sufficient for all groups to play the Universal Strategy in subgame perfect Nash equilibrium in the DPEM.

**Proof.** First note that if all players play the Universal Strategy, the game will end in the first stage with the chosen group including all others in its nominating coalition, and all groups accepting. The model’s analytic power comes from the requirement that the Universal Strategy implies the best choice in the many subgames off the equilibrium path. Groups must always prefer the larger nominating coalition, no matter what other groups might propose, or how previous groups have reacted.

Begin with the last possible decision: Two groups (call them Group 1 and 2) remain unaffiliated, one (say, Group 1) has been chosen to propose a nominating coalition and has proposed a coalition that includes Group 2. Should Group 2 join or not? The expected pay-off from joining is

$$
\left( \frac{n-2}{n} \right) Y_{n-2} + \frac{2}{n} \left( B - \frac{2c}{n} \right)
$$

where $Y_{n-2}$ is the expected pay-off conditional on the event that a party supported by any of the already-affiliated groups wins.\(^2\) Group 2’s expected pay-off from refusing to join Group 1’s coalition is

$$
\left( \frac{n-2}{n} \right) Y_{n-2} + \frac{1}{n} \left( B - \frac{c}{n} \right) + \frac{1}{n} \left( - \frac{c}{n} \right).
$$

Group 2 is better off joining Group 1’s coalition whenever $B > \frac{2c}{n}$. Given $B > c$ and $n > 2$, this condition will always be met.

\(^2\)The precise form of $Y_{n-2}$ depends on the extent to which the already-affiliated groups have joined together. Its value does not affect the difference in expected pay-offs.
Precisely same reasoning implies that Group 1 will indeed include Group 2 in its coalition proposal. Given that Group 2 will accept if included, Group 1 has the choice of being in a nominating coalition with one other group or being along.

Moving backward up the tree, knowing that if two groups are unaffiliated, they will coalesce, how should groups behave at a stage when there are only 3 left? More generally, if \( k \) groups remain unaffiliated, how should they behave, given that any remaining groups will consolidate?

Analysis is greatly simplified if we note that when \( B > 2c \), the expected utility of each of \( k \) groups is strictly increasing in the size of the nominating coalition they join.

To see this, first observe that since \( n-k \) groups have already affiliated and nominated candidates, the pay-offs of the remaining unaffiliated groups do not differ when one of the previously-nominated candidates wins. Nor is the probability that this occurs affected by actions of the \( k \) unaffiliated. Thus we can condition expected pay-offs on the event that a previously nominated candidate does not, which we will call \( \tilde{Y} \), roughly consistent with above.

Focusing on expected utility conditional on \( \tilde{Y} \) allows us to ignore the analogs of the \( Y_{n-2} \) terms in expressions (3) and (4). If all players follow the Universal Party Strategy at this stage, conditional expected pay-offs will be

\[
EU_k|\tilde{Y} = B - \frac{kc}{n}.
\] (5)

Compare (5) to conditional expected pay-offs to a nominating coalition of \( j < k \) groups, assuming (as we are) that the remaining \( k-j \) groups will form a single party at the next stage. We have

\[
EU_j|Y = \frac{j}{k} \left( B - \frac{jc}{n} \right) + \frac{k-j}{k} \left( \frac{(k-j)(-c)}{n} \right)
\] (6)

\[
= \frac{jB}{k} - \frac{c}{nk} (2j^2 - 2kj + k^2)
\] (7)

Taking the derivative with respect to \( j \) (ignoring for the moment the fact that \( j \) is not continuous), we have

\[
\frac{\partial EU_j|Y}{\partial j} = \frac{B}{k} - \frac{c(4j - 2k)}{nk}.
\] (8)

Given that \( j < k < n \), (8) is greater than zero whenever \( B > 2c \). This tells us that, under the conditions we are interested in, all groups will take
the action that puts them in the largest possible party. This means that a
group that has been included does better by accepting except when enough
other groups have been either excluded or declined inclusion that the group
in question would be able to join a larger party by declining the current
proposal. Specifically, this occurs when the number of groups who have
declined the proposal plus the number excluded exceeds $\frac{k}{2}$.

The result predicted by Proposition 1 clearly does not describe what
most people think of as partisan electoral competition. It may apply to some
electoral arenas, in particular those not conventionally considered partisan,
such as in many municipalities. That said, it is natural to wonder whether
the restriction to distributive policy demands, which seemed innocuous in
the legislative arena, is appropriate for modeling parties.

3 The Redistributive Model: Policy Enemies

Now consider a cost structure that is completely redistributive, in the sense
that the benefits and the costs are concentrated. Opposition to redistributive
policy demands is organized and focused, in the same way that support is.
A more accurate term might be “divisive policy demands,” as costs need
not be financial. The “cost” of gay marriage, for example, is concentrated
on those who oppose it on religious grounds. We now turn to the question
of how divisive, redistributive issues affect the incentives to form electoral
parties.

In the Redistributive Model of Electoral Parties (RMEP) each policy
demand is supported by one group and opposed by another. Specifically,
we can divide the $n$ groups into two subsets, so that the enemy of each
group $i \leq \frac{n}{2}$ is group $i + \frac{n}{2}$. If group $i$’s policy demand is implemented,
group $i$ receives benefits $B_i$, and $i$’s enemy (group $i + \frac{n}{2}$) receives $-C_i + \frac{n}{2}$.
Conversely, the policy demand of $i$’s enemy (group $i + \frac{n}{2}$) imposes costs $C_i$
on group $i$ and gives benefit $B_{i+\frac{n}{2}}$ to group $i + \frac{n}{2}$. Note $C_i$ represents the
cost imposed on group $i$ by its enemy’s project, that is by project $i + \frac{n}{2}$. In
the RMEP, there are no diffuse costs.

Except for the cost structure, the RMEP’s assumptions about strategies
and pay-offs are identical to the DMEP. In particular, we retain the assumption
that the candidate who wins the election implements all of the policy
demands of the groups in the party that supported her. This assumption
is less innocuous in the RMEP, however. We must now allow for the possi-
bility of an “unnatural party,” that is, a party that contains at least one
pair of enemy groups. The assumption implies that an unnatural party will implement the policy demands of *both* enemies.

On the surface this seems unrealistic. Parties are famously credited with suppressing issues (e.g. slavery, Civil Rights) that would split their coalitions. The impact of our assumption, however, is simply to define payoffs for enemies in an unnatural party as $B_i - C_i$. That is, from group $i$’s point of view, the policy position of a candidate supported by both itself and its enemy $(B_i - C_i)$ is worse than the position of candidate supported by itself and not its enemy $(B_i)$ and better than the position of a candidate supported by its enemy and not itself $(-C_i)$.

The point of the RMEP is to focus on redistributive conflict. We thus assume that $B_i < C_i$ for all groups — the strength of each group’s opposition to its enemy’s project outweighs the intensity of its demand for its own project. Put another way, each group cares more about keeping its enemy’s agents out of power than about electing its own agents. The assumption implies that a group is actually worse off given this assumption, a pair of enemies who find themselves together in an unnatural party will actually be worse off if their candidate is elected than if not. In one sense, this seems puzzling: why would groups ever support a candidate whose electoral success makes them worse off? The puzzle is resolved if we remember that each group would do *even worse* if the winning candidate was supported by its enemy and not itself. It does not seem unreasonable to think that members of a party coalition might find themselves in a Prisoners’ Dilemma-like situation as they maneuver to pull a politician in opposite directions. Indeed, the costs of such maneuvering might make the net pay-off from being in a party with one’s enemy negative even if $B_i > C_i$.

This seemingly simple change to the cost structure makes the model much more complicated to analyze, and we offer a preliminary and incomplete analysis here.

First, note that since we have now peppered the players with a dependent cost structure, the players are no longer interchangeable. Two coalitions of equal size are also no longer interchangeable. Thus the sequence of who is recognized now matters greatly. Even with strategies to address the combinatorics, the game tree is complicated. However, we can take two approaches here. One is to analyze closely the $n = 4$ version of the game. The other is to relax some of the assumptions about sequence.

For the first approach, the equilibrium outcome is a party system in which two groups are in a natural coalition, and the other two groups are isolated. But this an artifact of our stopping rule. Groups benefit from
their policy enemy being isolated, and so a group who is offered a spot in a 
collection in the first round will decline, because accepting the spot will allow 
its policy enemy to coalesce with the remaining group in the next round. 
Declining increases the probability the group itself will be isolated, but it 
also increases the probability that its enemy will be isolated. Since $B < C$, 
this works out to still be preferable in expectation to the sure thing of a 
party system with both the group and its enemy in separate parties. 

This result is not satisfying, however, since the remaining isolated groups 
would surely form a party if given the opportunity. They are merely pre-
vented by the structure of the game, which ends when there are no more 
unaffiliated groups who have not yet had a chance to make a proposal. But 
if we were to relax that stopping rule, the equilibrium is a party system 
in which all groups are isolated. As advantageous as a coalition is, it is 
better to keep your enemy isolated, even if that means you yourself are iso-
lated. That, too, is not satisfying, because two groups could improve by 
coordinating, and they only do not because they know the others will also 
coordinate.

Two things are driving these outcomes, and both are artifacts of the 
model that we do not think necessarily reflect the phenomena we are study-
ing. The first is, as noted, the stopping rule. It would thus be useful to 
relax the structure of the game. 

Second, the result depends on the perfect symmetry of the groups, and 
thus the symmetry of the expected outcomes from various arrangements. 
One group’s being in a coalition with one non-enemy is the same as any 
other non-enemy. This is unrealistic for a variety of reasons. For one, some 
groups are more powerful than others. It may make sense for groups to 
seek out those more valuable allies to ensure that their enemies do not ally 
with them. And that could make the strategy of isolating one’s enemy less 
promising.

We describe these two approaches in turn.

3.1 Relaxing the structure of the game.

Instead of selecting a proposer and allowing groups to respond in sequence, 
we propose that the same actors are already arranged into some sort of party 
system, $P$. That could be universalism, a natural or unnatural two party 
system, a multiparty system or all groups unaffiliated. These of course in-
clude the outcomes that were equilibria in the four-group sequential version 
of the game.

It will be useful to have notation to refer to coalitions and party systems.
Let $P_{ij-kl}$ denote the party system in which $i$ and $j$ are in one party and $k$ and $l$ in another. Thus, $P_{13-2-4}$ denotes a party system in which Groups 1 and 3 are in a party together and Groups 2 and 4 are each alone in separate parties. Similarly, let $ijk$ denote the party (or the proposal for a party) that contains Groups $i$, $j$ and $k$, so that $P_{13-24}$, for example is the party system composed of parties 13 and 24.

Given $P$, we ask which party systems would change to new party systems, following two simple rules.

1. Any group can unilaterally leave a coalition. This is the only thing that one group can unilaterally do.

2. Any group can propose to join an existing coalition (or join with another isolated group). They will be allowed to join if all members of that coalition agree to accept the new member.

### 3.1.1 Leaving Coalitions: Which coalitions are Nash Equilibria?

First, we consider which constellations of coalitions are Nash Equilibria, in the sense that no group will want to unilaterally abandon them. In any arrangement, any one group that is part of a coalition can unilaterally defect from that coalition. When would they want to.

Perhaps not surprisingly, every arrangement can be supported as a Nash Equilibrium, depending on the values of $B$ and $C$. However, there are some that are always in equilibrium. Groups never wish to unilaterally leave coalitions that do not include their natural enemies. Coalitions with natural enemies are sometimes not preferable to going it alone.

**Case 1:** A group considers leaving a coalition in which its natural enemy is *not* a member.

Coalitions without natural enemies are always Nash Equilibria. More precisely, no group is better off by leaving a coalition in which their natural enemy is not a member.

Such a deviation could occur in a number of cases. Using the four-group scenario as a model and focusing on Group 1, for example, we might have:

- Group 1 could leave its coalition in $P_{124-3}$, creating $P_{24-1-3}$.
- Group 1 could leave its coalition in $P_{12-34}$, creating $P_{34-1-2}$.
- Group 1 could leave its coalition in $P_{12-3-4}$, creating $P_{1-2-3-4}$. 
In all of these cases, the new configuration is not preferred to the old configuration. The group reduces the probability that they will be in the winning coalition, thereby reducing the probability that their policy is enacted. But their abandoning their coalition does not affect the probability that the enemy wins, either alone or in another coalition. If $B$ and $C$ are positive, this cannot be preferred.

Let $m_1$ be the size of the coalition the group is considering leaving, and let $m_E$ be the size of the coalition that the groups’ enemy is in. The group should leave if the utility from going it alone is superior to that of staying in the coalition, which is when:

$$\frac{m_1}{n}B - \frac{m_E}{n}C < \frac{1}{n}B - \frac{m_E}{n}C$$

Equation 9 implies $m_1 < 1$, which is not possible if $m_1$ is the size of a coalition. Thus, a group will never wish to unilaterally leave a coalition in which its enemy is not a member.

**Case 2:** A group considers leaving a coalition in which its natural enemy is a member.

The result in Case 1 is not surprising. There is no downside to being in a coalition that does not include an enemy, and larger coalitions are ceteris paribus better. But coalitions which contain natural enemies are less stable. A group may be better off by leaving a coalition in which their natural enemy benefits from their own contribution to the probability of the coalitions’ victory.

Again, such a deviation could occur in a number of cases, and again, we illustrate with the four-group scenario, focusing on Group 1:

- Group 1 could leave its coalition in $P_{1234}$, creating $P_{234-1}$.

- Group 1 could leave its coalition in $P_{123-4}$, creating $P_{23-4-1}$.

- Group 1 could leave its coalition in $P_{13-24}$, creating $P_{1-3-24}$.

- Group 1 could leave its coalition in $P_{13-2-4}$, creating $P_{1-2-3-4}$.

In these cases, the group reduces the probability that their own policy wins, possibly by a lot. But they also reduce the probability that their
enemy wins, by no more than and usually less than their own reduction. So if $C_i$, the cost of letting the enemy win, is sufficiently greater than $B_i$, the benefits of winning, abandoning the coalition is preferred.

What “sufficiently greater” is depends on how large the original coalition is, and thus how much one group’s exit hurts the rest of the coalition.

Again, let $m_1$ be the size of the coalition a group considers leaving. The group should leave if:

$$\frac{m_1}{n}(B - C) < \frac{1}{n}B - \frac{m_1 - 1}{n}C$$

Equation is satisfied under the following condition:

$$m_1 - 1 < \frac{C}{B}$$

This condition will not be often met. It requires that the costs of the enemy’s policy being enacted must be $m_1 - 1$ times larger than the benefits from winning, which is a big difference for large coalitions. This may explain why some unnatural coalitions hold together for so long. The New Deal coalition included both northern liberals and southern racial conservatives. But neither group wanted to abandon a winning coalition and go alone. This may also explain why the New Deal coalition eventually broke apart the way that it did. For segregationists, the costs of advancing the Civil Rights agenda were great, perhaps many times greater than the benefits southern Democrats received from being in the coalition. Once the groups that wanted Civil Rights began to get serious about it, the costs of their success became to much for segregationists to take.

3.1.2 Joining a new coalition

The previous section considered when one group would like to unilaterally leave a coalition. But groups that consider abandoning a coalition will not necessarily have no place to go. Another rival coalition may accept them.

In this section, we consider which coalitions accept a new member:

**Case 1:** A group wishes to join a coalition that does not include its policy enemy.

Here, we again illustrate with Group 1, now as the outsider wishing to join the coalition.

- Group 1 wishes to join the existing coalition of Groups 2 and 4, creating $P_{124-3}$ from $P_{24-1-3}$
• Group 1 wishes to join Group 2 (or Group 4), creating $P_{12-3-4}$ or $P_{12-34}$ (or $P_{14-3-2}$ or $P_{14-32}$) from $P_{1-2-3-4}$ or $P_{1-2-34}$

If the petitioning group is not the natural enemy of any group in the coalition, then the problem is essentially the same from the point of view of every member in the group. The decision of the members of that group depend on whether their own enemy is also in that coalition or not. For those groups whose own enemy is not in the coalition, let $m_E$ be the number of groups in their enemy’s coalition. For them, the new group should be admitted if

$$\frac{m_2}{n} B - \frac{m_E}{C} < \frac{m_2 + 1}{n} B - \frac{m_E}{C}$$  \hspace{1cm} (12)

For those groups whose enemy is in their same coalition, they should admit the new group if

$$\frac{m_2}{n} (B - C) < \frac{m_2 + 1}{n} (B - C)$$  \hspace{1cm} (13)

Equation 12 is straightforwardly always true, since $m_2 < m_2 + 1$. Groups should always admit non-enemies into a natural party. But equation 13 is never true if $B < C$, which we have assumed it is. When a group shares a party with its enemy, it actually prefers for its party to fail, and bringing in more resources doesn’t help that.

While we do not model any internal dynamics in a coalition, we note that this obviously creates tensions within an unnatural party. Not only do groups that are enemies not want their enemy to be in the same party with them, but the other groups in that party also will not want enemy pairs within their party, as they would oppose efforts to expand it.

**Case 2:** A group wishes to join a coalition that does include its policy enemy.

We now consider cases wherein the coalition the group is interested in includes its own policy enemy, such as

• Group 1 wishes to join the existing coalition of Groups 2 and 3 (or, Groups 4 and 3) creating $P_{123-4}$ (or $P_{134-2}$)

• Group 1 wishes to join Group 2 (or Group 4), creating $P_{12-3-4}$ or $P_{12-34}$ (or $P_{14-3-2}$ or $P_{14-32}$) from $P_{1-2-3-4}$ or $P_{1-2-34}$
From the point of view of the groups that are not the petitioning group’s policy enemy, the situation is the same as in Case 1. But the policy enemy (Group 3 in the four-group example) will want to admit the new group if

$$
\frac{m_2}{n} B - \frac{m_1}{n} C < \frac{m_2 + 1}{n} (B - C)
$$

(14)

This condition is met when

$$
\frac{1}{m_2 + 1 - m_1} < \frac{C}{B}
$$

(15)

That is, a group will allow a policy enemy into the group if the benefits from their own project, which are more likely to be realized with the resources brought by the new member, are enough greater than the costs of the new member’s project. But since we have stipulated $B < C$, this will not be the case.

3.1.3 Defecting to a new party

The treatment of a group abandoning their coalition to nothing in section 3.1.1 is unrealistic, as a group is unlikely to defect from an existing party to go it alone, but they may abandon it for a new one. This section puts the analysis from section 3.1.1 together with section 3.1.2 to address when a group will be able to leave one party and join another.

Two conditions must be met for this to occur. A group must want to move, and the coalition they’d like to move to must be willing to take them. The first condition is similar to section 3.1.1 above. The second condition is the same as outlined in section 3.1.2.

There are now three cases of interest. (1) Is the group leaving a coalition with its policy enemy? (2) Is the group interested in joining a coalition with its enemy? and (3) Is the group’s enemy in neither their old nor proposed new coalition.

**Case 1:** A group wishes to leave a coalition with its policy enemy and join one without the enemy.

**Petitioning group’s perspective:** The group will want to make the move if:

$$
\frac{m_1}{n} (B - C) < \frac{m_2 + 1}{n} B - \frac{m_1 - 1}{n} C
$$

(16)
This condition is met when

\[ m_2 + 1 - m_1 > \frac{C}{B} \tag{17} \]

Equation 17 is of course not satisfied whenever the new coalition, after the group has joined it, is still smaller than the old one. In such a case, the group reduces their own chances of victory while hurting their enemy’s only by the same amount. Since \( C > B \), that is not enough. How much larger the new coalition needs to be depends on how much larger \( C \) is than \( B \), in a relationship similar to the incentives in the unilateral defection case above.

Coalition’s perspective: As noted above, if the coalition contains any internal policy enemies, some of its members will not want to get bigger so long as \( B < C \). But if it contains no policy enemies, everyone in the coalition will want to accept the new member. So the only coalitions that a group can join will be natural parties.

Case 2: A group wishes to leave a coalition that does not include its policy enemy and join one that does.

Petitioning group’s perspective: Here, the group will want to join the party if the following holds:

\[
\frac{m_1}{n} B - \frac{m_2}{n} C < \frac{m_2 + 1}{n} (B - C) \tag{18}
\]

which is satisfied by

\[ m_1 - (m_2 + 1) > \frac{C}{B} \tag{19} \]

So the group would only wish to join a coalition with its enemy if the new coalition is so much larger than the relative difference between \( C \) and \( B \) that what the group makes up for in its increase in \( B \) that the small contribution the group makes to its enemy is offset by the big gains in the probability of enacting its own coalition.

The condition for the group to be accepted is given in section 3.1.2.

Case 3: A group considers a move from one coalition to another, neither of which contain its natural enemy.

First, note that Group 1 would prefer to be in the proposed coalition.
Let \( m_1 \) be the size of Group 1’s original coalition, and \( m_2 \) be the size of the new coalition, before the group joins. If the group’s policy enemy is not in either coalition, then its payoff from switching is

\[
\frac{m_1}{n} (B) - \frac{m_E}{C} < \frac{m_2 + 1}{n} (B) - \frac{m_E}{C}
\]

(20)

which is satisfied whenever the new coalition is bigger than the old one \( (m_1 < m_2 + 1) \).

Meanwhile, the condition for acceptance is the same as in Case 1. Groups without their enemies in the coalition will welcome the newcomer, while groups with their enemies will not want their coalition to grow.

3.1.4 Discussion

The results here suggest that natural parties are more stable and more likely to form than unnatural parties, but there are conditions under which we would see unnatural parties. That is consistent with the observation that we sometimes do observe unnatural parties, and that they do tend to collapse. See for example the unnatural New Deal coalition in the United States.

The conditions for unnatural parties depend a great deal on the value of \( \frac{C}{B} \). When the costs of the enemy winning are high relative to the benefits of a group’s own project, then that group will not want to contribute to the victory of their enemy, and will wish to leave the party. Similarly, when \( \frac{C}{B} \) is large, groups will not want to accept enemies into their own party. The conditions for not wanting to leave, for wanting to join and for accepting enemies into an unnatural coalition are that \( B \) must be much larger than \( C \), which we have ruled out.

It is reasonable to imagine that there are many situations in which \( C \) is not only larger than \( B \), but that it is much larger. There are a number of cases where groups are largely fighting to prevent other groups from weakening the status quo, be it through gay marriage or the erosion of labor regulations.

We also observe that there are perverse incentives for groups in coalitions that contain enemy pairs. As noted, this raises issues that are unmodeled here. First, enemy pairs within the same party have incentives that are at odds with the incentives of the rest of the party, as they prefer not to win, while the rest of the party prefers to win. Second, such a situation creates incentives to drive out one of the two groups, even for the coalition members that have no policy conflicts with that group.
3.2 Allowing the strengths of the groups to vary

In this section, we vary the impact that each group’s support has on its candidate’s chance of winning. Let $r_i$ denote the proportion of electorally relevant resources controlled by group $i$. Because the $r_i$ are proportions, they sum to one. We thus assume that the probability that a candidate supported by coalition $S$ wins is $\sum_{i \in S} r_i$.

For simplicity, consider a simple scenario in which $n = 4$, so that groups 1 and 3 are enemies, as are groups 2 and 4. We will also assume that groups 1 and 2 are equal in strength, and each stronger than their (equally weak) enemies, that is $r_1 = r_2 = r_s$, $r_3 = r_4 = r_w$ and $r_s > r_w$. Thus, each enemy pair is composed of a strong and a weak group. Note that these assumptions imply that $r_s + r_w = 0.5$.

3.2.1 Case 1: Strong group makes first proposal

Suppose that a strong group is recognized in the first round. Let this be Group 1; the analysis for Group 2 will be identical. Group 1’s ideal party system is $P_{124-3}$: Groups 2 and 4 will both contribute resources to Group 1’s candidate without demanding policies that impose costs on Group 1.\footnote{Note that Group 1’s expected pay-off from the 1-2-4 coalition, $(1 - r_w)B - r_wC$, is always greater than its pay-off from universalism, $B_i - C_i$.}

Moreover, Group 1’s enemy, Group 3 is alone in a resource-poor party. The question is whether 2 and 4 would accept 1’s proposal of 124. Would enemy Groups 2 and 4 prefer to join a large party together, or remain available for a different coalition in the next round?

<table>
<thead>
<tr>
<th>Party System</th>
<th>$EU_1$</th>
<th>$EU_2$</th>
<th>$EU_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{124-3}$</td>
<td>$(2r_s + r_w)B - r_wC$</td>
<td>$(2r_s + r_w)(B - C)$</td>
<td>$(2r_s + r_w)(B - C)$</td>
</tr>
<tr>
<td>$P_{12-3-4}$</td>
<td>$2r_sB - r_wC$</td>
<td>$2r_sB - r_wC$</td>
<td>$r_wB - 2r_sC$</td>
</tr>
<tr>
<td>$P_{14-2-3}$</td>
<td>$(r_s + r_w)B - r_wC$</td>
<td>$r_sB - (r_s + r_w)C$</td>
<td>$(r_s + r_w)B - r_wC$</td>
</tr>
</tbody>
</table>

Table 1: Pay-offs for subgame after Group 1 has proposed 124.

Suppose that Group 1 has proposed 124 and that Group 4 responds first, as depicted in panel (a) of Figure 1, with pay-offs given in Table 1. Note that if both Groups 2 and 4 decline Group 1’s offer, the game will progress to the next round with Groups 2, 3 and 4 unattached. The outcome of this subgame is $P_{1-23-4}$ with probability 0.5 and $P_{1-234}$ with probability...
0.5 when \( \frac{C}{B} < \frac{2r_w}{r_s} \) and \( P_{1-23-4} \) with probability 1 when \( \frac{C}{B} > \frac{2r_w}{r_s} \). (See Appendix A.1.)

At node II in Figure 1(a), if Group 2 accepts, the party system will be \( P_{124-3} \) and Group 2’s pay-off will be

\[
EU_2(P_{124-3}) = (2r_s + r_w)(B - C). \tag{21}
\]

If Group 2 declines, it will coalesce in the next round with Group 3. The resulting party system of 1-4 vs 2-3 offers Group 2

\[
EU_2(P_{14-23}) = (r_s + r_w)(B - C). \tag{22}
\]

Given \( B_i < C_i \), \( EU_2(P_{14-23}) > EU_2(P_{124-3}) \), so Group 2 declines at node II. This means that if Group 4 accepts at node I, the outcome will be \( P_{14-23} \).

At node III, if Group 2 accepts, 3 and 4 will coalesce in the next round, the party system will be \( P_{12-34} \), and Group 2 will get

\[
EU_2(P_{12-34}) = 2r_sB - 2r_wC. \tag{23}
\]

If Group 2 declines, the game will move on to the next round with groups 2, 3 and 4 not yet attached to any candidate, a subgame which we can denote as \( S_{234} \). As shown in the Appendix, \( S_{234} \) will either result in \( P_{1-23-4} \) with probability 1, or in \( P_{1-23-4} \) or \( P_{1-23-4} \) each with equal probability.

From Group 4’s point of view, all three of the possible outcomes from declining at Node I are worse than \( P_{14-23} \), the consequence of accepting.
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Thus, if Group 4 responds first, Group 1’s 124 proposal will result in $P_{14-23}$.

The other possibility is that Group 2 responds first, as depicted in Figure 1(b). At node II, if Group 4 accepts, the party system is $P_{124-3}$, and 4’s pay-off is

$$EU_4(P_{124-3}) = (2r_s + r_w)(B - C).$$

(24)

If Group 4 declines, the party system is $P_{12-34}$, yielding

$$EU_4(P_{12-34}) = 2r_wB - 2r_sC.$$  

(25)

Group’s 4 choice depends on the specific parameter values. If

$$\frac{2r_s - r_w}{r_w} < \frac{C}{B},$$

(26)

then Group 4 prefers the overall weaker coalition ($P_{12-34}$) without its enemy to the stronger coalition with the enemy ($P_{124-3}$), and will therefore decline.

Suppose the condition in (26) holds, so that Group 4 will decline if 2 has accepted, leading to a party system in which both strong groups are in one party and both weak groups in the other. This outcome is better for Group 2 than any outcome that could result from node III.

Thus if condition (26) holds, Group 2 will accept, Group 4 will decline, and the party system will be $P_{12-34}$.

Next, suppose that (26) does not hold, so that Group 4 will accept if Group 2 accepts, with $P_{124-3}$ as the result. If Group 2 declines, Group 4 will accept, because the pay-off from $P_{14-23}$

$$EU_4(P_{14-23}) = (r_s + r_w)(B - C).$$

(27)

exceeds Group 4’s pay-off for both $P_{234}$ and $P_{1-23-4}$, the possible outcomes for $S_{234}$. Thus at node III, 4 will accept. Group 2’s choice at node 1 is thus between $P_{124-3}$ (Accept) and $P_{14-23}$ (Decline). Previous comparison of equations (21) and (22) established that Group 2 prefers $P_{14-23}$, so will decline, with $P_{14-23}$ as the result.

We have now established that if Group 1 proposes its unconstrained optimal coalition of 124, the result will be either $P_{14-23}$ with certainty if (26) does not hold, or a 50-50 chance of $P_{14-23}$ and $P_{12-34}$ if (26) does hold.

Group 1 prefers $P_{12-34}$ to either of the outcomes from proposing 124: if it proposes 12, will Group 2 accept? If 2 accepts, Groups 3 and 4 will coalesce in the next round, and the party system will be $P_{12-34}$. If 2 declines,
we move to \( S_{234} \). Analysis in Appendix 1 shows that the outcome of \( S_{234} \) is either \( P_{1-23-4} \) with probability 1 when

\[
\frac{2r_w}{r_s} < \frac{C}{B}
\]  

(28)

or \( P_{1-23-4} \) with probability 0.5 and \( P_{1-234} \) with probability 0.5 otherwise.

Thus when (28) holds, the question is whether Group 2 prefers \( P_{12-34} \) to \( P_{1-23-4} \). Group 2 will accept Group 1’s proposal of 12 in this case when

\[
\frac{C}{B} < \frac{r_s - r_w}{r_w}.
\]  

(29)

That is, when the resources of a strong group \( (r_s) \) are relatively high and the damage done by the enemy’s project \( (C) \) are small relative to the benefits of its own project, strong Group 2 prefers the party system in which it allies with the other strong group to the system in which it has weak group 3 as its ally and its enemy, weak group 4 is unallied. The latter could only happen if the other strong group (Group 1) had marginalized itself with an unsuccessful proposal.

When (28) does not hold, the question is whether Group 2 prefers \( P_{12-34} \) to a 50-50 chance of \( P_{1-23-4} \) or \( P_{1-234} \). In this latter case, Group 2 accepts Group 1’s proposal of 12 when

\[
\frac{3r_w - 2r_s}{r_s - r_w} < \frac{C}{B}.
\]  

(30)

To summarize, when

\[
\frac{2r_w}{r_s} < \frac{C}{B} < \frac{r_s - r_w}{r_w}
\]  

(31)

or

\[
\frac{3r_w - 2r_s}{r_s - r_w} < \frac{C}{B} < \frac{2r_w}{r_s},
\]  

(32)

Group 1 successfully proposes 12, Group 2 accepts and the party system is \( P_{12-34} \).

In the opposite cases, Group 2 should clearly not propose 12 because it will be declined, and Group 1 will be marginalized. When

\[
\max \left( \frac{2r_w}{r_s}, \frac{r_s - r_w}{r_w} \right) < \frac{C}{B}
\]  

(33)

a proposal of 124 or 14 will both lead to \( P_{14-23} \).
Table 2: Payoffs relevant to Group 3’s proposal of 234

When

\[
\frac{C}{B} < \min \left( \frac{2r_w}{r_s}, \frac{3r_w - 2r_s}{r_s - r_w} \right),
\]

(34)

Group 1’s best choice is to propose 124, which (as noted above) will lead to
\(P_{12-34}\) or \(P_{14-23}\), each with probability 0.5.

3.2.2 Case 2: Weak group recognized

Now suppose that the first group recognized to propose an electoral coalition
is Group 3, a weak group. (The analysis for Group 4 is identical.) Being
weak, Group 3 is not an attractive coalition partner. Given that Group 1
could not achieve its ideal outcome, \(P_{12-34}\), it is reasonable to conjecture
that Group 3 will not be able to achieve its own ideal, \(P_{1-234}\) either.
consequences of \( S_{124} \) (either \( P_{12-3-4} \) with probability 1 or a 50-50 chance of \( P_{12-3-4} \) or \( P_{124-3} \), depending on parameter values) are derived in the Appendix.

If Group 4 responds first, we will be in panel (a) of Figure 2, in which Group 2’s dominant strategy is to decline, and coalesce with Group 1 in the next round. Group 4 thus accepts and the result is \( P_{12-34} \). If Group 2 responds first (panel (b)), Group 4 will accept if 2 declines and decline if 2 accepts. Group 2 prefers to decline, and again the outcome is \( P_{12-34} \). Thus, the consequence to Group 3 of making its ideal proposal is to end up a weak ally while its enemy has a strong ally.

Alternatively, Group 3 could propose 23. If Group 2 declines, we move to \( S_{124} \), which (as shown in the Appendix) results in \( P_{12-3-4} \) when

\[
\frac{C}{B} < \frac{r_w}{2r_s}
\]  

(35)

and a 50-50 chance of \( P_{12-3-4} \) or \( P_{124-3} \) when not.

When (35) does not hold, then, weak Group 3’s optimal proposal is 23, resulting in party system \( P_{14-23} \).

When (35) does hold, the best that Group 3 can do is propose 34, leading to \( P_{12-34} \).

To summarize: The outcome of the RMEP with groups of different strengths will be a natural two-party system.

### 3.2.3 Discussion

By varying the strength of the groups, the RMEP gives the result we observe in the world. Varying the strength makes the model more realistic, but it is important to understand why the change matters.

Varying group strength breaks the natural symmetry among groups, so that groups are no longer indifferent between which group is their ally.

### 4 Implications and future research

We began by noting that the distributive model used in the Schwartz-Aldrich model of party formation, with its focus on the legislative arena, may not be applicable to parties formed in elections. But we observe parties forming in elections. We thus attempted to move the model to the electoral arena, where it behaves differently.

When taken to the electoral arena, the distributive model (DMEP) gives us universalism in elections. However, when we introduce policy enemies,
universalism is no longer the only expected outcome. The model with policy
enemies (RMEP) is somewhat less tractable than the distributive model, but
it does lead to multiple parties, and generally toward natural parties.

Still, we think we have evidence that the key to understanding multiple
parties in the electorate is through policy enemies. It’s notable that extreme
enmity (when $C_B$ is large) is particularly conducive to multiple parties.

There are still other directions we may wish to move, particularly since
the analysis here is still tentative. For one, we would like to expand the
analysis in section 3.2 to $n > 4$.

We may also wish to become more rigorous about the long-term results
from the analysis in section 3.1. We show that party systems will tend to
transition toward natural party systems under some conditions, but it would
be useful to model the results of such transitions after many iterations.

Finally, we may also wish to explore differences in risk attitudes. Since
uncertainty is such a major element of the model, it may make sense to
account for it.

In any event, we think policy enemies are an important part of the
explanation for parties forming in the electorate.
A Appendix: Decline-decline subgames

This appendix analyzes the subgames that would result when a three group party is proposed in the first round and both included groups decline. These subgames are never on the equilibrium path, but establishes the constraints the first proposer must observe in order to avoid a failed proposal.

Let $S_{ijk}$ denote the subgame in which groups $i$, $j$ and $k$ remain.

A.1 One strong and two weak groups remain.

First, consider what would happen in $S_{234}$, as a step toward understanding the choices at Node III in Figures 1 and 2. This is the subgame in which strong group 1’s proposal has failed to attract allies. Table 2 depicts relevant expected pay-offs, excluding those from clearly dominated party systems like $P_{1-3-24}$ and $P_{1-2-3-4}$.

For Group 3, we clearly have $P_{1-234} >_3 P_{1-23-4} >_3 P_{1-2-34}$.

Group 2 clearly prefers $P_{1-23-4}$ to both $P_{1-234}$ and $P_{1-2-34}$, but the ranking of the last two alternatives depends on the relative magnitudes of $r_s$ and $r_w$. Group 2 prefers $P_{1-2-34}$ to $P_{1-234}$ when

$$\frac{2r_w}{r_s} < \frac{C}{B}. \quad (36)$$

When condition (36) holds, strong Group 2 prefers being isolated to being part of a party that includes its enemy and the other weak group. Similarly, of the options available in this subgame, Group 4 most prefers the party system in which it has an ally and its enemy is isolated ($P_{1-2-34}$.) Group 4 prefers $P_{1-2-34}$ to $P_{1-234}$ when

$$\frac{r_s + r_w}{r_w} < \frac{C}{B}. \quad (37)$$

Note that the left-hand side of (37) is greater than the left-hand side of (36), so that condition (37) implies (36). Conversely, if 36 does not hold, then neither does (37). Thus, whenever strong Group 2 prefers the 3-group unnatural party to isolation, Group 4 will also.

Given these preferences, we can now consider best proposals given that each group has an equal chance of being allowed to make a proposal.

If Group 2 is recognized, it will propose a coalition with Group 3, 3 will accept and the outcome will be $P_{1-23-4}$. 
If condition (36) does not hold, Group 3 can propose the 234 coalition and it will succeed, because, as noted, the condition under which Group 4 prefers $P_{1-234}$ to $P_{1-2-34}$, is implied by the converse of (36.). If (36) does hold, Group 3 will propose a coalition with Group 2 and the outcome will be $P_{1-23-4}$.

Group 4 cannot successfully propose $P_{1-2-34}$ under any relevant parameter values, because Group 3 would decline and coalesce with Group 2 in the next round. Group 4’s best proposal depends on whether (36) holds or not.

If (36) holds, $P_{1-234}$ will not succeed because Group 2 prefers isolation. Knowing that Group 2 will decline, Group 3 does too. So if Group 4 proposed $P_{1-234}$, the outcome would be $P_{1-23-4}$. This is same outcome that, given (36), results from a proposal by Group 2 or Group 3.

In the case where (36) does not hold, (36) does not either, and Group 4 prefers $P_{1-234}$ to $P_{1-23-4}$. If Group 4 proposes $P_{1-234}$, the proposal will succeed if Group 3 responds first. Group 3 will agree to its own first choice, and 2 (given that (36) holds) will then also join. If Group 2 responds first, it will decline, and Group 3, at that point facing a choice between coalescing immediately with weak Group 4 or waiting to coalesce with strong Group 2, will choose the latter, and the result will be $P_{1-23-4}$.

Thus if (36) holds, the outcome is $P_{1-23-4}$ with probability 0.5, occurring either because Group 2 made a proposal, or responded first to Group 4’s proposal. The outcome is $P_{1-234}$ with probability 0.5, either because Group 3 made a proposal, or responded first to Group 4’s proposal.

Thus we have demonstrated:

**Claim A1:** When $\frac{2w}{r_s} < \frac{C}{b}$, $S_{234}$ results in $P_{1-23-4}$ with probability 1. When $\frac{2w}{r_s} > \frac{C}{b}$, $S_{234}$ results in $P_{1-23-4}$ with probability 0.5 and $P_{1-234}$ with probability 0.5.

### A.2 One weak and two strong groups remain.

The analysis for the subgame in which one weak and two strong groups remain is similar. Consider $S_{124}$, the subgame that would occur if a proposal by Group 3 failed to gain allies. Expected pay-offs of relevant proposals are depicted in Table 1.

Parallel to Group 3 in the previous case, Group 1 has clear preferences

$$P_{124-3} \succ_1 P_{12-3-4} \succ_1 P_{14-2-3}.$$
Groups 2 and 4 each have a clear first choice, $P_{12-3-4}$ and $P_{14-2-3}$, respectively, but may or may not prefer the three-way coalition with their enemy to running in isolation. In particular, Group 2 prefers $P_{124-3}$ to $P_{14-2-3}$ when

$$\frac{C}{B} < \frac{1}{2r_s}. \quad (38)$$

Group 4 prefers $P_{3-124}$ to $P_{3-12-4}$ when

$$\frac{C}{B} < \frac{r_w}{2r_s}. \quad (39)$$

The right-hand side of (38) is larger than the right-hand side of (39), implying that whenever Group 4 prefers coalescing to isolation, Group 2 does also.

As in the previous case, when Group 2 is recognized, it can successfully propose its first choice, $P_{12-3-4}$, and Group 1 will accept, since the alternative is $P_{12-3-4}$.

When Group 1 is recognized, it can successfully propose its first choice, $P_{124-3}$ when (39) holds; otherwise it proposes $P_{12-3-4}$.

When Group 4 is recognized, its best option is to successfully propose $P_{124-3}$ when (39) holds. Otherwise, it can cannot successfully propose any coalition; $P_{12-3-4}$ is the result.

To summarize

**Claim A2:** When $\frac{C}{B} < \frac{r_w}{2r_s}$, $S_{124}$ results in $P_{12-3-4}$ with probability 0.5 and $P_{124-3}$ with probability 0.5. When $\frac{C}{B} > \frac{r_w}{2r_s}$, $S_{124}$ results in $P_{12-3-4}$ with probability 1.
References


