DOES WEALTH INEQUALITY HELP INFORMAL INSURANCE?

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ABSTRACT. This paper investigates the effects of inequality in the presence of voluntary risk-sharing. In any period, an agent's resources are composed of his share of a secure endowment (wealth, land) and a random component (labor income). To be sure, the distribution of wealth does not affect the Pareto optimal payoff vectors but, by changing the autarchic utilities, affects the set of these payoff vectors that are self-enforcing. When risk-sharing is not perfect, a transfer of wealth from an agent to the other is shown to cause the frontier of the self-enforcing payoff vectors to pivot. The more power an agent has, the larger the change in his utility from an increase in his share of wealth.

Surprisingly, inequality is shown to help risk-sharing in a large range of cases. Regressive transfers of wealth increase the likelihood of perfect insurance all utility functions displaying hyperbolic absolute risk aversion (HARA) at the exception of the constant absolute risk aversion for which, as it is well known, wealth effects are absent. Moreover, introducing wealth inequality between the agents can increase the sum of utilities of the agent, thereby being desirable for a social planner maximizing a symmetric and additive welfare function.

These results have important welfare and policy implications for redistribution programs such as land reform.

We also show that when risk aversion is decreasing individuals tend to prefer sharing risk with poorer partners.

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VERY PRELIMINARY DRAFT. Comments welcome. Please address correspondence to gg58@georgetown.edu.

1. INTRODUCTION

In most of the developing world, people are exposed to substantial, even catastrophic, risk and mitigating risk is a central concern. A high degree of dependence on agricultural production, widespread poverty and the lack of access to formal insurance and credit, make the need for consumption smoothing particularly acute. As a result, most households in low-income countries deal with adverse economic events through informal insurance, arrangements arising between individuals and communities on a personalized basis (Morduch 1999). Individuals typically respond to the large fluctuations in their income by engaging in informal risk-sharing: providing each other with help, gifts and transfers with some expected reciprocity.

There is considerable evidence of the presence of *some* but *limited* insurance in village communities (Deaton 1992, Townsend 1994, Udry 1994, Attanasio and Davis 1996, Jalan and Ravallion 1999, Grimard 1997, Gertler and Gruber 2002, Ligon, Thomas and Worrall 2000, Foster and Rosenzweig 2002). These limitations could result from various incentive constraints. Asymmetry of information, moral hazard and lack of enforceability are all potential impediments to risk sharing. The most important constraint appears to arise from the lack of enforceability of risk-sharing agreements.[1] Explicit, legally binding, and credibly enforceable contracts not being available, these agreements must be designed to elicit voluntary participation. This constraint often seriously limits the extent of insurance informal risk-sharing agreements can provide.

There is a growing theoretical literature on self-enforcing risk-sharing agreements. Some important contributions are Posner (1981), Kimball (1988), Coate and Ravallion (1993), Kocherlakota (1996), Kletzer and Wright (2000), Ligon et al. (2000), and Genicot and Ray (2003). However, most studies focus on risk-sharing among identical agents. Two exceptions are Krueger and Perri (2002) and Sadoulet (2001) who consider different sources of heterogeneity than us. We will discuss the parallels in more details in Sections 4 and 5.

In this paper, we study risk-sharing between individuals facing the same risk but differing in their mean income, and investigate the impact of this inequality on the agreement they engage in. Consider two agents whose resources, at any date, are composed of their share of a secure endowment, that we shall call their permanent income or wealth, and a random component. If it were not for their wealth, the two agents would be identical. Changes in the distribution

[1]Udry (1994), for instance, finds this constraint to be the most important in describing the structure of reciprocal agreements in rural northern Nigeria.

of wealth do not affect the aggregate resources at any time, and therefore the Pareto set is unchanged. However, by changing their autarchic utilities, the wealth distribution affects the set of *self-enforcing* payoff vectors.

Redistribution of wealth from an agent to the other is shown to cause a pivot of the frontier of the self-enforcing payoff vectors. The more power an agent has, the larger the change in his utility from an increase in his share of wealth.

Inequality is actually shown to help risk-sharing in a large range of cases. Regressive transfers of wealth increase the likelihood of perfect insurance for all utility functions displaying hyperbolic absolute risk aversion (HARA) — which encompasses all the utility functions typically used — at the exception of the constant absolute risk aversion for which, as it is well known, wealth effects are absent. Moreover, introducing wealth inequality between the agents often increase the transfers made in the constrained optimal agreement. Hence, though the net welfare effect is in general ambiguous, regressive transfers have the potential to *increase* the sum of the agent utilities.

These results are interesting in several dimensions. First, they highlight a potential cost of redistribution in the form of a reduction in risk-sharing or informal insurance. The implications of such finding are clearly important for the design of redistribution policies such as land reform.

There is a large literature identifying potential costs or benefits of inequality. Various reasons have been advanced showing that inequality can be beneficial such as convex savings (Bourguignon 1981), changes in the balance of power in the political system (Bertola 1993, Alesina and Rodrik 1994, Persson and Tabellini 1994), credit constraints and investment thresholds (Banerjee and Newman 1993, Galor and Zeira 1993, Murgai, Winters, Sadoulet and de Janvry 2002, Aghion and Bolton 1997). However, as far we know, the effect of inequality on informal insurance and its welfare consequences has not been studied before.

Second, the paper shows that income levels are an important determinant of choice of partner in risk-sharing agreements. We show that, when risk aversion is decreasing, individuals tend to prefer sharing risk with poorer agents. This is because the latter are willing to transfer more in exchange for transfers in time of need. However this does not necessarily imply that, if agents could form pairs to share risk, we would see assortative matching. In fact, checking for the stability of match is a complex object.

The rest of the paper is organized as follows. The next Section lays out the basic model and describes the optimal allocation. Section 3 then presents the main results regarding the impact of inequality on risk-sharing. These results are illustrated by some numerical examples. In Section 4, we briefly discuss the question of choice of a risk sharing partner. Section 5 discusses some important implications of the paper and its limitations. Section 6 concludes. All proofs are relegated to Section 7.

2. Informal Risk-Sharing

Before stating and answering the questions of interest, we need to lay out the basic setup of the risk-sharing problem without commitment.

2.1. **Premises of the Model.** Two agents, indexed by $i \in \{1, 2\}$, are engaged in the production and consumption of a perishable good at any date t. Each agent i's income at the beginning of each period $\widetilde{y}^i$ is random. It is composed of a secure endowment w_i, what we shall call their permanent income or *wealth*, and a random element $\widetilde{\epsilon}^i$ so that

$$\widetilde{y}^i = w_i + \widetilde{\epsilon}^i.$$

We think of these two components as two different sources of revenue: a *wealth* component, for instance the ownership of trees would produce a regular endowment; and a *labor* income that fluctuates. Alternatively, in each period, the agents could experience a loss or fall sick, and this risk is additive.

The two agents are identical in *all* relevant dimensions but their wealth. Let z be agent 1's share of the aggregate wealth w such that

$$w_i = z_i w, \quad z_1 = z \quad \text{and} \quad z_2 = 1 - z, \quad z \in [0, 1].$$

Without loss of generality, we normalize w at 1.

The distribution of the random components in the agent incomes $(\widetilde{\epsilon}^1_t, \widetilde{\epsilon}^2_t)$ is *symmetric* and independent over time. $\widetilde{\epsilon}^i_t$ can take on a finite number of values $\epsilon_1 < \epsilon_2 ... < \epsilon_N$. Let S be the set of possible realizations for $(\widetilde{\epsilon}^1, \widetilde{\epsilon}^2)$ and p_s be the probability of a particular state of the world $s \in S$ ($p_s \geq 0$ and $\sum_{s \in S} p_s = 1$). Assume that there the state of the world (ϵ_1, ϵ_N) has a positive probability. For each state s, denote by $\boldsymbol{y}(s) = (y_1(s), y_2(s))$ the vector of *realized incomes* and by $y_s = \sum_i y_i(s)$ the aggregate resources.

Our two agents have the same one-period von Neumann-Morgenstern utility function u defined on consumption. Their lifetime expected utility from any date t onwards is given by

$$\mathbf{E}\sum_{j=0}^{\infty}\delta^j u(c^i_{t+j}) \qquad \forall i \in \{1,2\}.$$

where u is increasing, smooth and strictly concave, and $\delta \in (0,1)$. is the discount rate.

2.2. **Consumption Allocations.** There is no storing technology available but individuals can make transfers to each other in order to smooth their consumption. In the main part of the paper, we consider stationary transfers schemes (where transfers do depend only on the state of nature) but in a later section, we point out the results that can be extended to the case in which individuals make non-stationary (history-dependent) transfers.

A *consumption allocation* $\mathbf{c} \equiv \{c_1(s), c_2(s)\}_{s\in S}$ gives, for each state of nature s, a nonnegative vector of consumptions $\mathbf{c}(s)$ that is *feasible* $\sum_{i\in\{1,2\}} c_1(s) = \sum_{i\in\{1,2\}} y_1(s)$.

An allocation generates a *vector of expected payoffs* $\mathbf{v} = (v_1, v_2)$ (these are discounted expected payoffs for each individual). Let $\widetilde{V}$ be the upper boundary of the collection of all feasible payoff vectors. It is the set of all *Pareto optimal* allocations. Each point on the frontier corresponds to the preferred allocation of a planner maximizing a weighted sum of the lifetime expected utilities of the individuals for some welfare weights $\boldsymbol{\alpha} = (\alpha, 1-\alpha)$ where $\alpha \in [0,1]$.

2.3. **Voluntary Risk-Sharing.** To be sure, most of, if not all, the Pareto optimal allocations in $\widetilde{V}$ are generally not *voluntary implementable*. To be voluntary implementable, at each date, not only both individuals must *ex-ante* prefer the allocation to autarchy, before nature picks a state of the world, but they must also prefer it *ex-post*, once they know their realized income for the period.

In autarchy, each agent simply consumes his income at each date and enjoys a (discounted) lifetime expected utility of

$$u^a(z_i) = \frac{1}{1-\delta}\sum_{s\in S} p(s)u(y_i(s)). \tag{1}$$

where $y_i(s) = z_i + \epsilon^i(s)$.

Hence, an allocation $\mathbf{c}$ is *voluntary implementable* given a division of wealth z if the following two conditions are satisfied:

[PARTICIPATION.] For all i

$$\sum_s p_s u(c_i(s)) \geq u^a(z_i) \tag{2}$$

[INCENTIVE.] For all i and every state of nature s,

$$(1-\delta)u(c_i(s)) + \delta \sum_{s'} p_{s'} u(c_i(s')) \geq (1-\delta)u(y_i(s)) + \delta u^a(z_i) \tag{3}$$

To be sure, the set of feasible risk-sharing agreements and therefore the set of Pareto optimal allocations are *independent* of z since the aggregate income is unaffected by z. However, the division of wealth affects the autarchic utility and thereby *does* affect the set of implementable allocations.

2.4. **Constrained Optimal Allocations.** Given a level of utility v_2 promised to 2, the following incentive-constrained optimization problem describes the utility that 1 can reach:

[THE $\mathbf{z}$-PROBLEM.]

$$\max_{\mathbf{c}} v(v_2, \mathbf{z}) \equiv \sum_s p_s u(c_1(s)) \tag{4}$$

subject to the incentive and participation constraints:

$$\sum_s p_s u(y_s - c_1(s)) \geq v_2; \tag{5}$$

$$(1-\delta)u(c_1(s)) + \delta \sum_{s'} p_{s'} u(c_1(s')) \geq (1-\delta)u(y_1(s)) + \delta u_1^a(\mathbf{z}); \tag{6}$$

$$(1-\delta)u(y_s - c_1(s)) + \delta \sum_{s'} p_{s'} u(y_{s'} - c_1(s')) \geq (1-\delta)u(y_2(s)) + \delta u_2^a(\mathbf{z}). \tag{7}$$

with $u_i^a(\mathbf{z}) = \sum_s p_s u(y_i(s))$, $y_i(s) = z_i + \epsilon_s$, $y_s = \sum_i y_i(s)$. Notice that the incentive constraints imply that the participation constraints are satisfied.

It is easy to check that the objective function is strictly concave in $\mathbf{c}$ and the set of constraints convex. Hence, for any v_2 for which a solution exists, this problem has a unique solution that we shall denote $\mathbf{c}^*(v_2, \mathbf{z})$. Let $V^*(\mathbf{z})$ be the set of the constrained optimal payoffs vectors given $\mathbf{z}$. These are generated by solving the $\mathbf{z}$-problem for all possible values of v_2. Abusing slightly notation we shall denote by z, in these expressions, the distribution of wealth corresponding to $z_1 = z$ and $z_2 = 1 - z$.

For a given z and v_2, the constrained optimal allocation $\mathbf{c}^*$ is characterized by the first-order conditions

$$\frac{u'(c_1(s))}{u'(y_s - c_1(s))} = \frac{\chi + \frac{\mu_s^2}{p_s}(1-\delta) + \delta \sum_{s'} \mu_{s'}^2}{1 + \frac{\mu_s^1}{p_s}(1-\delta) + \delta \sum_{s'} \mu_{s'}^1} \tag{8}$$

where μ_s^1 and μ_s^2 are the multiplier on the incentive constraints (6) and (7) when realized state is s, and χ is the multiplier on the promise keeping constraint. This condition characterizes the way the ratio of marginal utility of the agents — their relative needs — responds to income shocks.

Denote as $\theta(s)$ the equilibrium ratio of marginal utilities (RMU) of the agents in state s

$$\theta(s) = \frac{u'(c_1^*(s))}{u'(c_2^*(s))}$$

whose law of motion is given by equation (8).

If no incentive constraint binds in a state s, then the *ratio of marginal utilities* $\theta(s)$ of the agents is kept constant at

$$\mathcal{M} = \frac{\chi + \delta \sum_{s'} \mu_{s'}^2}{1 + \delta \sum_{s'} \mu_{s'}^1}.$$

If no incentive constraint ever binds then the ratio of marginal utilities of the agents is kept constant across all state of nature. This is perfect insurance. The particular level of RMU is given by the agents' relative welfare weights. When some constraints do bind in other states then the importance of the welfare weights in determining the RMU in state s decreases. In in state s one of the agents' incentive constraint binds, equation (8) tells us that the ration of marginal utilities of the agents $\theta(s)$ is adjusted so as to increase that agent's consumption level.

Let $\overline{\theta}_s(z)$ be the largest RMU such that 1's incentive constraint is not violated in state s, given the efficient risk-sharing agreement and wealth distribution z. Similarly, define $\underline{\theta}_s(z)$ as the smallest RMU such that 2's incentive constraint is not violated in state s given the efficient risk-sharing agreement and given z. Let $\Theta_s(z) \equiv [\underline{\theta}_s(z), \overline{\theta}_s(z)]$ for all $s \in S$.[2]. The (stationary) constrained efficient allocation is such that

$$\theta(s) = \begin{cases} \overline{\theta}_s(z) \\ \mathcal{M} \\ \underline{\theta}_s(z) \end{cases} \text{for all } s \text{ such that } \mathcal{M} \begin{cases} > \overline{\theta}_s(z) \\ \in \Theta_s(z). \\ < \underline{\theta}_s(z) \end{cases} \tag{9}$$

[2]Note that these concepts can be defined since the constraints are forward looking.

We define the *autarchic RMU* as the ratio of marginal utility that would have been observed in the absence of any risk sharing between the agents:

$$\theta_s^a(z) = \frac{u_1'(z+\epsilon_s^1)}{u_2'(1-z+\epsilon_s^2)} \text{ for all } s \in S.$$

Naturally, $\theta_s^a(z) \in \Theta_s(z)$ for all s, since autarchic RMUs are always implementable.

There exists a first-best consumption allocation *iff*

$$\widetilde{\Theta}(z) = \bigcap_{s \in S} \Theta_s(z) \neq \emptyset. \tag{10}$$

We shall call $\widetilde{\Theta}(z)$ the set of *steady state RMU* whose elements are RMUs associated with first-best risk-sharing agreements for some welfare weights given z. If $\widetilde{\Theta}(z) \neq \emptyset$, then it is clear from the law of motion (9) that for any $\theta = \chi \in \widetilde{\Theta}(z)$, the equilibrium RMU stays constant at θ. To be sure, if $\theta(s) = \theta_s^a$ for all s.

3. Inequality and Risk-Sharing

Our main interest lies in examining the relationship between *inequality* and the extent of *risk-sharing*. We first look at how wealth inequality affects the frontier of the self-enforcing payoff vectors. We then proceed in studying the impact of inequality on the likelihood of full insurance and on the level of insurance when risk-sharing is imperfect.

3.1. **Inequality and the Constrained Pareto Frontier.**

Without loss of generality assume that $z \geq 1/2$, individual 1 is at least as well off as individual 2. The first question that we are interested in is the effect on the set of constrained efficient payoff vector of a regressive transfer of wealth. That is we are looking at a transfer from 2 to 1 such that $z' > z$. Let $V^*(z)$ and $V^*(z')$ be the sets of constrained efficient payoffs respectively before and after the wealth redistribution.

The following Proposition states that, starting from a situation in which the set of steady state RMU is empty, a permanent redistribution of wealth from 2 to 1 increases the slope of the frontier of the set of implementable payoff vectors to pivot to the right and that there is a *single crossing* point. This is in contrast with the set of optimal payoff vectors $\widetilde{V}$ that is unaffected by such redistribution.

Proposition 1. [Single-Crossing] *If* $v_1(v_2, z') \geq v_1(v_2, z)$, *for* $z' > z$ *then* $v_1(v_2', z') > v_1(v_2', z)$ *for* $v_2' < v_2$.

This claim is very intuitive. If an increase in individual 1's share of wealth raises 1's utility for a given level of utility for 2, then it will do so for any lower utility promised to 2.

Consider an initial situation in which both agents are identical, that is $z = 1/2$, and the first best cannot be reached for any welfare weights, $\widetilde{\Theta}(1/2) = \emptyset$. Proposition 2 states that, starting from a situation in which the two agents are identical, and in which there is no steady state RMU, a permanent redistribution of small amount of wealth from one agent to the other causes the frontier of the set of implementable payoff vectors to pivot around it's center point.

Proposition 2. $\dfrac{dv_1(v_2, 1/2)}{dz} \begin{cases} > \\ = \\ < \end{cases} 0$ *for all* $v_2 \begin{cases} > \\ = \\ < \end{cases} v_1$.

$c^*(v_2, 1/2)$ is the constrained efficient allocation under equal division of wealth for a promise v_2. Since the set of steady state RMU is empty, the incentive constraints of the agents binds in some states. Now, transferring a small amount of individual 2's wealth to individual 1 affects their autarchic utilities. For all state s, the redistribution decreases individual 2's utility in autarchy and increases 1's. It follows that under initial allocation $c^*(v_2, 1/2)$ all of 2's constraints are slack while 1's previously binding constraints are now violated. The net welfare impact weights these two effects. When the agents are identical and 1's welfare weight is higher that 2's not only 2's constraint is more likely to bind but the welfare cost of 2's constraint binding is higher, while it is exactly the opposite if 2's welfare weight is higher than 1's. The two effects balance each other exactly when $v_1 = v_2$. Hence, for a small increase in z the frontier of the set of implementable payoff vectors pivots to the right around its central point. Note that for larger changes in z other effects may come into play as the number of states under which the constraints bind may change too.

Figure $1b$ provides a graphical description of Proposition 2. The set of optimal payoff vectors $\widetilde{V}$ is represented with a dashed line while the sets of constrained efficient payoff vectors V^* for two different wealth allocations $z = 1/2$ and $z' > z$ are represented with solid lines. $\widetilde{V}$ lies everywhere above $V^*(1/2)$, since the set of steady state RMU is empty at $z = 1/2$, and the lowest possible implementable expected utility for an agent at date 0 corresponds to his autarchic utility. Now consider a small transfer of wealth from 2 to 1 such that $z' > 1/2$. We see that as z increases the frontier of the set of implementable payoff vectors pivots to the right.

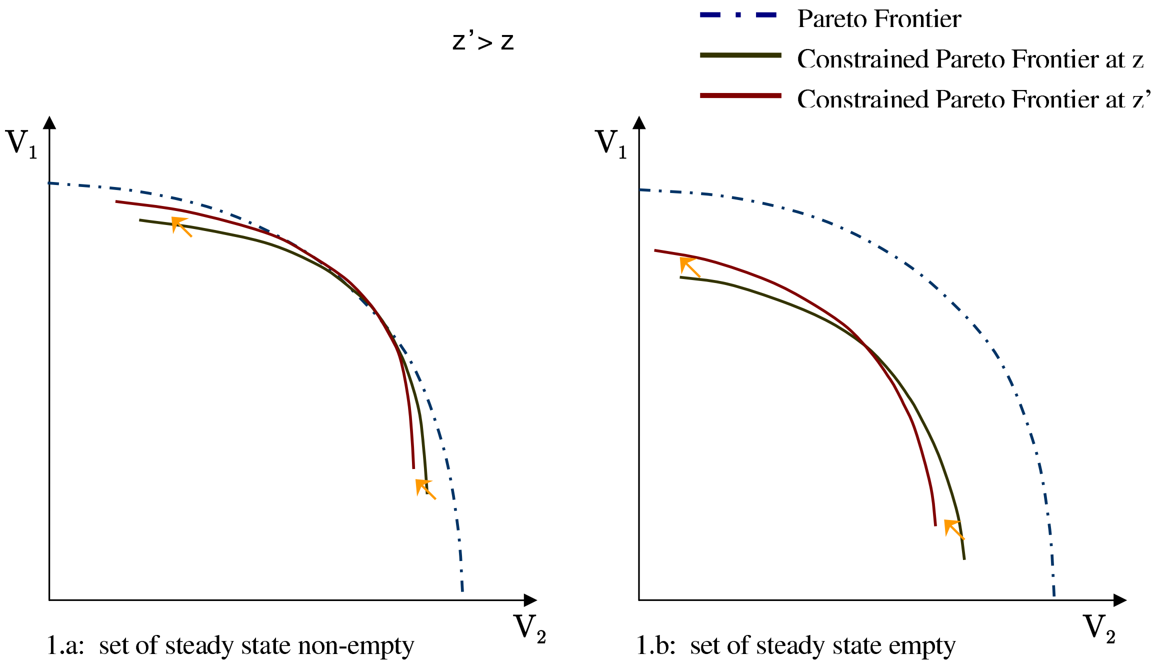

FIGURE 1. Regressive Wealth Transfers and the Pareto Frontier.

In contrast, if given the initial welfare weights and z the incentive constraints are not binding, small changes in z will not affect the agents payoff. This is illustrated in Figure $1a$ where the set of steady state RMU is non-empty. Wherever the slope of the contract curve belongs to $\widetilde{\Theta}$, the set of optimal payoff vectors $\widetilde{V}$ and the set of constrained efficient payoff vector V^* coincide and a small change in the wealth distribution does not affect the points at the interior of this set.

3.2. **The Incentive Compatible Equivalent.** In this section, we show that inequality makes first best risk sharing more likely for all utility functions with hyperbolic absolute risk aversion (HARA) but the constant absolute risk aversion utility for which wealth distribution does not matter. While restrictive an assumption, HARA utility functions do include as special cases most of the utility functions that are commonly used such as the logarithmic and other constant relative risk aversion utility functions, the constant absolute risk aversion (exponential) utility, and utility functions with increasing risk aversion such as the quadratic utility.

Utility functions of the HARA class take the following shape:

$$u(c) = \begin{cases} \frac{1-k}{(2-k)\gamma}[\gamma c + \beta]^{\frac{2-k}{1-k}} + C, & \text{if } k \neq 1, 2; \\ ln(\gamma c + \beta), & \text{if } k = 2. \end{cases}$$

where $\frac{\gamma}{1-k} < 0$ and $\beta > -\gamma c$ for all c. Among these we can distinguish (a) utilities exhibiting decreasing risk aversion, $k > 1$ and $\gamma > 0$; and (b) utilities with increasing risk aversion, $k < 1$ and $\gamma < 0$. If $k = 1$, the utility corresponds to the case of constant absolute risk-aversion in which clearly there is no wealth effect, and therefore redistribution of wealth will not affect the extent of risk-sharing. Hence, in what follows we focus on $k \neq 1$.

We claim that in this case, redistributing wealth from the poorest to the richest agent makes it more likely that the set of steady state RMU is non empty. It is in this sense that inequality makes perfect insurance more likely. To make this claim more precise, it will help to define the concept of *incentive compatible equivalent*.

We saw that perfect insurance consists in keeping the ratio of marginal utilities – the relative needs – of the agents constant across all state of nature. Hence, to know whether perfect insurance is possible or not, we can compare the highest constant RMU that would satisfy all of individual 1's incentive constraints with the smallest constant RMU that individual 2 would accept.

A ratio of marginal utility θ and aggregate resources y imply the following consumption level for i:

$$c_i = g_i(\theta, y) = x_i(\theta)y + (x_i(\theta) - x_{-i}(\theta))\frac{\beta}{\gamma} \quad \text{for} \quad i \in \{1, 2\} \tag{11}$$

for a pair of *shares*

$$x_1(\theta) = \frac{1}{1 + \theta^{k-1}} \qquad \text{and} \quad x_2(\theta) = \frac{\theta^{k-1}}{1 + \theta^{k-1}}(= 1 - x_1(\theta)). \tag{12}$$

Clearly, it is when an agent's income is the largest (her realized income shock of ϵ_N) while the other agent's income is the lowest (his income shock of ϵ_1) that her incentive constraint is the hardest to satisfy. Let $\widetilde{y} = 1 + \epsilon_1 + \epsilon_N$ and $\overline{y}_i = z_i + \epsilon_N$. The largest constant RMU that individual 1 would accept $\overline{\theta}$ is therefore so that

$$(1 - \delta)u(g_1(\overline{\theta}, \widetilde{y})) + \delta E_{s'} u(g_1(\overline{\theta}, y_{s'})) = (1 - \delta)u(\overline{y}_1) + \delta u^a(z_1). \tag{13}$$

Similarly, the smallest constant RMU that satisfies all of 2's incentive constraint $\underline{\theta}$ is defined by

$$(1-\delta)u(g_2(\underline{\theta},\widetilde{y}))+\delta E_{s'}u(g_2(\underline{\theta},y_{s'}))=(1-\delta)u(\overline{y}_2)+\delta u^a(z_2). \tag{14}$$

Perfect insurance is implementable if the highest RMU that 1 requires is less than the largest RMU that 2 wants: $\overline{\theta}\leq\underline{\theta}$.

Individual i's *incentive compatible equivalents* (ICE) is the constant share that just satisfies all incentive constraints for this agent:

$$\widehat{x}_1=x_1(\overline{\theta})\text{ and }\widehat{x}_2=x_2(\underline{\theta}).$$

Notice that for the commonly used utility with constant relative risk aversion, as for utility functions where $\beta=0$, the shares $\mathbf{x}$ correspond to the shares of aggregate income allocated to the agents.

For the utilities of type (a) (decreasing risk aversion), an increase in x_i translates into an increase in i's consumption. Hence, any $x\geq\widehat{x}_1$ would satisfy 1's constraints and 2 would accept any share x for 1 lower or equal to $1-\widehat{x}_2$. It follows that the set of steady state RMUs is non empty if and only if $\widehat{x}_1<1-\widehat{x}_2$. In contrast, for utilities of type (b) an increase in x_i decreases i's consumption. Agent 1 would accept any $x\leq\widehat{x}_1$ and 2 can credibly commit to any share x for 1 not smaller than $1-\widehat{x}_2$. In this case, a steady state RMU exists if and only if $\widehat{x}_1>(1-\widehat{x}_2)$.

Hence, define

$$\Delta\widehat{x}=\begin{cases}\widehat{x}_1-(1-\widehat{x}_2) & \text{if } k>1 \text{ and } \gamma>0\\ (1-\widehat{x}_2)-\widehat{x}_1 & \text{if } k<1 \text{ and } \gamma<0\end{cases}$$

The following proposition says that increasing the richest agent's share of wealth decreases $\Delta\widehat{x}$. In other words, a more equal wealth distribution makes perfect insurance less likely.

Proposition 3. *If* $z\begin{cases}<\\=\\>\end{cases}1/2$ *then* $\dfrac{d(\Delta\widehat{x})}{dz}\begin{cases}>\\=\\<\end{cases}0$

Proposition 3 implies that if the set of steady state RMU is non-empty at $z\geq 1/2$ then it is non-empty at $z'>z$. And if the set is empty at some $z\geq 1/2$ then transferring wealth from 2 to 1 makes $\widehat{x}_1$ and $(1-\widehat{x}_2)$ converge. Once they have converged then the set of steady state RMU is non-empty, that is perfect insurance is possible and, for any initial welfare weights, the ratio of marginal utilities of the agent converges to a constant. The following example and Figure 2 illustrate this result.

Example 1. *The incentive compatible equivalent.*

In this example we assume a very simple income distribution for the agents that we will use again later. The agents' fluctuating income can take only one of two values, h or ℓ with $h > \ell \geq 0$, with probability $1/2$ each. The individuals' fluctuating income are perfectly negatively correlated (when one get h the other gets ℓ), such that the aggregate income stays constant.

Assume that our two individuals have constant relative risk aversion utility (CRRA):

$$u(c) = \frac{c^{1-\rho}}{1-\rho}$$

where ρ is the Arrow-Pratt coefficient of relative risk aversion. The following parameters are set through the example: $\delta = 0.65$, $\rho = 2$, $w = 20$, $h = 50$, and $\ell = 20$. The aggregate income is therefore $y = 90$.

We consider several values of $z \geq \frac{1}{2}$, progressively raising the level of wealth inequality between the two agents. The resulting change in the incentive compatible equivalents are reported in Table 1. The second column represents the minimum level of consumption $\widehat{x}_1 y$ that individual 1 wants and the third column represents $\widehat{x}_2 y$, the maximum consumption that individual 2 is willing to leave him given z. The difference between the minimum consumption that 1 wants and the maximum that individual 2 is willing to leave him $\Delta\widehat{x}$ decreases with regressive transfers from individual 1 to 2 as shown in Figure 2 and column 4 of Table 1.

z	$\widehat{x}_1 y$	$(1-\widehat{x}_2)y$	$\Delta\widehat{x}y$
0.5	45.28	44.72	0.57
0.6	47.57	47.03	0.54
0.7	49.83	49.37	0.46
0.8	51.85	51.50	0.34
0.9	54.07	53.92	0.15
0.96	55.44	55.4	0.00

TABLE 1. Inequality & Incentive Compatible Equivalents.

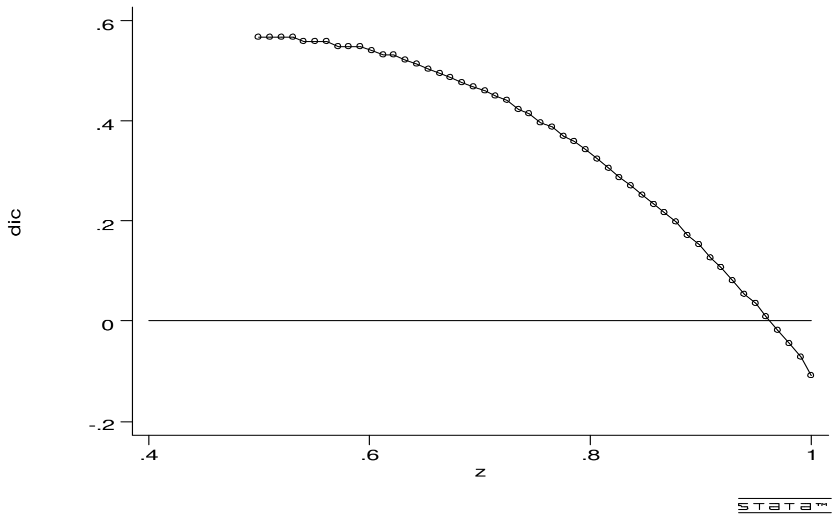

FIGURE 2. Inequality & Incentive Compatible Equivalents

3.3. **Inequality and Optimal Risk-Sharing.** The preceding sections made clear that inequality makes perfect insurance more likely for all utility functions of the HARA class. A related but different question is whether, when perfect insurance is not possible, inequality increases the level of insurance provided by the risk-sharing agreement. This section provides a condition guaranteeing that this is the case and evaluates this condition in a particular context.

Assume that the agents' fluctuating income can take only one of two values, h or ℓ. The terminology h and ℓ naturally suggests the ordering $h > \ell > 0$. Let $\mathbf{c}^*(v, z)$ be the *constrained efficient allocation* for division of wealth z and utility v to agent 2. Let's decompose the individual's consumption into a mean term, m_i, and a random component $\eta_i(s)$, that is $c^i(s) = m_i + \eta_i(s)$ for all s. We use IC_i to refer to agent i's incentive constraint when an agent's income is high while the other agent's income is low. Hence, $\frac{dm_i}{dz_i}|_{IC_i}$ is the change in mean consumption of agent i that leaves IC_i constant for a small increase in z_i.

A sufficient condition for the introduction of inequality to improve risk-sharing is that, keeping the transfer scheme unchanged

$$\frac{dm_1}{dz_1}|_{IC_1} \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} \frac{dm_2}{dz_2}|_{IC_2} \text{ for all } z \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} 1/2.$$

The above condition requires the (spread-preserving) increase in mean consumption m_i that keeps i's incentive constraint unchanged for an increase in z_i to be smaller for the richest agent.

Clearly this condition is difficult to evaluate in the most general case. Hence, in this section, we study the relationship between wealth inequality and risk sharing in the case where the aggregate income is constant, as in Example 1, since this case can be explicitly solved for. The agent's labor income being symmetric, this implies a probability $1/2$ for each state. The following table summarizes the labor income distribution of the two agents.[3]

individual\state	$state1$	$state2$
1	h	ℓ
2	ℓ	h
probability	$1/2$	$1/2$

It is well known that with this simple distribution, when no first best allocation is incentive compatible, the constrained optimal agreement with the most insurance is fully characterized by two values, that we shall denote as $\overline{c}$ and $\underline{c}$, individual 1's consumption when his income is high and when his income is low. These consumption levels are such that the incentive constraints hold with equality,

$$\begin{aligned} (1-\frac{\delta}{2})u(\overline{c})+\frac{\delta}{2}u(\underline{c}) &= (1-\frac{\delta}{2})u(z+h)+\frac{\delta}{2}u(z+\ell) \\ (1-\frac{\delta}{2})u(y-\overline{c})+\frac{\delta}{2}u(y-\underline{c}) &= (1-\frac{\delta}{2})u((1-z)+h)+\frac{\delta}{2}u((1-z)+\ell) \end{aligned} \tag{15}$$

where $y = 1+\ell+h$ is the aggregate income.

Proposition 4. *For all utility functions of the HARA class with $k \geq 2$ or k sufficiently close to 0, introducing some inequality between the agents when risk-sharing is not perfect improves informal insurance by reducing $\overline{c}-\underline{c}$.*

[3] Note that the states are labeled for the agent whose income is high.

3.4. **Welfare Impact.** The effects of introducing inequality on welfare are in general ambiguous. Surprisingly enough, introducing some inequality between the two agents can actually *increase* the sum of the utility of the agents. In the following example, for instance, for small inequalities the positive effect of inequality on the informal insurance of redistribution dominates such that the sum of the utilities of the agents increases with inequality. But at large levels of inequality, the negative effect dominates.

Example 2. *Welfare and Inequality*

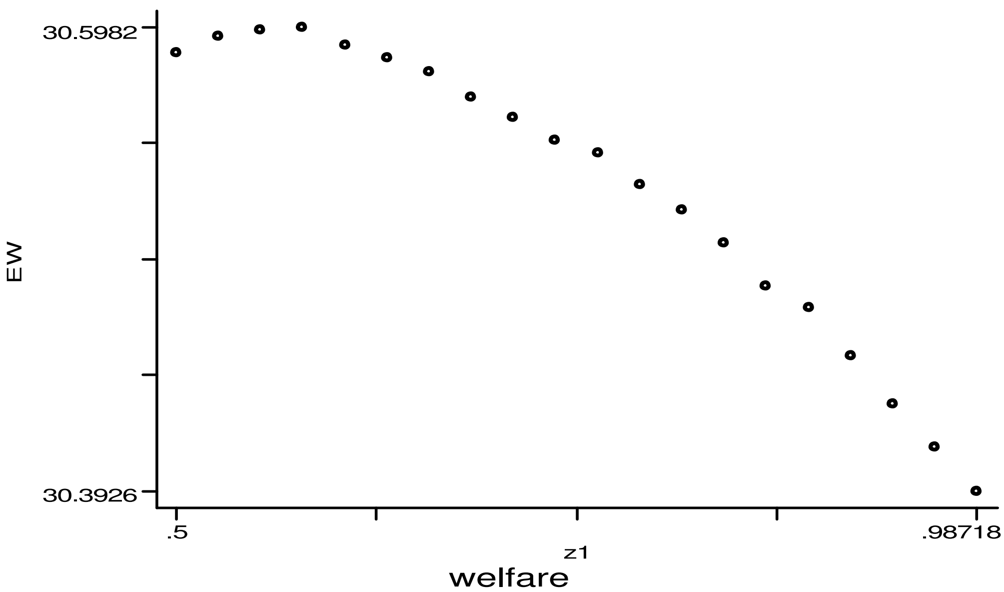

FIGURE 3. Welfare and Redistribution

Assume that our two individuals have the following CRRA utility function

$$u(c) = \frac{1}{1-\rho} c^{1-\rho} - 1$$

where ρ is the Arrow-Pratt coefficient of relative risk aversion. Their incomes can take one of two values $h = 50$ and $l = 20$ with probability $1/2$ and are independently distributed, and the total wealth in this economy is $w = 20$. The following parameters are set through the example: $\delta = 0.75$, $\rho = 0.99$. We progressively increase z from $.5$ to $.78$ thereby increasing the level of wealth inequality between the agents. The results are illustrated in Table 2 and Figure 3.

$w1$	$w2$	V^1	V^2	$V^1 + V^2$
10	10	15.2932	15.2936	30.5867
10.5128	9.4872	15.343	15.2515	30.5944
11.0256	8.9744	15.3962	15.2008	30.5971
11.5385	8.4615	15.449	15.1492	30.5982
12.0513	7.9487	15.4986	15.0915	30.5902
12.5641	7.4359	15.5486	15.0362	30.5848
13.0769	6.9231	15.5985	14.98	30.5784
13.5897	6.4103	15.6464	14.9206	30.567
14.1026	5.8974	15.6952	14.8632	30.5584
14.6154	5.3846	15.7432	14.8051	30.5483
15.1282	4.8718	15.792	14.7506	30.5426
15.641	4.359	15.8388	14.6897	30.5286

TABLE 2. Inequality and Welfare.

4. CHOICE OF RISK-SHARING PARTNER

In the previous sections, we studied the effect of inequality on voluntary risk-sharing. A different but related question concerns the choice of risk-sharing partner people make. To keep things simple assume that the agents' income shocks can take on only two values high h or low ℓ. Consider an individual who can pick a risk-sharing partner among different individuals, some richer and some poorer in terms of permanent income. Who do we expect her to pick? Proposition 5 suggests that poorer partners are preferred to richer partners when risk aversion is decreasing.

Proposition 5. *If two agents with wealth z_1 and z_2 respectively are sharing risk with each other then the higher z_2*
(i) *the lower the transfers, if their incomes are perfectly negatively correlated;*
(ii) *the higher the transfers for $z_2 \geq z_1$ if their incomes are perfectly positively correlated*

and the opposite for $z_2 < z_1$;

Proposition 5 shows that when the labor incomes are perfectly negatively correlated, larger transfers are possible when agent 1 is paired with poorer agents if risk aversion is decreasing. Keeping the transfers, between 1 and 2 the same, lowering 2's wealth relaxes the incentive constraint. That is, a poorer 2 is willing to give more to 1 when high in exchange of the same transfer from 1 when he is low. The level of insurance is higher and they receive more surplus.

When incomes are perfectly positively correlated, agents with the same level of permanent income are unable to provide insurance to each other. Larger transfers are possible only if agent 1 is matched with someone either sufficiently richer or sufficiently poorer that some transaction occurs. If risk aversion is decreasing the poorest of the two agents would borrow when both are low and repay when both are high.

We could push the inquiry further and ask the question of which matches would form among different individuals, some rich and some poor. Assuming that individuals share risk in pairs (see Genicot and Ray 2003, 2005 for reasons for which small groups form), who do we expect to see sharing risk with each other? This is related to Ghatak (1999) and Sadoulet (2001) who look at pairwise matching in group lending in the presence of heterogeneity in risk.[4]

Consider a very simple example: two sets of agents M and F of same size – male and female say – can form pairs to share risk with each other. Assume that each set contains two agents, a *poor* with wealth z_p and a *rich* with wealth z_r ($z_r > z_p$), and that individuals have decreasing risk aversion. In this case, the matching that we expect to observe depends on the correlation between the incomes of the two set of agents.

Proposition 6. *[i] Positive assortative matching is stable if incomes are perfectly positively correlated; and [ii] negative assortative matching is stable with perfectly positively correlated incomes;*

Notice that this proposition relies on stationary contracts. Allowing for history dependent contract – described in details in Section – we would not necessarily get positive assortative matching when incomes are perfectly negatively correlated. Proposition 5 tells us that poorer individuals are willing to make larger transfers when low in exchange of a given transfer when rich. This is

[4]Also clearly relevant is Legros and Newman (2002) who study monotonic matching in the context of non-transferable utilities.

why we expect that the poorer agents would be better off forming a risk sharing agreement among themselves. However this is not necessarily the case. It is relatively easier for richer agents to *temporarily* demand a lower transfer from the poorer agent if the latter is high as long as he has not received ℓ. As soon as the poorer agent receives ℓ, the agreement consists in the larger transfers that the two agents incentive constraints allow. Hence, the stability of a match is a complex object to check for as the following example illustrates.

Example 3. *Endogenous Matching*

Individuals have CRRA utilities $u(c) = \frac{1}{1-\rho}c^{1-\rho}$ with risk aversion $\rho = 0.8$ and discount rate $\delta = 0.8$. Their labor incomes can be low $\ell = 1$ or high $h = 3$ with probability $1/2$ and are perfectly negatively correlated between the two M and F.

Let the poor's wealth z_p be 1. Denote as u_{pp} the expected utility (in per period term) that a poor person has when sharing risk in a symmetric agreement with someone of same wealth and as u_{rr} the same object for a rich person. A match where poor and rich are sharing risk in separate groups is *stable* if there is no risk-sharing agreement between a poor and rich that could give to at least u_{pp} to the poor and u_{rr} to the rich.

In our example, this the case for relatively small difference between poor and rich. If $z_r = 1.5$, there is no agreement between a rich and a poor that can give at least $u_{pp} = 6.42$ to the poor while guaranteeing $u_{rr} = 6.59$. In contrast, if the rich are twice as wealthy as the poor $z_r = 2$, then by giving the poor a break early on in the relationship, the rich can guarantee an expected utility of at least $u_{pp} = 6.42$ to the poor while earning more than $u_{rr} = 6.75$.

5. Implications, Limitations and Extensions

The remainder of the paper explores some important extensions and policy implications of the previous results.

5.1. **Income Inequality and Consumption Inequality.**

Krueger and Perri (2002) investigate the relationship between the cross-sectional distribution of income and consumption in the United States between 1980 and 1997. They present evidence that the Gini coefficient of after-tax labor income did increase while the consumption Gini remained roughly constant.

The authors suggest that an increase in the spread in the agent's income distribution is a potential explanation for this observation. They consider the specific two-agent example with perfectly negatively correlated income, similar to Example 1. The agents have constant relative risk aversion and their income distributions are multiple of each other. They show that in this case, when risk-sharing is not perfect, an increase in the dispersion in the agents' income increases both the Gini measure of income inequality and the level of risk-sharing among the agents.[5] An increase in income dispersion would then result in a lower increase in the consumption Gini than in the income Gini.

Evidences show that the US income distribution experienced both a permanent increase in inequality and an increase in the dispersion in the personal income distribution (see Katz and Autor (1999) for a review of these evidences). Hence, this paper suggests another channel through which higher inequality in income, by increasing the level of voluntary risk-sharing, could actually decrease the inequality in consumption.

5.2. **Redistribution Program.** To be sure, these results have policy implications regarding redistribution programs. One of the most important redistribution policy being land reform.

Redistribution is of course a goal in itself, quite apart from any efficiency gains that might result from a more equitable land distribution. However as many recent papers have argued, inequality can have positive as well as negative effects of efficiency. In this paper, we identified a new potential cost of redistribution. In many cases, redistribution reduces the level of risk-sharing or informal insurance to which people have access.

In example 2, we saw a situation where introducing some inequality actually increased the sum of the utilities of the agents. In such situation, even a government that slightly favors the poorest agent may actually side with the richest agent and oppose a land reform. This is because the loss of efficiency due to a decrease in risk-sharing dominates the benefits from redistribution. If the initial inequality is very large, then clearly the overall benefits or redistribution will dominate but it is still important to realize that the level of insurance may have decreased in the process.

Naturally, if the redistribution that takes place, such as land reform, effectively increases the access of the poor to formal forms of credit then this would

[5]Note that with two income shocks, this result can be shown to hold whenever stationary schemes are feasible and self-enforcing.

mitigate the consequences of the loss of informal insurance. Otherwise redistribution initiatives should be accompanied by safety net policies.

6. Conclusion

Need to provide better insurance

7. Appendix

7.1. **The Constrained Optimal Allocation.** Denote *t-history* as $s^t \in S^t$, that is the history of all past and current realization of the state of nature up to time t. If we allow for history dependent risk-sharing arrangement, then an *allocation* is a list of functions $\sigma = \{\mathbf{c}_t\}_{t=0}^{\infty}$ such that for all $t \geq 0$, $\mathbf{c}_t$ maps the product of t-histories and current income realizations to feasible consumption vectors. (non negative and for all t-history $s^t \in S^t$, $\sum_i c_t^i(s^t) \leq \sum y_{s_t}^i \; \forall t, s^t$).

Kocherlakota (1996) showed that the constrained optimal scheme depends on history in a very simple way. At any point in time, the current promise utility to agent 2 summarizes all past history. For a given wealth distribution $\mathbf{z}$, the constrained optimal allocations solve the following maximization for different values of v_2,[6]

$$v_1(v_2, z) = \max_{\mathbf{c}_s, v_s'} E_s\left[(1-\delta)u(c^1(s)) + \delta v_1(v_s', z)\right], \tag{16}$$

subject to the promise keeping constraint

$$E_s\left[(1-\delta)u(c^2(s)) + \delta v_s'\right] \geq v_2, \tag{17}$$

feasibility and the individual's incentive constraints:

$$(1-\delta)u(c^1(s)) + \delta v_1(v_s', z) \;\geq\; (1-\delta)u(y^1(s)) + \delta u^a(z_1) \tag{18}$$

$$(1-\delta)u(c^2(s)) + \delta v_s' \;\geq\; (1-\delta)u(y^2(s)) + \delta u^a(z_2) \tag{19}$$

for every state s and history v_2.

Hence, denote as $\theta(s^t)$ the equilibrium ratio of marginal utilities (RMU) given s^t

$$\theta(s^t) = \frac{u'(c_t^1(s^t))}{u'(c_t^2(s^t))}.$$

[6] See Kocherlakota (1996) and Ligon et al. (2000).

The first order condition for the problem tells us that

$$\theta(s^t) = \frac{\chi(s^{t-1}) + \psi^2(s_t)}{1 + \psi^1(s^t)} \tag{20}$$

$$\frac{dv_1(v'_s, z)}{dv_2} = \theta(s^t) \tag{21}$$

where $\chi(s^{t-1})$ is the multiplier on the promise keeping constraint (17), and $p(s_t)\psi^i(s_t)$ is the multiplier on i's incentive constraint (18) or (19)given the realized state s_t.

It follows that the optimal contract is characterized by a simple *updating rule* for the evolution of the ratio of marginal utilities along the equilibrium path. If no incentive constraint binds then the RMU at time t stays the same as last period realized RMU. Alternatively, if one individual's incentive constraint binds the the ratio of marginal changes in the direction that relaxes the constraint, the least as possible but enough to satisfy the binding incentive constraint.

Let $\overline{\theta}_s(z)$ be the largest implementable RMU in state s — that is the largest RMU such that 1's incentive constraint is not violated — given the efficient risk-sharing agreement and wealth distribution z. Similarly, define $\underline{\theta}_s(z)$ as the smallest implementable RMU in state s — that is the smallest RMU such that 2's incentive constraint is not violated. Let $\Theta_s(z) \equiv [\underline{\theta}_s(z), \overline{\theta}_s(z)]$ for all $s \in S$.[7]

Given a realized RMU $\theta(s^{t-1})$ a time $t-1$ the constrained efficient allocation is such that today's RMU $\theta(s^t)$ follows the following law of motion

$$\theta(s^{t-1}, s) = \begin{cases} \overline{\theta}_s(z) \\ \theta(s^{t-1}) \\ \underline{\theta}_s(z) \end{cases} \text{for all } s \text{ such that } \theta(s^{t-1}) \begin{cases} > \overline{\theta}_s(z) \\ \in \Theta_s(z) \equiv [\underline{\theta}_s(z), \overline{\theta}_s(z)] \\ < \underline{\theta}_s(z) \end{cases}$$

with $\theta(s^{-1}) = \frac{1-\alpha}{\alpha}$.

Note that the current RMU is the absolute value of the slope of the contract curve at the continuation utilities. A given $\theta(s^{t-1})$ and the above updating rule perfectly identifies the level of expected utility of the agents. When an agent's constraint binds, his continuation utility increases and the position on the contract curve is modified in a direction more favorable to this agent.

[7]Note that these concepts can be defined since the constraints are forward looking.

7.2. **Proofs .**

Proof of Proposition 1.

Proof. Because the planner's problem has a unique solution for every v_2, this solution must be continuous in the parameter z. It therefore suffices to consider only changes in z from z to z' such that exactly the same constraints bind before and after. The absolute value of that slope of the constrained Pareto frontier at v_2 is given by the multiplier on the promise keeping constraint χ. To prove the Proposition, we claim that following an increase from z to z', the slope of the constrained Pareto frontier χ has strictly increased for all v_2 so that v_1 has not increased, $v_1(v_2, z') \leq v_1(v_2, z')$. Assume that this claim is not true and that χ has (weakly) decreased.

Since an increase in z increases the right-hand-side of 1's incentive constraint (6) and that $v_1(v_2, z)$ has not increased, it must be that $c_1(s)$ strictly increase in all state s in which 1's constraint binds. Similarly, an increase in z decreases the right-hand-side of 2's incentive constraint (7) and $v_2(\alpha, z)$ has increased, so that $c_1(s)$ must strictly increase in all state s in which 2's constraint binds.

Finally, in all state s in which no incentive constraint binds, the first order condition (8) tells us that

$$\frac{u'(c_1(s))}{u'(y_s - c_1(s))} = \mathcal{M} \equiv \frac{\chi + \delta M_2}{1 + \delta M_1}.$$

where $M_i = \sum_{s'} \mu^i_{s'}$. Let ΔM_i be the change in M_i and $\Delta\chi$ be the change in χ from z to z'.

Since v_2 is the same and v_1 has not increased, it must be the case that, in states where no constraint binds, $\mathcal{M}$ has increased so that $c_1(s)$ decreases and $c_2(s)$ increases. Hence,

$$(\Delta\chi + \delta\Delta M_2)(1 + \delta M_1) - \delta\Delta M_1(\chi + \delta M_2) \geq 0. \tag{22}$$

Since $\Delta\chi \leq 0$, this implies that either $[a]$ $\Delta M_2 \geq 0$ or $[b]$ $dM_1 \leq 0$ or $[c]$ both.

For all states s in which 2's constraint binds, this, along with the first order condition (8) and the fact that $c_1(s)$ strictly increases, also implies that $\mu^2_s/(1 + \delta M_1)$ decreases. Hence,

$$\delta\Delta M_2(1 + \delta M_1) - \delta\Delta M_1(\delta M_2) < 0. \tag{23}$$

Similarly, the first order condition (8) and the fact that $c_1(s)$ strictly increases imply that $\mu^1_s/(\chi + \delta M_2)$ increases for all states in which 1's constraint binds. It follows that,

$$(\Delta\chi + \delta\Delta M_2)\delta M_1 - \delta\Delta M_1(\chi + \delta M_2) < 0. \tag{24}$$

Notice that hypothesis $[c]$, both $\Delta M_2 \geq 0$ and $dM_1 \leq 0$, would immediately contradict inequalities (23) and (24) and therefore cannot be true.

Now, suppose that $[b]$ holds so that $\Delta M_2, \Delta M_1 \leq 0$. Notice that (22) and 24 imply that

$$(\Delta\chi + \delta\Delta M_2) \geq \delta\Delta M_1(\chi + \delta M_2) - (\Delta\chi + \delta\Delta M_2)\delta M_1 > 0.$$

This is a contradiction, since under $[a]$ the left-hand side is strictly negative.

Finally, assume that $[a]$ holds so that $\Delta M_2, \Delta M_1 \geq 0$. Inequalities (22) and 23 jointly imply that

$$\Delta\chi(1 + \delta M_1) - \chi\delta\Delta M_1 \geq \delta\Delta M_1(\delta M_2) - \delta\Delta M_2(1 + \delta M_1) > 0.$$

But, $[a]$ implies that the left-hand-side is negative, a contradiction.

This contradicts the initial hypothesis that χ has not increased. Hence, the slope of the constrained Pareto frontier must strictly increase whenever when $v_1(v_2, z') \leq v_1(v_2, z)$ so that we have a single crossing. ■

Proof of Proposition 2. The effect of a small change in z on the Lagrangian $\mathcal{L}(v_2; z)$ is given by

$$\begin{aligned} \frac{d\mathcal{L}(v_2; z)}{dz} \quad = \quad & -\textstyle\sum_s \ \mu_1(s)[u'(z + \epsilon^1(s)) + \frac{\delta}{1-\delta}\sum_{s'} p(s')u'(z + \epsilon^1(s'))] \quad + \\ & \textstyle\sum_s \mu_2(s)[u'((1-z) + \epsilon^2(s)) + \frac{\delta}{1-\delta}\sum_{s'} p(s')u'((1-z) + \epsilon^2(s'))] \end{aligned}$$

since by the envelope theorem all derivatives of $\mathcal{L}$ with respect to $\mu^i(s)$ and $c^i(s)$ are null.

At $z = 1/2$, both individuals are exactly identical and the income distributions are symmetric so that for each state s where $1'$s constraint is binding there is a symmetric state $\widetilde{s}$ with $\mu_1(s) = \mu_1(s')$ and $u'(y_1(s)) + \frac{\delta}{1-\delta}Eu'(y_1(s')) = u'(y_2(\widetilde{s})) + \frac{\delta}{1-\delta}\sum_{s'} p(s')u'(y_2(s'))$. Hence $dv_1(v_1, 1/2)/dz = 0$.

This in combination with Proposition 1, imply that $\dfrac{dv_1(v; \frac{1}{2})}{dz} < 0$ for $v > v_1$ while $\dfrac{dv_1(v; \frac{1}{2})}{dz} > 0$ for $v < v_1$. ■

Proof of Proposition 3.
Recall that given our utility function,

$$u(g_i(\theta, Y)) = \frac{(1-k)}{(2-k)\gamma} \left[x_i(\theta)(\gamma Y + 2\beta)\right]^{\frac{2-k}{1-k}}.$$

Using this in the definition of $\overline{\theta}$ in (13) and $\underline{\theta}$ in (14), we can rewrite $\widehat{x}_1 = x_1(\overline{\theta})$ and $\widehat{x}_2 = x_2(\underline{\theta})$ as

$$\widehat{x}_i = \left[\frac{(1-\delta)[\widetilde{x}_i^a(\gamma\widetilde{y} + 2\beta)]^{\frac{2-k}{1-k}} + \delta E_s[x_i^a(s)(\gamma Y_s + 2\beta)]^{\frac{2-k}{1-k}}}{(1-\delta)(\gamma\widetilde{y} + 2\beta)^{\frac{2-k}{1-k}} + \delta E_s(\gamma Y_s + 2\beta)^{\frac{2-k}{1-k}}}\right]^{\frac{1-k}{2-k}} \tag{25}$$

where $\widetilde{x}_i^a = \frac{\gamma(z_i+\epsilon_N)}{\gamma\widetilde{y}+2\beta}$ and $x_i^a(s) = \frac{\gamma(y^i(s))}{\gamma Y_s+2\beta}$ are the autarchic shares.

Since $\frac{dx_i^a(s)}{dz_i} = \frac{\gamma}{\gamma y_s+2\beta}$, the effect of a small increase in z on $\widehat{x}_1$ is given by

$$\frac{d\widehat{x}_i}{dz_i} = \frac{\gamma B_i}{[O_i]^{\frac{1-k}{2-k}}[A_i]^{\frac{1}{2-k}}} \tag{26}$$

where

$$\begin{aligned}
A_i &= [\widetilde{x}_i^a(\gamma\widetilde{y} + 2\beta)]^{\frac{2-k}{1-k}} + \tfrac{\delta}{1-\delta} E_s[x_i^a(s)(\gamma Y_s + 2\beta)]^{\frac{2-k}{1-k}};\\
B_i &= [\widetilde{x}_i^a(\gamma\widetilde{y} + 2\beta)]^{\frac{1}{1-k}} + \tfrac{\delta}{1-\delta} E_s[x_i^a(s)(\gamma Y_s + 2\beta)]^{\frac{1}{1-k}};\\
O_i &= (\gamma\widetilde{y} + 2\beta)^{\frac{2-k}{1-k}} + \tfrac{\delta}{1-\delta} E_s(\gamma Y_s + 2\beta)^{\frac{2-k}{1-k}}.
\end{aligned}$$

Two observations directly follow from this equation. First, an increase in 1's share of wealth increases the levels of consumption corresponding to 1's ICE and decreases the level of consumption corresponding to 2's ICE. So the question is which of these effects dominates. Second, at $z = 1/2$, the effects on 1's ICE and 2's ICE just cancel out, that is $\frac{d(\widehat{x}_1+\widehat{x}_2)}{dz}$ just equals zero.

Hence, to prove our claim it suffices to show that $\widehat{x}_i$ is concave in z for the utility functions of type (a) and convex for the utilities of type (b).

Taking the derivative of (26), we find that
(27)

$$\frac{d^2\widehat{x}_i}{d(z_i)^2} = \frac{[A_i]^{\frac{1-k}{2-k}-2}}{[O_i]^{\frac{1-k}{2-k}}}\gamma^2 \tfrac{1}{1-k}\left[\widetilde{x}_i^a(\gamma\widetilde{y} + 2\beta)]^{\frac{k}{1-k}} + \tfrac{\delta}{1-\delta} E_s[x_i^a(s)(\gamma y_s + 2\beta)]^{\frac{k}{1-k}}\right] A_i - B_i^2$$

Since $\frac{1}{1-k}$ is negative for the utility of class (a) but positive for the utilities of class b, it follows that the proposition is true if

$$\left[\widetilde{x}_i(\gamma\widetilde{y}+2\beta)]^{\frac{k}{1-k}}+\tfrac{\delta}{1-\delta}E_s[x_i(s)(\gamma y_s+2\beta)]^{\frac{k}{1-k}}\right]\left[[\widetilde{x}_i(\gamma\widetilde{y}+2\beta)]^{\frac{2-k}{1-k}}+\tfrac{\delta}{1-\delta}E_s[x_i(s)(\gamma y_s+2\beta)]^{\frac{2-k}{1-k}}\right]$$
$$\geq\left[[\widetilde{x}_i(\gamma\widetilde{y}+2\beta)]^{\frac{1}{1-k}}+\tfrac{\delta}{1-\delta}E_s[x_i(s)(\gamma y_s+2\beta)]^{\frac{1}{1-k}}\right]^2$$

Expanding both sides of the inequality, we see that

$$[\widetilde{x}_i(\gamma\widetilde{y}+2\beta)]^{\frac{2}{1-k}} + \tfrac{\delta}{1-\delta}E_s[x_i(s)(\gamma y_s+2\beta)]^{\frac{2}{1-k}} + \tfrac{\delta}{1-\delta}[\widetilde{x}_i(\gamma\widetilde{y}+2\beta)]^{\frac{k}{1-k}}E_s[x_i(s)(\gamma y_s+2\beta)]^{\frac{2-k}{1-k}}$$
$$+ [\widetilde{x}_i(\gamma\widetilde{y}+2\beta)]^{\frac{2-k}{1-k}}\tfrac{\delta}{1-\delta}E_s[x_i(s)(\gamma y_s+2\beta)]^{\frac{k}{1-k}} + 2(\tfrac{\delta}{1-\delta})^2E_sE_{s'>s}[x_i(s)(\gamma y_s+2\beta)]^{\frac{k}{1-k}}$$
$$[x_i(s')(\gamma Y_{s'}+2\beta)]^{\frac{2-k}{1-k}} \geq \widetilde{x}_i(\gamma\widetilde{y}+2\beta)]^{\frac{2}{1-k}} + \tfrac{\delta}{1-\delta}E_s[x_i(s)(\gamma y_s+2\beta)]^{\frac{2}{1-k}} + 2$$
$$[\widetilde{x}_i(\gamma\widetilde{y}+2\beta)]^{\frac{1}{1-k}}\tfrac{\delta}{1-\delta}E_s[x_i(s)(\gamma y_s+2\beta)]^{\frac{1}{1-k}} + 2(\tfrac{\delta}{1-\delta})^2E_sE_{s'>s}[x_i(s)(\gamma y_s+2\beta)]^{\frac{1}{1-k}}$$

Canceling some terms, we can rewrite this last inequality as

$$E_s[x_i(s)(\gamma y_s+2\beta)]^{\frac{k}{1-k}}\left[x_i(s)(\gamma y_s+2\beta)-\widetilde{x}_i(\gamma\widetilde{y}+2\beta)\right]^2\tfrac{\delta}{1-\delta}[\widetilde{x}_i(\gamma\widetilde{y}+2\beta)]^{\frac{k}{1-k}} + (\tfrac{\delta}{1-\delta})^2$$
$$E_sE_{s'>s}[x_i(s)(\gamma y_s+2\beta)x_i(s')(\gamma Y_{s'}+2\beta)]^{\frac{k}{1-k}}\left[x_i(s)(\gamma y_s+2\beta)-x_i(s')(\gamma Y_{s'}+2\beta)\right]^2 \geq 0$$

which is clearly satisfied. ■

Proof of Proposition 4. Before we proceed to a proof of Proposition 4, the following preliminary lemma is useful.

Lemma 1. *For all utility functions of the HARA class with $k \geq 2$ or k sufficiently close to 0, and for any two values $\overline{c}$ and $\underline{c}$ such that $\overline{c} > \underline{c}$, increasing the insurance along the incentive constraint decreases the following expression:*

$$\frac{-(1-\frac{\delta}{2})u''(\overline{c})-(\frac{\delta}{2})u''(\underline{c})}{\left[(1-\frac{\delta}{2})u'(\overline{c})+(\frac{\delta}{2})u'(\underline{c})\right]^2}. \tag{28}$$

Proof. Moving along the incentive constraint means that $(1-\frac{\delta}{2})u(\overline{c})+(\frac{\delta}{2})u(\underline{c})$ remains constant. Hence, increasing the level of insurance along the constraint means that a unit decrease in $\overline{c}$ is compensated by an increase in $\underline{c}$ of $\frac{(1-\frac{\delta}{2})}{\frac{\delta}{2}}\frac{u'(\overline{c})}{u'(\underline{c})}$.

A simple differentiation tells us that, for utility functions of the HARA class, the effect of such changes on (28) is negative if

$$
\begin{aligned}
&\tfrac{-\gamma}{1-k}2(1-\tfrac{\delta}{2})\left[[\gamma\underline{c}+\beta]^{\frac{k}{1-k}}\left[\tfrac{\gamma\bar{c}+\beta}{\gamma\underline{c}+\beta}\right]^{\frac{1}{1-k}}-[\gamma\bar{c}+\beta]^{\frac{k}{1-k}}\right]\left[(1-\tfrac{\delta}{2})[\gamma\bar{c}+\beta]^{\frac{k}{1-k}}+\tfrac{\delta}{2}[\gamma\underline{c}+\beta]^{\frac{k}{1-k}}\right]\\
<\;&\tfrac{-\gamma k}{1-k}(1-\tfrac{\delta}{2})\left[[\gamma\underline{c}+\beta]^{\frac{k}{1-k}-1}\left[\tfrac{\gamma\bar{c}+\beta}{\gamma\underline{c}+\beta}\right]^{\frac{1}{1-k}}-[\gamma\bar{c}+\beta]^{\frac{k}{1k}-1}\right]\left[(1-\tfrac{\delta}{2})[\gamma\bar{c}+\beta]^{\frac{1}{1-k}}+\tfrac{\delta}{2}[\gamma\underline{c}+\beta]^{\frac{1}{1-k}}\right].
\end{aligned}
$$

Simplifying this expression we get

$$
\begin{aligned}
&2\tfrac{[\gamma\bar{c}+\beta]^{\frac{k}{1-k}}}{[\gamma\underline{c}+\beta]}\left[[\gamma\bar{c}+\beta]-[\gamma\underline{c}+\beta]\right]\left[(1-\tfrac{\delta}{2})[\gamma\bar{c}+\beta]^{\frac{k}{1-k}}+\tfrac{\delta}{2}[\gamma\underline{c}+\beta]^{\frac{k}{1-k}}\right]\\
<\;&k\tfrac{[\gamma\bar{c}+\beta]^{\frac{k}{1-k}-1}}{[\gamma\underline{c}+\beta]^2}\left[[\gamma\bar{c}+\beta]^2-[\gamma\underline{c}+\beta]^2\right]\left[(1-\tfrac{\delta}{2})[\gamma\bar{c}+\beta]^{\frac{1}{1-k}}+\tfrac{\delta}{2}[\gamma\underline{c}+\beta]^{\frac{1}{1-k}}\right],
\end{aligned}
$$

which, on rearrangement, yields

$$
\begin{aligned}
&(2-k)[\gamma\bar{c}+\beta][\gamma\underline{c}+\beta]\gamma(\bar{c}-\underline{c})\left[(1-\tfrac{\delta}{2})[\gamma\bar{c}+\beta]^{\frac{k}{1-k}}+\tfrac{\delta}{2}[\gamma\underline{c}+\beta]^{\frac{k}{1-k}}\right]\\
<\;&k\gamma(\bar{c}-\underline{c})\left[(1-\tfrac{\delta}{2})[\gamma\bar{c}+\beta]^{\frac{1}{1-k}}+\tfrac{\delta}{2}[\gamma\underline{c}+\beta]^{\frac{1}{1-k}}\right].
\end{aligned}
\tag{29}
$$

Remember that $\frac{\gamma}{1-k}$ is negative such that either $\gamma > 0$ and $k > 1$ decreasing absolute risk aversion) or $\gamma < 0$ and $k < 1$ (increasing absolute risk aversion), while the special case of $k = 1$ corresponds to constant absolute risk aversion. It is easy to check that for all $k \geq 2$ and at $k = 0$ (quadratic utility), the inequality (29) is satisfied. This establishes the lemma. Note that this inequality is violated for values of k close to 1. ■

We now complete the proof of the proposition. To this end, differentiating (15), we see that

$$
\begin{bmatrix} (1-\frac{\delta}{2})u'(\bar{c}) & \frac{\delta}{2}u'(\underline{c}) \\ -\frac{\delta}{2}u'(y-\bar{c}) & -(1-\frac{\delta}{2})u'(y-\underline{c}) \end{bmatrix}\begin{bmatrix} d\bar{c}/dz \\ d\underline{c}/dz \end{bmatrix} = \begin{bmatrix} \omega(z) \\ -\omega(1-z) \end{bmatrix}
$$

where

$$
\omega(z_i) = (1-\tfrac{\delta}{2})u'(z_i+h) + \tfrac{\delta}{2}u'(z_i+\ell).
$$

and therefore,

$$\frac{d\overline{c}}{dz} = \frac{1}{D}\left[-(1-\frac{\delta}{2})u'(y-\underline{c})\,\omega(z)+\frac{\delta}{2}u'(\underline{c})\,\omega(1-z)\right] \tag{30}$$

$$\frac{d\underline{c}}{dz} = \frac{1}{D}\left[-(1-\frac{\delta}{2})u'(\overline{c})\,\omega(1-z)+\frac{\delta}{2}u'(y-\overline{c})\,\omega(z)\right] \tag{31}$$

where $D = -(1-\frac{\delta}{2})^2u'(\overline{c})u'(y-\underline{c}) + (\frac{\delta}{2})^2u'(\underline{c})u'(y-\overline{c})$.

It is easy to show that $D < 0$ since otherwise it would be possible to find a vector of consumption providing more insurance to the agents while satisfying their incentive constraints. Note also that either $\overline{c}$ or $\underline{c}$ or both increases as 1's share of wealth z increases, and therefore at $z = 1/2$ both $\overline{c}$ and $\underline{c}$ increase.

Inequality improves insurance, that is to decrease the spread between $\overline{c}$ and $\underline{c}$, if the following is true:

$$\frac{d(\overline{c}-\underline{c})}{dz} \qquad \begin{cases} > 0 & \text{if } z < \frac{1}{2}, \\ = 0 & \text{if } z = \frac{1}{2}, \\ < 0 & \text{if } z > \frac{1}{2} \end{cases} \tag{32}$$

The sign of $\frac{d(\overline{c}-\underline{c})}{dz}$ is given by the sign of

$$\frac{\omega(z)}{(1-\frac{\delta}{2})u'(\overline{c})+\frac{\delta}{2}u'(\underline{c})} - \frac{\omega(1-z)}{(1-\frac{\delta}{2})u'(y-\underline{c})+\frac{\delta}{2}u'(y-\overline{c})}$$

Hence, our claim requires that the (spread preserving) increase in mean consumption that keeps agent 1's incentive constraint unchanged after an increase in her wealth be less that the decrease in mean consumption that agent 2's incentive constraint would allow following a decrease in his wealth.

A sufficient condition for (32) to be true is that

$$\frac{(1-\frac{\delta}{2})u'(z+h)+\frac{\delta}{2}u'(z+\ell)}{(1-\frac{\delta}{2})u'(\overline{c})+\frac{\delta}{2}u'(\underline{c})}$$

decreases in z (note that $\overline{c}$ and $\underline{c}$ both depend on z). That is,

$$\frac{-(1-\frac{\delta}{2})u''(z+h)-\frac{\delta}{2}u''(z+\ell)}{(1-\frac{\delta}{2})u'(z+h)+\frac{\delta}{2}u'(z+\ell)} > \frac{-(1-\frac{\delta}{2})u''(\overline{c})\frac{d\overline{c}}{dz}-\frac{\delta}{2}u''(\underline{c})\frac{d\underline{c}}{dz}}{(1-\frac{\delta}{2})u'(\overline{c})+\frac{\delta}{2}u'(\underline{c})}. \tag{33}$$

At $z = 1/2$, it is easy to see that

$$\frac{d\overline{c}}{dz} = \frac{d\underline{c}}{dz} = \frac{(1-\frac{\delta}{2})u'(z+h)+\frac{\delta}{2}u'(z+\ell)}{(1-\frac{\delta}{2})u'(\overline{c})+\frac{\delta}{2}u'(\underline{c})}$$

Hence, starting from a situation of perfect equality, a regressive transfer in permanent income necessarily improves insurance if

$$\frac{-(1-\frac{\delta}{2})u''(z+h)-\frac{\delta}{2}u''(z+\ell)}{\left[(1-\frac{\delta}{2})u'(z+h)+\frac{\delta}{2}u'(z+\ell)\right]^2} > \frac{-(1-\frac{\delta}{2})u''(\overline{c})\frac{d\overline{c}}{wdz}-\frac{\delta}{2}u''(\underline{c})\frac{d\underline{c}}{wdz}}{\left[(1-\frac{\delta}{2})u'(\overline{c})+\frac{\delta}{2}u'(\underline{c})\right]^2}$$

This inequality together with lemma (1) completes the proof of the proposition. ∎

Proof of Proposition 5: To prove this claim, the following lemma is useful.

Lemma 2. *Pick two consumption level c and $\widetilde{c}$ and let $v(c,\widetilde{c}) = pu(c)+qu(\widetilde{c})$. If $c > \widetilde{c}$, a small reduction in c compensated by an increase in $\widetilde{c}$ keeping $v(c,\widetilde{c})$ constant decreases (increases) the following expression,*

$$pu'(c)+qu'(\widetilde{c}), \tag{34}$$

for all utility functions with decreasing (increasing) risk aversion. The converse is true if $c < \widetilde{c}$.

Proof. Consider a small decrease in c compensated by an increase in $\widetilde{c}$ that keeps $v(c,\widetilde{c})$ constant. That is $-pu'(c)dc+qu'(\widetilde{c})d\widetilde{c} = 0$ or $\frac{d\widetilde{c}}{dc} = \frac{pu'(c)}{qu'(\widetilde{c})}$. A simple differentiation shows that the effect of such changes on the expression (34) is negative (positive) if

$$\begin{aligned} -pu''(c)dc+qu''(\widetilde{c})d\widetilde{c} &< (>) \quad 0 \\ -\frac{u''(c)}{u'(c)}+\frac{u''(\widetilde{c})}{u'(\widetilde{c})} &< (>) \quad 0 \end{aligned}$$

If the utility function exhibits decreasing risk aversion, the left hand side of this inequality is negative (positive) if $c > \widetilde{c}$ ($c < \widetilde{c}$). Clearly the converse is true if risk aversion is increasing. ∎

Consider part (i) of the claim. When the agents' incomes are perfectly negatively correlated, they are recipients of transfers when their income is low and give transfers when their income is high. Let $\overline{c}$ and $\underline{c}$ be agent 2's consumption within the risk-sharing agreement in the state where his income is high and low respectively. If the incentive constraint binds $z_2+\ell < \underline{c} < \overline{c} < z_2+h$ and

$$(1-\tfrac{\delta}{2})u(\overline{c})+(\tfrac{\delta}{2})u(\underline{c}) = (1-\tfrac{\delta}{2})u(z_2+h)+(\tfrac{\delta}{2})u(z_2+\ell).$$

Notice that we can find get from $(z_2+h, z_2+\ell)$ to $(\overline{c}, \underline{c})$ using a sequence of small changes in c_h and c_l that decreases the spread and keeps $(1-\frac{\delta}{2})u(c_h) + (\frac{\delta}{2})u(c_l)$ constant. Hence, together with Lemma 2, it implies that

$$(1-\tfrac{\delta}{2})u'(\overline{c}) + (\tfrac{\delta}{2})u'(\underline{c}) < (1-\tfrac{\delta}{2})u'(z_2+h) + (\tfrac{\delta}{2})u'(z_2+\ell).$$

So that, following an increase in z_2, the same transfers between 1 and 2 do not satisfy 2's incentive constraint. The increase in z_2 reduces the transfer that agent 2 is willing to make when his income is high for any given transfer that agent 1 makes in return. As a result, the higher z_2 the lower the level of insurance.

Now, let's turn to part (ii) of the claim. It is easy to see that a necessary and sufficient condition for i making a non-zero transfers to j when $\ell\ell$ occurs is that

$$\left(\frac{1-\frac{\delta}{2}}{\frac{\delta}{2}}\right)^2 \leq \frac{\theta^{aut}_{hh}}{\theta^{aut}_{\ell\ell}} \tag{35}$$

where θ^{aut} is the ratio of marginal utilities of i to j in autarchy.

Indeed, (35) is condition for the maximum price for a unit transfer when low that j is willing to pay when high $\frac{\frac{\delta}{2}}{1-\frac{\delta}{2}}\frac{u'(z_j+\ell)}{u'(z_j+h)}$ to be greater or equal to the minimum that i would want to make such transfer $\frac{1-\frac{\delta}{2}}{\frac{\delta}{2}}\frac{u'(z_i+l)}{u'(z_i+h)}$.

Clearly, a necessary condition for (35) to hold is that $\theta^{aut}_{hh} > \theta^{aut}_{\ell\ell}$. Clearly, no transfer is possible when $z_1 = z_2$. When risk aversion is decreasing (increasing), $\theta^{aut}_{hh}/\theta^{aut}_{\ell\ell}$ requires $z_i > z_j$ ($z_i < z_j$) to be larger than 1 and is increasing in z_i and decreasing in z_j (decreasing in z_i and increasing in z_j). With decreasing (increasing) risk aversion, the poorer (richer) agent would receive a transfer when $\ell\ell$ occurs and gives a transfer when hh is realized. Moreover, the difference in wealth between the two agents needs to be sufficiently large for transfers to be possible.

Assume that a non-zero transfer is possible and $z_i > z_j$. Let $\underline{c}_j$ and $\overline{c}_j$ be j's consumption when $\ell\ell$ and hh respectively. If risk aversion is decreasing then $z_j + \ell < \underline{c}_j < \overline{c}_j < z_j + h$ while $\underline{c}_i < z_i + \ell < z_i + h < \overline{c}_i$. Moreover

$$\begin{aligned}
(1-\tfrac{\delta}{2})u(\overline{c}_j) + (\tfrac{\delta}{2})u(\underline{c}_j) &= (1-\tfrac{\delta}{2})u(z_j+h) + (\tfrac{\delta}{2})u(z_j+\ell);\\
(1-\tfrac{\delta}{2})u(\underline{c}_i) + (\tfrac{\delta}{2})u(\overline{c}_i) &= (1-\tfrac{\delta}{2})u(z_i+\ell) + (\tfrac{\delta}{2})u(z_i+h).
\end{aligned}$$

Using lemma 2, we see that

$$\begin{array}{rcl}(1-\frac{\delta}{2})u'(\overline{c}_j)+(\frac{\delta}{2})u'(\underline{c}_j) & < & (1-\frac{\delta}{2})u'(z_j+h)+(\frac{\delta}{2})u'(z_j+\ell);\\ (1-\frac{\delta}{2})u'(\underline{c}_i)+(\frac{\delta}{2})u'(\overline{c}_i) & > & (1-\frac{\delta}{2})u'(z_i+\ell)+(\frac{\delta}{2})u'(z_i+h).\end{array}$$

It follows that given the same transfers between 1 and 2, a decrease in z_j or an increase in z_i relaxes the incentive constraints. A similar argument can be used for utility functions with increasing risk aversion. ■

Proof of Proposition 6.
Part [ii] is obvious in the view of Proposition 5 as homogenous agents are unable to provide each other any insurance. Hence, individuals can only be made better off by pairing up with someone of different wealth.

Now consider part [i]. Following Legros and Newman (2006), to check whether positive assortative matching is stable, we can focus on individuals of symmetric arrangement between individuals of same wealth. Let u_{zz} be the utility of an individual, agent 1, with wealth z in an homogenous match. Consider agent 2 with wealth $z' > z$. In order to attract agent 1 to form a match with her, agent 2 needs to offer a contract $(\underline{c}, \overline{c})$ that guarantees 1 at least u_{zz}. Let $t_1 = z + h - \overline{c}$ be the transfer that 1 gives when high and $t_2 = \underline{c} - (z + \ell)$ the transfer she receives when low. We saw in Proposition 5 that for any t_1, a richer agent 2 cannot offer as high a t_2 than a poorer agent. This implies that all contracts that between 1 and 2 that are incentive compatible will have lower transfers than the transfers that 2 would give and receive in a homogenous match. Since 1's utility must be at least u_{zz} this implies that $t_1 > t_2$ in a match between 1 and 2. Hence, agent 2 would prefer the highest transfers (t_1, t_2) that gives u_{zz} to 1 and satisfies his incentive compatibility constraint if there is any. In this case, it follows that his incentive constraint is binding

$$(1-\frac{\delta}{2})u(z'+h-t_1)+\frac{\delta}{2}u(z'+\ell+t_2) = (1-\frac{\delta}{2})u(z'+h)+\frac{\delta}{2}u(z'+\ell).$$

Since $t_1 > t_2$, then $t_1, t_2 < t_{z'}$ where $t_{z'}$ is the transfer that 2 would make and receive in a homogenous match

$$(1-\frac{\delta}{2})u(z'+h-t_{z'})+\frac{\delta}{2}u(z'+\ell+t_{z'}) = (1-\frac{\delta}{2})u(z'+h)+\frac{\delta}{2}u(z'+\ell).$$

Therefore, agent 2 could not guarantee u_{zz} to 1 and receive a higher utility than his own utility in a homogenous match $u_{z'z'}$. ■

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