Global Imbalances, Risk and the Great Recession

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ABSTRACT

This paper describes a new analytical framework for the quantitative assessment of international external positions. The framework links each country's current net foreign asset position to its current trade flows, forecasts of future trade flows, and expectations concerning future returns on foreign assets and liabilities in an environment where countries cannot run Ponzi schemes or exploit arbitrage opportunities in world financial markets. It provides guidance on how external positions should be measured in the data, and on how the sustainability of a country's current position can be assessed. To illustrate its usefulness, I study the external positions of 12 countries (Australia, Canada, China, France, Germany, India, Italy, Japan, South Korea, Thailand, The United States and The United Kingdom) between 1970 and 2011. In particular, I examine how changes in the perceived risk associated with future returns across world financial markets contributed to evolution of external positions before the 2008 financial crisis, and during the ensuing Great Recession.

"Global imbalances are probably the most complex macroeconomic issue facing economists and policy makers. They reflect many factors, from saving to investment to portfolio decisions, in many countries. These cross-country differences in saving patterns, investment patterns, and portfolio choices are in part "good" - a natural reflection of differences in levels of development, demographic patterns, and other underlying economic fundamentals. But they are also in part "bad," reflecting distortions, externalities, and risks, at the national and international level. So it is not a surprise that the topic is highly controversial, and that observers disagree on the diagnosis and thus on the policies to be adopted." Blanchard and Milesi-Ferretti (2009)

Introduction

This paper proposes an analytical framework for the quantitative assessment of international external positions. The framework links each country's current net foreign asset position to its current trade flows, forecasts of future trade flows, and expectations concerning future returns on foreign assets and liabilities in an environment where countries cannot run Ponzi schemes or exploit arbitrage opportunities in world financial markets. As such, it allows researchers and policy makers to quantify the contribution of the many potential factors (both the "good" and "bad", as Blanchard and Milesi-Ferretti (2009) note) determining imbalances in net foreign asset positions and trade flows across countries and through time. The framework also provides guidance on how external positions should be measured in the data, and on how the sustainability of a country's current position can be assessed. In short, it is a diagnostic tool that can help researchers and policy makers work through the complex issues associated with global imbalances. To illustrative its usefulness, I use the framework to study the external positions of 12 countries (Australia, Canada, China, France, Germany, India, Italy, Japan, South Korea, Thailand, The United States and The United Kingdom) between 1970 and 2011. In particular, I examine how changes in the perceived risk associated with future returns across world financial markets contributed to evolution of external positions before the 2008 financial crisis, and during the ensuing Great Recession.

The framework I present incorporates several key features. First it accommodates the secular increase in international trade flows and gross asset/liability positions that have taken place over the past 40 years. The secular growth in both trade flows and positions greatly exceeds the growth in GDP on a global and country-by-country basis. Over the past 40 years, the annual growth in trade and positions exceeds the growth in GDP by an average of 2.6 and 4.8 percent, respectively, across the countries I study. This feature of the data has proved to be a challenge for researchers studying the determinants of global imbalances. For example, Gourinchas and Rey (2007) derive an expression for a country's net foreign asset position from a "de-trended" version of the consolidated budget constraint (that governs the evolution of a country's net foreign asset position from trade flows and returns), that filters out the secular growth in trade flows and positions. Thus their analysis focuses on the "cyclical" variations in net foreign asset positions, rather than the "total" variations. Similarly, Corsetti and Konstantinou (2012) use the consolidated budget constraint to derive an approximation to the current account that includes deterministic trends in the log ratios of consumption, gross assets and gross liabilities to output to accommodate the long-term growth in trade flows and positions (relative to GDP). In contrast, I develop an expression for a country's total

net foreign asset position from the consolidated budget constraint and show how it can be evaluated empirically without counterfactual assumptions concerning the growth in trade and positions. This approach has an important empirical advantage relative to the alternatives cited above. It allows us to study the source of the persistent changes in many country's external positions rather than just their short-term variations around a secular trend.

The second key feature of my framework concerns the identification of expected future returns. As a matter of logic (based on the consolidated budget constraint), expected future returns on a country's asset and liability portfolios *must* affect the value its current net foreign asset position, so pinning down these expectations is unavoidable in analyzing external positions. This is easily done in textbook models where the only internationally traded asset is a risk free bond with a constant interest rate (see, e.g., the intertemporal approach to the current account), but in the real world countries' asset and liability portfolios comprise equity, FDI, bonds and other securities, with risky and volatile returns. Pinning down the expected future returns on these portfolios requires forecasts for the future returns on different securities and the composition of the portfolios. The need for multilateral consistency further complicates this task: Expected returns in one country's foreign asset portfolio must be matched by the expected return in others' liability portfolios. To avoid these complications, I use no-arbitrage conditions to identify the impact of expected future returns on net foreign asset positions via forecasts of a single variable, the world Stochastic Discount Factor (SDF). SDFs play a central role in modern finance theory (linking security prices and cash flows) and appear in theoretical examinations of the determinants of net foreign asset positions (see, e.g., Obstfeld, 2012). A key step in my analysis is to show how the world SDF can be constructed from data on returns and then used to pin down expectations of future returns that affect net foreign asset positions.

Since SDF's are much less commonly used in macroeconomics than in finance, it is worth highlighting the benefits of incorporating the world SDF into my analytical framework. First, its use imposes multilateral consistency. No country's can unilaterally benefit from expected future return differentials between its foreign asset and liability holdings. Second, the use of the SDF does not require any assumption about how the composition of a particular country's asset or liability portfolio are determined. They may represent, in aggregate, the optimal portfolio decisions of private sector agents, or they may not. So, to the extent that capital controls affect the composition of portfolios, the presence, absence or change in controls doesn't invalidate the use of the world SDF in the determination of a particular country's net foreign asset position. Third, although expected future returns on foreign assets and liabilities may differ from the forecasts of the world SDF under special circumstances, it is easy to test empirically whether these circumstance apply to a particular country. Fourth, the use of the SDF allows us to distinguish between the effects of changing expectations concerning the future path of the risk free rate on global imbalances, and the effects of changes in perceived (systematic) risk that is reflected in the expected returns on risky assets and liabilities. Finally, I use the SDF to focus on external positions that are *not* supported by Ponzi-schemes. This analytical focus is important. Any external position must be supported by agents willing to hold the country's asset/liability positions, but no rational agent would willingly participate (i.e. hold the country's liabilities) in a Ponzi-scheme. Consequently, any analysis of external positions that allows for the presence of Ponzi-schemes implicitly relies on the fragile assumption that (some) agents are acting against their own best interests. It is straightforward to exclude external positions supported

by Ponzi-schemes with a condition that involves the world SDF.

Traditionally, researchers and policy makers concerned with global imbalances have focused their attention on current account balances. For example, Lane and Milesi-Ferretti (2012) examine how changes in current account balances between 2008 and 20010 relate to pre-crisis current account gaps estimated from a panel regression model. Similar empirical models of current account determination can be found in Chinn and Prasad (2003), Gruber and Kamin (2007), Lee et al. (2008), Gagnon (2011) and others. Current accounts also remain a focus in current multilateral surveillance frameworks used by the International Monetary Fund and the European Commission (see, e.g., IMF, 2012 and EU, 2010). Nevertheless, there are reasons to question whether this attention is warranted. First, current account imbalances are simply not that informative about the changes in net foreign asset positions, or equivalently, cumulated past current account imbalances produce only an approximation to the current net foreign asset position valued at market prices. These discrepancies arise because the Balance of Payments methodology ignores the capital gains and losses on existing foreign asset and liability positions that arise from exchange rate variations and changes in security prices, but the gains and losses are reflected in the net foreign asset positions. Second, as Obstfeld (2012) notes, by focusing on the current account we run the risk of neglecting potential balance sheet vulnerabilities to unexpected changes in exchange rates and security prices that could significantly alter the market values of foreign assets and liabilities. Researchers and policy makers are, of course, well aware of these issues. The problem is the lack of an analytic framework that allows for a more comprehensive quantitative assessment of global imbalances.

The current account is not the focus of the framework I present. When one starts from a minimal set of assumptions concerning international transactions (budget constraints and no-arbitrage conditions), the current account does not appear as an important economic measure of a country's external position. What emerges, instead, is a measure that combines the country's current net foreign asset position and trade flows. Specifically, I measure each country's external position as the gap between its current net foreign asset position and the steady state present value of the current trade deficit, where the latter is computed at the point where expected future growth in imports and exports are equal and the expected future returns on all securities are constant (but not necessarily equal). The framework also shows us how to normalize this measure across countries. We simply divide by the current trade flow (i.e., the sum of exports and imports). This is a departure from the standard practice of normalizing current account imbalances and net foreign asset positions by GDP. Normalizing by trade rather than GDP avoids problems associated with the secular growth in trade relative to GDP discussed above. Moreover, the measure provides a natural way to identify external imbalances. Market clearing insures that the measure aggregates across countries to give a world external position of zero. The measure also differs from zero for an individual country when expectations for future trade flows and returns differ from their unconditional (steady state) values. So the analysis of how different factors (both the "good" and "bad") affect these expectations is the key to understanding the source of global imbalances across countries and through time.

In the second half of the paper I study the external positions of 12 countries (Australia, Canada, China, France, Germany, India, Italy, Japan, South Korea, Thailand, The United States and The United Kingdom). I first show how the world SDF can be estimated from data on returns and discuss how the estimates can be tested for specification errors. Next I turn to the identification of expectations. In theory, each country's external position is determined by agents' expectations

concerning future growth in exports, imports and the world SDF. For the purpose of this paper I identify these expectations from VAR forecasts. Following Campbell and Shiller (1987), this is a very common approach in academic research, but it is not without its limitations. I discuss how alternative identification methods could (and should) be used by policy makers when the framework is used for multilateral surveillance.

My empirical analysis takes three perspectives. First I examine the implications of my framework for the cross-country distribution of external positions each year between 1970 and 2011. In this analysis, each country faces the same set of world financial conditions as summarized by the expected path for the future world SDF. Cross-country differences in the positions are thus attributable to differences in expectations concerning future trade flows and differences in each country's exposure to expected changes in future financial conditions. Second, I consider the dynamics of external positions on a country-by-country basis. This analysis provides evidence on the different channels through which adjustment in net foreign asset positions and trade flows takes place. As in Gourinchas and Rey (2007), my framework identifies two adjustment channels: the trade and valuation channels. Over the entire sample period (1970-2011), the trade channel appears to be the most important adjustment channel for the majority of countries I study. The one notable exception is The United States, where adjustment via the valuation channel dominates. My third perspective focuses on global imbalances in the past decade. Here I examine how changes in financial conditions affected imbalances before the 2008 financial crises and during the following Great Recession. I find evidence of large swings in systemic risk (measured by the difference between the expected future path for the world SDF and the risk free rate), with a large rise occurring between 2006 and 2009. This change in risk produced significant adjustments in the external positions of countries running large trade imbalances (e.g. Australia, China and the United States). However, overall, most of the adjustment in external positions between 2006 and 2009 appears to have taken place through the trade channel via revisions in expected future trade flows.

The remainder of the paper is structured as follows: Section 1 describes the data and documents the secular variations in international trade flows and positions. Sections 2 and 3 develop the analytical framework. I first discuss the problem of determining the value a country's net foreign asset position without the use of an SDF. I then show how the world SDF is used to determine net foreign asset positions that are not supported by Ponzi schemes. Section 4 discusses empirical implementation. The results of my empirical analysis are reported in Sections 5 and 6. Section 7 concludes.

1   Data

I study the external positions of twelve countries: the G7 (Canada, France, Germany, Italy, Japan, the United States and the United Kingdom) together with Australia, China, India, South Korea and Thailand. Data on each country's foreign asset and liability portfolios and the returns on the portfolios come from the databased constructed by Lane and Milesi-Ferretti (2001) and updated in Lane and Milesi-Ferretti (2009) available via the IMF's International Financial Statistics database. These data provide information on the market value of the foreign asset and liability portfolios at the end of each year together with the returns on the portfolios from the end of one year to the next. A detailed discussion of how these data series are constructed can be found in Lane and Milesi-Ferretti

Figure 1: Net Foreign Assets and Net Exports

A: Net Foreign Assets (% of GDP)
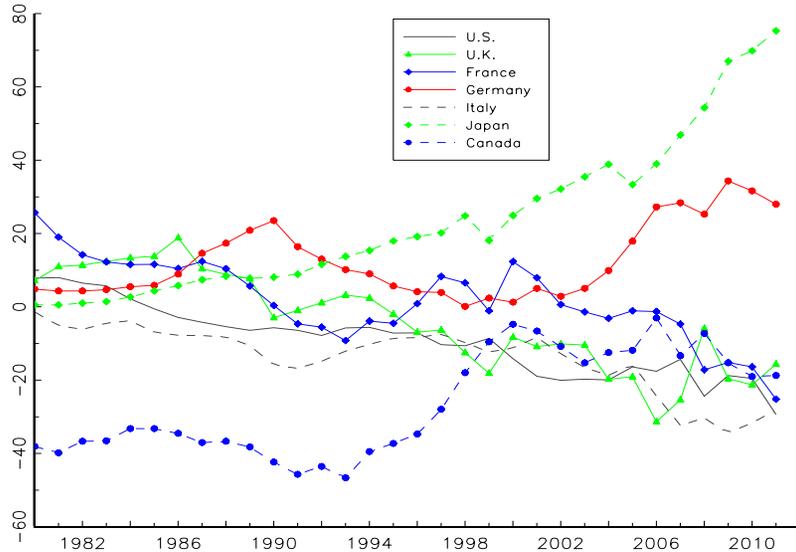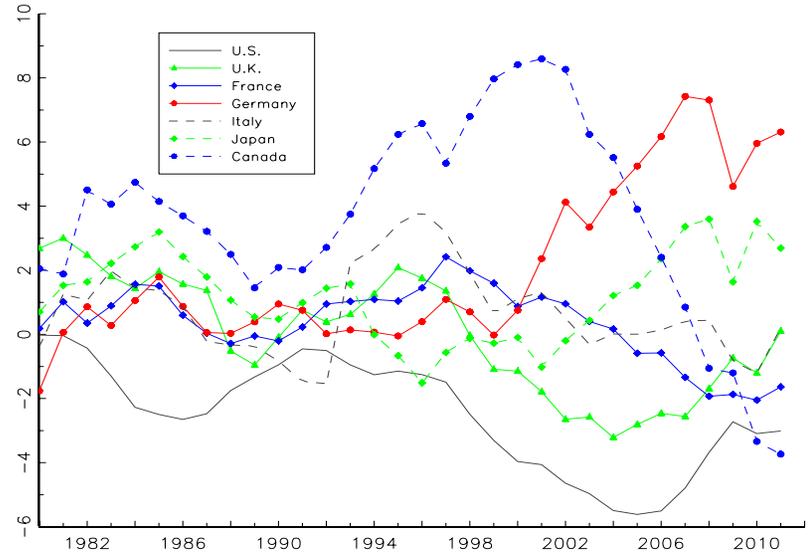
B: Net Exports (% of GDP)

C: Net Foreign Assets (% of GDP)
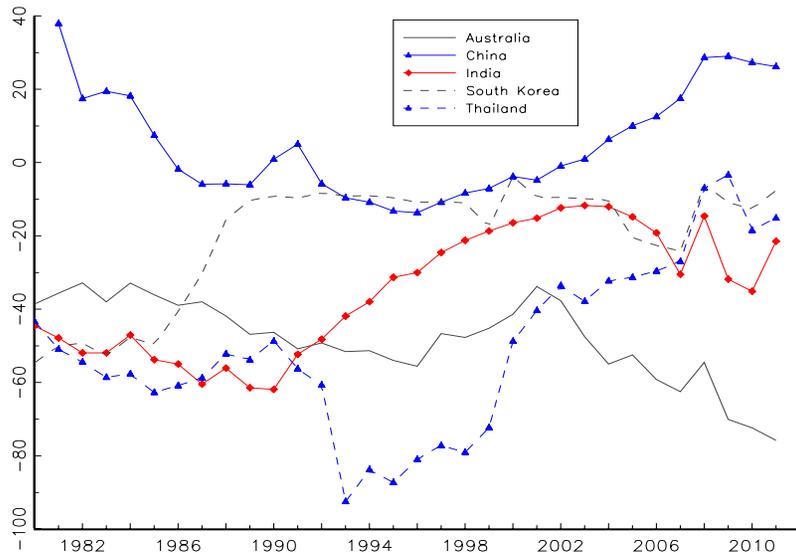
B: Net Exports (% of GDP)
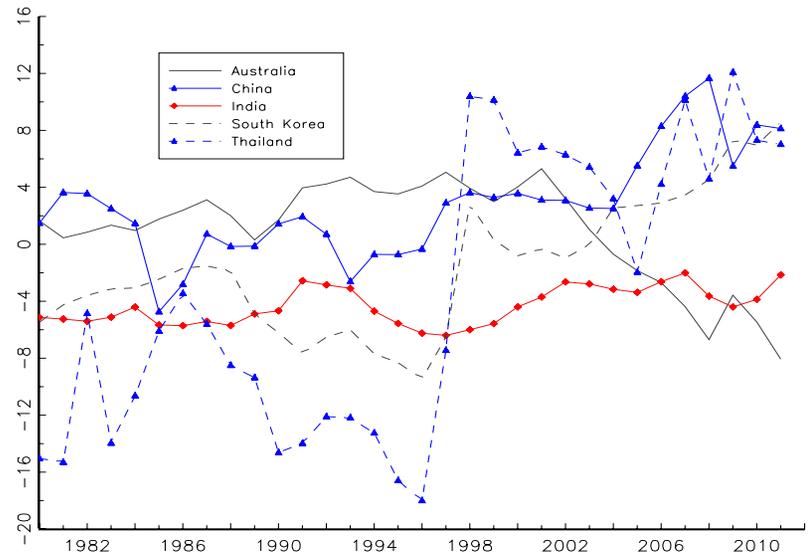
(2009). I also use data on exports, imports and GDP for each country and data on the one year U.S. T-bill rate, 10 year U.S. T-bond rate and U.S. inflation. All asset and liability positions, trade flows and GDP levels are transformed into constant 2005 U.S. dollars using the prevailing exchange rates and U.S. price deflators. All portfolio returns are similarly transformed into real U.S. returns. The Lane and Milesi-Ferretti position data is constructed on an annual basis, so my analysis below is conducted at an annual frequency.[1] Although the span of individual data series differs from country to country, most of my analysis uses data spanning 1970-2011.

Figure 1 provides a visual perspective on the task of understanding the behavior of external positions and trade flows across the world's major economies. Panels A and C plot the ratio of each country's net foreign asset (NFA) position (i.e., the difference between the value of its foreign asset and liability portfolios) to GDP between 1980 and 2011. These plots display two noteworthy features. First, they clearly show that variations in the NFA/GDP ratios of many countries are highly persistent, with significant movements often lasting decades. This means that any analysis of the drivers of the NFA/GDP ratios must focus on the source of movements below business-cycle frequencies. The second feature concerns the dispersion of the ratios across countries. Panel A shows that the dispersion has increased markedly across the G7 in the last decade, with ratios ranging from -20 to 80 percent of GDP in 2011. With the notable exception of Canada, imbalances between the value of foreign assets and liabilities have been steadily growing across the G7 for the past 30 years. Panel C shows that the dispersion in NFA/GDP ratios also increased across the non-G7 countries in the last decade. Panels B and D plot the ratios of net exports (exports minus imports) to GDP for the comparable countries over the same sample period. Again, we can see that these ratios display a good deal of time series persistences. Among the G7, the ratios have become most dispersed since the early 1990s, while there is no clear change in the dispersion of the ratios among the other countries.

The plots in Figure 1 follow the standard practice of measuring NFA positions and net exports relative to GDP. This normalization facilitates comparisons of external positions and trade flows across countries with economies of different sizes at a point in time, but is less useful for intertemporal comparisons. To understand why, Figure 2 plots the sum of foreign asset and liability positions as a fraction of GDP and the sum of exports and imports as a fraction of GDP for each of the countries on the dataset between 1980 and 2011. Clearly, both trade and gross foreign positions have been growing persistently relative to GDP in every country. Moreover, it is clear that gross positions rose particularly rapidly in the last decade. The plots in Figure 2 also illustrate how the cross-country differences in the degree of openness (both in terms of trade flows and gross positions) have increased over time. These trends complicate intertemporal comparisons of NFA and net export positions. For example, should a fall in a country's NFA position from -20 to -30 percent of GDP be viewed as a significant deterioration in its external position when the gross asset position has risen from 100 to 200 percent of GDP? Similarly, does a constant net export-to-GDP ratio really indicate stability in a country's trade position when total trade is steadily rising relative to GDP?

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[1] Ideally, we would like to track international positions and returns at a higher (e.g. quarterly) frequency, but constructing the market value of foreign assets and liabilities for a large set of countries is a herculean task. For the United States, Gourinchas and Rey (2005) compute quarterly market values for four categories of foreign asset and liabilities: equity, foreign direct investment, debt and other, by combining data on international positions with information on the capital gains and losses. In Evans (2012b) I revise and update their data to 2012:IV. Corsetti and Konstantinou (2012) also work with quarterly U.S. position data which they impute from the annual Milesi-Ferretti data using quarterly capital flows. For a discussion of the different methods used to construct return data, see Gourinchas and Rey (2013).

Figure 2: Total Assets and Trade

A: Foreign Assets and Liabilities (% of GDP)
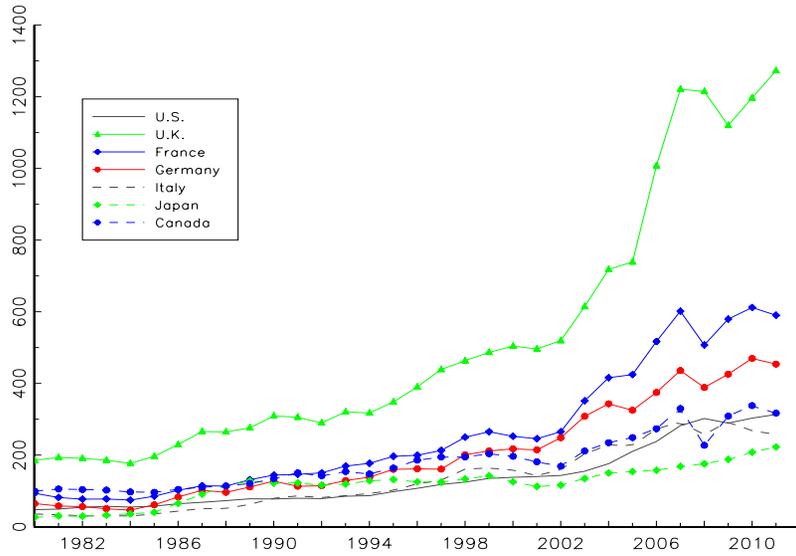
B: Exports and Imports (% of GDP)
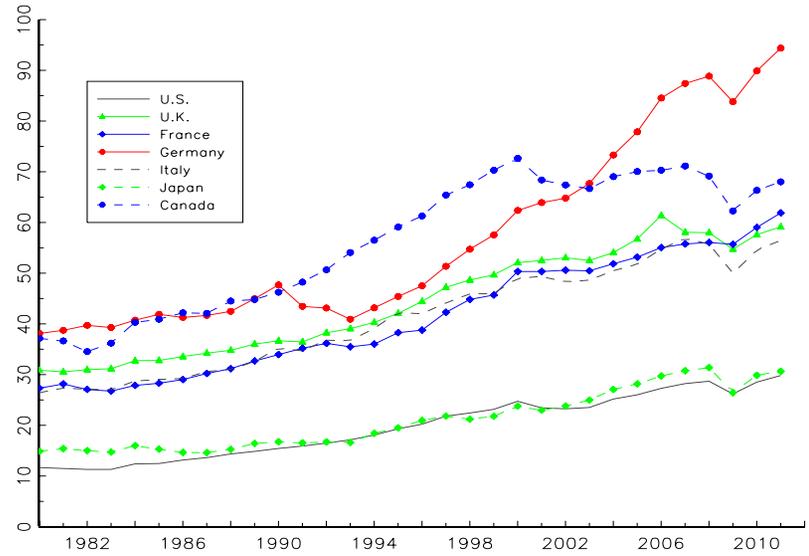
C: Foreign Assets and Liabilities (% of GDP)
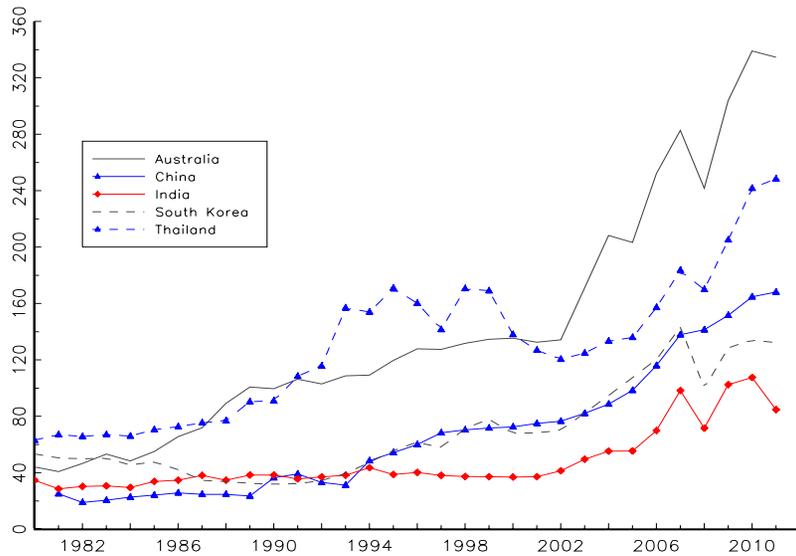
D: Exports and Imports (% of GDP)
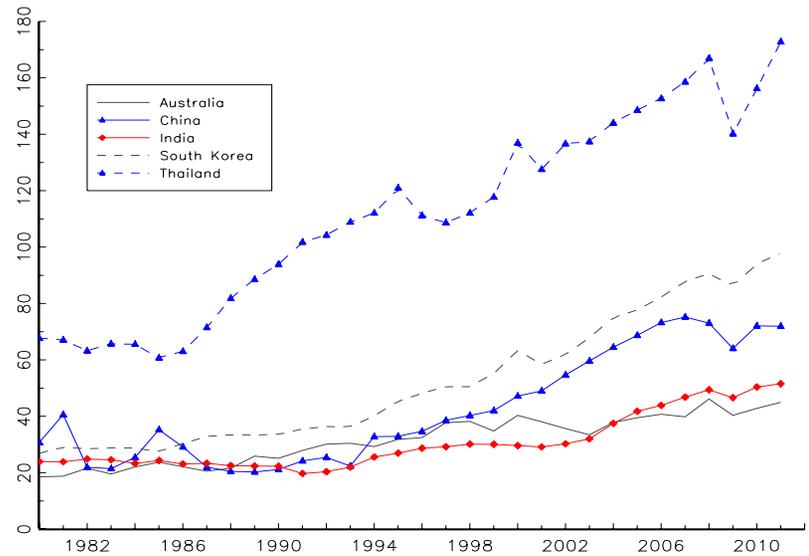
Table 1: Growth in Trade and Foreign Positions

	Trade Growth			Position Growth			Export-Import Differential		
	Mean	Std.	AR(1)	Mean	Std.	AR(1)	Mean	Std.	AR(1)
A:									
Canada	4.660	5.557	0.138	6.083	10.153	-0.289	-0.836	4.764	0.208
France	4.790	4.238	0.083	8.343	9.318	0.148	0.276	3.641	-0.039
Germany	5.057	4.573	0.031	9.099	11.186	0.226	0.560	3.827	0.053
Italy	4.190	5.290	-0.107	6.920	11.988	0.307	0.657	5.552	0.063
Japan	5.081	6.986	-0.132	9.711	12.926	0.472	1.047	9.263	0.129
United Kingdom	4.200	4.142	0.148	9.348	8.775	0.282	-0.116	3.770	0.204
United States	5.652	5.356	0.047	8.446	5.430	0.368	0.379	7.368	0.538
Australia	5.481	4.837	-0.310	9.016	13.770	0.069	-1.253	10.461	-0.017
China	12.090	10.131	0.140	16.850	7.890	-0.043	0.157	13.208	-0.017
India	7.771	6.718	0.249	8.924	12.694	0.199	1.274	7.035	0.071
South Korea	11.529	8.635	0.058	11.661	12.475	-0.018	2.842	11.106	0.068
Thailand	8.172	9.239	0.022	9.778	10.858	0.259	1.736	12.911	-0.292
Average	6.552	4.456	-0.117	8.751	7.175	0.144	0.453	2.799	0.140
B: Relative to GDP Growth									
Canada	1.787	4.075	0.162	3.210	10.447	-0.294			
France	2.579	3.086	-0.023	6.131	9.157	0.080			
Germany	2.759	3.807	0.144	6.800	11.015	0.173			
Italy	2.177	3.834	-0.177	4.907	12.135	0.222			
Japan	2.477	5.854	-0.236	7.107	12.032	0.402			
United Kingdom	2.042	2.987	-0.114	7.191	7.961	0.243			
United States	2.877	3.758	-0.071	5.671	4.971	0.312			
Australia	2.271	8.438	-0.327	5.806	11.887	-0.017			
China	1.900	17.822	0.049	6.660	14.096	-0.042			
India	2.554	6.759	0.067	3.708	15.679	-0.042			
South Korea	5.316	6.549	0.103	5.448	12.693	-0.064			
Thailand	2.503	7.398	-0.189	4.109	10.152	0.048			
Average	2.624	3.827	-0.189	4.822	6.863	0.064			

Notes: Panel A reports the sample mean and standard deviation (in annual percent) and first order autocorrelation coefficient for: (i) trade growth $\frac{1}{2}(\Delta x_t + \Delta m_t)$, (ii) the position growth $\frac{1}{2}(\Delta f a_t + \Delta m f l_t)$, and (iii) the export-import growth differential $\Delta x_t - \Delta m_t$; where x_t, m_t $f a_t$ and $f l_t$ denote the logs of exports, imports, the value of foreign assets and foreign liabilities, respectively (in constant U.S. dollars). Panel B reports statistics for (i) the relative growth in trade $\frac{1}{2}(\Delta x_t + \Delta m_t) - \Delta y_t$ and (ii) the relative growth in positions liabilities $\frac{1}{2}(\Delta f a_t + \Delta m f l_t) - \Delta y_t$; where y_t denotes the log of real GDP. All statistics are computed in annual data over the sample period of 1971-2011.

Table 1 provides statistical evidence complimenting the plots in Figure 2. Panel A reports sample statistics for the annual growth in trade, gross positions, and the export-import growth differential. Trade growth is computed as the average growth rate for real exports and imports $\frac{1}{2}(\Delta x_t + \Delta m_t)$, position growth by the average growth in foreign assets and liabilities $\frac{1}{2}(\Delta fa_t + \Delta fl_t)$, and the export-import differential as the difference between the growth in exports and imports, $\Delta x_t - \Delta m_t$; where x_t, m_t, fa_t and fl_t denote the logs of exports, imports, the value of foreign assets and foreign liabilities, respectively; and Δ is the first-difference operator. (Throughout I use lowercase letters to denote the natural log of a variable.) As the table shows, the mean trade growth and mean position growth are similar across the G7 countries, with mean position growth roughly two to four precent higher. Cross-country difference in mean trade growth and position are more pronounced across the other countries. The mean export-import growth differentials shown in the right-hand columns are small by comparison. Some of the cross-country differences in the mean trade and position growth rates reflect differences in the degree of economic development that in turn are reflected in GDP growth. This can be seen in Panel B where I report statistics for trade growth and position growth relative to GDP growth, measured as $\frac{1}{2}(\Delta x_t + \Delta m_t) - \Delta y_t$ and $\frac{1}{2}(\Delta fa_t + \Delta fl_t) - \Delta y_t$, respectively; where y_t is the log of real GDP. Here the cross-country differences in mean growth rates are much smaller. Notice, however, that mean rates are all positive. Averaging across all the countries, trade grew approximately 2.6 percent faster than GDP, while foreign asset and liability positions grew 4.8 percent faster.

Figure 3: Global Growth Rates
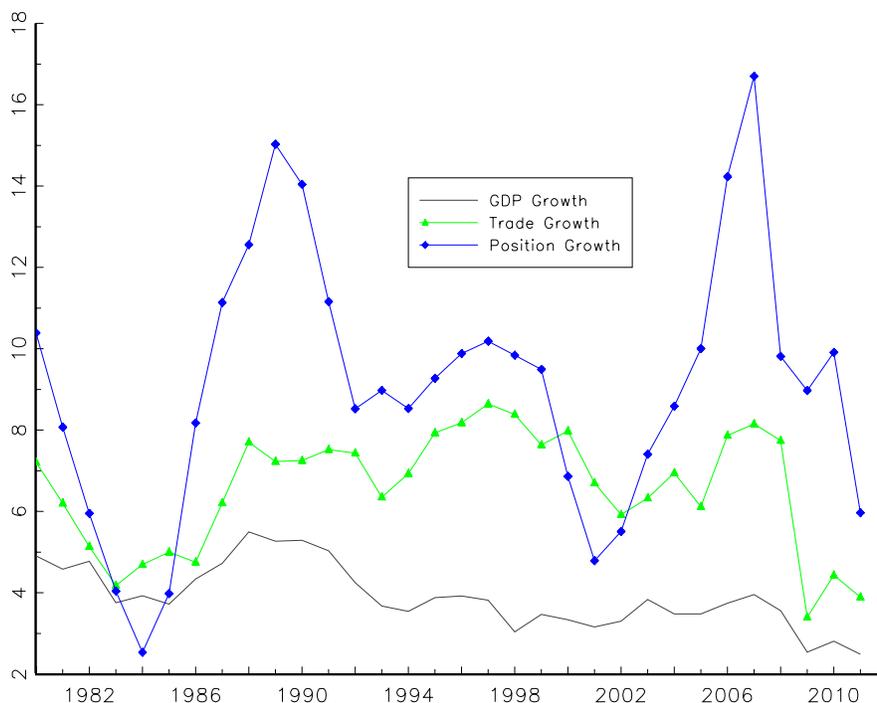
Notes: The figure plots the five-year moving average of the cross-country averages for: (i) GDP growth $\frac{1}{N}\sum_n \Delta y_{n,t}$, (ii) trade growth $\frac{1}{2N}\sum_n(\Delta x_{n,t} + \Delta m_{n,t})$ and (iii) position growth $\frac{1}{2N}\sum_n \Delta fa_{n,t} + \Delta fl_{n,t})$ all in annual percent.

Figure 2 and Table 1 show that, on average, the growth in global trade and financial positions have greatly exceeded global output growth in the last three decades. Year-by-year, the picture is more complicated. Figure 3 plots the five-year moving average of the cross-country average for GDP growth, trade growth and position growth between 1980 and 2011. These growth rates are computed as $\frac{1}{N} \sum_n \Delta y_{n,t}$, $\frac{1}{2N} \sum_n (\Delta x_{n,t} + \Delta m_{n,t})$ and $\frac{1}{2N} \sum_n \Delta fa_{n,t} + \Delta fl_{n,t})$, respectively; from the trade and position data of each country $n = \{1, 2, ...N\}$ in the dataset. The plots reveal that swings in global trade growth and position growth have been much larger than global business cycles represented by the growth in GDP. The size and timing of the swings in position growth are even more striking. The last three decades witnessed two episodes of increasingly rapid growth in foreign asset and liability positions; the first in the mid-1980's and the second between 2000 and 2006. Conversely, growth declined markedly in three episodes; the first in the early 1980's, the second following the 1997 Asian crises, and the third starting in 2007. The first and third episodes also witnessed a significant fall in trade growth.

The growth in both trade and positions relative to GDP present a challenge in studying countries' NFA positions because standard models describe a world where these features are absent. For example, in standard open-economy models consumer's preferences tie exports and imports to relative prices and domestic consumption (see, e.g. Evans, 2011). In these models relative prices are constant in the steady state so exports and imports share the same trend as output. This means that trade growth cannot exceed output growth in the long run. Similarly, open economy models with many financial assets predict that position growth equals output growth in the long run. Here the growth in the value of a country's foreign asset and liability positions are determined by aggregating individuals' steady state portfolio choices. In standard models these choices imply that individual's foreign asset and liability holdings are constant fraction of wealth, so a country's position shares the same long run trend as GDP.[2] Clearly, these models could not generate the global growth plots in Figure 3.

2 Net Foreign Assets, Trade and Returns

The framework I develop contains three elements: (i) the consolidated budget constraint that links a country's foreign asset and liability positions to exports, imports and returns; (ii) a condition that rules out international Ponzi schemes; and (iii) a no-arbitrage condition that restricts the behavior of returns. In this section I introduce the first two elements and explain why they are not sufficient for constructing the framework we need. Section 3 combines all three elements into the framework I will use.

I begin with country's $n's$ consolidated budget constraint:

$$FA_{n,t} - FL_{n,t} = X_{n,t} - M_{n,t} + R^{\text{FA}}_{n,t} FA_{n,t-1} - R^{\text{FL}}_{n,t} FL_{n,t-1}. \tag{1}$$

Here $FA_{n,t}$ and $FL_{n,t}$ denote the value of foreign assets and liabilities of country n at the end of year t, while $X_{n,t}$ and $M_{n,t}$ represent the flow of exports and imports during year t, all measured in real terms (constant U.S. dollars). The gross real return on the foreign asset and liability portfolios of country n between the end of years $t-1$ and t are denoted by $R^{\text{FA}}_{n,t}$ and $R^{\text{FL}}_{n,t}$, respectively.

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[2]See, e.g., Evans (2012a), or the models surveyed in Coeurdacier and Rey (2012).

Equation (1) is no more than an accounting identity. It should hold true for any country provided the underlying data on positions, trade flows and returns are accurate. Notice, also, that $FA_{n,t}$ and $FL_{n,t}$ represent the values of portfolios of assets and liabilities comprising equity, bond and FDI holdings, and that $R_{n,t}^{\text{FA}}$ and $R_{n,t}^{\text{FL}}$, are the corresponding portfolio returns. These returns will generally differ across countries in the same year because of cross-country differences in the composition of asset and liability portfolios.

It proves useful to rewrite (1) in terms of a reference (gross) real return, R_t, and excess portfolio returns returns, $ER_{n,t}^{\text{FA}} = R_{n,t}^{\text{FA}} - R_t$ and $ER_{n,t}^{\text{FL}} = R_t^{\text{FA}} - R_t$:

$$NFA_{n,t} = X_{n,t} - M_{n,t} + R_t NFA_{n,t-1} + ER_{n,t}^{\text{FA}} FA_{n,t-1} - ER_{n,t}^{\text{FL}} FL_{n,t-1}, \qquad (2)$$

where $NFA_{n,t} = FA_{n,t} - FL_{n,t}$ is the net foreign asset position at the end of year t. Re-arranging this expression as

$$NFA_{n,t} = \frac{1}{R_{t+1}}\left(M_{n,t+1} - X_{n,t+1}\right) + \frac{1}{R_{t+1}}\left(ER_{n,t+1}^{\text{FL}} FL_{n,t} - ER_{n,t+1}^{\text{FA}} FA_{n,t}\right) + \frac{1}{R_{t+1}} NFA_{n,t+1},$$

dividing by the country's GDP, $Y_{n,t}$, and iterating forward produces

$$\frac{NFA_{n,t}}{Y_{n,t}} = \sum_{i=1}^{\infty} \mathfrak{D}_{n,t+i}\left\{\frac{M_{n,t+i} - X_{n,t+i}}{Y_{n,t+i}}\right\}$$
$$+ \sum_{i=1}^{\infty} \mathfrak{D}_{n,t+i}\left\{\frac{ER_{n,t+i}^{\text{FL}} FL_{n,t+i-1}}{Y_{n,t+i}} - \frac{ER_{n,t+i}^{\text{FA}} FA_{n,t+i-1}}{Y_{n,t+i}}\right\}$$
$$+ \lim_{i \to \infty} \mathfrak{D}_{n,t+i}\left\{\frac{NFA_{n,t+i}}{Y_{n,t+i}}\right\}, \qquad (3)$$

where

$$\mathfrak{D}_{n,t+i} = \prod_{j=1}^{i}\left\{\frac{Y_{n,t+j}}{R_{t+j} Y_{n,t+j-1}}\right\}$$

is the year t discount factor for year $t+i$. The final step is to take expectations on both sides of (3) conditioned on year t information (that includes the value of $NFA_{n,t}/Y_{nt}$):

$$\frac{NFA_{n,t}}{Y_{n,t}} = \mathbb{E}_t \sum_{i=1}^{\infty} \mathfrak{D}_{n,t+i}\left\{\frac{M_{n,t+i} - X_{n,t+i}}{Y_{n,t+i}}\right\}$$
$$+ \mathbb{E}_t \sum_{i=1}^{\infty} \mathfrak{D}_{n,t+i}\left\{\frac{ER_{n,t+i}^{\text{FL}} FL_{n,t+i-1}}{Y_{n,t+i}} - \frac{ER_{n,t+i}^{\text{FA}} FA_{n,t+i-1}}{Y_{n,t+i}}\right\}$$
$$+ \mathbb{E}_t \lim_{i \to \infty} \mathfrak{D}_{n,t+i}\left\{\frac{NFA_{n,t+i}}{Y_{n,t+i}}\right\}. \qquad (4)$$

Equation (4) is little more that an accounting identity that follows from the budget constraint in (1) and the consistent application of the conditional expectations operator, $\mathbb{E}_t$. It implies that *any* NFA/GDP ratio we observe reflects a set of expectations concerning future trade flows, excess returns, discount factors and the long-horizon NFA/GDP ratio. In the absence of any restrictions on these expectations it is impossible to conduct meaningful cross-country comparisons of NFA/GDP ratios at a point in time, or make sense of their dynamics through time.

The restrictions implied by simple textbook models are a natural place to start. Consider the third term on the right-hand-side of (4). This term is equal to expected present value of the country's net asset position as the horizon rises without limit relative to current GDP.[3] In a perfect foresight model the term could only be negative if foreign agents were willing to foregone some of their lifetime resources by lending indefinitely to agents in country n, something they would never find optimal to do. Conversely, country-n agents would have to be willing to foregone some of their lifetime resources if the term were positive. In sum, therefore, optimal behavior in a perfect foresight model ensures that the third term disappears. In models with uncertainty things are more complicated because lifetime resources are unknown ex ante. Under these circumstances the third term disappears if agents are unwilling to lend to entities that intend running a Ponzi scheme of rolling over their debt indefinitely into the future (see, e.g. Bohn, 1995). I return to the implications of Ponzi-schemes in Section 3 below.

Textbook models also place restrictions on the remaining terms on the right-hand-side of (4). In a model where all international borrowing and lending takes place via a single risk free bond, countries either have positive foreign asset or liability positions depending on whether they are international lenders with positive bond holdings or borrowers with negative holdings. Under theses circumstances the returns on assets and liabilities are both equal to the risk free rate, which identifies the reference rate, R_t. This means that $ER_{n,t}^{\text{FA}} = ER_{n,t}^{\text{FL}} = 0$ for all t so, imposing the no-Ponzi restriction, we are left with

$$\frac{NFA_{n,t}}{Y_{n,t}} = \mathbb{E}_t \sum_{i=1}^{\infty} \mathfrak{D}_{n,t+i}\left\{ \frac{M_{n,t+i} - X_{n,t+i}}{Y_{n,t+i}} \right\}. \tag{5}$$

In contrast to (4), this expression provides a well-defined framework for considering both cross-country NFA/GDP ratios at a point in time, and their dynamics through time. The equation states the ratio for country n equals the expected present discounted value of future trade deficits measured relative to future GDP, discounted at the cumulated risk free rate minus the GDP growth rate. So cross-country differences in the NFA/GDP ratios at a point in time must either reflect differences in prospective future trade deficits, and/or differences in prospective future GDP growth, $\Delta y_{n,t+i}$, that affect the discount factor $\mathfrak{D}_{n,t+i} = \exp(\sum_{j=1}^{i} \Delta y_{n,t+j} - r_{t+j})$. Through time, changes in the NFA/GDP ratio must reflect news about future trade deficits and/or news concerning future GDP growth and risk free rate. Moreover, in a world where all international borrowing and lending occurs via a risk free bond, these changes in the NFA/GDP ratio are accomplished via changes in domestic consumption relative to GDP (because there are not capital gains or losses on existing NFA positions).

Equation (5) is unsuitable for studying actual NFA/GDP ratios for a couple of reasons. First, the average rate of GDP growth exceeds reasonable estimates of the risk free rate for all the countries under study. Thus, the discount factor $\mathfrak{D}_{n,t+i}$ would often be increasing in the horizon i making the present value term sensitive to long-horizon forecasts of trade deficits, which are inherently imprecise. Of course one way to alleviate this problem is to choose a reference rate R_t such that $\mathfrak{D}_{n,t+i}$ is always declining in the horizon i given any prospect for future GDP growth, but it unclear how this choice should actually be made. Alternatively we could rewrite (5) without reference to

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[3]Formally, we can rewrite the term as $Y_{n,t}^{-1}\mathbb{E}_t \lim_{i\to\infty}\{\prod_{j=1}^{i} R_{t+j}^{-1}\}NFA_{n,t+i}$.

GDP as

$$\frac{NFA_{n,t}}{Y_{n,t}} = \mathbb{E}_t \sum_{i=1}^{\infty} \left(\prod_{j=1}^{i} R_{t+j}^{-1}\right) \left\{\frac{M_{n,t+i} - X_{n,t+i}}{Y_{n,t}}\right\}.$$

This formulation avoids the discount factor problem, but it now requires forecasts for future trade deficits normalized by current rather than future GDP. In view of the secular increase in trade relative to GDP shown in Figure 2, such forecasts are again likely to be imprecise.

The second reason concerns the composition of foreign asset and liability portfolios. In reality, most countries' portfolios include equities, FDI holdings, long and short-term bonds and other securities (in time-varying proportions). Consequently, there are cross-country differences in the returns on foreign asset portfolios and foreign liability portfolios and differences between the returns on assets and liabilities for individual countries. It is thus impossible to choose a reference return such that the excess portfolio returns, $ER_{n,t}^{\text{FA}}$ and $ER_{n,t}^{\text{FL}}$, are zero across countries in every year.

To illustrate the empirical relevance of this issue, I consider how excess returns contribute to the dynamics of the NFA positions when the real return on U.S. T-bills is used as the reference rate. From (2) we can write the change in the NFA position as

$$\Delta NFA_{n,t} = X_{n,t} - M_{n,t} + (R_t - 1) NFA_{n,t-1} + \left(ER_{n,t}^{\text{FA}} FA_{n,t-1} - ER_{n,t}^{\text{FL}} FL_{n,t-1}\right). \qquad (6)$$

If the country only uses the U.S. T-bill market for international borrowing and lending $ER_{n,t}^{\text{FA}} = ER_{n,t}^{\text{FL}} = 0$ so changes in its NFA position arise from the current account balance identified by the first three terms on the right-hand-side ($(R_t - 1) NFA_{n,t-1}$ identifies the net investment income balance).[4] We can therefore gauge the importance of the excess portfolio returns as a driver of NFA dynamics by computing the contribution of $X_{n,t} - M_{n,t} + (R_t - 1) NFA_{n,t-1}$ to the variance of $\Delta NFA_{n,t}$ in the data.[5]

Panel I of Table 2 reports estimates of these variance contributions together with the upper and lower bounds of the 95 percent confidence interval. Panel II reports estimates using the average real return on U.S. T-bills as the reference rate for comparison. As the table shows, excess returns on existing asset and liability positions are the dominant driver of NFA changes across all but one of the the countries in the dataset. The exception is China, where the current account balances account for close to 100 percent of the variance in NFA changes (indeed 100 lies within the confidence interval).[6] In all the other countries, current account imbalances account for less than 30 percent of the variance in the NFA changes, in some cases very much less. These results are robust to the time-series variation in the reference rate. The estimated variance contributions in Panel II using a constant rate are very similar to the estimates in Panel I.

The results in Table 2 show that excess returns on existing asset liability positions played a

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[4]For the sake of clarity, this discussion abstracts from the effects of the capital account balance, unilateral transfers and the statistical discrepancy on NFA dynamics.

[5]Equation (6) implies that

$$\mathbb{V}[\Delta NFA_{n,t}] = \mathbb{CV}\left[X_{n,t} - M_{n,t} + (R_t - 1) NFA_{n,t-1}, \Delta NFA_{n,t}\right]$$
$$+ \mathbb{CV}\left[\left(ER_{n,t}^{\text{FA}} FA_{n,t-1} - ER_{n,t}^{\text{FL}} FL_{n,t-1}\right), \Delta NFA_{n,t}\right],$$

so by least squares the variance contribution can be estimated as the slope coefficient from the regression of $X_{n,t} - M_{n,t} + (R_t - 1) NFA_{n,t-1}$ on $\Delta NFA_{n,t}$; i.e. $\widehat{\mathbb{CV}}\left[X_{n,t} - M_{n,t} + (R_t - 1) NFA_{n,t-1}, \Delta NFA_{n,t}\right] / \widehat{\mathbb{V}}[\Delta NFA_{n,t}]$.

[6]This finding arises from the fact that U.S. Treasury securities comprised a large fraction of China's foreign asset portfolio and that the variations in excess returns on long-term U.S. bonds have been small relative to the current account balances over the sample period.

Table 2: ΔNFA Variance Contributions

	I			II		
	Lower	Estimate	Upper	Lower	Estimate	Upper
Canada	-8.59%	15.59%	39.77%	-7.71%	17.11%	41.92%
France	-1.10%	7.59%	16.29%	-0.41%	8.41%	17.24%
Germany	4.47%	24.45%	44.44%	4.18%	25.79%	47.39%
Italy	-0.34%	8.14%	16.61%	-3.50%	6.52%	16.54%
Japan	3.46%	18.87%	34.27%	2.87%	21.27%	39.67%
United Kingdom	-8.99%	-0.87%	7.24%	-8.28%	-0.30%	7.67%
United States	-10.53%	1.75%	14.04%	-10.58%	1.81%	14.20%
Australia	-20.75%	7.19%	35.13%	-16.17%	11.77%	39.71%
China	71.46%	94.57%	117.67%	69.51%	94.01%	118.51%
India	-6.12%	2.87%	11.86%	-7.19%	2.47%	12.13%
South Korea	-21.16%	5.69%	32.54%	-19.80%	6.45%	32.70%
Thailand	-3.86%	26.87%	57.61%	-2.38%	28.17%	58.73%

Notes: The table reports the contribution of $X_{n,t} - M_{n,t} + (R_t - 1) NFA_{n,t-1}$ to the variance of $\Delta NFA_{n,t}$ computed as the slope coefficient from the regression of $X_{n,t} - M_{n,t} + (R_t - 1) NFA_{n,t-1}$ on $\Delta NFA_{n,t}$. The columns headed Lower and Upper report the lower and upper bounds of the 95% confidence interval around the slope estimate computed from the OLS standard error for the estimate. The results in panel I use the real return on U.S. T-bills as the reference rate, those in panel II use the average real return on U.S. T-bills.

significant ex post role in driving NFA dynamics for most countries over the past forty years. The key question is this: Is it reasonable to assume that these excess returns were all completely unanticipated at the time? If they were, the second term in (4) can be safely ignored, so the NFA ratio remains pinned down by the present discounted value of future trade surpluses (in the absence of Ponzi schemes). On the other hand, if the realized excess returns in part represent compensation for risk that was anticipated ex ante, then (absent Ponzi-schemes) the NFA ratio is pinned down by

$$\frac{NFA_{n,t}}{Y_{n,t}} = \mathbb{E}_t \sum_{i=1}^{\infty} \mathcal{D}_{n,t+i} \left\{ \frac{M_{n,t+i} - X_{n,t+i}}{Y_{n,t+i}} \right\}$$
$$+ \mathbb{E}_t \sum_{i=1}^{\infty} \mathcal{D}_{n,t+i} \left\{ \frac{ER^{FL}_{n,t+i} FL_{n,t+i-1}}{Y_{n,t+i}} - \frac{ER^{FA}_{n,t+i} FA_{n,t+i-1}}{Y_{n,t+i}} \right\}.$$

Here financial considerations play a direct role in the determination of the NFA ratio via the second expected present value term on the right-hand-side.

To illustrate the economic intuition behind this term, imagine that some news leads investors across the world to change their perception of the risk associated with holding a particular class of country's n liabilities, say equity. If the equilibrium equity risk premium rises in response, and there are no offsetting changes in the risk premia on the country's other assets or liabilities, the expected excess return on the liability *portfolio* increases producing a rise in the present value term. At the

same time, the rise in the equity risk premium induces a drop in the current price the country's equity liabilities so that current value of the country's liability position falls and its NFA position rises. Thus, changes in the risk premia on the securities present in a country's asset and liability portfolio can alter NFA ratios by producing capital gains and losses on existing asset and liability holdings. Quantifying these financial effects is a complicated proposition. Even if historical data were available, simple time series forecasts will not generally produce paths for $ER^{\text{FL}}_{n,t+i}FL_{n,t+i-1}/Y_{n,t+i}$ and $ER^{\text{FA}}_{n,t+i}FA_{n,t+i-1}/Y_{n,t+i}$ that are consistent with the future values of the NFA ratio that satisfy the present value expression.

In sum, while equation (4) appears a natural starting point, we quickly run into problems using it as an analytic framework for studying NFA positions across countries and through time.

3 Analytic Framework

This section presents the analytic framework I use to the study the NFA positions of the countries in the dataset. As above, the positions are pinned down by an expected present value expression that is derived from the country's consolidated budget constraint. I also make use of a no-arbitrage condition that restricts the behavior of the returns on each country's foreign asset and liability portfolios. This additional condition is key to identifying the determinants of the NFA positions in an analytic framework that is amenable to empirical analysis.

The World SDF

In a world where financial assets with the same payoffs have the same prices and there are no restrictions on the construction of portfolios (such as short sales constraints), there exists a positive random, $\mathcal{K}_{t+1}$, such that

$$1 = \mathbb{E}_t[\mathcal{K}_{t+1}R^i_{t+1}], \tag{7}$$

where R^i_{t+1} is the (gross real) return on any freely traded asset i. As above, $\mathbb{E}_t[.]$ denotes expectations conditioned on common period-t information. The variable $\mathcal{K}_{t+1}$ is known as the pricing kernel or Stochastic Discount Factor (SDF). This condition is very general. It does not rely on the preferences of investors, the rationality of their expectations, or the completeness of financial markets.[7]

To illustrate the economic intuition behind (13), consider first a world where all investors have the same time-separable utility defined over consumption, $\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i})$. The first-order condition governing the investors' optimal holding of asset i is $1 = \mathbb{E}_t[\beta\{U'(C_{t+1})/U'(C_t)\}R^i_{t+1}]$, so under these circumstances the SDF equals the intertemporal marginal rate of substitution (IMRS) common to all investors. Next, suppose that the world is populated by investors with different preferences and a complete set of financial markets. In this environment any asset can be represented as a portfolio of contingent claims. In particular, the price of asset i is $\mathcal{P}^i_t = \sum_{z \in \mathcal{Z}} \mathcal{P}_t(z)\mathcal{X}^i_{t+1}(z)$, where $\mathcal{X}^i_{t+1}(z)$ is the payoff on asset i when the period- $t+1$ state of the world is z, and $\mathcal{P}_t(z)$ is the period-t price of contingent claim to state z. By definition the return on asset i in state z is given by $R^i_{t+1}(z) = \mathcal{X}^i_{t+1}(z)/\mathcal{P}^i_t$, so this condition can be rewritten as $1 = \sum_{z \in \mathcal{Z}} \pi_t(z)\mathcal{K}_{t+1}(z)\mathcal{R}^i_{t+1}(z) = \mathbb{E}_t[\mathcal{K}_{t+1}R^i_{t+1}]$, where $\mathcal{K}_{t+1}(z) = \mathcal{P}_t(z)/\pi_t(z)$ and $\pi_t(z)$ denotes the conditional probability of state z occurring in period $t+1$. Thus, when markets are complete, the SDF is equal to the ratio of the contingent claims

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[7]For a textbook discussion of SDFs, see Cochrane (2001).

price of future state z to the probability of state z. Finally, in a world with incomplete markets and heterogeneous investors (13) still holds but there may exist more than one SDF that satisfies the no-arbitrage condition. In this case there is a first-order condition involving R_{t+1}^i and the IMRS for each investor, so each investor's IMRS works as a SDF for pricing all the freely traded assets. Unlike the complete markets world, there may be idiosyncratic differences between the IMRS of different investors because they cannot completely share the risks they face, but any one will work as an SDF that satisfies the no-arbitrage relation in (13).

If condition in (13) applies to the returns on every security in a country's asset and liability portfolios, it also applies to the returns on the portfolios themselves; i.e.

$$1 = \mathbb{E}_t[\mathcal{K}_{t+1} R_{n,t+1}^{\text{FA}}] \qquad \text{and} \qquad 1 = \mathbb{E}_t[\mathcal{K}_{t+1} R_{n,t+1}^{\text{FL}}]. \tag{8}$$

The present value expression for the NFA position I develop below assumes that these conditions hold for the returns on the portfolios of assets and liabilities of all countries. This is not a particularly strong assumption. For example, it does not require one to take a stand on how the composition of asset and liability portfolios are determined. To see why, note that $R_{t+1}^{\text{F}} = \sum_j \alpha_{j,t} R_{t+1}^{\text{F}j}$ where $R_{t+1}^{\text{F}j}$ denotes the return on $\text{F} = \{\text{FA}, \text{FL}\}$ (asset or liability) security j and $\alpha_{j,t}$ are the ex ante portfolio shares (determined in period t) with $\sum_j \alpha_{j,t} = 1$. As long as the no-arbitrage condition applies to the returns on the individual securities, then $\mathbb{E}_t[\mathcal{K}_{t+1} R_{n,t+1}^{\text{F}}] = \mathbb{E}_t[\sum_j \alpha_{j,t} \mathcal{K}_{t+1} R_{t+1}^{\text{F}j}] = \sum_j \alpha_{j,t} \mathbb{E}_t[\mathcal{K}_{t+1} R_{t+1}^{\text{F}j}] = 1$ for $\text{F} = \{\text{FA}, \text{FL}\}$ and any set of portfolio shares $\alpha_{j,t}$. Nor is it necessary to assume that a particular security is freely traded throughout the world. While the presence of restrictions such as capital controls and short sales constraints may inhibit trade in security j for investors in some countries, $\mathbb{E}_t[\mathcal{K}_{t+1} R_{t+1}^{\text{F}j}]$ will still equal unity *if* the security's price is determined by the actions of the investors who can trade freely (as seems likely for many widely-held securities; e.g., U.S. Treasury securities).

The conditions in (13) have two important implications that I exploit below. First, notice that the SDF is common to the condition involving the portfolio returns on assets and liabilities. As I noted above, changes in expected future returns affect a country's NFA position via capital gains and losses on existing asset and liability positions. Condition (13) links changes in expected future asset returns and liability returns, which tie down the possible capital gains and losses. The second implication concerns the cross country behavior of expected future returns. Although the composition of asset and liability portfolios differs across countries, condition (13) applies simultaneously to the returns for all countries n. Put differently, expected variations in the SDF not only affect expected returns on the asset and liability returns of an individual country, they also affect expected returns across countries. In short, $\mathcal{K}_{t+1}$ is a world SDF than ties the behavior of returns together across countries.

At this point it may appear that little has been achieved by imposing the restrictions in (13) because the exact determinants of the world SDF have yet to be specified. However, these restrictions greatly simplify deriving an expression for a country's NFA position that can be used empirically. I derive this expression next, before considering the determinants of the SDF.

Net Foreign Asset Positions

To determine a simple expression for a country's' NFA position, I first multiply both sides of the budget constraint in (1) by the SDF and then take conditional expectations. This produces

$$
\begin{aligned}
\mathbb{E}_t\left[\mathcal{K}_{t+1} NFA_{n,t+1}\right] &= \mathbb{E}_t\left[\mathcal{K}_{t+1}(X_{n,t+1} - M_{n,t+1})\right] \\
&\quad + \mathbb{E}_t\left[\mathcal{K}_{t+1} R_{n,t+1}^{\text{FA}}\right] FA_{n,t} - \mathbb{E}_t\left[\mathcal{K}_{t+1} R_{n,t+1}^{\text{FL}}\right] FL_{n,t} \\
&= \mathbb{E}_t\left[\mathcal{K}_{t+1}(X_{n,t+1} - M_{n,t+1})\right] + NFA_{n,t}.
\end{aligned}
$$

Rearranging this expression and solving forward using the Law of Iterated Expectations we obtain

$$
NFA_{n,t} = \mathbb{E}_t \sum_{i=1}^{\infty} \mathcal{D}_{t+i}\left(M_{n,t+i} - X_{n,t+i}\right) + \mathbb{E}_t \lim_{i \to \infty} \mathcal{D}_{t+i} NFA_{n,t+i}, \tag{9}
$$

where $\mathcal{D}_{t+i} = \prod_{j=1}^{i} \mathcal{K}_{t+j}$. Equation (9) differs from the forward-looking expression for the NFA ratio derived above in Section 2 in several important respects. First the discount factor used to discount future trade deficits depends only on the world SDF, and so is the same for all countries at a point in time. Second, there are no longer any terms involving (excess) portfolio returns. Expectations concerning future returns still affect a country's NFA position, but the effects work through the SDF, as I explain below.

The last term on the right-hand-side on (9) identifies the expected present value of the country's net asset position as the horizon rises without limit using a discount factor determined by the world's SDF. To rule out Ponzi-schemes, I assume that

$$
\mathbb{E}_t \lim_{i \to \infty} \mathcal{D}_{t+i} NFA_{n,t+i} = 0, \tag{10}
$$

for all countries n.

To understand the economics behind this restriction, suppose a debtor country (i.e. a country with $NFA_{n,t} < 0$) decides to simply roll over existing asset and liability positions while running zero future trade balances. Under these circumstances, the country's asset and liability portfolios evolve as $FA_{n,t+i} = R_{n,t+i}^{\text{FA}} FA_{n,t+i-1}$ and $FL_{n,t+i} = R_{n,t+i}^{\text{FL}} FL_{n,t+i-1}$ for all $i > 0$. Since $\mathbb{E}_t[\mathcal{K}_{t+1}\mathcal{X}_{t+1}]$ identifies the period$-t$ value of any period $t+1$ payoff $\mathcal{X}_{t+1}$, the value of claim to the country's net assets next period is

$$
\begin{aligned}
\mathbb{E}_t\left[\mathcal{K}_{t+1} NFA_{n,t+1}\right] &= \mathbb{E}_t\left[\mathcal{K}_{t+1}(R_{n,t+1}^{\text{FA}} FA_{n,t} - R_{n,t+1}^{\text{FL}} Fl_{n,t})\right] \\
&= \mathbb{E}_t\left[\mathcal{K}_{t+1} R_{n,t+1}^{\text{FA}}\right] FA_{n,t} - \mathbb{E}_t\left[\mathcal{K}_{t+1} R_{n,t+1}^{\text{FL}}\right] FL_{n,t} \\
&= NFA_{n,t}
\end{aligned}
$$

This same reasoning applies in all future periods, i.e., $\mathbb{E}_{t+i}\left[\mathcal{K}_{t+i+1} NFA_{n,t+i+1}\right] = NFA_{n,t+i}$ for all

$i > 0$, so the value of a claim to the foreign asset position τ periods ahead is

$$
\begin{aligned}
\mathbb{E}_t \left[\mathcal{D}_{t+\tau} NFA_{n,t+\tau} \right] &= \mathbb{E}_t \left[\mathcal{D}_{t+\tau-1} \mathbb{E}_{\tau-1} \left[\mathcal{K}_{t+\tau} NFA_{n,t+\tau} \right] \right] \\
&= \mathbb{E}_t \left[\mathcal{D}_{t+\tau-1} NFA_{n,t+\tau-1} \right] \\
&= \ldots \\
&= NFA_{n,t}.
\end{aligned}
$$

Taking the limit as $\tau \to \infty$ gives $NFA_{n,t} = \mathbb{E}_t \lim_{i \to \infty} \left[\mathcal{D}_{t+i} NFA_{n,t+i} \right] < 0$. Thus, the country's current net foreign asset position must be equal to the value of a claim on rolling the asset and liability positions forward indefinitely into the future. Clearly then, no country n can unexpectedly initiate a Ponzi scheme in period t when $\mathbb{E}_t \lim_{i \to \infty} \mathcal{D}_{t+i} NFA_{n,t+i} \geq 0$. Moreover, since $\sum_n NFA_{n,t} = 0$ by market clearing, if $\mathbb{E}_t \lim_{i \to \infty} \mathcal{D}_{t+i} NFA_{\tilde{n},t+i} > 0$ for any one country, $\tilde{n}$, then at least one other must be involved in a Ponzi scheme. Thus, the restriction in (10) prevents any country from unexpectedly adopting a Ponzi scheme in period t.

What about contingent future Ponzi schemes? Suppose, for example, that the country intends to start a scheme in $t+1$ if $NFA_{n,t+1} < 0$. In this situation, $R^{\text{FA}}_{n,t+1} FA_{n,t} - R^{\text{FL}}_{n,t+1} FL_{n,t} + X_{n,t+1} < M_{n,t+1}$ so the funds available in period $t+1$ from exports and liquidating asset and liability positions are insufficient to pay for imports. Thus, the scheme can only be implemented if foreigners can be induced to hold newly issued debt. This will be impossible if the foreigners are rational because the value of a claim on country's future net assets is $\mathbb{E}_{t+1} \lim_{i \to \infty} \left[\mathcal{D}_{t+i} NFA_{n,t+i} \right] = NFA_{n,t+1} < 0$. Clearly, this argument applies to contingency plans at all future dates. In sum, therefore, the no-arbitrage conditions in (8) ensure that no rational foreigners would be willing participants in any contingent future Ponzi scheme.

We can now identify the determinants of a county's NFA position by combining (9) and the no-Ponzi restriction (10):

$$
NFA_{n,t} = \mathbb{E}_t \sum_{i=1}^{\infty} \mathcal{D}_{t+i} \left(M_{n,t+i} - X_{n,t+i} \right). \tag{11}
$$

This expression states that in the absence of Ponzi schemes and arbitrage opportunities, the NFA position of any country n should equal the expected present discounted value of future trade deficits, discounted at the cumulated world SDF. Notice, also, that (11) is exact (i.e., it contains no approximations). It must hold under the stated conditions for accurate NFA and trade data given market expectations and the world SDF.

Equation (11) has several important economic implications. First, it implies that *small* countries can only be current net international debtors if they are expected to run trade surpluses at some point in the future. In particular, when $NFA_{n,t} < 0$, there must be some future horizons j for which $\mathbb{E}_t[\mathcal{D}_{t+j}(M_{n,t+j} - X_{n,t+j})] < 0$. This condition simplifies to $\mathbb{E}_t[\mathcal{D}_{t+j}]\mathbb{E}_t[M_{n,t+j} - X_{n,t+j}] < 0$ in small countries where the trade balance is driven by idiosyncratic factors that are uncorrelated with the world SDF. Consequently, $\mathbb{E}_t[X_{n,t+j} - M_{n,t+j}] > 0$ for some horizon j because $\mathcal{D}_{t+j}$ is a positive random variable. In contrast, large countries can be debtors without prospective future trade surpluses provided the surpluses are negatively correlated with the world SDF. In these cases the requirement that $\mathbb{E}_t[\mathcal{D}_{t+j}(M_{n,t+j} - X_{n,t+j})] < 0$ holds for some horizon j can be satisfied if $\mathbb{E}_t[M_{n,t+j} - X_{n,t+j}]$ has an upper bounded of $-\mathbb{CV}_t[M_{n,t+j} - X_{n,t+j}, \mathcal{D}_{t+j}]/\mathbb{E}_t[\mathcal{D}_{t+j}] > 0$.

The second implication concerns the effects of portfolio choice. In Section 2 we saw that composition of a country's asset and liability portfolios could affect its NFA position via the expected future excess portfolio returns on assets and liabilities. Equation (11) limits these financial affects. If the equilibrium returns on individual securities are unaffected by a particular country's choice of portfolio, then the choice cannot affect the world SDF. Under these circumstances, (11) implies that choice of portfolio has no effect on the country's net foreign asset position. Conversely, if a country's choice of asset portfolio (say) affects world returns in a manner that is reflected in the world SDF, those choices will affect NFA positions across the world because the same world SDF is used to discount future trade deficits of every country.

Equation (11) also takes explicit account of risk. It states that a country's NFA position is equal to the value of a claim to the future stream of trade deficits in a world where those deficits are uncertain. This is not the same a discounting future trade deficits by the expected path of the risk free rate. By definition, the risk free return, R_t^{RF} is equal to $1/\mathbb{E}_t\mathcal{K}_{t+1}$, so $\mathcal{K}_{t+1} = \frac{1}{R_t^{\mathrm{RF}}} + (\mathcal{K}_{t+1} - \mathbb{E}_t\mathcal{K}_{t+1})$. We can therefore rewrite the discount factor in (11) as

$$\mathcal{D}_{t+i} = \prod_{j=1}^{i} \left\{ \frac{1}{R_{t-1+j}^{\mathrm{RF}}} + (\mathcal{K}_{t+j} - \mathbb{E}_{t+j-1}\mathcal{K}_{t+j}) \right\}.$$

In a world without risk future deficits are discounted by the risk free rate because $\mathcal{K}_{t+1} = \mathbb{E}_t\mathcal{K}_{t+1}$. However, (11) allows for the fact that variations in the world SDF are unpredictable so that uncertainty about future deficits, risk free rates and the SDF must be jointly accounted for in the determination of a country's NFA position.

Identifying the World SDF

In a fully specified theoretical model of the world economy the world SDF would be identified from the equilibrium conditions governing investors' portfolio and savings decisions. Fortunately, for our purposes, we can avoid such a complex undertaking. Instead, I adopt a "reverse-engineering" approach in which I construct a specification for the SDF that explains the behavior of a set of returns; the returns on the asset and liability portfolios for the G7 countries. This approach is easy to implement and allows us to transform (11) into an equation amenable to empirical analysis.

Let $\mathbf{er}_{t+1}$ denote a $k \times 1$ vector of log excess portfolio returns, $er_{t+1}^i = r_{t+1}^i - r_{t+1}^{\mathrm{TB}}$, where r_{t+1}^i denotes the log return on portfolio i and r_{t+1}^{TB} is the log return on U.S. T-bills. I assume that the log of the SDF, $\kappa_{t+1} = \ln\mathcal{K}_{t+1}$, is determined as

$$\kappa_{t+1} = a - r_{t+1}^{\mathrm{TB}} - b'(\mathbf{er}_{t+1} - \mathbb{E}[\mathbf{er}_{t+1}]), \tag{12}$$

where $\mathbb{E}[.]$ denotes the unconditional expectations operator. This specification for the SDF contains $k + 1$ parameters: the constant a and the $k \times 1$ vector b. In the "reverse-engineering" approach values for these parameters are chosen to ensure that the no-arbitrage conditions are satisfied for the specified SDF. More specifically, here I find values for a and b such that the portfolio returns for the asset and liability portfolios of the G7 countries and the U.S. T-bill rate all satisfy the no-arbitrage conditions.

Consider the condition for the $i'th$ portfolio return: $1 = \mathbb{E}_t[\exp(\kappa_{t+1} + r_{t+1}^i)]$. Taking uncondi-

tional expectations we can rewrite this condition as

$$
\begin{aligned}
1 &= \mathbb{E}[\exp(\kappa_{t+1} + r^i_{t+1})] \\
&\simeq \exp\left(\mathbb{E}[\kappa_{t+1} + r^i_{t+1}] + \tfrac{1}{2}\mathbb{V}[\kappa_{t+1} + r^i_{t+1}]\right),
\end{aligned}
\tag{13}
$$

where $\mathbb{V}[.]$ denotes the unconditional variance. When the log returns are normally distributed the second line holds with equality because (12) implies that κ_{t+1} and r^i_{t+1} are jointly normal. Otherwise, the second line includes an approximation error.[8]

Next, I substituting for the log SDF from (12) in (13) and take logs to give

$$
0 = a + \mathbb{E}\left[r^i_{t+1} - r^{\mathrm{TB}}_{t+1}\right] + \tfrac{1}{2}\mathbb{V}\left[r^i_{t+1} - r^{\mathrm{TB}}_{t+1} - b'(\mathbf{er}_{t+1} - \mathbb{E}[\mathbf{er}_{t+1}])\right],
$$

or, after some re-arrangement

$$
a + \mathbb{E}\left[er^i_{t+1}\right] + \tfrac{1}{2}\mathbb{V}\left[er^i_{t+1}\right] + \tfrac{1}{2}b'\mathbb{V}\left[\mathbf{er}_{t+1}\right]b = \mathbb{CV}\left[er^i_{t+1}, \mathbf{er}'_{t+1}\right]b,
\tag{14}
$$

where $\mathbb{CV}[.,.]$ denotes the unconditional covariance. This equation must hold for the T-bill rate (i.e., when $r^i_{t+1} = r^{\mathrm{TB}}_{t+1}$, or $er^i_{t+1} = 0$) so

$$
a + \tfrac{1}{2}b'\mathbb{V}\left[\mathbf{er}_{t+1}\right]b = 0.
\tag{15}
$$

Imposing this restriction on (14) gives

$$
\mathbb{E}\left[er^i_{t+1}\right] + \tfrac{1}{2}\mathbb{V}\left[er^i_{t+1}\right] = \mathbb{CV}\left[er^i_{t+1}, \mathbf{er}'_{t+1}\right]b.
$$

This equation holds for each of the k portfolio returns. So stacking the k equations we obtain

$$
\mathbb{E}\left[\mathbf{er}_{t+1}\right] + \tfrac{1}{2}\Lambda = \Omega b,
\tag{16}
$$

where $\Omega = \mathbb{V}[\mathbf{er}_{t+1}]$ and Λ is a $k \times 1$ vector containing the leading diagonal of Ω.

Finally, we can solve (15) and (16) to give

$$
a = -\tfrac{1}{2}b'\Omega b \qquad \text{and} \qquad b = \Omega^{-1}\mu \qquad \text{with} \qquad \mu = \mathbb{E}\left[\mathbf{er}_{t+1}\right] + \tfrac{1}{2}\Lambda.
$$

Substituting the solutions for a and b in (12) produces the following expression for the log SDF:

$$
\kappa_{t+1} = -\tfrac{1}{2}\mu'\Omega^{-1}\mu - r^{\mathrm{TB}}_{t+1} - \mu'\Omega^{-1}(\mathbf{er}_{t+1} - \mathbb{E}[\mathbf{er}_{t+1}]).
\tag{17}
$$

By construction, equation (17) identifies a specification for the log SDF such that the unconditional no-arbitrage condition, $1 = \mathbb{E}[\exp(\kappa_{t+1} + r_{t+1})]$, holds for the k log portfolio returns and the return on U.S. T-bills. This specification would also satisfy the conditional no-arbitrage condition, $1 = \mathbb{E}_t[\exp(\kappa_{t+1} + r_{t+1})]$, if log returns were independently and identically distributed. However, since this is not the case, we need to amend the specification to incorporate conditioning information. Fortunately, this is quite straightforward.

---

[8] For the sake of clarity, I ignore the approximation error in the discussion below. I consider its empirical significance in Section 5.

Consider the no-arbitrage condition (with conditional expectations) for portfolio return i:

$$1 = \mathbb{E}_t[\exp(\kappa_{t+1} + r_{t+1}^i)].$$

Let z_t be a valid instrument known to market participants in period t. Multiplying both sides of the no-arbitrage expression above by $\exp(z_t)$ and taking unconditional expectations produces

$$\mathbb{E}[\exp(z_t)] = \mathbb{E}\left[\exp(\kappa_{t+1} + r_{t+1}^i + z_t)\right],$$

or, after some re-arrangement

$$1 = \mathbb{E}\left[\exp(\kappa_{t+1} + r_{t+1}^{i,z})\right], \tag{18}$$

where $r_{t+1}^{i,z} = r_{t+1}^i + z_t - \ln \mathbb{E}[\exp(z_t)]$. Notice that (18) takes the same form as (13) used in the constructions of the log SDF in (17). The only difference is that (18) contains the adjusted log return on portfolio i, $r_{t+1}^{i,z}$, rather than the unadjusted return r_{t+1}^i. This means that we can reverse engineer a specification for the log SDF that incorporates the conditioning information if we add adjusted log returns to the set of returns. Specifically, let $er_{t+1}^{i,z^j} = r_{t+1}^i - r_{t+1}^{\mathrm{TB}} + z_t^j - \ln \mathbb{E}[\exp(z_t^j)]$ denote the log excess adjusted return on portfolio i using instrument z_t^j. If $\mathbf{er}_{t+1}$ now represents a vector containing er_{t+1}^i and er_{t+1}^{i,z^j}, the log SDF identified in (17) will satisfy the non-arbitrage condition

$$1 = \mathbb{E}\left[\exp(\kappa_{t+1} + r_{t+1}^i)|z_t^j\right],$$

for all the portfolio returns i and instruments z_t^j included in $\mathbf{er}_{t+1}$.

Three aspects of this reverse engineering procedure deserve comment. First, equation (17) doesn't necessarily identify a unique SDF that satisfies the no-arbitrage conditions for a set of returns. Indeed, we know as a matter of theory that many SDF exist when markets are incomplete. Rather the specification in (17) identifies *one* specification for the SDF that satisfies the no-arbitrage conditions.

Second, this reverse engineering approach makes no attempt to relate the SDF to underlying macro factors. Obviously this would be a fatal drawback if our aim was to relate the behavior of returns to macro variables (as general equilibrium asset pricing models do), but here we have a much more modest aim of incorporating information from asset and liability returns with data from trade flows to identify the determinants of a country's NFA position. The present value expression in (11) showed that information concerning prospective future returns only affect a country's NFA position via the SDF, while (17) identifies a link between the SDF and returns that satisfy the no-arbitrage conditions. If our goal is to understand how changes in prospective future returns affect NFA positions across the world, this specification for the SDF is all we need.

The third aspect concerns the use of instrumental variables to control for conditioning information. In principle the conditional expectations of market participants that appear in the no-arbitrage conditions equal expectations conditioned on every instrumental variable in their information set. In practice, there is a limit to the number of instruments we want to incorporate into the log SDF specification. Below I chose instruments that have forecasting power for log excess portfolio returns and I consider the robustness of my results to alternative specifications of the log SDF based on different instrument choices.

4 Empirical Implementation

I now combine the present value expression for the NFA position in (11) with the specification for the log SDF in (17) to produce an equation that is amenable to empirical analysis. The challenge here is in computing forecasts for $\mathcal{D}_{t+i}M_{n,t+i}$ and $\mathcal{D}_{t+i}X_{n,t+i}$ with $\mathcal{D}_{t+i} = \prod_{j=1}^{i}\exp(\kappa_{t+j})$ for all $i > 0$ from data on imports, exports and the returns that constitute the log SDF, κ_t. To meet this challenge, I use a standard approximation.

Approximating Net Foreign Asset Positions

To approximate the present value expression for each country's NFA position, I first rewrite (11) as

$$NFA_{n,t} = M_{n,t}\mathbb{E}_t\sum_{i=1}^{\infty}\exp\left(\sum_{j=1}^{i}\Delta m_{n,t+j} + \kappa_{t+j}\right) - X_{n,t}\mathbb{E}_t\sum_{i=1}^{\infty}\exp\left(\sum_{j=1}^{i}\Delta x_{n,t+j} + \kappa_{t+j}\right). \quad (19)$$

This transformation simply relates the NFA position to the current levels of imports and exports and their future growth rates, $\Delta m_{n,t+i}$ and $\Delta x_{n,t+i}$, rather than the future levels of exports and imports shown in (11).

Next, I approximate to the two terms involving expectations. If δ_t is a random variable with mean $\mathbb{E}[\delta_t] = \delta < 0$, then a first-order approximation to δ_{t+j} around δ produces

$$\mathbb{E}_t\sum_{i=1}^{\infty}\exp\left(\sum_{j=1}^{i}\delta_{t+j}\right) = \mathbb{E}_t\exp(\delta_{t+1}) + \mathbb{E}_t\exp(\delta_{t+1} + \delta_{t+2}) + ...$$

$$\simeq \frac{\rho}{1-\rho} + \rho\mathbb{E}_t(\delta_{t+1} - \delta) + \rho^2\mathbb{E}_t(\delta_{t+1} - \delta) + \rho^3\mathbb{E}_t(\delta_{t+2} - \delta) + ...$$

$$= \frac{\rho}{1-\rho} + \frac{\rho}{1-\rho}\mathbb{E}_t(\delta_{t+1} - \delta) + \frac{\rho^2}{1-\rho}\mathbb{E}_t(\delta_{t+2} - \delta) + ...$$

$$= \frac{\rho}{1-\rho} + \frac{1}{1-\rho}\mathbb{E}_t\sum_{i=1}^{\infty}\rho^i(\delta_{t+i} - \delta), \quad (20)$$

where $\rho = \exp(\delta) < 1$.

To apply this approximation, I make two assumptions:

$$\mathbb{E}[\Delta m_{n,t}] = \mathbb{E}[\Delta x_{n,t}] = g, \qquad \text{and} \quad\quad\quad\quad (A1)$$

$$g + \kappa = \delta < 0, \qquad \text{with} \quad \mathbb{E}[\kappa_t] = \kappa. \quad\quad\quad\quad (A2)$$

Under assumption A1 the mean growth rate for imports and exports are equal. This will be true of any economy on a balanced growth path. It also appears consistent with the empirical evidence for the the G7 countries. Table 1 showed that their sample means for $\Delta x_{n,t} - \Delta m_{n,t}$ are close to zero. To interpret the assumption A2, note that in the steady state the log risk free rate r satisfies $1 = \mathbb{E}[\exp(\kappa_t)]\exp(r)$. Thus $\delta = g + \kappa \simeq g - r - \frac{1}{2}\mathbb{V}[\kappa_t]$, so A2 will hold provided $\mathbb{V}[\kappa_t] > 2(g - r)$. Recall the mean growth rate for trade across the countries in the dataset is approximately 6.5 percent, which is well above any reasonable estimate of the mean risk free rate of close to 1 percent. Clearly then, A2 will only hold if the variance of the log SDF exceeds roughly $0.11 = 2(0.065 - 0.01)$. This volatility bound is easily exceeded by estimates of the log SDF based on (17) derived below.

Applying the approximation in (20) to the expectations terms in (19) and simplifying the result gives

$$NFA_{n,t} = \tfrac{\rho}{1-\rho}\left(M_{n,t} - X_{n,t}\right) + \tfrac{1}{2(1-\rho)}\left(M_{n,t} + X_{n,t}\right)\mathbb{E}_t \sum_{i=1}^{\infty} \rho^i \left(\Delta m_{n,t+i} - \Delta x_{n,t+i}\right)$$

$$+ \tfrac{1}{1-\rho}\left(M_{n,t} - X_{n,t}\right)\mathbb{E}_t \sum_{i=1}^{\infty} \rho^i \left(\Delta\tau_{n,t+i} - g\right)$$

$$+ \tfrac{1}{1-\rho}\left(M_{n,t} - X_{n,t}\right)\mathbb{E}_t \sum_{i=1}^{\infty} \rho^i \left(\kappa_{t+i} - \kappa\right), \tag{21}$$

where $\Delta\tau_{n,t} = \tfrac{1}{2}(\Delta m_{n,t} + \Delta x_{n,t})$. This expression identifies the three sets of factors determining a country's NFA position in a clear fashion. The first term on the right-hand-side identifies the influence of the current trade balance. This would be the only factor determining the NFA position in the stochastic steady state where import growth, export growth and the log SDF followed i.i.d. processes because the terms involving expectations would equal zero. As such, this first term identifies the *atemporal* influence of trade flows in the NFA position. The remaining terms of the right-hand-side identify the intertemporal factors that were present in (11). Consider, for example, news that leads agents to revise their forecasts for future trade deficits upwards. If trade is currently balance (i.e., $M_{n,t} = X_{n,t}$) the news must also raise their forecasts of future import growth relative to export growth so the second term on the right of (21) increases. The news may also induce a revision in expected future trade growth, $\Delta\tau_{n,t+i}$, if trade is currently unbalanced, producing a change in the third term as well.

The last term on the right-hand-side of (21) identifies how news concerning the future SDF affects a country's NFA position. To illustrate the economic intuition behind this term, consider the effect of news that lowers agents' forecasts of the future SDF but leaves their forecasts for future trade flows unchanged. Under these circumstances, future trade deficits are discounted more heavily so the country's current NFA position is more closely tied to the value of a claim on its near term deficits. Thus the NFA positions of countries currently currently running trade deficits deteriorates (i.e., their positive NFA positions fall towards zero), while the NFA positions of those running current trade surpluses improve (i.e. their negative positions rise towards zero) as indicated by the last term in (21). Notice, also, that such news affects the NFA positions of all countries because (in the absence of arbitrage) they all use the same SDF to discount future trade deficits, but the change in each country's NFA position depends on their current trade balance.

We can use (21) to identify a measure of a country's external position that is comparable across countries. For this purpose, I define country $n's$ external position by

$$NXA_{n,t} = \frac{NFA_{n,t}}{M_{n,t} + X_{n,t}} - \frac{\rho}{1-\rho}TD_{n,t} \qquad \text{where} \qquad TD_{n,t} = \frac{M_{n,t} - X_{n,t}}{M_{n,t} + X_{n,t}}.$$

In words, the country's NXA position is defined as the gap between its current NFA position and the steady state present value of the future trade deficits, all normalized by the current volume of international trade. The rationale behind this definition is easily seen by substituting for $NFA_{n,t}$

from (21):

$$NXA_{n,t} = \frac{1}{2(1-\rho)}\mathbb{E}_t \sum_{i=1}^{\infty} \rho^i \left(\Delta m_{n,t+i} - \Delta x_{n,t+i}\right)$$

$$+ \frac{1}{1-\rho}TD_{n,t}\mathbb{E}_t \sum_{i=1}^{\infty} \rho^i \left(\Delta \tau_{n,t+i} - g\right) + \frac{1}{1-\rho}TD_{n,t}\mathbb{E}_t \sum_{i=1}^{\infty} \rho^i \left(\kappa_{t+i} - \kappa\right). \qquad (22)$$

Notice that the none of the terms on the right-hand-side should be systematically related to country $n's$ level of GDP. For example, the second and third terms include the ratio of the country's current trade deficit to the volume of trade, $TD_{n,t}$, a ratio that must lie between -1 and +1. This means that cross-country comparisons of $NXA_{n,t}$ can be meaningfully interpreted in terms of the differences in current trade ratios and forecasts for future growth in trade flows. Equation (22) also makes clear that the NXA measure of any country's external position will be (approximately) zero in the steady state where agents' forecasts for future import growth, export growth and the log SDF are equal to their unconditional means. Of course market clearing ensures that $\sum_n NFA_{n,t} = 0$ and $\sum_n (M_{n,t} - X_{n,t}) = 0$ so aggregating the NXA measure across countries gives a world external position of zero; i.e., $\sum_n NXA_{n,t} = 0$.

Figure 4 plots the NFA positions and trade deficits as a fraction of annual trade for each of the countries in the data set between 1980 and 2011. The left-hand panels show that NFA positions vary between $\pm$ 300 percent of trade. This is more than twice the size of the range for the NFA to GDP ratios shown in Figure 1. The plots in the right-hand panels show that trade deficits vary between $\pm$ 20 per cent of trade - a range comparable to the net export to GDP ratios. Figure 5 plots the NXA positions for each country in the dataset between 1980 and 2011. These NXA values are computed using a value for ρ of 0.586. I describe how this value for ρ is estimated from the data in Section 5. The left-hand panel in Figure 5 shows that the NXA measures for all but one of the G7 countries have remained between ± 1 during the past 30 years. The one exception is the Japanese NXA series, which persistently increased from 0.1 to 2.6 during the period. Variations in the NXA positions of countries outside the G7 are generally larger. The plots in the right-hand panel of Figure 5 show large improvements in the external positions of India and South Korea while Australia's NXA position has remained largely unchanged. It is also interesting to note that the steady improvement in the NXA position of China in the last twenty years is not nearly as pronounced as the improvement in Japan's position.

In Section 5 I use (21), (22) and time series forecasts for trade flows and the log SDF to study the evolution of the NXA positions shown in Figure 5. Before turning to this analysis, it is worth emphasizing that these equations are not derived from counterfactual assumptions concerning the evolution of the world economy. More specifically, the accuracy of the approximations in (21) and (22) are not compromised by the fact that on average both trade growth and position growth far exceed the rate of GDP growth across countries. Moreover (21) and (22) apply to each country's whole NFA and NXA positions, not the positions relative to a secular trend. As was clear from the NFA plots in Figure 1, most movements in NFA positions are very persistent, so there is a significant advantage in studying the dynamics of whole NFA and NXA positions rather than their de-trended components. That said, accounting for the diverse NXA paths plotted in Figure 5 represents a significant challenge.

Figure 4: Net Foreign Assets and Net Exports

A: Net Foreign Assets (% of Trade)
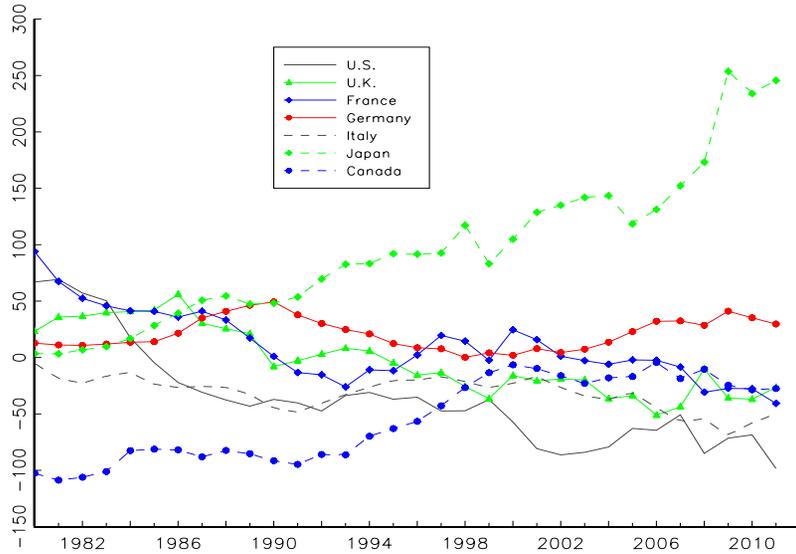
B: Trade Deficits (% of Trade)
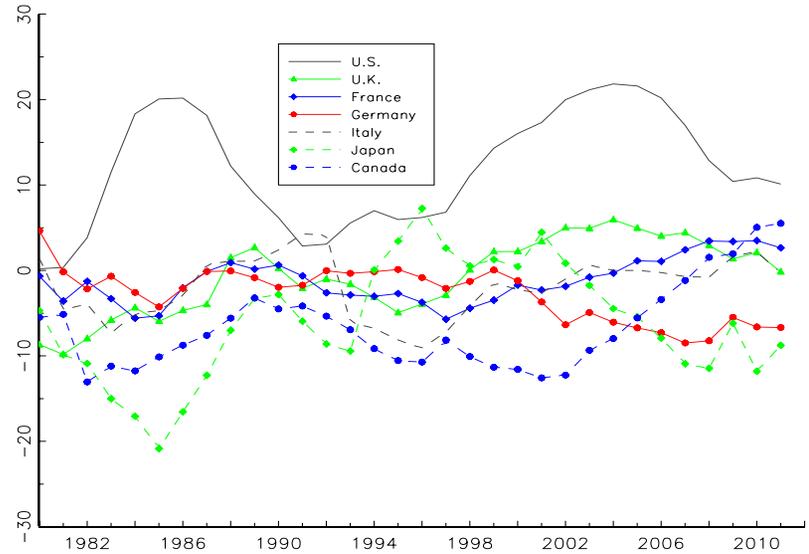
C: Net Foreign Assets (% of Trade)
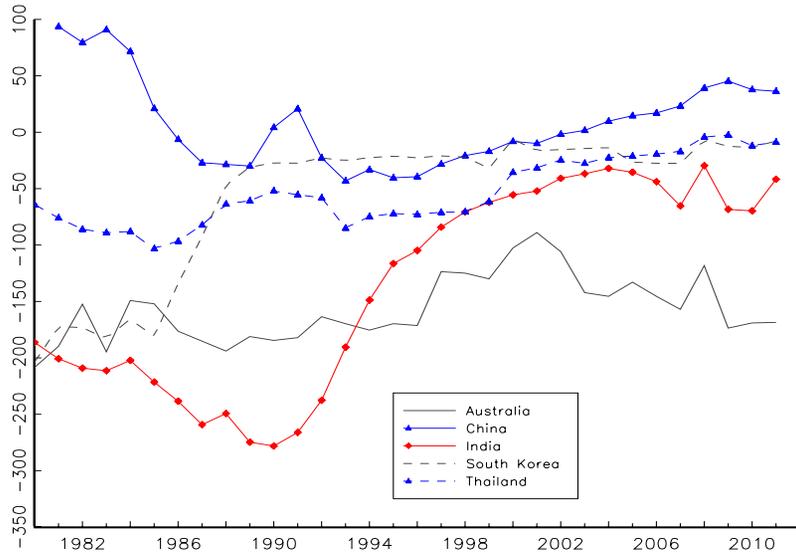
B: Trade Deficits (% of Trade)
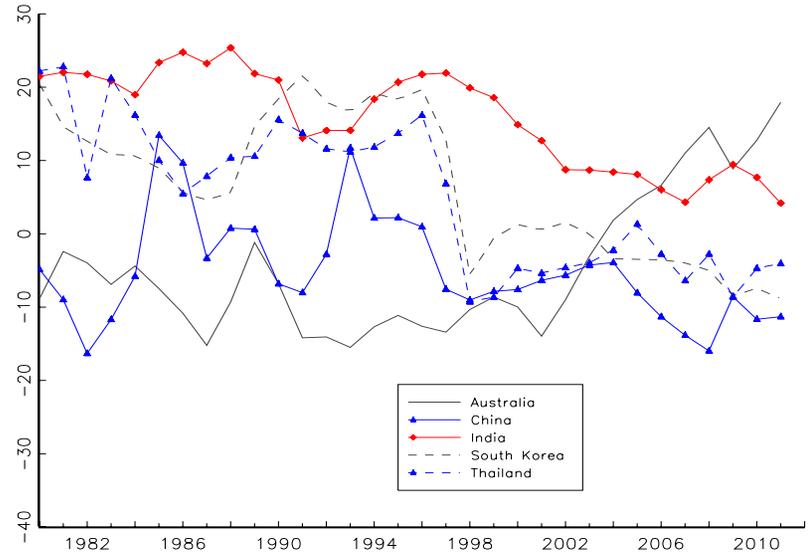
Figure 5: NXA Positions
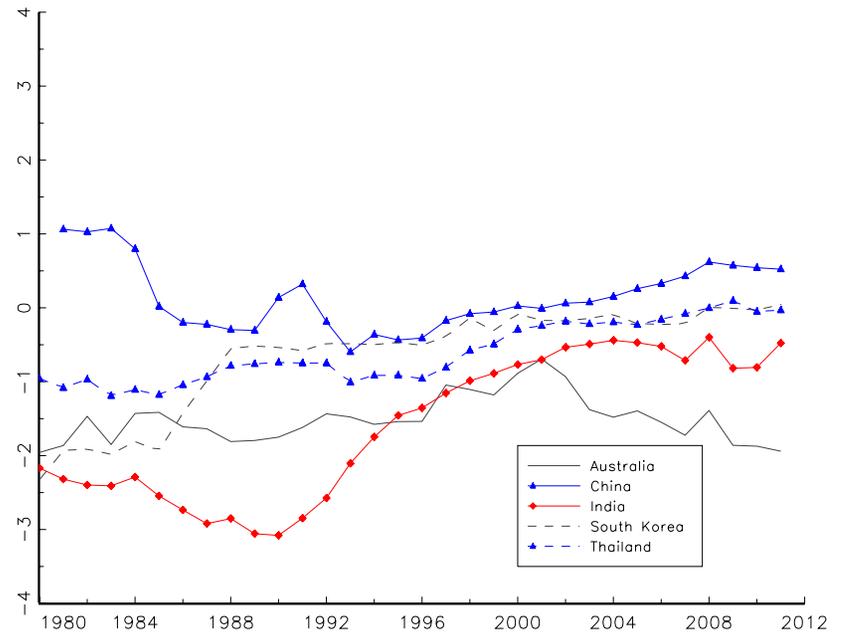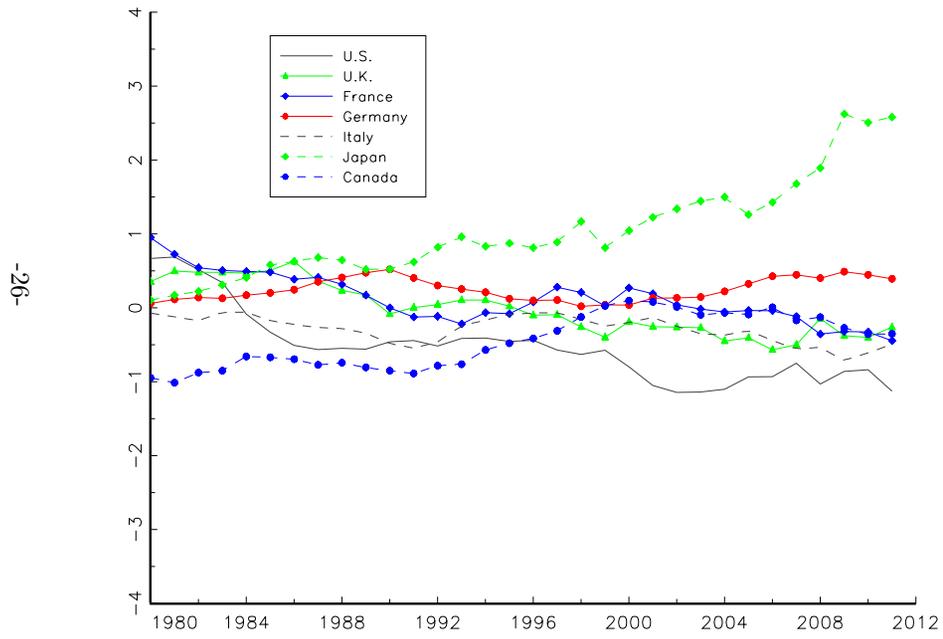
Sustainability

The question of whether a country's current external position is sustainable occupies a central position in international policy discussions. Equation (11) and the approximations in (21) and (22) enable us to make the notion of sustainability precise.

Equation (11) and the approximations in (21) and (22) are based on two economic assumptions: (i) that portfolio returns on assets and liabilities satisfy the no-arbitrage conditions; and (ii) that investors are unwilling to participate in Ponzi schemes. These two assumptions are necessary conditions for any meaningful definition of sustainability. To see why, suppose that a country's NXA position was based on the assumption that it could make a riskless profit in the future from issuing liabilities and investing the proceeds in a foreign asset market. Clearly this is an implausible assumption unless there is a credible reason for the asset market to remain closed to all but the country's investors. Lacking such a reason, the prospect of future riskless profits sustaining the current NFA positions would disappear as the asset market opened to investors from around the world attempting to profit from the arbitrage opportunity. Similarly, a country's NFA position could only be supported by running a Ponzi scheme while there are enough foreign investors willing to purchase the country's liabilities (at a positive price). In this case sustaining such a position indefinitely is implausible because the foreign investors are acting against their own self-interest.

Not all NXA positions consistent with (22) need be sustainable. In principle there exists a set of expectations concerning future paths for trade flows and the SDF that make the right-hand-side of (22) equal to the value of any NXA position. Thus, any precise notion of sustainability requires restrictions on these expectations in addition to the no-arbitrage and on-Ponzi conditions. Of course these restrictions arise naturally in theoretical models with rational expectations. Here agents' expectations are restricted to conditional expectations based on the true distribution of future equilibrium trade flows and SDFs, so they guarantee sustainability: Unexpected future changes in the NXA position are driven entirely by shocks that induce revisions in equilibrium expectations. And, in the absence of shocks, the NXA position converges to its steady state value of zero.

Judging the sustainability of an actual country's external position is more complicated because any judgement must be conditioned on a particular set of expectations concerning the future SDF and trade flows. To understand the issues involved, consider judgements that utilize (22). This equation contains expectations conditioned on the common information set of agents in period t. Since much of this information is unavailable to analysts, we must "condition down" from agents' common information before the equation can be used.

Let Φ_t denote a subset of agents' information at t that includes $NXA_{n,t}$ and $TD_{n,t}$. Taking expectations conditioned on Φ_t on both sides of (22) and applying the Law of Iterated Expectations, we find that

$$NXA_{n,t} = \frac{1}{2(1-\rho)} \sum_{i=1}^{\infty} \rho^i \mathbb{E}\left[(\Delta m_{n,t+i} - \Delta x_{n,t+i}) \,|\Phi_t\right]$$

$$+ \frac{1}{1-\rho} TD_{n,t} \sum_{i=1}^{\infty} \rho^i \mathbb{E}\left[(\Delta \tau_{n,t+i} - g) \,|\Phi_t\right] + \frac{1}{1-\rho} TD_{n,t} \sum_{i=1}^{\infty} \rho^i \mathbb{E}\left[(\kappa_{t+i} - \kappa) \,|\Phi_t\right],$$

or, more compactly,

$$NXA_{n,t} = \tfrac{1}{2}\mathcal{PV}(\Delta m_{n,t} - \Delta x_{n,t}) + TD_{n,t}\mathcal{PV}(\Delta \tau_{n,t} - g) + TD_{n,t}\mathcal{PV}(\kappa_t - \kappa), \qquad (23)$$

where $\mathcal{PV}(\upsilon_t) = \frac{1}{1-\rho}\sum_{i=1}^{\infty}\rho^i\mathbb{E}\left[\upsilon_{t+i}|\Phi_t\right]$. This equation takes the same form as (22) except the agents' expectations are replaced by expectations conditioned on Φ_t. Conditioning down in this manner doesn't affect the link between the country's external position and the expectations because information used by agents is effectively contained in Φ_t via the presence of $NXA_{n,t}$ and $TD_{n,t}$.

Judging sustainability with the aid of (23) is conceptually straightforward. All we need do is compare the actual value of $NXA_{n,t}$ with the valued implied by the right-hand-side that use estimates of the conditional expectations terms and its associated confidence band that accounts for estimation (and approximation) error. If the value for $NXA_{n,t}$ falls within this band, there is no evidence against the sustainability of the country's external position. Alternatively, if $NXA_{n,t}$ falls outside the band, there is a *prima facie* case the the country is on an unsustainable path. In these circumstances the question of whether the country's external position is truly sustainable requires further judgement. In particular, we would want to asses whether the confidence band computed for the right-hand-side of (23) covers the range of economically plausible expectations agents could hold concerning future trade flows and the SDF.

Three features of (23) simplify such an assessment. First, the right-hand-side involves expectations concerning the export-import differential, $\Delta x_{n,t} - \Delta m_{n,t}$, and trade growth, $\Delta \tau_{n,t} = \tfrac{1}{2}(\Delta m_{n,t} + \Delta x_{n,t})$. These variables display little serial correlation (see Table 1) and are hard to forecast using historical data, so the plausible range of agents' expectations for these terms is tightly bound by historical norms. Second, agents' expectations concerning the future log SDF affect the NXA positions of all countries. If the NXA positions of other countries fall with the confidence bands computed from estimates of $\mathbb{E}[\kappa_{t+i} - \kappa|\Phi_t]$, it is unlikely that agents' expectations differ significantly from these estimates. Finally, all the estimated expectations on the right-hand-side of (23) are discounted by $\rho = \exp(g + \kappa)$, where $g = \mathbb{E}[\Delta m_{n,t}] = \mathbb{E}[\Delta x_{n,t}]$ and $\kappa = \mathbb{E}[\kappa_t]$. I estimate the value for ρ to be approximately 0.6 using sample moments from 12 countries over 40 years. This estimate implies that agents' short-horizon expectations concerning future trade flows and the SDF are quantitatively far more important than their medium- or long-horizon expectations in determining $NXA_{n,t}$. Thus, when contemplating the plausible range for agents' expectations we can focus primarily on their short-term expectations.

A country's external position should be view as unsustainable in cases where the value of $NXA_{n,t}$ falls outside the confidence band that is judged wide enough to cover the range of economically plausible expectations agents could hold. To be clear, in these cases the current value of the country's asset and liability portfolios are viewed as inconsistent with the plausible prospects for future trade flows and the SDF. For example, in the case of a net debtor country, the value of its liabilities may reflect overoptimism concerning future export growth; i.e. agents expectations $\mathbb{E}_t\Delta x_{n,t+i}$ are implausibly high. Such a country would be judged to be in an unsustainable position because at some point the overoptimism will evaporate and the price of the country's liabilities will collapse (including the possibility of a default on its debt). This "adjustment" process will raise the future value of $NXA_{n,t}$ to a sustainable level, i.e., a level consistent with economically plausible expectations for trade and the SDF going forward.

Forecasts

In principle, the analysis described above can be conducted using estimates of expectations computed in a variety of ways. For example, policymakers might want to combine forecasts from several policy models and/or statistical forecasting models. In this paper I compute estimates of the present value terms on the right-hand-side of (23) from VARs. This approach follows a large literature initiated by the work of Campbell and Shiller (1987).

Specifically, let the vector $z_{n,t} = [\ \Delta m_{n,t} - \Delta x_{n,t}, \Delta \tau_{n,t} - \hat{g}, \quad NXA_{n,t}, TD_{n,t}, .. \ ]'$ follow a p'th. order VAR:

$$z_{n,t} = a_1 z_{n,t-1} + a_2 z_{n,t-2} + ...a_k z_{n,t-p} + u_{n,t},$$

where a_i are matrices of coefficients from each of the VAR equations, and $u_{n,t}$ is a vector of mean-zero shocks. $\hat{g}$ denotes the pooled sample mean for trade growth across countries. To compute the first two present value terms on the right-hand-side of (23), the estimated VAR is written in companion form:

$$\begin{bmatrix} z_{n,t} \\ \vdots \\ \vdots \\ z_{n,t-p+1} \end{bmatrix} = \begin{bmatrix} \hat{a}_1 & \cdots & \cdots & \hat{a}_p \\ I & & & \\ & \ddots & & \\ & & I & 0 \end{bmatrix} \begin{bmatrix} z_{n,t-1} \\ \vdots \\ \vdots \\ z_{n,t-p} \end{bmatrix} + \begin{bmatrix} u_{n,t} \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

or, more compactly,

$$Z_{n,t} = \hat{A}_n Z_{n,t-1} + U_{n,t},$$

where $\hat{a}_i$ are the matrices of estimated coefficients. The present value terms are then computed as

$$\begin{align} \widehat{\mathcal{PV}}(\Delta m_{n,t} - \Delta x_{n,t}) &= \tfrac{\rho}{1-\rho} \imath_1 \hat{A}_n (I - \rho \hat{A}_n)^{-1} Z_{n,t} \qquad \text{and} \tag{24} \\ \widehat{\mathcal{PV}}(\Delta \tau_{n,t} - g) &= \tfrac{\rho}{1-\rho} \imath_2 \hat{A}_n (I - \rho \hat{A}_n)^{-1} Z_{n,t} \tag{25} \end{align}$$

where $\imath_1$ and $\imath_2$ are vectors that pick out the first and second rows of $Z_{n,t}$: i.e., $\Delta m_{n,t} - \Delta x_{n,t} = \imath_1 Z_{n,t}$ and $\Delta \tau_{n,t} - \hat{g} = \imath_2 Z_{n,t}$. These calculations are computed from VAR's estimated country-by-country, and thus allow for cross-country differences in the present value terms. The present value term involving the log SDF is common to all countries and so is calculated in an analogous fashion from a single VAR for that includes $\kappa_t - \hat{\kappa}$, as

$$\widehat{\mathcal{PV}}(\kappa_t - \hat{\kappa}) = \tfrac{\rho}{1-\rho} \imath_1 \hat{A} (I - \rho \hat{A})^{-1} Z_t, \tag{26}$$

where $\kappa_t - \hat{\kappa} = \imath_1 Z_t$ and $\hat{\kappa}$ is the sample average of the log SDF. The calculations in (24)-(26) use a value for ρ equal to $\exp(\hat{g} + \hat{\kappa})$.

5 Empirical Analysis

Estimating the World SDF

I consider two specifications for the log SDF. The first, denoted by $\hat{\kappa}_t^{\mathrm{I}}$, is estimated from (17) using the portfolio returns on assets and liabilities for the G7 and the real return on U.S. T-bills as the set of returns. This specification doesn't incorporate conditioning information. To assess whether the estimates satisfy the no-arbitrage condition, $1 = \mathbb{E}[\exp(\hat{\kappa}_{t+1}^{\mathrm{I}} + r_{t+1}^i)|z_t^j]$, I estimate regressions of the form:

$$\exp(\hat{\kappa}_{t+1}^{\mathrm{I}} + r_{t+1}^i) - 1 = b_1(fa_{n,t} - fl_{n,t}) + b_2(x_{n,t} - m_{n,t}) + v_{t+1}. \tag{27}$$

Panel A of Table 3 reports the estimation results for the log returns on the asset and liability portfolios of the G7 countries. Notice that the log ratios of assets-to-liabilities and export-to-imports are valid instruments so the estimates of b_1 and b_2 should be statistically insignificant under the null of a correctly specified SDF. As Panel A shows, this is not the case for the portfolio returns of four countries. The log asset-to-liability ratio has predictive power for German, U.K. and U.S. returns, while the log export-to-import ratio has power for the returns on Japanese assets.

Table 3: Forecasting Returns

	Asset Returns			Liability Returns		
	b_1	b_2	R^2	b_1	b_2	R^2
A: $\hat{\kappa}^{\mathrm{I}}$						
France	0.059	-0.210	-0.001	0.117	-0.205	0.003
Germany	-0.428*	0.669	0.124	-0.442**	0.594	0.129
Italy	-1.031	2.436	0.135	-1.009	2.667*	0.143
Japan	0.299	2.304**	0.098	0.327	2.374	0.106
United Kingdom	-5.852**	0.324	0.183	-5.843**	0.437	0.177
United States	-1.108**	0.216	0.132	-1.059**	0.252	0.115
B: $\hat{\kappa}^{\mathrm{II}}$						
France	-0.188	-0.636	0.023	-0.116	-0.610	0.017
Germany	-0.083	2.824	0.057	-0.091	2.862	0.059
Italy	-0.653	-0.668	0.018	-0.653	-0.453	0.016
Japan	0.742	1.809	0.050	0.774	1.874	0.055
United Kingdom	-4.595	2.237	0.052	-4.698	2.529	0.054
United States	-0.229	0.515	0.022	-0.163	0.558	0.023

Notes: The table reports the OLS estimates of the regression (27) using the SDFI in panel A and SDFII in panel B. "**" and "*" indicate statistical significance at the 5% and 10% levels, respectively. All regression estimated in annual data between 1971 and 2011.

In the light of these results, I incorporate conditioning information in my second specification for the log SDF, denoted by $\hat{\kappa}_t^{\mathrm{II}}$. Specifically, I now add the adjusted log return on U.S. assets, $r_{t+1}^{i,z} = r_{\mathrm{US},t+1}^{\mathrm{A}} + (fa_{\mathrm{US},t} - fl_{\mathrm{US},t}) - \ln \mathbb{E}[\exp(fa_{\mathrm{US},t} - fl_{\mathrm{US},t})]$, where $r_{\mathrm{US},t+1}^{\mathrm{A}}$ is the log return on U.S. assets, to the set of returns used to estimate the log SDF in (17). This specification incorporates information concerning the future value of the SDF that is correlated with variations in the U.S.

NFA position. Thus, $fa_{\text{US},t} - fl_{\text{US},t}$ should not have forecasting power for $\exp(\hat{\kappa}^{\text{II}}_{t+1} + r^i_{t+1}) - 1$ by construction. To check whether the other instruments retain their forecasting power, I then re-estimate regression (27) with $\hat{\kappa}^{\text{II}}_{t+1}$ replacing $\hat{\kappa}^{\text{I}}_{t+1}$. Panel B of Table 3 reports these regression results. In contrast to Panel A, none of the b_1 and b_2 coefficient estimates are statistically significant. Notice, also, that the R^2 statistics are (in most cases) an order of magnitude smaller than their counterparts in Panel A. The asset-to-liability and export-to-import ratios do not account for an economically meaningful fraction of the variation in $\exp(\hat{\kappa}^{\text{II}}_{t+1} + r^i_{t+1}) - 1$. These findings appear robust to the choice of estimation period and instruments. Re-estimating (27) over a sample period that ends in 2007 gives essentially the same results. I also find statistically insignificant coefficients in regressions using $\hat{\kappa}^{\text{II}}_{t+1}$ as the log SDF when GDP growth rates and/or lagged returns are used as alternate instruments.[9]

Figure 6 plots the two estimated SDFs, $\hat{\mathcal{K}}^{\text{I}}_t = \exp(\hat{\kappa}^{\text{I}}_t)$ and $\hat{\mathcal{K}}^{\text{II}}_t = \exp(\hat{\kappa}^{\text{II}}_t)$, together with the inverse of the real return on U.S. T-bills, $1/R^{\text{TB}}_t$. In the special case where the expected excess portfolio returns on assets and liabilities are zero, equation (17) implies that the SDF is equal to $1/R^{\text{TB}}_t$. Thus differences between $1/R^{\text{TB}}_t$ and the estimated SDF's arise because the SDFs must account for the expected excess portfolio returns. As the plots clearly show, both estimates of the SDF are more volatile than $1/R^{\text{TB}}_t$. In fact, variations in the log real return on U.S. T-bills contribute less than one percent to the sample variance of $\hat{\kappa}^{\text{I}}_t$ and $\hat{\kappa}^{\text{II}}_t$. Thus, changes in U.S. T-bill returns do not appear to have an economically significant impact on estimates of the SDF that "explain" excess returns on asset and liability portfolios in the G7. The plots in Figure 6 also show that there are numerous episodes where the estimates SDFs are well above one. Ex ante, the conditionally expected value of the SDF, $\mathbb{E}_t\mathcal{K}_{t+1}$, identifies the value of a claim to one real dollar next period. So safe dollar assets sold at a premium during periods where these high values for the SDF were forecast ex ante.

While the time series for $\hat{\mathcal{K}}^{\text{II}}_t$ and $1/R^{\text{TB}}_t$ in Figure 6 look very different, the unconditional moments of $\hat{\mathcal{K}}^{\text{II}}_t$ and R^{TB}_t are closely related. Let r denote the log risk free rate in the steady state that satisfies the no-arbitrage condition $1 = \mathbb{E}[\exp(\kappa_t + r)] = \mathbb{E}[\exp(\kappa_t)]\exp(r)$. After substituting for κ_t from (17) and evaluating the expectation with a log-normal approximation, we can rewrite this condition as

$$r = -\mathbb{E}[\kappa_t] - \tfrac{1}{2}\mathbb{V}[\kappa_t] = \mathbb{E}\left[r^{\text{TB}}_t\right] - \tfrac{1}{2}\mathbb{V}\left(r^{\text{TB}}_t\right) - \mathbb{CV}\left[r^{\text{TB}}_t, \mathbf{er}'_t\right]\Omega^{-1}\mu.$$

Thus, the steady state risk free rate is equal to the unconditional expected real return on U.S. T-bills and a risk premium that accommodates variations in real returns and their co-variation with excess portfolio returns.[10] When the log SDF is identified by $\hat{\kappa}^{\text{II}}_t$, I estimate that the steady state risk free rate equals 1.84 percent. By comparison, the average real return on U.S. T-bills is 1.54 percent, 30 basis points lower. Intuitively, the average return on U.S. T-bills is lower than the risk free rate because the bonds provide unexpectedly large real returns when the realized value of the SDF is high; i.e., a hedge against "bad" states of the world where agents are willing to pay a premium for safe dollar assets.

Finally, the estimates of the log SDF, $\hat{\kappa}^{\text{II}}_t$, allow us to pin down the discount rate $\rho = \exp(g +$

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[9]Recall that specification for κ_t in (17) was derived using a log normal approximation to evaluate expected future returns. Based of these regression estimates, there is no evidence to suggest that the approximation is a significant source of specification error for $\hat{\kappa}^{\text{II}}_t$.

[10]Strictly speaking, the variance term arises from Jensen's inequality because we are working with log returns on T-bills, so is not part of the risk premium per se. Nevertheless, I follow the common practice of including the variance term when referring to the risk premium.

Figure 6: SDF Estimates
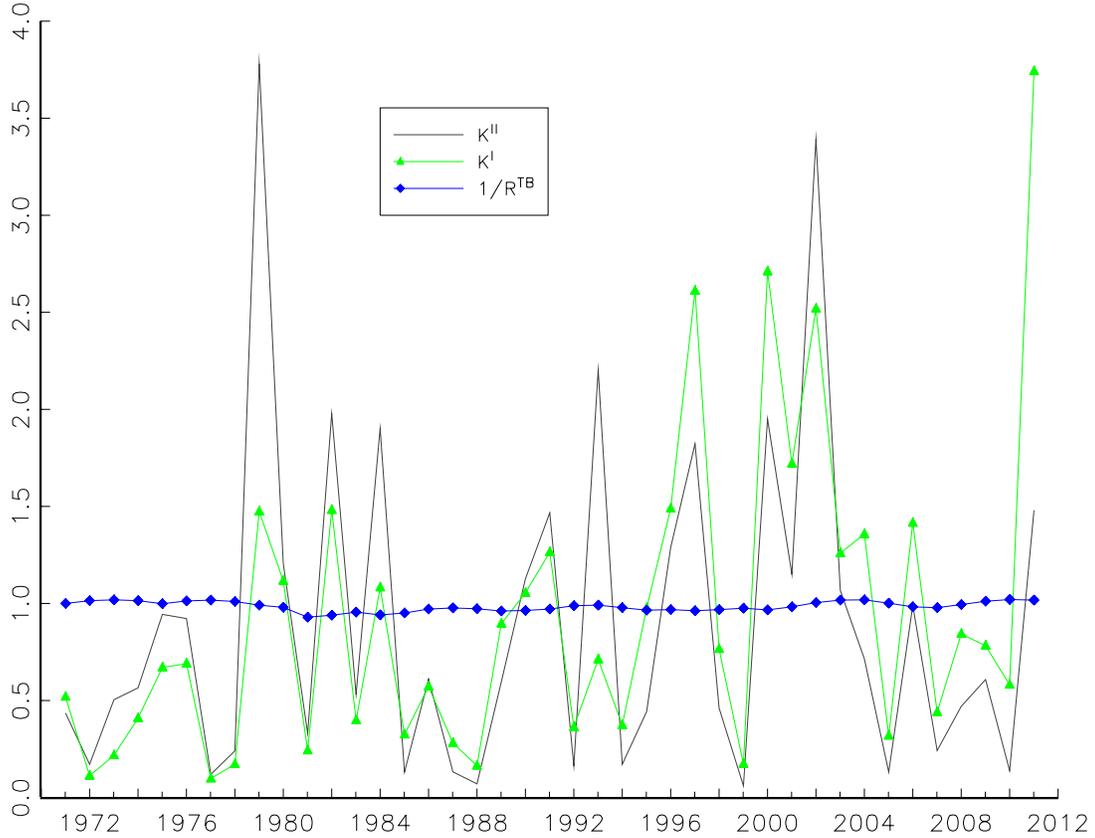
Notes: The figure plots two estimates of the world SDF, $\hat{\mathcal{K}}_t^{\mathrm{I}} = \exp(\hat{\kappa}_t^{\mathrm{I}})$ and $\hat{\mathcal{K}}_t^{\mathrm{II}} = \exp(\hat{\kappa}_t^{\mathrm{II}})$, with κ_t determined in (17); and the inverse of the real return on U.S. T-bills, $1/R_t^{\mathrm{TB}}$.

κ) used in computing the present value terms in the NXA equation (23). Recall that g is the unconditional growth rate for exports and imports, estimated to be 0.064 from the pooled average of import and export growth across countries. My estimate of κ computed from the average value of $\hat{\kappa}_t^{\mathrm{II}}$ is approximately -0.59. Together, these estimates imply a discount rate of $\rho = 0.586$. This is the value I use below when constructing the NXA measures of each country's external position and computing the present value expressions in the NXA equations.

Forecasting Trade and the SDF

According to the analytic framework developed in Section 3, forecasts of future trade flows are embedded in each country's external position. In particular, equation (23) showed how the NXA position of a country was related to the present value of the import-export growth differential, $\mathcal{PV}(\Delta m_{n,t} - \Delta x_{n,t})$, and trade growth, $\mathcal{PV}(\Delta \tau_{n,t} - g)$. Evidence concerning the time series pre-

dictability of these variables is presented in Table 4. Here I report the estimates from two regressions:

$$\Delta m_{n,t+1} - \Delta x_{n,t+1} = c_0 + c_1(x_{n,t} - m_{n,t} - \widehat{\mu}_n) + c_2(\Delta m_{n,t} - \Delta x_{n,t}) + v_{n,t+1} \qquad (28)$$

and
$$\Delta \tau_{n,t+1} - g = d_0 + d_1(x_{n,t} - m_{n,t} - \hat{\mu}_n) + d_2(\Delta \tau_{n,t} - g) + v_{n,t+1}, \qquad (29)$$

where $\hat{\mu}_n$ denotes the sample average of $x_{n,t} - m_{n,t}$. The left-hand-panel of the table shows that there is a good deal of time series predictability in the import-export growth differential. In all but four countries, the estimates of c_1 are positive and statistically significant. Thus, future imports tend to grow at a faster rate than exports when the log export-to-import ratio is above its historical norm (i.e., $\hat{\mu}_n$). This pattern of predictability is consistent with the presence of cointegration between $x_{n,t}$ and $m_{n,t}$. Lagged import-export growth also has predictive power in the case of the United States and China. The estimates of regression (29) reported in the right-hand panel show much less evidence of predictability in trade growth. In only two countries, Australia and India, are any of the slope coefficients statistically significant at the 5 percent level.

Based on these results, I estimate the present value terms involving future trade flows in the NXA equation from VARs that include the import-export growth differential, $\Delta m_{n,t} - \Delta x_{n,t}$, trade growth, $\Delta \tau_{n,t}$ and the log export-to-import ratio, $x_{n,t} - m_{n,t}$. Consistent with equation (23), I also include $NXA_{n,t}$ and the U.S. real T-bill rate, r_t^{TB}, although the results are robust with respect to the presence or absence of these variables. Below I report results based on first-order VARs estimated separately for each country, n. Alternatively, one could estimate the VAR coefficients from the pooled VAR, and then use the estimates in conjunction with the country-specific values for current import-import growth, trade growth, etc..

I also use a VAR to compute the present value of the log SDF, $\widehat{\mathcal{PV}}(\hat{\kappa}_t^{\text{II}} - \hat{\kappa}^{\text{II}})$. In this case the VAR includes $\hat{\kappa}_t^{\text{II}}$, the U.S. T-bill rate, r_t^{TB}, the U.S. inflation rate, π_t^{US}, the yield spread between the yield on ten and one year U.S. T-bonds, spr_t^{US}, and the average rate of real GDP growth across the G7, Δy_t^{G7}. Among these variables, most of the predictive power comes from the U.S. yield spread. Higher values for the spread predict lower future values for the log SDF. I also use the VAR to compute the present value of the log return on U.S. T-bills, $\widehat{\mathcal{PV}}(r_t^{\text{TB}} - \hat{r}^{\text{TB}})$, where $\hat{r}^{\text{TB}}$ is the sample average of r_t^{TB}. Here most of the predictive power of the VAR for future U.S. T-bill returns comes from current returns.

Figure 7 plots the VAR-based estimates of the present value for the log SDF and minus one times the present value of the log return on U.S. T-bills over the sample period. The VAR is estimated using data between 1971 and 2011. Alternative estimates of the present value derived from a VAR estimated on pre-crisis data (1971-2006) follow a similar pattern. As the figure clearly shows, time series variations in the present value for the log SDF follow a cyclical pattern and are much larger in magnitude than the changes in the present value of the log return on U.S. T-bills. The differences between these series reflect the effects of time-varying risk. To see why, consider the no-arbitrage condition governing the return on U.S. T-bills: $1 = \mathbb{E}_t[\exp(\hat{\kappa}_{t+1}^{\text{II}} + r_{t+1}^{\text{TB}})]$. Using a log-normal approximation to evaluate the conditional expectation, we can rewrite this condition as $\mathbb{E}_t[\hat{\kappa}_{t+1}^{\text{II}} + r_{t+1}^{\text{TB}}] = -\frac{1}{2}\mathbb{V}_t[\hat{\kappa}_{t+1}^{\text{II}} + r_{t+1}^{\text{TB}}]$, where $\mathbb{V}_t[.]$ denotes the conditional variance. Subtracting

Table 4: Forecasting Trade Flows

	Export-Import Growth			Trade Growth		
	c_1	c_2	R^2	d_1	d_2	R^2
Canada	9.095	0.275	0.108	10.845	0.089	0.080
France	19.963**	0.015	0.136	4.349	0.113	0.015
Germany	12.903**	0.119	0.103	-5.917	0.036	0.016
Italy	17.201**	0.134	0.126	-1.743	-0.045	0.003
Japan	15.729**	0.195	0.114	-9.487	-0.185	0.076
United Kingdom	10.169	0.229	0.091	1.890	0.165	0.028
United States	26.553**	0.663**	0.510	-4.950	0.031	0.018
Australia	8.058	0.039	0.033	4.276	-0.284**	0.130
China	46.757**	0.272**	0.309	14.897	0.282*	0.106
India	13.477**	0.121	0.129	11.936**	0.153	0.147
South Korea	13.852**	0.070	0.151	-6.968	0.014	0.062
Thailand	11.297	-0.244	0.135	1.930	0.034	0.004

Notes: The left- and right-hand panels reports the OLS estimates of the slope coefficients and the R^2 statistics from regressions (28) and (29), respectively. Each row reports estimates for country n. "**" and "*" indicate statistical significance at the 5% and 10% levels, respectively. All regressions estimated in annual data between 1971 and 2011.

unconditional expectations from both sides and re-arranging using (17) gives

$$\mathbb{E}_t[\hat{\kappa}_{t+1}^{\text{II}} - \hat{\kappa}] = -\mathbb{E}_t\left[r_{t+1}^{\text{TB}} - \hat{r}^{\text{TB}}\right] - \tfrac{1}{2}\left\{\mathbb{V}_t[b'\boldsymbol{er}_{t+1}] - \mathbb{E}\left[\mathbb{V}_t[b'\boldsymbol{er}_{t+1}]\right]\right\}.$$

Thus a fall in the present value of the log SDF must reflect either a rise in the present value of the log real return on U.S. T-bills and/or a rise in the conditional variance of future excess portfolio returns on asset and liabilities across the G7. Similarly, a rise in the present value of the log SDF must reflect a fall in the present value of the log real return on U.S. T-bills and/or a fall in the conditional variance. With this perspective, the plots in Figure 7 clearly indicate that changes in risk (as measured by the conditional variance) are the primary driver behind the cyclical variations in the present value of the log SDF. For example, the sizable swings in the log SDF between 1998 and 2008 appear to reflect, in turn, an large rise, fall, and rise again in expectations concerning the level of risk well into the future.

Cross-Country External Positions

The NXA equation derived in Section 4 has implications for both the cross-country distribution of external positions at a point in time, and the variation in the positions of individual countries over time. Here I examine the cross-country implications.

To begin, I consider the importance of intertemporal factors. Recall from equation (21) that the net foreign asset position of any country will be proportional to its current trade balance when agents' expect the future growth in exports, imports and trade to be at their steady state rates and they expect the future log SDF to be equal to a constant. Under these circumstances intertemporal factors

Figure 7: The Present Value of the log SDF
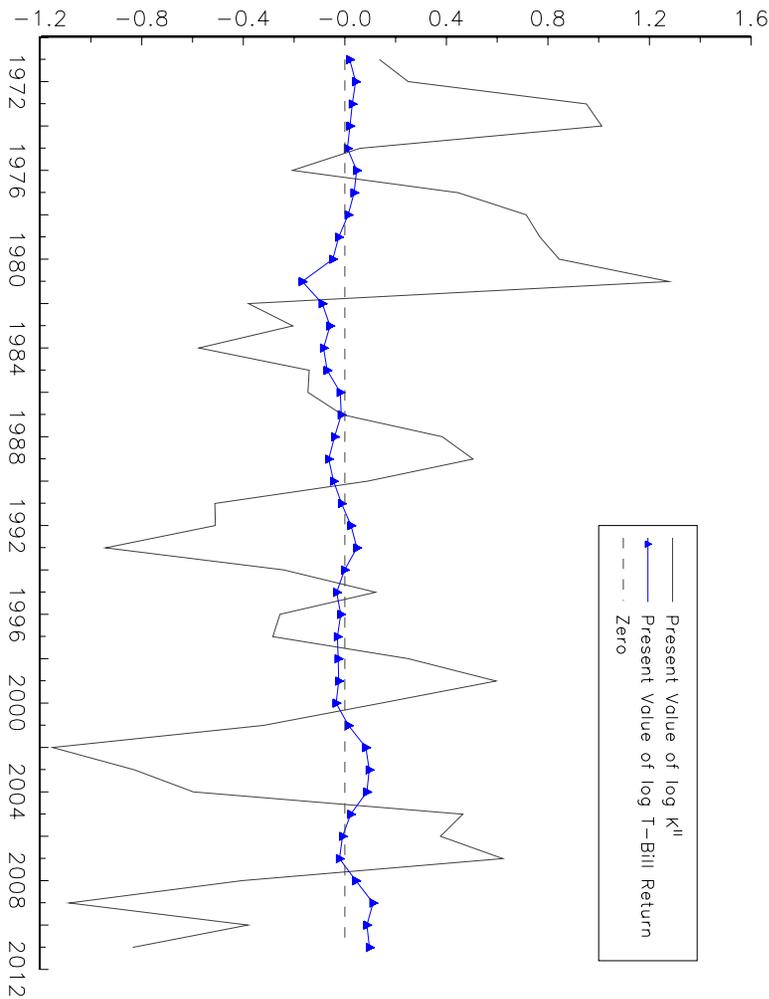
are absent and the cross-country distribution in trade balances fully accounts for the distribution of NFA positions. To examine the empirical relevance of this restriction, I estimate a series of cross-country regressions for each year t in the sample period:

$$NFA_{n,t}/(X_{n,t} + M_{n,t}) = \beta_{0,t} + \beta_t TD_{n,t} + \xi_{n,t}. \tag{30}$$

Under the null hypothesis that intertemporal factors have no impact on the cross-country distribution of NFA positions in year t, β_t will equal $\rho/(1-\rho)$, and the R^2 statistic from the regression should equal one.[11] These implications are not supported by the estimates of (30). Panels A and B in Figure 8 plot the estimates of β_t and the R^2 statistics for each year between 1971 and 2012. The plots show that the estimates of β_t are generally negative, and the R^2 statistics only rise above 0.5 after 2007. These findings imply the current trade imbalances account for very little of the cross-country NFA distribution. Indeed, even though the R^2 are higher at the end of the sample, this is the period where the estimates of β_t are most strongly negative. These findings suggest that intertemporal factors have been the dominant determinants of NFA across countries, rather than current trade flows since the onset of the Great Recession

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[11] I include an intercept $\beta_{0,t}$ in cross-country regressions to allow for the fact that the aggregate NFA position and trade balance for countries in the dataset differs from zero each year.

Figure 8: Intertemporal Trade and the Cross-Country Distribution of NXA
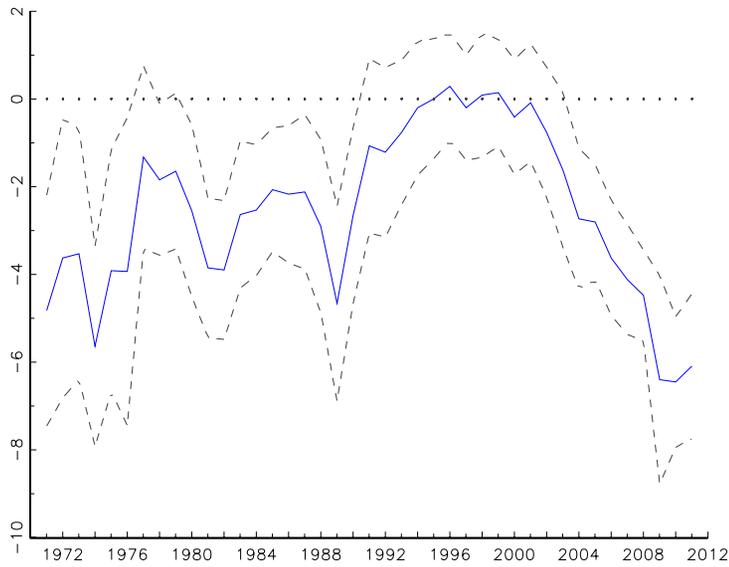

A: OLS estimates of the slope coefficient β_t from regression (30). Dashed lines indicate a two standard error bound around the estimates.
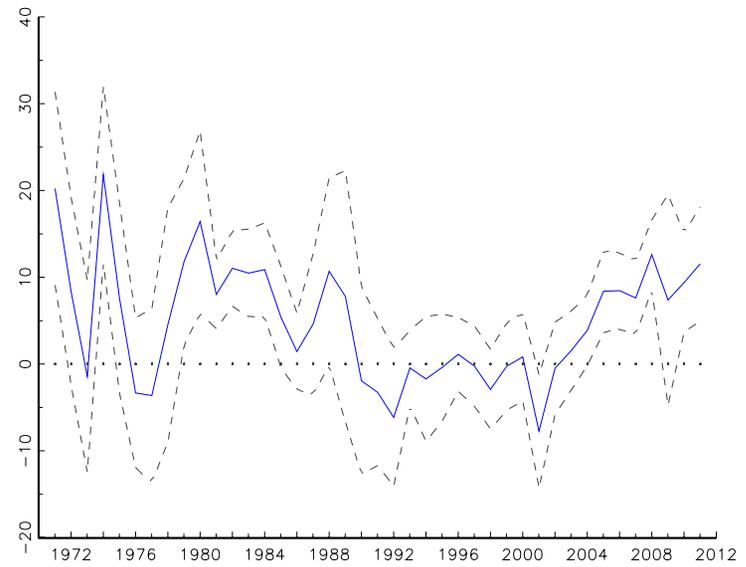
C: OLS estimates of the slope coefficient β_t from regression (31). Dashed lines indicate a two standard error bound around the estimates.
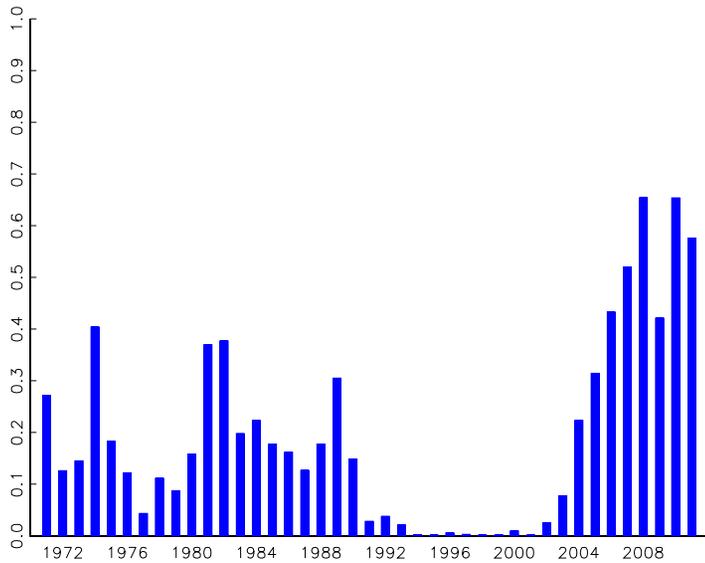
B: R^2 from regression (30).
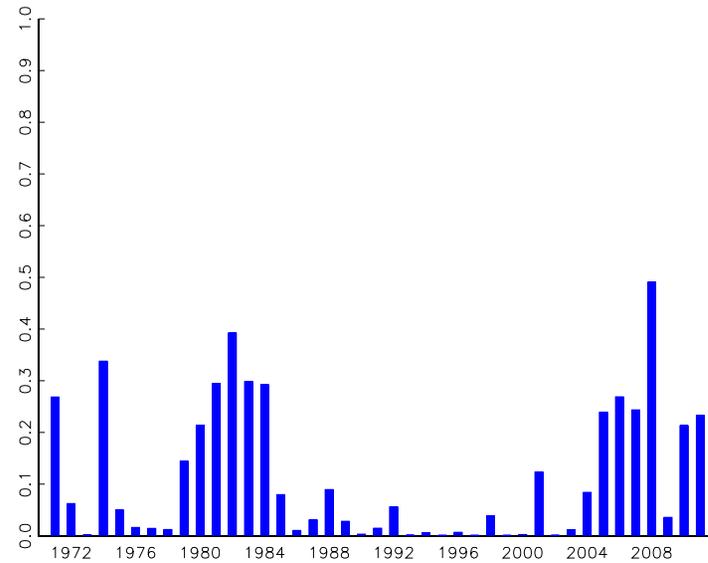
D: R^2 from regression (31).

Next, I consider the role of intertemporal trade on the cross-country distribution of external positions. For this purpose I estimate a series of cross-country regressions for the NXA positions on the present value of the growth differentials between imports and and exports:

$$NXA_{n,t} = \beta_{0,t} + \beta_t \left(\tfrac{1}{2}\widehat{\mathcal{PV}}(\Delta m_{n,t} - \Delta x_{n,t}) \right) + \xi_{n,t}, \tag{31}$$

where $\widehat{\mathcal{PV}}(\Delta m_{n,t} - \Delta x_{n,t})$ are the VAR-based estimates of the present value described above. If agents expectations concerning future trade growth and the log SDF equal their steady state values (i.e. when $\mathbb{E}_t \Delta \tau_{t+i} = g$ and $\mathbb{E}_t \kappa_{t+i} = \kappa$,) the cross-country distribution of NXA should only reflect differences in the present value of the import-export growth differentials. Under this null, β_t will equal unity, and the R^2 statistic from the regression should equal one for each year t. Panels C and D of Figure 8 plot the estimates of β_t and the R^2 statistics from estimating (31). In this case the estimates of β_t are mostly positive or statistically insignificant, but the R^2 statistics are generally very small. These results suggest the cross-country differences in the prospective growth of imports relative to exports typically account for only a small fraction of the cross-country distribution in external positions.

In Figure 9 I examine how agents' expectations concerning the future path for the log SDF contribute to the cross-country distribution of returns. According to equation (23), $NXA_{n,t} - \tfrac{1}{2}\mathcal{PV}(\Delta m_{n,t} - \Delta x_{n,t}) = TD_{n,t}[\mathcal{PV}(\Delta \tau_{n,t} - g) + \mathcal{PV}(\kappa_t - \kappa)]$. So, once we control for differences in the present value of import-export growth differentials, any remaining cross-country differences in external positions depend on current trade deficits, $TD_{n,t}$, and agents' expectations about trade growth and the log SDF. Panels A and B of Figure 9 plot the estimates of β_t and the R^2 statistics from regressions of the form:

$$NXA_{n,t} - \tfrac{1}{2}\widehat{\mathcal{PV}}(\Delta m_{n,t} - \Delta x_{n,t}) = \beta_{0,t} + \beta_t TD_{n,t} + \xi_{n,t}. \tag{32}$$

If agents expectations concerning future trade growth are the same across countries, β_t will identify $\mathcal{PV}(\Delta \tau_{n,t} - g) + \mathcal{PV}(\kappa_t - \kappa)$. In addition, we should expect to find R^2 statistics close to one if the VAR estimates of $\mathcal{PV}(\Delta m_{n,t} - \Delta x_{n,t})$ accurately identify agents expectations for the future import-export growth differentials. Panels A and B reveal two interesting features. First, the estimates of β_t are generally positive and rise steadily during the last decade of the sample. Second, there is a marked rise in the R^2 statistics after 2004, with a peak value of 0.72 in 2008. These results indicate that a sizable portion of recent cross-country differences in external positions can be attributed to expectations of higher-than-normal trade growth and/or the future path of log SDFs.

The plots in Panel C and D Figure 9 help disentangle these effects. Here I plot the estimates of β_t and the R^2 statistics from regressions:

$$NXA_{n,t} - \tfrac{1}{2}\widehat{\mathcal{PV}}(\Delta m_{n,t} - \Delta x_{n,t}) - TD_{n,t}\widehat{\mathcal{PV}}(\Delta \tau_{n,t} - g) = \beta_{0,t} + \beta_t TD_{n,t} + \xi_{n,t}. \tag{33}$$

Notice that in this case the estimates for β_t identify the present value of agents' expectations for the future path of the SDF because the dependent variable controls for their expectations concerning trade growth with the inclusion of $TD_{n,t}\widehat{\mathcal{PV}}(\Delta \tau_{n,t} - g)$. Comparing the left and right-hand panels of Figure 9 we see similar patterns for the estimates of β_t and the R^2 statistics in the last decade of the sample. In both cases, the estimates of β_t are close to 10 by 2011 while the R^2 statistics are

**Figure 9: Financial Factors and the Cross-Country Distribution of NXA**
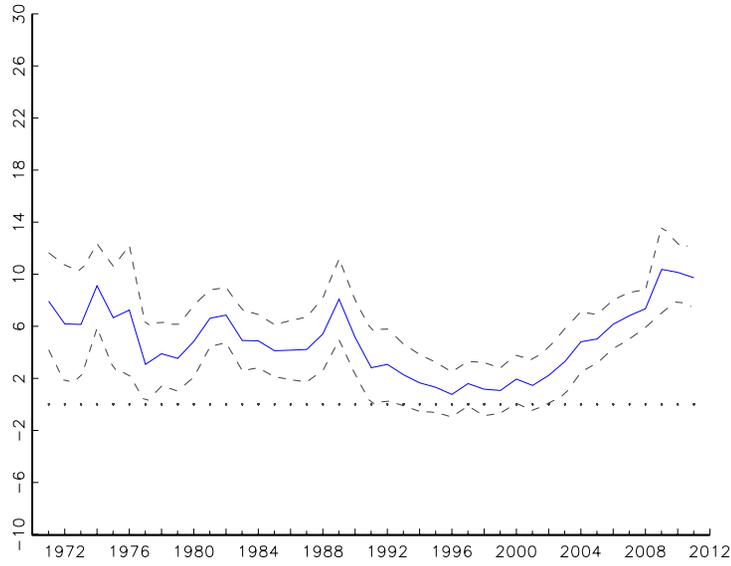
A: OLS estimates of the slope coefficient β_t from regression (32). Dashed lines indicate a two standard error bound around the estimates.
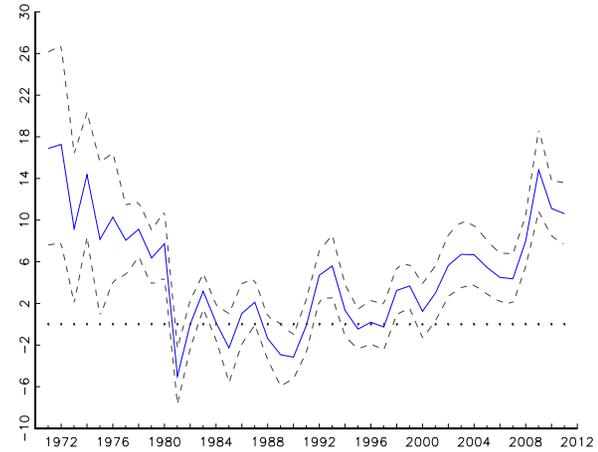
C: OLS estimates of the slope coefficient β_t from regression (33). Dashed lines indicate a two standard error bound around the estimates.
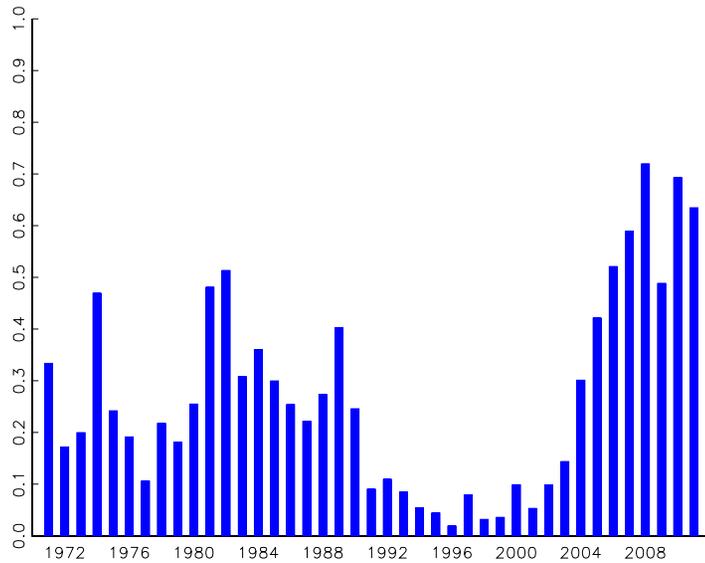
B: R^2 from regression (32).
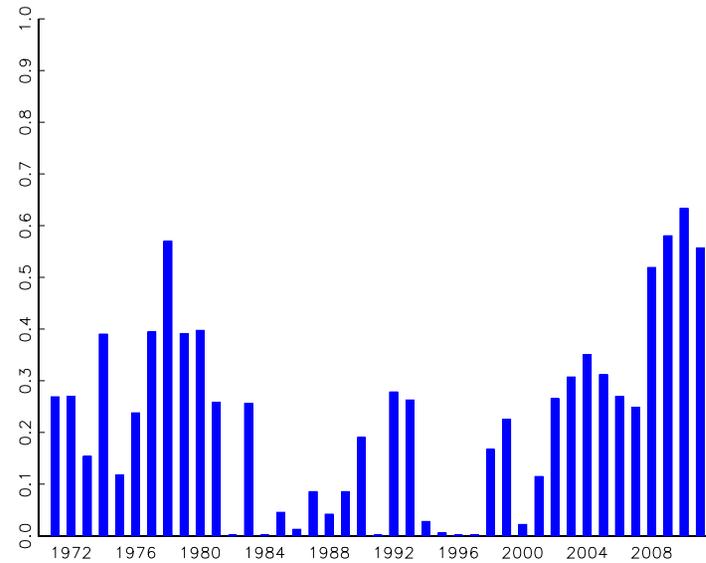
D: R^2 from regression (33).

near 0.6. These results suggest that agents expectations concerning the future path of the log SDF contribute significantly to the recent cross-country distribution of external positions.

External Position Dynamics

I now turn to the dynamic implications of the NXA equation in (23). Recall from Table 4 that the future growth differential between imports and exports could be forecast by the current differential and the log of the export-to-import ratio. Since these forecasts appear in the first present value term on the right-hand-side of (23), the forecasting variables should also account for some of the time series variations in $NXA_{n,t}$. To investigate this possibility, I estimate time series regressions,

$$NXA_{n,t} = a_0 + a_1(x_{n,t} - m_{n,t} - \hat{\mu}_n) + a_2(\Delta m_{n,t} - \Delta x_{n,t}) + \zeta_{n,t}, \tag{34}$$

for each country n over the entire sample period (1971-2011). If variations in the forecasting variables on the right-hand-side are correlated with changes in agents' expectations in the present value, $\mathcal{PV}(\Delta m_{n,t} - \Delta x_{n,t})$, both a_1 and a_2 should be statistically significant. Panel A of Table 5 shows that this is the case for the majority of countries in the dataset. Notice, also, that the estimates of a_1 are all positive except in the case of France. To interpret this finding, recall from Table 4 that future imports tend to grow at a faster rate than exports when the log export-to-import ratio is above its historical norm. Thus positive estimates of a_1 are consistent with the idea that agents' real-time expectations concerning future import and export growth in $\mathcal{PV}(\Delta m_{n,t} - \Delta x_{n,t})$ adjust in an analogous manner.

Table 5: External Positions with Trade Forecasts

	A:				B:	
	a_1	a_2	R^2		α_1	R^2
Canada	0.740	1.400	0.040		5.979**	0.251
France	-5.885**	-6.342**	0.290		-15.588**	0.131
Germany	1.260**	0.801	0.411		5.820**	0.550
Italy	0.079	-0.328	0.008		2.751**	0.171
Japan	1.709**	1.418	0.177		6.785**	0.216
United Kingdom	2.940**	2.166**	0.653		9.434**	0.377
United States	2.011**	0.386	0.176		1.635	0.030
Australia	0.658**	0.446	0.104		0.578	0.007
China	2.622**	1.132**	0.580		4.222**	0.471
India	6.728**	4.551**	0.685		17.525**	0.324
South Korea	4.581**	3.714**	0.713		18.903**	0.650
Thailand	0.983**	0.455	0.405		3.517**	0.416

Notes: Panels A and B report OLS estimates of the slope coefficients and R^2 statistics from regressions (34) and (35), respectively. Each row reports estimates for country n. "**" and "*" indicate statistical significance at the 5% and 10% levels, respectively. All regressions estimated in annual data between 1971 and 2011.

Next, I compare the time series variation in each country's NXA position with the variations in the estimated present values of the future import-export growth differential, $\widehat{\mathcal{PV}}(\Delta m_{n,t} - \Delta x_{n,t})$. These estimates are derived from (country-specific) VARs that include $\Delta m_{n,t} - \Delta x_{n,t}$, $\Delta \tau_{n,t}$, $x_{n,t} - m_{n,t}$, r_t^{TB} and $NXA_{n,t}$ so they incorporate information about the future growth in trade flows from a larger set of variables than regression (34). In Panel B of Table 4 I report the results from estimating

$$NXA_{n,t} = \alpha_0 + \alpha_1 \widehat{\mathcal{PV}}(\Delta m_{n,t} - \Delta x_{n,t}) + \zeta_{n,t}, \tag{35}$$

for each country n over the sample period. The estimates of α_1 are positive and appear statistically significant in nine of the twelve countries.[12] The three exceptions are France, Australia and the United States. In the French case the estimate of α_1 is large and negative. This is a counterintuitive result, but it is consistent with the estimates in Panel A. In the Australian and United States' cases, the estimates of α_1 are small and positive but the R^2 statistics are very close to zero. For the other countries changing expectations concerning the future growth in imports and exports act as economically important driver of external positions. For example, the regressions for Germany, China, South Korea and Thailand produce R^2 statistics above 40 percent.

Table 6 shows how changes in expected future trade flows and the log SDF contribute to the time series variation in external positions. Here I report the results from estimating

$$NXA_{n,t} = \alpha_0 + \alpha_1 \widehat{\mathcal{PV}}(\Delta m_{n,t} - \Delta x_{n,t}) + \alpha_2 TD_{n,t} \widehat{\mathcal{PV}}(\Delta \tau_{n,t} - g)$$
$$+ \alpha_3 TD_{n,t} \widehat{\mathcal{PV}}(\kappa_t - \kappa) + \zeta_{n,t}, \tag{36}$$

for each country n, over the sample period. Panel A reports results from estimating (36) with the first two present value terms on the right-hand-side that identify the effects of changing expectations concerning future trade flows. Panel B reports the results from estimating the regression estimates with all three present value terms and so includes the influence of changing expectations about the future path for the SDF.

Table 6 contains several interesting results. First, variations in the expected future path of the log SDF only appear to make a significant incremental contribution to the NXA dynamics of three countries: Australia, Italy and the United States. In the Australian and Italian regressions, the estimated α_i coefficients are positive and statistically significant and adding $\widehat{\mathcal{PV}}(\kappa_t - \kappa)$ raises the R^2 statistics by an economically meaningful amount (i.e., 0.31 to 0.38 and 0.51 to 0.66, respectively). In the U.S. case, variations in the expected future path of the log SDF appear as the most significant driver of NXA dynamics. The estimates in panel A imply that expected future trade flows alone account for almost none of the variations in NXA, whereas those in Panel B show that when $\widehat{\mathcal{PV}}(\kappa_t - \kappa)$ is included in the regression the R^2 rises to 0.34. Across the other countries, the R^2 statistics indicate that changing expectations concerning future trade flows account for sizable fractions of the time series variations in the NXA positions. Moreover, since the estimates of α_1 and α_2 are mainly positive and statistically significant, these variations are generally in the direction consistent with the NXA expression in equation (23). South Korea and Thailand are exceptions. Here the estimates of α_2 are negative and significant.

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[12]Accurate statistical inference here (and in Table 6 below) should account for the fact that $\widehat{\mathcal{PV}}(\Delta m_{n,t} - \Delta x_{n,t})$ is a "generated regressor" derived from the VAR estimates. This requires computing bootstrap standard errors for the estimates of α_1, which I have not done.

Table 6: External Positions with Trade and SDF Forecasts

	A:			B:			
	α_1	α_2	R^2	α_1	α_2	α_3	R^2
Canada	5.807**	0.417**	0.472	5.540**	0.417**	0.556	0.475
France	-3.944	3.959**	0.590	-3.940	3.898**	1.128	0.591
Germany	5.588**	-0.144	0.553	5.498**	-0.194	-0.881	0.572
Italy	3.256**	0.715**	0.508	4.041**	0.782**	2.954**	0.660
Japan	6.467**	-0.108	0.226	6.484**	-0.101	-0.215	0.226
United Kingdom	9.728**	0.201	0.381	9.649**	0.235	-0.952	0.389
United States	1.344	0.093	0.034	1.331	0.468**	5.138**	0.343
Australia	2.037**	0.410**	0.309	2.184**	0.402**	1.569**	0.380
China	4.129**	0.019	0.473	3.908**	0.032	1.212	0.489
India	7.532**	0.424**	0.665	6.294	0.419**	-1.128	0.672
South Korea	10.612**	-0.287**	0.869	10.084**	-0.271**	-1.813*	0.881
Thailand	3.939**	-0.118**	0.507	4.072**	-0.110**	0.481	0.521

Notes: Panels A and B report OLS estimates of the slope coefficients and R^2 statistics from regression (36). Each row reports estimates for country n. "**" and "*" indicate statistical significance at the 5% and 10% levels, respectively. All regressions estimated in annual data between 1971 and 2011.

The estimates of regression (36) reported in Table 6 provide quantitative but informal evidence on the drivers of NXA dynamics. That is to say, the estimates do not provide a formal econometric test of the link between NXA positions and agents exceptions described by equation (23), i.e.

$$NXA_{n,t} = \tfrac{1}{2}\mathcal{PV}(\Delta m_{n,t} - \Delta x_{n,t}) + TD_{n,t}\mathcal{PV}(\Delta \tau_{n,t} - g) + TD_{n,t}\mathcal{PV}(\kappa_t - \kappa).$$

Even if the sample dynamics of $\Delta x_{n,t}$, $\Delta m_{n,t}$, $\Delta \tau_{n,t}$ and κ_t are accurately represented by the estimated VARs, the forecasts from the VARs only represent the best forecasts using linear combinations of current variables in Φ_t. It is possible that forecasts based on both linear and nonlinear combination of the variables in Φ_t have a lower mean squared forecast error. Under these circumstances the estimated present value terms used in (36) would be inaccurate thereby affecting the estimated α_i coefficients and the R^2 statistics. In sum, therefore, any formal econometric evaluation of the link between NXA positions and agents exceptions requires testing a joint hypothesis concerning the restrictions in (23) and the identification of agents' conditional expectations.

6   Global Imbalances in the Great Recession

The results in Table 6 provide information on the sources of NXA dynamics over the past four decades. In this section I focus on the factors that have driven recent changes in external positions. In particular, I examine how conditions before, during and after the 2008 financial crisis affected countries' NXA positions via their impact on expectations future trade flows and the SDF.

Figure 10: Net Foreign Assets and Net Exports

A: Net Foreign Assets (% of Trade)
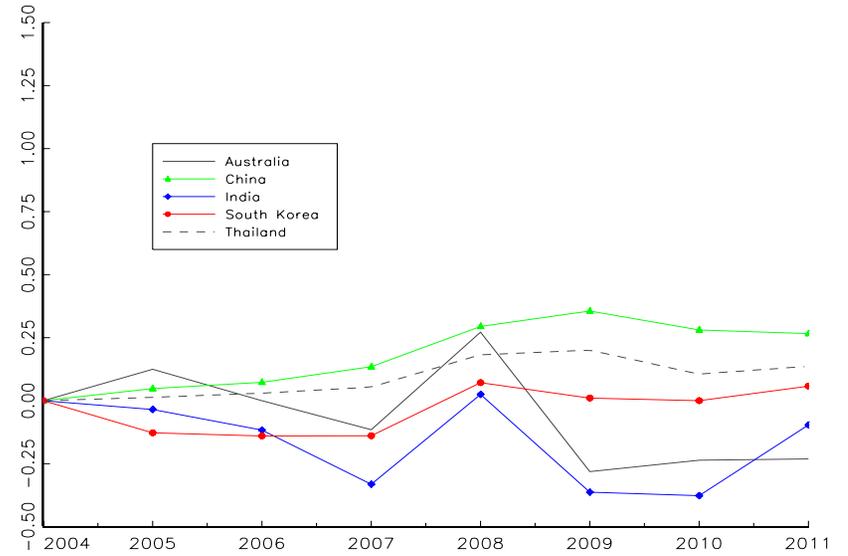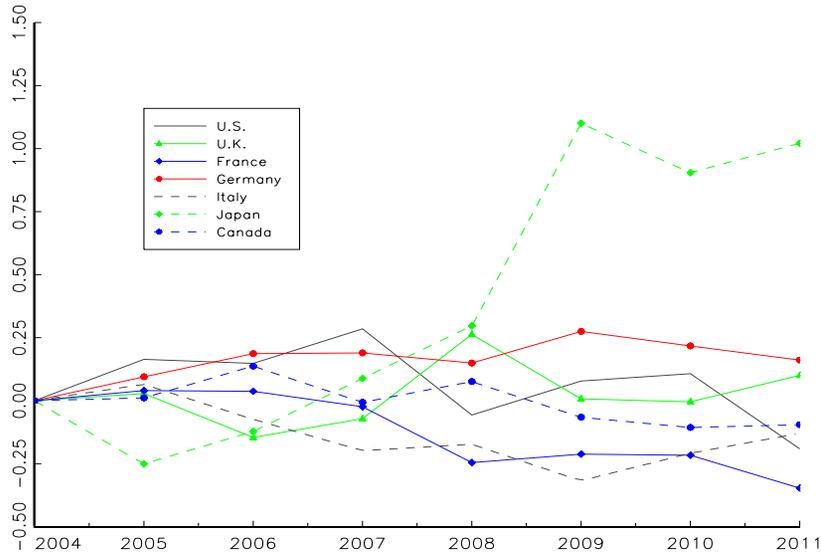

B: Trade Deficits (% of Trade)
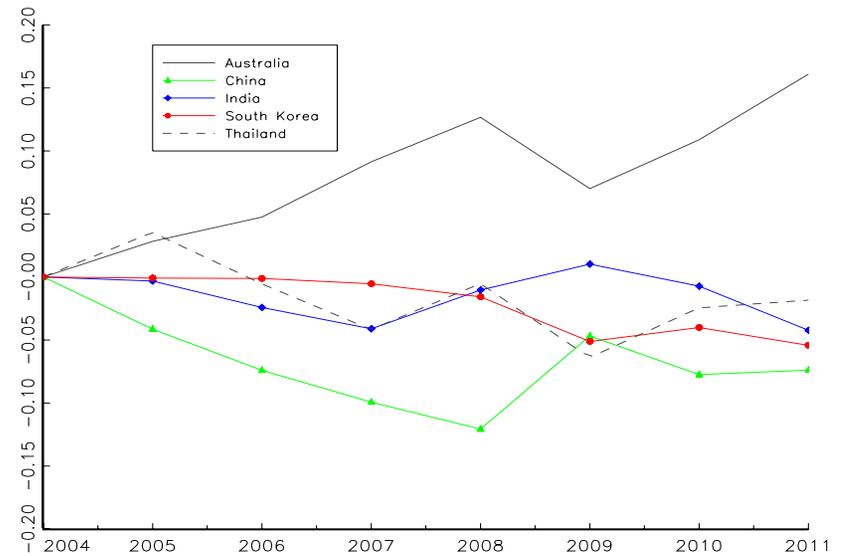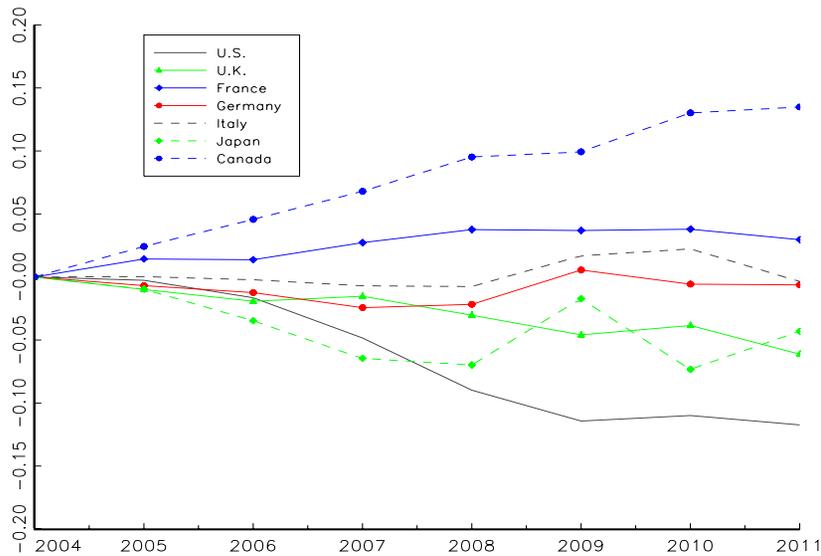
Figure 10 plots the evolution of the NFA positions and trade deficits (i.e., $NFA_{n,t}/(X_{n,t}+M_{n,t})$ and $TD_{n,t}$) between 2004 and 2011 relative to their 2004 values. Recall from Figure 7 that the estimated present value for the future log SDF, $\widehat{\mathcal{PV}}(\kappa_t - \kappa)$, was close to zero in 2004. This year is thus a natural starting point for assessing how changing financial conditions surrounding the 2008 crisis and the ensuing Great Recession affected external positions. I plot positions relative to their 2004 values (rather than their absolute values) to emphasize the cross-county differences in how the NFA and net export positions evolve.

Figure 10 reveals two interesting features. First, despite its severity, the Great Recession did not have a uniform and significant affect on either the NFA positions or trade deficits. Some countries, most notably Australia, India and the United Kingdom, experienced a marked improvement and then deterioration in their NFA positions between 2007 and 2009. By contrast, Japan experienced a substantial improvement in its NFA position (continuing a twenty-year trend), while changes in the NFA positions of many others were unexceptional. The second feature appears in the plots of the trade deficits shown in Panel B. Here we see that the crisis didn't disrupt the divergent paths for trade balances that existed between 2004 and 2007 for most countries. For example, the trade balances of Canada and Australia continued to deteriorate, while those of the United States and South Korea improved. However, the crisis does appear to have curtailed the rapid increase in China's trade surplus. These features contrast with the sizable effects of the crisis on trade and positions growth. As Figure 3 showed, trade growth fell from roughly 8 to 4 percent after 2007, while position growth fell even more from 16 to 6 percent.

Next, I examine how changing expectations concerning future trade flows and the SDF affected the NXA positions between 2004 and 2011. For this purpose, I combine the present value terms containing expectations of trade flows on the right-hand-side of (23) to give

$$NXA_{n,t} = NXA_{n,t}^{\text{TR}} + NXA_{n,t}^{\text{VAL}} \tag{37}$$

where $NXA_{n,t}^{\text{TR}}$ and $NXA_{n,t}^{\text{VAL}}$ are the trade and valuation components of the NXA position identified by

$$NXA_{n,t}^{\text{TR}} = \tfrac{1}{2}\mathcal{PV}(\Delta m_{n,t} - \Delta x_{n,t}) + TD_{n,t}\mathcal{PV}(\Delta \tau_{n,t} - g) \qquad \text{and}$$

$$NXA_{n,t}^{\text{VAL}} = TD_{n,t}\mathcal{PV}(\kappa_t - \kappa).$$

Figure 11 plots estimates of these components for each of the countries in the dataset between 2004 and 2011. The trade component for each county use forecasts for $\Delta m_{n,t}$, $\Delta x_{n,t}$ and $\Delta \tau_{n,t}$ computed from country-specific VARs while the valuation component uses forecasts for the future log SDF computed from a single VAR. As in Figure 10, I plot the estimated trade components relative to their 2004 values.

The upper panel of Figure 11 reveals sizable cross-country differences in the evolution of the trade components, $NXA_{n,t}^{\text{TR}}$. In principle variations in $NXA_{n,t}^{\text{TR}}$ can reflect revisions in expectations concerning the relative growth rates for imports and exports and/or revisions in expectations concerning trade growth. Among the G7 countries, revisions in expectations concerning future trade growth produce the largest changes in the U.S. trade component. Between 2004 and 2007 expecta-

Figure 11: Trade and Valuation Components 2004-2011

A: Trade Components $\widehat{NXA}_{n,t}^{\mathrm{TR}}$
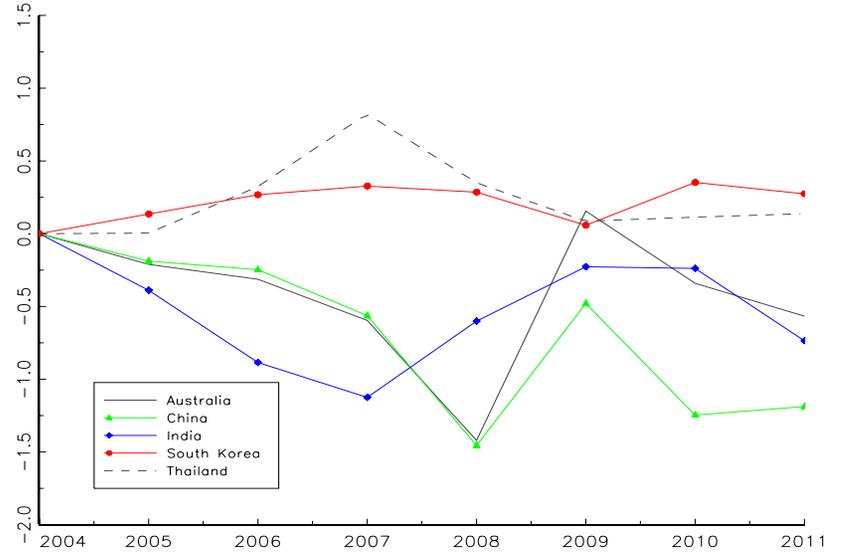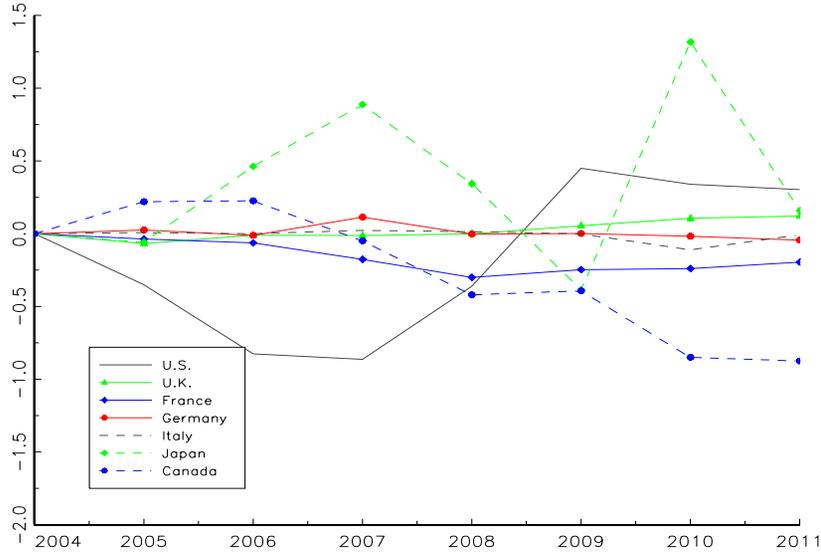
B: Valuation Components $\widehat{NXA}_{n,t}^{\mathrm{VAL}}$
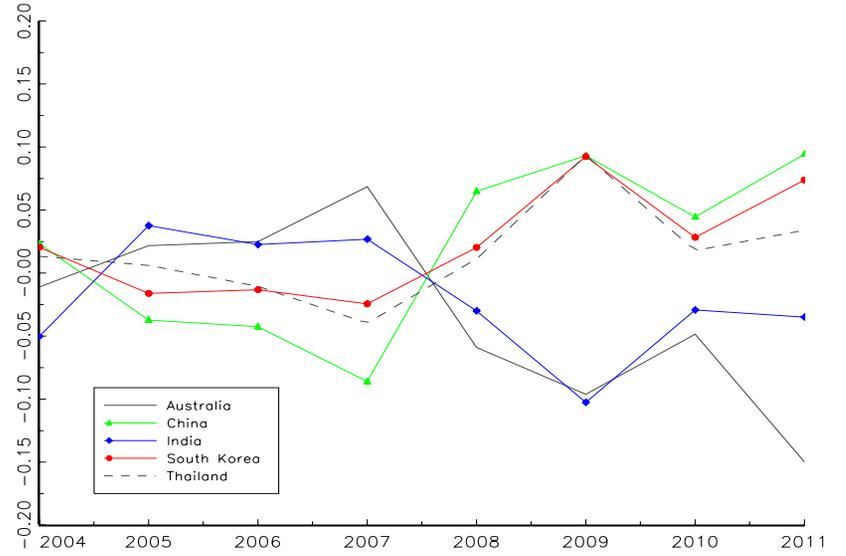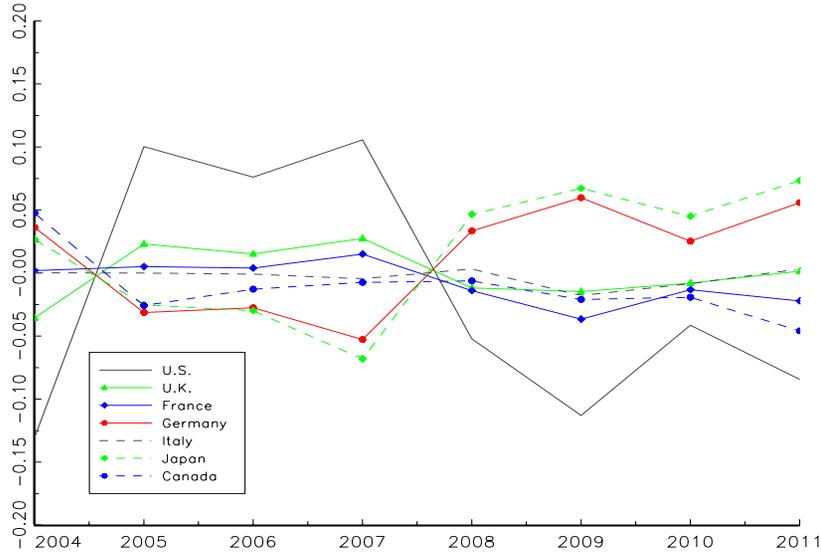
tions of higher than average future trade growth (i.e. $\mathbb{E}[\Delta \tau_{t+i}|\Phi_t] > g$ for $i > 0$) lowered the present value of future U.S. trade deficits, whereas expectations of lower than average future trade growth after 2007 pushed the present value back above its 2004 level. These are sizable variations. In the absence of other factors, the rise in the trade component between 2007 and 2009 would have brought the United States close to external balance. There are also sizable variations in the Japanese and Canadian trade components. Because Japan ran persistent trade surpluses during the period, expectations of higher than average trade growth produce a rise and then fall in the present value of future trade deficits that almost mirror the movements in the United States between 2004 and 2009. In the Canadian case the trade component falls steadily from 2006 until 2010 because expected future trade growth is below normal as the current trade balance moved further into deficit. Variations in the trade components are even more variable outside the G7. Changing expectations concerning trade growth produce particularly large swings in Australia, China, India and Thailand. For example, expectations of higher trade growth coupled with large trade surpluses in China produced a fall in the trade component between 2004 and 2008, that was followed by a temporary bump reflecting a lowering of expectations for trade growth in the aftermath of the crisis.

The lower panel if Figure 11 plots the valuation components, $NXA_{n,t}^{\text{VAL}}$. Recall from Figure 7 that the estimates of $\mathcal{PV}(\kappa_t - \kappa)$ turned positive between 2004 and 2005, rose to a peak of 0.5 in 2007 and then fell to -1.2 by 2009. These swings appear largely driven by changes in expectations concerning the future risk because the present value of future returns on U.S. T-bills remain comparatively stable. The changes in $\mathcal{PV}(\kappa_t - \kappa)$ had the largest impact on the United States NXA position among the G7. As perceptions of future risk declined below normal between 2005 and 2006 future U.S. trade deficits were discounted less heavily so the expected present value of the deficits rose reflecting a smaller weight on the current deficit. This patten was reversed between 2007 and 2009. As perceptions of future risk rose, future U.S. deficits were discounted more heavily and the present value fell. Among the other G7 countries, the German and Japanese value components show the greatest movement. Since these countries ran persistent trade surpluses during the period, changing perceptions of risk produced variations in the value components that mirror the United States. Outside the G7, the valuation patterns follow either the United States in the case of trade deficit countries (India and Australia), or Germany and Japan in the case of the surplus countries (China, South Korea and Thailand).

Figure 11 also provides a useful perspective on the relative importance of changing expectations concerning trade and the SDF as drivers of NXA positions. Comparing the plots for $NXA_{n,t}^{\text{TR}}$ and $NXA_{n,t}^{\text{VAL}}$ on a country-by-country basis, it is clear that revisions in trade expectations play the dominant role. For example, the rise in the U.S trade component between 2007 and 2009 is approximately six times the size of the fall in the U.S. valuation component even though this period witnessed the largest financial crisis since the Great Depression. This is not to say that variations in the SDF are unimportant. As is clear from the plots in panel B, at times expectations concerning the future SDF can contribute significantly to the cross-country distribution of NXA positions.

Figure 12 compares the evolution of each country's NXA position with the path of the estimated trade component, $\widehat{NXA}_{n,t}^{\text{TRD}}$, and the sum of the trade and value components, $\widehat{NXA}_{n,t} = \widehat{NXA}_{n,t}^{\text{TRD}} + \widehat{NXA}_{n,t}^{\text{VAL}}$, between 2004 and 2011. Several features stand out from these plots. First, variations in the estimated trade components account for most of variations in $\widehat{NXA}_{n,t}$. Changing expectations concerning the SDF are just not as economically important as those concerning future trade flows.

Figure 12: $NXA_{n,t}$ **and its components**
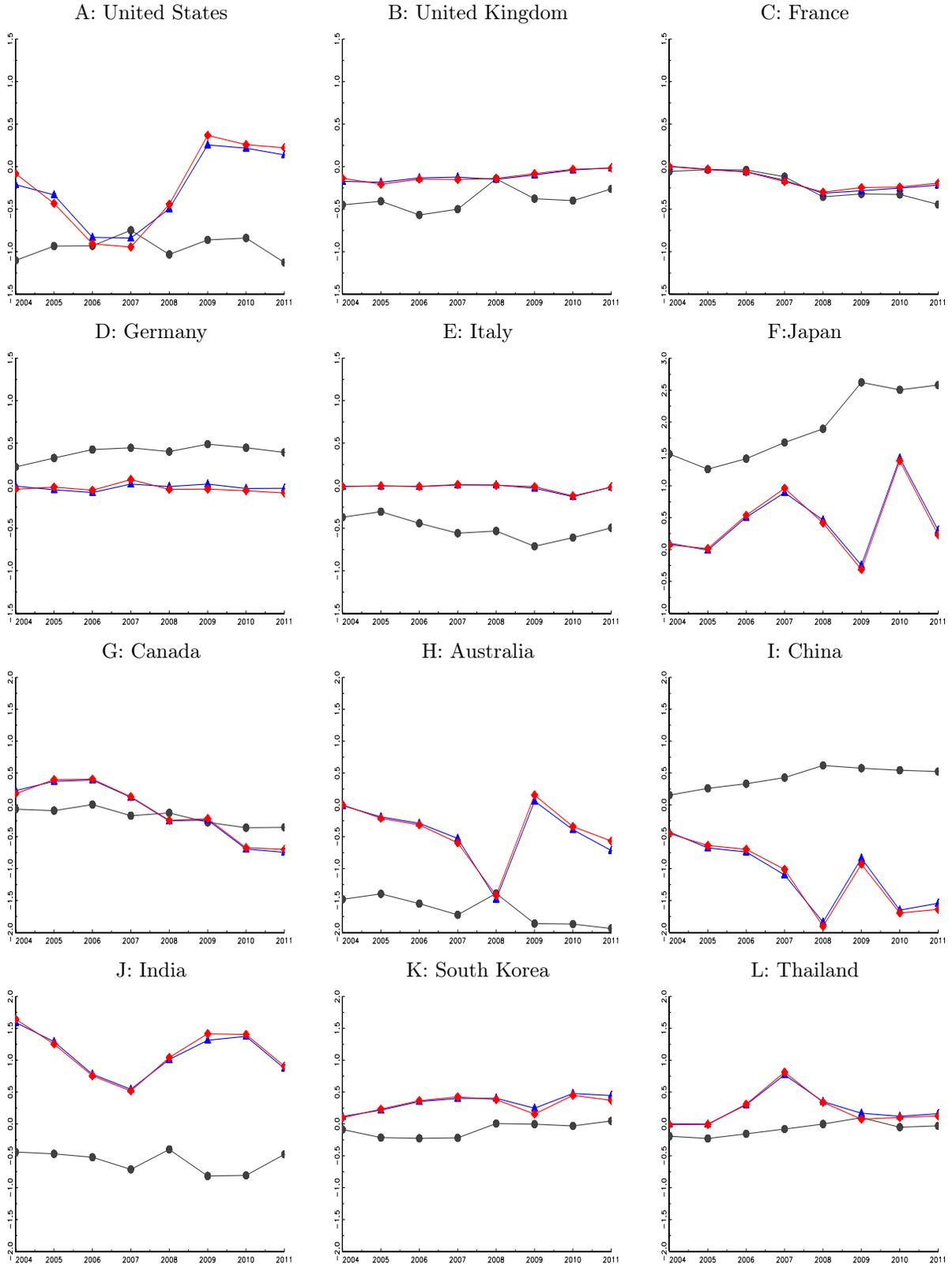
Notes: Each panel plots $NXA_{n,t}$ (black with bullets), $\widehat{NXA}_{n,t}^{\text{TRD}}$ (blue with triangles) and $\widehat{NXA}_{n,t}^{\text{TRD}} + \widehat{NXA}_{n,t}^{\text{VAL}}$ (red with diamonds).

Second, variations in actual NXA positions between 2007 and 2009 are generally unexceptional compared to the changes during other periods. In contrast, the onset of the Great Recession does coincide with significant movements in the estimated trade components for several countries (e.g. in the United States, Japan, Australia, China, India and Thailand). More generally, there are clearly significant discrepancies between the actual NXA positions and the estimated components for many countries. For example, the gaps between $NXA_{n,t}$ and $\widehat{NXA}_{n,t}$ in the United States, Japan, Australia, China and India represent more than 100 percent of their respective annual trade flows in 2011.

What should we make of these discrepancies? It is important to recognize that each $\widehat{NXA}_{n,t}$ series is constructed from trade and SDF forecasts that are consistent with time series evidence. In particular, the estimates of $\widehat{NXA}_{n,t}$ are based on the assumption that imports and exports grow at the state state rate of $g = 0.065$. This is the average growth rate for the countries in the dataset, but may be a poor approximation to the long run growth rates expected for some countries that have experienced more rapid growth in the past (e.g., China, India, South Korea and Thailand). If this is the case, then the estimated trade component will systematically under- or over-estimate the present value of future trade deficits based on agents' expectations. Differences between g and agents' long-horizon expectations for trade growth probably account for some of the persistent gaps between $NXA_{n,t}$ and $\widehat{NXA}_{n,t}$ in countries such as Germany and Italy. Accounting for the variable gaps is more challenging. Undoubtedly, the VAR-based estimates of expected future trade flows do not give perfectly precise values for how the trade component changed from year to year, so some of the discrepancy between $NXA_{n,t}$ and $\widehat{NXA}_{n,t}$ represents estimation (sampling) error. Moreover, it is possible that at times rational agents' conditional expectations for future trade flows changed in a way that was inconsistent with the VAR forecasts (because the latter are constructed from linear combinations of variables in agents' conditioning information sets). That said, it is hard to understand why in some counties $NXA_{n,t}$ and $\widehat{NXA}_{n,t}$ move significantly in opposite directions.

Discrepancies between $NXA_{n,t}$ and $\widehat{NXA}_{n,t}$ could also arise from the no-arbitrage condition. Recall that the estimates of $\widehat{NXA}_{n,t}$ are based on the assumption that the return on each country's foreign asset and liability portfolio satisfies the no-arbitrage condition. This is a conventional assumption with respect to the returns on securities that are freely traded in liquid markets. As such, applying the assumption to the returns on G7 foreign liabilities is uncontroversial. It also seems reasonable to apply the assumption to returns on G7 asset portfolios insofar as they are mainly comprised of freely traded securities. The assumption may be less tenable for the liabilities of countries that are not freely traded. *If* expected future returns on these liabilities systematically differ from return implied by the world SDF (i.e. the SDF implicitly identified by the no-arbitrage conditions applying to all freely trading securities), the country's net foreign asset position will differ from the discounted present value of future trade deficits (i.e. the right-hand-side of equation (11)) generating a gap between $NXA_{n,t}$ and $\widehat{NXA}_{n,t}$. Notice the mere existence of trading frictions is not sufficient to produce such gaps. The frictions need to be "large enough" to affect agents' expectations of future returns.

In the light of this discussion it is clearly inappropriate to interpret the differences between $NXA_{n,t}$ and $\widehat{NXA}_{n,t}$ as evidence that a country's external position is on an unsustainable path. As I noted in Section 4, a country's position should only be viewed as unsustainable when the value for $NXA_{n,t}$ falls outside the confidence band defined by the range of economically plausible

expectations agents could hold regarding future trade flows and the SDF. At best, the estimate of $\widehat{NXA}_{n,t}$ represents a point within this band so there is simply insufficient information in Figure 12 to make any meaningful judgement concerning sustainability.

7   Conclusion

In this paper I have proposed an analytical framework for the quantitative assessment of international external positions. The framework relies of two assumptions: the absence of Ponzi schemes and arbitrage opportunities in financial markets. These assumptions would appear in any modern macroeconomic model and are necessary to think consistently about the determinants of external positions across countries and through time. Importantly, the framework accommodates the secular increase in trade flows and asset positions that have taken place of the past 30 years, making it applicable to empirical analysis. It also produces a simple taxonomy of the factors determining both the cross-country distribution of the external positions each year, and those driving the dynamics of the positions of individual countries through time.

I used the framework to study the external positions of 12 countries (Australia, Canada, China, France, Germany, India, Italy, Japan, South Korea, Thailand, The United States and The United Kingdom) between 1970 and 2011. In particular, I explored its implications for the cross-country distribution of external positions at a point in time, and its implications for the dynamics of individual country's external positions. I also considered how economic conditions surrounding the 2008 financial crisis affected external positions via the trade and valuation channels. This analysis revealed that intertemporal effects (i.e. revisions in expectations concerning future trade flows and the SDF) are the dominate drivers of net foreign asset positions across countries and through time. Moreover, revisions in expected future trade flows appear to accounted for the lion's share of the time series variation in the external positions of many countries.

Perhaps the most novel aspect of my analysis concerns the role of financial risk. Systematic risk affects external positions via the world SDF that is used in the determination of a country's NFA position as the present value of future trade deficits. My estimates show that variations in the expected future path for the SDF produce significant cyclical variations in this present value even when the prospects for future deficits remain unchanged. These variations appear unrelated to changing forecasts for future U.S. real interest rates. Instead, they reflect revisions in the expected future volatility of the world SDF, a measure of systematic risk. I estimate that perceived risk increased significantly around the 2008 crisis, and that this affected the external positions of countries with large trade imbalances via the valuation channel. Nevertheless, external adjustment around the 2008 crisis and its aftermath appears to have mainly taken place via the trade channel.

Finally, one aspect of this work deserves special emphasis. The empirical analysis I present here combines the analytical framework with forecasts for future trade flows and the world SDF derived from VARs. These forecasts have the virtue of being consistent with the time series behavior of variables in the sample period, but they represent only one of many possible ways to estimate agents' conditional expectations. The real power of the proposed framework is that it can be combined with forecasts that cover the range of economically plausible expectations agents could hold to provide a robust assessment of any country's external position.

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