

# Forex Trading and the WMR Fix

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## Abstract

I examine the behavior of forex prices around the setting of the 4:00 pm WMR Fix. Numerous banks have been fined by regulators for their trading activities around the Fix, but the overall impact of their actions is not known. I first examine trading patterns around the Fix in a microstructure model of competitive trading. I then compare the model with the empirical behavior of forex prices across 21 currencies over a decade. My analysis reveals that forex price changes display extraordinary volatility and serial correlation around the Fix – features more consistent with collusive manipulation than competitive trading.

Keywords: Forex Trading, Order Flows, Forex Price Fixes, Microstructure Trading Models. JEL Codes: F3; F4; G1.

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## Introduction

Since 2013, law enforcement and regulatory authorities around the world have been investigating the forex trading activities of the world’s largest banks, particularly around the time that benchmark forex prices are determined. To date, these investigations have generated penalties and fines on the banks totaling more than \$5.6 billion and have led to the dismissal or suspension of numerous bank employees involved in forex trading.<sup>1</sup> The most widely used benchmarks are provided by the WM Company and Reuters, that were based on forex transactions during a one minute window around 4:00 pm (London time). These benchmarks are colloquially known as the “London 4 pm Fix”, “the WMR Fix” or just the “Fix” . They provide standardize forex prices that are used to value global equity and bond portfolios, to hedge currency exposure, to write and execute derivatives contracts, and administer custodial agreements.

This paper provides a detailed analysis of forex prices and trading around the 4:00 pm WRM Fix (hereafter, the “Fix”) and examines how the misconduct identified by the regulators affected the operation of the world-wide forex market. To this end, I first describe the unique institutional features of the Fix and its importance. In particular, I explain why some market participants have strong incentives to execute forex trades at the Fix benchmark via the submission of orders to dealer banks well before 4:00 pm.<sup>2</sup> These so-called Fix orders were the focus of regulators’ investigations and play a prominent role in my analysis. Next, I examine the behavior of prices and the trading patterns in a microstructure model of competitive forex trading. The model incorporates the key institutional features of the Fix and makes strong predictions concerning prices and trading patterns. I then use these predictions as benchmarks in the empirical analysis the covers a decade of trading data on 21 currency pairs. Here I examine how the behavior of forex prices around the Fix differs from their behavior under normal trading conditions that prevail at other times, and I compare their behavior with the predictions of the microstructure model.

My main findings are summarized as follows:

1. In the model’s equilibrium, Fix orders produce volatility in post-Fix price changes because they drive trades between dealers when the Fix orders are filled. By contrast, Fix orders do not drive dealers’ trades before the Fix. As a consequence, they have no price-effects, and do not contribute to any correlation between pre- and post-Fix price changes.
2. The observed behavior of forex prices around the Fix is highly atypical and inconsistent with

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<sup>1</sup>Details of these investigations are provided in an on-line appendix.

<sup>2</sup>Hereafter, all times are local London times unless otherwise indicated.

the predictions of the competitive trading model:

- (a) The volatility in spot rates observed immediately before the Fix is highly unusual – rates regularly jump by an amount that is very rarely seen under normal trading conditions. The incidence of these atypically large pre-Fix rate changes is particularly high at the end of each month. They appear to be pervasive across all currency pairs and throughout the decade covered by the sample.
- (b) The empirical correlation between pre- and post-Fix price changes is significantly negative for many currencies – particularly on the last trading day of each month, and are large enough to support economically attractive trading strategies.

The theoretical results in point 1 originate from the assumption that dealer-banks attempt to share risks efficiently when they quote forex prices. This is a key assumption in earlier multi-dealer models of forex trading (see, e.g., Lyons, 1997; Evans and Lyons, 2002 and Evans, 2011). It also provides the theoretical foundation for the fact dealers generally do not hold open forex positions overnight and the half-lives of the intraday positions are measured in minutes (see, e.g., Lyons, 1995 and Bjønnes and Rime, 2005). Risk-sharing plays a central role in how dealers determine the forex prices they quote before (and at the start of) the Fix window because they want to minimize the risk associated with filling a large aggregate imbalance in Fix orders to purchase and sell forex. Importantly, this imbalance is determined endogenously from the Fix orders submitted by investors who are price-sensitive, and “hedgers” who are not. In an efficient risk-sharing equilibrium, dealers quote prices so that the expected imbalances in Fix orders from investors and hedgers offset each other.

Dealers need to fill their Fix orders once the Fix benchmark is established; a task that necessitates trade between dealers. The order flow generated by this inter-dealer trade reveals the actual imbalance in Fix orders, which dealers then embed in their post-Fix price quotes (to again share risk efficiently). It is through this trade-based information process that the (unexpected) imbalance in Fix orders affects the post-Fix change in prices. In contrast, inter-dealer trading before the Fix reveals nothing about the aggregate imbalance in Fix orders because individual dealers have no incentive to trade based on the individual orders they have received. Consequently, information about the actual aggregate imbalance in Fix orders remains dispersed across dealers before the Fix is determined; and, as such, it has no impact on the prices dealers quote. This implication of the model counters the idea that dealers should “trade ahead of” or “font run” their fix orders (Levine, 2014).

The empirical results listed in point 2 are equally striking. Individual instances of “large” forex price movements before the Fix have been noted previously (see, e.g., Vaughan and Finch 2013 and Melvin and Prins, 2015). Here I take a more systematic approach that quantifies the degree to which volatility before the Fix exceeds volatility at other times. And, as a result, I show that the atypical pre-Fix volatility has been much more widespread across time and currencies than has been documented hitherto. It appears in all 21 currency pairs and every year covered by my data. Moreover, pre-Fix volatility is particularly high on the last trading day of the month when Fix orders from hedgers are known to be largest (Melvin and Prins, 2015). It appears likely that hedgers’ orders affect pre-Fix price changes, contrary to the predictions of the competitive trading model.

My empirical results are also at odds with the model concerning the correlation between pre- and post-Fix price changes. Dealers’ quotes in the model embed an intraday risk premium that generates a (small) positive serial correlation in price changes around the Fix. Furthermore, Fix orders only contribute to the volatility of post-Fix price changes, they do produce serial correlation. In contrast, my empirical analysis reveals a significant negative serial correlation across 18 currencies. These correlations are economically important enough to support attractive trading strategies for 15 of the currency pairs (inclusive of transactions costs).

The most plausible explanation for the differences between my empirical findings and the predictions of the trading model comes from regulators’ investigations. According to the U.K. Financial Conduct Authority and the U.S. Department of Justice, dealers shared information on their Fix orders in order to collusively trade before the Fix in a manner that would manipulate the benchmark to their advantage. If sufficiently widespread, these activities could have produced the atypical volatility in pre-Fix price changes and the pattern of serial correlation seen in the data.

My analysis connects with three strands of the literature. The first concerns the manipulation of securities prices; originating with Hart (1977), Vila (1989), Allen and Gale (1992), among others. Much of this literature’s focus is on the manipulation of equity prices, with the Vitale (2000) model of forex manipulation a notable exception. There are several important differences between equities and forex that limit the applicability of existing models to studying manipulation of the Fix. For example, manipulation via corners and squeezes is impractical for major currencies, while pump-and-dump schemes requiring the release credible but false information that move forex prices are implausible.<sup>3</sup> Similarly, the literature on closing equity price manipulation (see, e.g., Cushing and

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<sup>3</sup>The term “currency manipulation” is sometimes used to describe the actions of governments that affect forex prices. For example, Gagnon et al. (2012) defines currency manipulation as occurring “when a government buys or sells foreign currency to push the exchange rate of its currency away from its equilibrium value or to prevent the

Madhavan, 2000; and Hillion and Suominen, 2004, Comerton-Forde and Putniņš (2011)) applies in settings where trading (largely) stops, whereas forex trading takes place continuously. Importantly, I document that forex trading between 4:00 and 5:00 pm is comparable in terms of volume and liquidity to trading in the hours before the Fix. It is therefore quite inaccurate to characterize the Fix as a “closing forex price”. The relevance of LIBOR manipulation (Abrantes-Metz et al., 2012 and Eisl, Jankowitsch, and Subrahmanyam, 2014) to the Fix is also limited. LIBOR is based on banks’ reports of borrowing costs, whereas the Fix is determined by the forex prices for actual trades.<sup>4</sup>

This paper also connects to the literature on forex microstructure. The trading model I present extends the Portfolio Shifts (PS) model developed in Lyons (1997), Evans and Lyons (2002) and Evans (2011) to include a round of trading where the Fix benchmark price is determined. The model allows dealers to engage in inter-dealer trade after they have received Fix orders from hedgers and investors but before the Fix is determined. This feature enables us to study trading patterns and price dynamics before the Fix. As King, Osler, and Rime (2013) note in their recent survey, the PS model “has become the intellectual workhorse of the (forex) microstructure field”, so it is natural to extend its structure to accommodate a theoretical examination of the Fix. My theoretical analysis is also linked to the literature on the optimal execution of large trades (Bertsimas and Lo, 1998 and Almgren, 2012). As Saakvitne (2016) notes, a dealer with a large Fix order faces a similar optimal execution problem. Whereas many models in the optimal execution literature take the price-impact of trades as exogenous, this key feature is determined endogenously in the equilibrium of the PS model. Empirically, my results extend earlier microstructure findings on the intraday volatility of forex prices (see. e.g., Bollerslev and Melvin, 1994 and Ito, Lyons, and Melvin, 1998).<sup>5</sup>

The third strand of the literature explicitly focuses on the WMR Fix. Here the work of Melvin and Prins (2015) is most relevant. They describe how currency hedging by international equity portfolio managers generates a flow of Fix orders, and estimate a simple model for this flow at the end of each month. Melvin and Prins (2015) then present regressions showing that intraday returns are positively related to their estimated flows before the Fix, and negative related after the Fix (for some currencies). My analysis here compliments Melvin and Prins (2015) in two respects: First, the theoretical model includes Fix orders by “hedgers”, such as portfolio managers, who have a

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exchange rate from moving toward its equilibrium value”. The focus of this paper is on manipulation by private sector agents for profit.

<sup>4</sup>Similar to LIBOR, Japanese banks individually announce their benchmark forex prices at 10:00 am in Japan. These benchmark prices are called the Tokyo Fix. Unlike the WMR Fix, there are no formal rules governing how the banks choose these prices, see Ito and Yamada (2015).

<sup>5</sup>The patterns of high volatility and serial correlation I document are also similar to those found in equity prices at the market’s close (Cushing and Madhavan, 2000).

strong incentive to trade at the Fix benchmark price. It then shows how these orders interact with other Fix orders to affect forex prices in a fully optimizing model.<sup>6</sup> Second, my empirical analysis covers a wider range of currency pairs over a longer time period than has been undertaken hitherto, including the examination of intra-month and end-of-month data.<sup>7</sup> Finally, my empirical findings concerning the volatility and serial correlation of price changes around the WRM Fix have been confirmed in subsequent work by Ito and Yamada (2015).

The remainder of the paper is structured as follows: Section 1 describes the institutional details of the Fix and provides aggregate statistics on the importance of trading around 4:00 pm. Section 2 presents the microstructure model and examines its implications. The empirical analysis is contained in Section 3. Section 4 provides economic perspectives on the empirical results. Section 5 concludes. Mathematical details of the model and additional statistical results are contained in an on-line appendix.

## 1 Background

The WMR Fixes were established as a financial benchmark in 1993. Morgan Stanley Capital International (MSCI) announced that after December 31st. 1993 it would use the benchmark forex prices compiled at 4:00 pm by the WM Company and Reuters to value the foreign security positions in its MSCI equity indices.<sup>8</sup> Since then, the Fix benchmarks have become the de facto standard for construction of indices comprising international securities, and have been incorporated into numerous other tracking indices and derivatives.<sup>9</sup> They are also routinely used to compute the returns on portfolios that contain foreign-currency denominated securities as well as the value of foreign securities held in custodial accounts. Fixes are now computed every half-hour for 21 currency pairs and hourly for 160 currency pairs, but the 4:00 pm Fix remains the most prominent forex benchmark.

My empirical analysis covers forex trading from the start of 2004 until the end of 2013. This decade includes the period investigated by law enforcement and regulatory authorities. At the time, the WMR Fix benchmark was computed from the medians of the bid, offer and transaction

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<sup>6</sup>In contrast, Melvin and Prins (2015) assume that hedgers' forex orders affect forex prices via an exogenously given price-impact coefficient.

<sup>7</sup>Ideally, we would like to compare the theoretical model with bank trading data, that included information on their Fix orders. Unfortunately, this data is simply unavailable to researchers.

<sup>8</sup>Initially, the Fix benchmarks were used to compute the MSCI indices for all but the Latin American countries. After 2000, they were used for all the country indices.

<sup>9</sup>See, for example, Dow Jones Islamic Market, Global Real Estate (FTSE EPRA/NAREIT) and Global Coal (NASDAQ OMX) indices, the USD volatility warrants issued by Goldman Sacks; Wiener Borse AG financial futures and CME spot, forward and swaps.

rates sampled every second from the electronic trading platforms run by Reuters and Electronic Broking Services (EBS) over a one minute window starting 30 seconds before 4:00 pm.<sup>10</sup> While these platforms were the main venues for trades between dealer-banks, market participants could also trade on a variety of other platforms. Thus, the Fix benchmark was determined by subset of trade rates around 4:00 pm. The methodology also took no account of trading volume.

The economic importance of the Fix arises from its use in the valuation of other securities (e.g., equity portfolios and derivatives) because this creates strong incentives to trade in and around the Fix window. These incentives originate with two groups of market participants. The first comprises those wishing to hedge currency risk. As Melvin and Prins (2015) stress, fund managers with cross-boarder equity investments are important members of this group. Because the performance of their investments is often tracked against the returns on the MSCI indices, many managers will want to reduce the tracking error of their own portfolios by choosing to hedge some of their forex exposure to the Fix. In principle this hedging could take place continuously through the adjustment of forex forward positions, but in practice managers typically adjust their hedge positions at the end of each month. This hedging activity produces orders to purchase or sell forex. And, since the managers are concerned with tracking the MSCI indices, they want their forex orders to be filled at the Fix to minimize the tracking error in their own portfolio's performance. The use of Fixes in derivative contracts produces a similar incentive to submit forex orders to be filled at the Fix from others wishing to hedge their derivative positions. Thus, the use of Fixes in real-time valuation produces a hedging incentive for the submission of Fix orders to dealer-banks (particularly at the end of each month). By market convention, these orders must be submitted to dealer-banks before the 3:45 pm.

The second group of market participants affected by the Fix is the dealer-banks that accept Fix orders. These orders differ from standard currency orders because the dealer-banks agree to fill them at the Fix rate at least 15 minutes before that rate is determined. Thus, in effect, the dealer-banks are offering a guarantee that the order will be filled at a particular point in time whatever the prevailing Fix rate might be.<sup>11</sup> By contrast, in accepting a standard forex order, the dealer-bank undertakes to fill the order immediately at the best available prevailing rate.<sup>12</sup> Of course, such guarantees represent a source of risk to the dealer-bank. It is the desire to manage this risk that creates incentives for dealer-banks to trade in and around the Fix.

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<sup>10</sup>After the disclosure that regulators were investigating forex trading, the WM Company announced a change in its methodology in October 2014. The appendix contains a complete description of the old WMR methodology.

<sup>11</sup>While these are not legally binding guarantees, it is very rare for Fix orders not to be filled at the benchmark.

<sup>12</sup>Dealer-bank could also accept a limit order where price-contingency replaces the immediacy feature of the forex order.

Forex trading is heavily concentrated around the Fix. Panel A of Table 1 reports the ratio of trading volume per minute at the WMR and ECB Fixes relative to the average volume per minute between 7:00 am and 5:00 pm (excluding the Fix window) for three major currency pairs. These statistics are computed from EBS trading data spanning three months in each of 2007, 2010 and 2013. They show that trading volume is on average far higher during the WMR Fix than at other times, particularly at the end of the month. For comparison purposes, the table also reports volume ratios for the 12:15 pm ECB Fix. While trading volumes for the EURUSD and GBPUSD are above normal, they are well below the 4:00 pm WMR Fix levels. Trading volumes are even lower for the other hourly WMR fixes. These statistics confirm that the 4:00 pm Fix is by far the most important in terms of trading activity, so it is the focus of my analysis below.

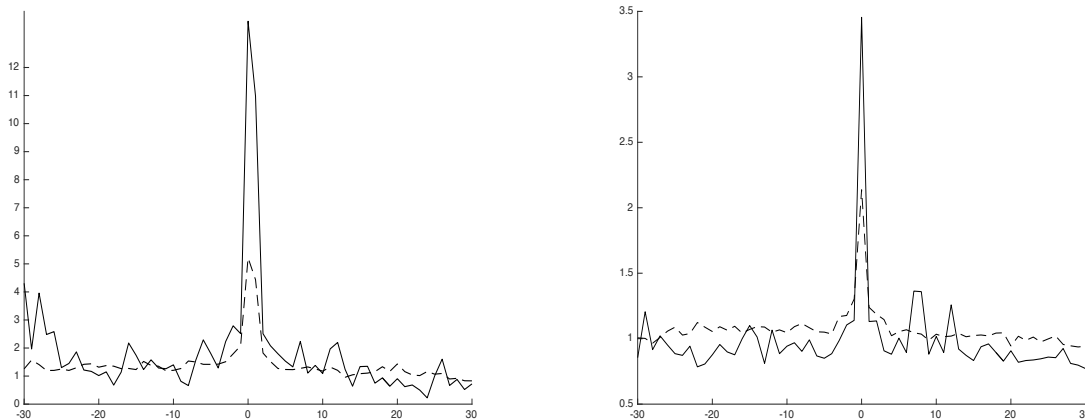
Table 1: Summary Trading Statistics

	EUR/USD		USD/GBP		JPY/USD	
	Intra	End	Intra	End	Intra	End
A: Fix Volume						
WMR	3.169	7.383	2.196	3.812	3.852	8.903
ECB	2.399	2.752	1.287	1.606	1.060	0.752
B: Post-WMR						
Volume	1.070	1.350	1.146	1.349	1.084	1.356
Spread	1.003	1.004	0.928	0.985	1.012	1.050
Depth	1.070	0.934	1.072	1.036	1.015	0.891

Notes: Panel A reports the ratio of the average trading volume per minute at the WMR and ECB Fixes relative to the average trading volume per minute between 7:00 am and 5:00 pm on intra-month and end-of-month trading days, under the columns headed Intra and End, respectively. Panel B reports analogous ratios for the trading volume, the spread between the best bid and offer prices, and the depth (total volume of outstanding limit orders) computed in the hour following the WMR Fix. All statistics are computed from EBS trading data in 2007, 2010 and 2013.



Figure 1: Volume and Depth around the WMR Fix



A: Average trading volume each minute from 30 minutes before to 30 minutes after the WMR Fix relative to average volume per minute between 7:00 am and 5:00: End of month trading days (solid); intra-month trading days (dashed).

B: Average depth in the EBS limit order book each minute from 30 minutes before to 30 minutes after the WMR Fix relative to average depth each minute between 7:00 am and 5:00 pm. End of month trading days (solid); intra-month trading days (dashed).

Panel B of Table 1 provides information on trading activity in the hour following the 4:00 pm Fix. Here I report the ratio of average trading volume, the average spread between the best EBS bid and offer rates, and the average depth of the EBS limit order book (measured by the volume of outstanding bid and offer limit orders) relative to their respective averages computed between 7:00 am and 5:00 pm (excluding the Fix window). All of these ratios are close to one. In terms of trading volume and liquidity, forex trading continues “as normal” for some time after the WMR Fix. This can also be seen in Figure 1, which plots the EUR/USD volume and depth ratios. These plots are similar to the plots for the other currency pairs. They provide clear evidence against the idea that the Fix occurs at (or close to) the end of active forex trading. Figure 1 also shows that both volume *and* depth rise sharply during the Fix window. The flow of limit orders from potential counterparties is more than sufficient to match the flow of market orders that produce the spike in volume during the fix window. The increase in depth is accompanied by narrowing spreads. The ratio of the average spread within the window to the average outside the window is 0.238, 0.413 and 0.419 for the EUR/USD, USD/GBP and JPY/USD, respectively. Ito and Yamada (2015) document similar patterns in their large sample of EBS data.<sup>13</sup>

<sup>13</sup>These trading patterns do not support the idea that forex price movements around the Fix reflect “price pressure”; the temporary absence of counterparties studied by Hendershott and Menkveld (2014). The behavior of forex spreads

In summary, there are four institutional facts about the WMR Fix that are important for the analysis that follows. First, the use of Fixes in real-time valuation produces a hedging incentive for the submission of Fix orders to dealer-banks. Second, Fix orders are quite different from the standard forex orders received by dealer-banks. Third, trading volumes around the WMR Fix are typically much higher than at other times (including other Fix times). Finally, The WMR Fix occurs *during* active trading hours for major currencies. Market participants wanting to trade well after the Fix face trading conditions (measured in terms of volumes, spreads and depth) that are similar to those found earlier in the day.

## 2 Model

This section studies the behavior of forex prices and trading patterns around the setting of a Fix benchmark in a microstructure model of competitive forex trading. For this purpose I extend the PS model to include a round of trading where a Fix benchmark is determined and used to fill previously submitted Fix orders. Otherwise, the structure of the model is identical to that in Evans (2011), so I focus on the models' predictions related to the Fix.

### 2.1 Overview

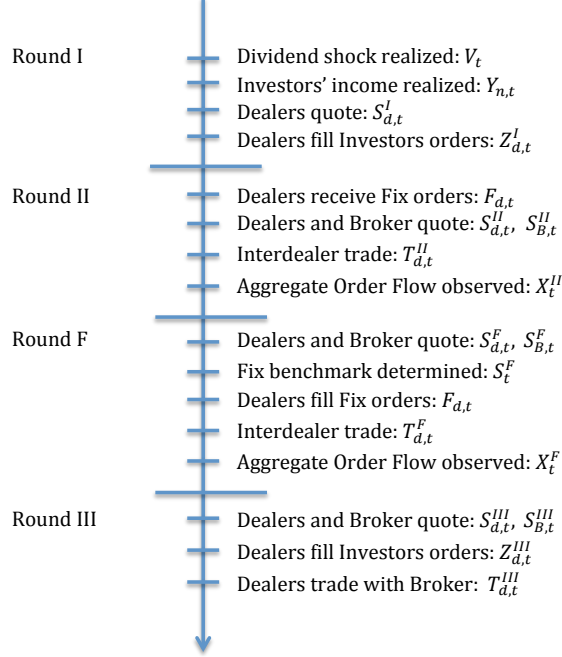
The model describes forex trading among a large number of dealers and a broker and between dealers and investors over a trading day that comprises four trading rounds; I, II, F and III, shown in Figure 1. Rounds I and III contain the same events as the original PS model. Here dealers quote forex prices at which they fill orders from investors, and investors place orders with individual dealers. Information about these trades is only known to the counter-parties.

The new elements of the model appear in the middle two trading rounds. At the start of round II dealers receive Fix orders from investors and “hedgers”, a group who have exogenous reasons for trading at the Fix. Fix orders are a source of private information to individual dealers. Round II then follows the PS model. The dealers and the broker quote prices for inter-dealer trades, and then the dealers trade with each other and the broker. All dealers observe aggregate inter-dealer order flow at the end of the round. Round F starts with dealers and the broker quoting prices for further inter-dealer trades. These quotes determine the Fix benchmark. Then dealers trade with each other and the broker, and fill the Fix orders they received at the start of round II. Aggregate inter-dealer order flow is again observed by all dealers at the end of the round.

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also stands in contrast with the finding that spreads tend to rise in the last minutes of trading before the close in equity markets; see, e.g., Hillion and Suominen (2004).

Figure 1: Daily Timing



This model incorporates three important features concerning the Fix. First, the Fix benchmark is established from transaction prices in round F, rather than at the end of the day in round III. So, consistent with the empirical evidence, the model allows for significant forex trading after the Fix benchmark is determined. Second, dealers have the opportunity to trade in round II knowing their own Fix orders, but before the benchmark is determined. Thus the model allows us to examine how dealers use private information on Fix orders to trade before and after the Fix benchmark is determined. Finally, by comparing equilibrium forex prices in round F with those in rounds II and III, we can examine how Fix orders contribute to both the volatility the serial correlation in pre- and post-Fix price changes.

## 2.2 Details

Consider a pure exchange economy with one risky asset representing forex and one risk-free asset with a daily return of  $1 + r$ . The economy is populated by a group of hedgers, a continuum of investors indexed by  $n \in [0, 1]$ ,  $D$  forex dealers indexed by  $d$  and a forex broker. Investors, dealers and the broker are risk-averse. All of their decisions in day  $t$  are derived optimally from maximizing

expected CARA utility defined over wealth on day  $t + 1$ , subject to their budget constraints and available information.

**Round I** At the start of round I on day  $t$ , everyone observes the dividend,  $D_t$ , paid to the current holders of forex that follows  $D_t = D_{t-1} + V_t$ , where  $V_t \sim i.i.d.N(0, \sigma_v^2)$ . Realizations of  $V_t$  represent the arrival of public macroeconomic information over time (e.g., changes in interest rates). At the same time each investor  $n$  receives forex income,  $Y_{n,t}$ . This is private information to each investor and motivates their forex orders. I assume that  $Y_{n,t} = Y_t + \xi_{n,t}$ , where  $Y_t \sim i.i.d.N(0, \sigma_y^2)$  and  $\xi_{n,t} \sim i.i.d.N(0, \sigma_\xi^2)$ . These terms represent the aggregate and idiosyncratic component of income, respectively. They are not directly observed by investors.

Next, each dealer simultaneously and independently quotes a scalar price at which they will fill investors' orders to buy or sell forex. The round-I price quoted by dealer  $d$  is  $S_{d,t}^I$ . Prices are observed by all dealers and investors and are good for orders of any size. Investors then place their orders. Orders may be placed with more than one dealer. If two or more dealers quote the same price, the customer order is randomly assigned among them. The customer orders received by dealer  $d$  are denoted by  $Z_{d,t}^I$ . Positive (negative) values of  $Z_{d,t}^I$  denote net customer purchases (sales) of forex and are only observed by dealer  $d$ .

**Round II** Round II begins with each dealer  $d$  receiving Fix orders  $F_{d,t}$  to be filled at the benchmark price determined in round F. Positive (negative) values of  $F_{d,t}$  denote net Fix purchases (sales) of forex, and are only observed by dealer  $d$ . I assume that Fix orders are randomly assigned across dealers so that  $F_{d,t} = \frac{1}{D} F_t + \xi_{d,t}$ , where  $\xi_{d,t}$  is a mean-zero random error with  $\sum_{d=1}^D \xi_{d,t} = 0$ .  $F_t$  represents the aggregate imbalance of Fix orders that comprises the orders from hedges and investors:

$$F_t = H_t + \int_n (A_{n,t}^F - A_{n,t}^I) dn. \quad (1)$$

Here  $H_t$  denotes the exogenous aggregate imbalance in Fix orders from hedgers. I assume that  $H_t \sim i.i.d.N(0, \sigma_H^2)$ . The second term identifies the imbalance in investors' Fix orders. It aggregates the difference between each investor's desired forex position in round F,  $A_{n,t}^F$ , and their position after round I,  $A_{n,t}^I$ . Each investor  $n$  chooses their round- $i$  positions  $A_{n,t}^i$  optimally conditioned on the contemporaneous information available to each of them,  $\Omega_{n,t}^i$ . Thus, the aggregate imbalance in Fix orders,  $F_t$ , is determined endogenously as part of the model's equilibrium.

Following the arrival of the Fix orders, events follow the PS model. In particular, the broker and

each dealer simultaneously and independently quotes a scalar price for forex,  $S_{B,t}^{\text{II}}$  and  $\{S_{d,t}^{\text{II}}\}_{d=1}^{\text{D}}$ . The quoted prices are observed by all dealers and are good for inter-dealer trades of any size. Each dealer then simultaneously and independently trades on other dealers' and the broker's quotes. I denote forex trades initiated by dealer  $d$  as  $T_{d,t}^{\text{II}}$  (positive values are forex purchases, negative values are sales) and orders received by dealer  $d$  as  $Z_{d,t}^{\text{II}}$ . Orders received by the broker are denoted by  $Z_{B,t}^{\text{II}}$ . Positive (negative) values of  $Z_{d,t}^{\text{II}}$  or  $Z_{B,t}^{\text{II}}$  denote purchases (sales) of forex initiated by another dealer. Trading with multiple dealers and the broker is feasible. If multiple agents quote the same price, trades are allocated equally between them. At the close of round II trading, all dealers and the broker observe aggregate inter-dealer order flow:  $X_t^{\text{II}} = \sum_{d=1}^{\text{D}} T_{d,t}^{\text{II}}$ .

**Round F** At the start of round F the broker and each dealer again simultaneously and independently quotes a scalar price for forex,  $S_{B,t}^{\text{F}}$  and  $\{S_{d,t}^{\text{F}}\}_{d=1}^{\text{D}}$ . The average of these prices determines the Fix benchmark,  $S_t^{\text{F}}$ . Dealers fill their Fix orders at this price, and trade with each other and the broker as in round II. Once again, aggregate inter-dealer order flow is observed by all dealers and the broker at the end of the round:  $X_t^{\text{F}} = \sum_{d=1}^{\text{D}} T_{d,t}^{\text{F}}$ , where  $T_{d,t}^{\text{F}}$  denote the trades initiated by dealer  $d$ .<sup>14</sup>

**Round III** Round III follows the PS model. The dealers quote prices,  $\{S_{d,t}^{\text{III}}\}_{d=1}^{\text{D}}$ , at which they will fill investors' orders, and the broker quotes a price  $S_{B,t}^{\text{III}}$  at which he will fill dealers' orders. Investors then place their orders with dealers. As above, orders may be placed with more than one dealer and are randomly assigned to dealers quoting the same prices. The round III customer orders received by dealer  $d$  are denoted by  $Z_{d,t}^{\text{III}}$ . Once each dealer has filled his customer orders, he can trade with the broker.

## 2.3 Equilibrium

An equilibrium in this model comprises: (i) investors' trades in rounds I and III, and their Fix orders in round II; (ii) the forex price quotes by dealers and the broker; and (iii) dealers' trading decisions in rounds II, F and III. All these decisions must be optimal in the sense that they maximize the expected utility of the respective agent given available information and they must be consistent with the following market clearing conditions:

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<sup>14</sup>Clearly, this sequencing of events is much simpler than the continuous process of price-quotes and trades that takes place during the actual Fix window. To check the robustness of the equilibrium to this simple structure, I also considered a version of the model where the Fix benchmark was determined by the average of the round F and III prices. In this equilibrium  $X_t^{\text{F}}$  affects the determination of the benchmark price, but other features of the equilibrium are unchanged.

$$\sum_{d=1}^D Z_{d,t}^I = \int_0^1 (A_{n,t}^I - A_{n,t-1}^{III} - Y_{n,t}) dn. \quad (2a)$$

$$\sum_{d=1}^D Z_{d,t}^j + Z_{B,t}^j = \sum_{d=1}^D T_{d,t}^j \quad \text{for } j = \{II, F\} \quad (2b)$$

$$\sum_{d=1}^D Z_{d,t}^{III} = \int_0^1 (A_{n,t}^{III} - A_{n,t}^F) dn, \quad \text{and} \quad Z_{B,t}^{III} = \sum_{d=1}^D T_{d,t}^{III}. \quad (2c)$$

Condition (2a) states that in aggregate incoming forex orders received by dealers in round I equal investors desired change in forex holdings:  $A_{n,t}^I - A_{n,t-1}^{III} - Y_{n,t}$ . Market clearing in rounds II and F requires that aggregate incoming orders received by dealers and the broker equal the aggregate forex purchases initiated by dealers as shown in (2b). In round III condition (2c) shows that dealers' incoming orders must match investors desired change in forex holdings, and the broker's order must match dealer-initiated trades. The market clearing condition for Fix orders was given in equation (1).

As in the standard PS model, equilibrium forex prices and dealers' trades are identified from the Bayesian-Nash Equilibrium of a simultaneous-game, while investors' forex orders are determined optimally given the equilibrium sequence of price quotes. The equilibrium of the model is summarized below.

**Equilibrium** *In and efficient risk-sharing equilibrium: (i) All dealers quote same forex price in each round, i.e.  $S_{d,t}^i = S_t^i$  for  $i = \{I, II, F, III\}$ . (ii) The broker quotes the same price as dealers in rounds II, F, and III. (iii) Common prices follow*

$$S_t^I = S_{t-1}^{III} - (\lambda_A^{II} + \lambda_A^F) A_{t-1} + \frac{1}{r} V_t, \quad (3a)$$

$$S_t^{II} = S_t^I, \quad (3b)$$

$$S_t^F = S_t^{II} + \lambda_A^{II} A_{t-1} + \lambda_X^{II} (X_t^{II} - \mathbb{E}[X_t^{II} | \Omega_{D,t}^{II}]), \quad \text{and} \quad (3c)$$

$$S_t^{III} = S_t^F + \lambda_A^F A_{t-1} + \lambda_X^F (X_t^F - \mathbb{E}[X_t^F | \Omega_{D,t}^F]), \quad (3d)$$

with  $A_{t-1} \equiv \int_0^1 A_{n,t-1}^{III} dn$ , where  $\Omega_{D,t}^j$  denotes common information of dealers and the broker at the start of round  $j$ . (iv) Aggregate inter-dealer order flows in rounds II and F are  $X_t^{II} =$

$\sum_{d=1}^D T_{d,t}^{\text{II}}$  and  $X_t^{\text{F}} = \sum_{d=1}^D T_{d,t}^{\text{F}}$ , where dealers' individual trades are

$$T_{d,t}^{\text{II}} = \alpha_Z^{\text{II}} Z_{d,t}^{\text{I}} + (\alpha_A^{\text{II}}/D) A_{t-1} \quad \text{and} \quad (4a)$$

$$T_{d,t}^{\text{F}} = \alpha_Z^{\text{F}} Z_{d,t} + (\alpha_A^{\text{F}}/D) A_{t-1} + \alpha_F^{\text{F}} F_{d,t} + (\alpha_X^{\text{F}}/D) X_t^{\text{II}}. \quad (4b)$$

(iv) The investor orders, and Fix orders received by dealer  $d$  are in rounds I and II are

$$Z_{d,t}^{\text{I}} = (\beta/D) Y_t + \varepsilon_{d,t}, \quad (5a)$$

$$F_{d,t} = (1/D) H_t + \xi_{d,t} \quad (5b)$$

where  $\sum_{d=1}^D \varepsilon_{d,t} = 0$  and  $\sum_{d=1}^D \xi_{d,t} = 0$ .

A detailed derivation of these equations is provided in the on-line appendix.

This equilibrium shares several features found in the standard PS model. In particular, forex prices incorporate information from two sources. First, public information concerning future dividends (i.e.,  $V_t$  shocks) is immediately impounded into dealers' round I quotes, as shown in equation (3a). Second, dealers' quotes in rounds F and III incorporate information about aggregate foreign income  $Y_t$  and the hedging demand  $H_t$  that is conveyed by inter-dealer order flow from rounds II and F. Equation (5) shows that dealers obtain private but imprecise information about  $Y_t$  and  $H_t$  from the forex orders they receive from investors in round I and their Fix orders in round II. They then optimally trade on this information in rounds II and F (see equation 4), with the result that the inter-dealer order flows,  $X_t^{\text{II}}$  and  $X_t^{\text{F}}$ , convey information on  $Y_t$  and  $H_t$  across all dealers in the market. This is a more complex version of the trade-based information aggregation process found in the PS model.

**Prices Dynamics around the Fix** The behavior of prices around the Fix reflect the factors driving dealers' quote decisions. As in the PS model, dealers' quote prices to share risk efficiently. To this end, their round III quote is chosen so that investors are willing to hold the aggregate available stock of forex overnight, i.e.,  $A_t \equiv \int_0^1 A_{n,t}^{\text{III}} dn$ .<sup>15</sup> These round III holdings follow  $A_t = A_{t-1} + Y_t - H_t$  because  $Y_t - H_t$  represents the additional forex available net of the hedgers Fix orders. Inverting investors' round III demand, and aggregating across investors, gives

$$S_t^{\text{III}} = \frac{1}{r} D_t - \frac{1}{r} (\gamma + \lambda_A^{\text{II}} + \lambda_A^{\text{F}}) (A_{t-1} + Y_t - H_t). \quad (6)$$

<sup>15</sup>This is an efficient risk-sharing allocation because there are finite number of dealers and a continuum of investors. The implication that dealers do not hold open forex positions overnight is also consistent with actual dealer behavior.

Dealers are able to compute this price by the start of round III because they learn the value of  $D_t$  in round I, and the values for  $Y_t$  and  $H_t$  from the inter-dealer order flows in rounds II and F. Similarly, dealers quote the same price in rounds I and II so that in aggregate investors have an incentive to retain their overnight forex holdings. Inverting investors' aggregate demand in this case gives

$$S_t^I = \mathbb{E}[S_t^{\text{III}}|\Omega_{D,t}^I] - (\lambda_A^{\text{II}} + \lambda_A^{\text{F}})A_{t-1}. \quad (7)$$

As in the PS model, dealers' quotes include an intraday risk premium,  $(\lambda_A^{\text{II}} + \lambda_A^{\text{F}})A_{t-1}$ .

The same risk-sharing principle applies to determination of the Fix benchmark,  $S_t^{\text{F}}$ . In this case dealers choose their round-F quotes so that  $\mathbb{E}[F_t|\Omega_{D,t}^{\text{F}}] = 0$ . In words, they quote a price that eliminates any expected imbalance in aggregate Fix orders because it would contribute to their intraday forex holdings. Recall from (1) that  $F_t$  comprises the Fix orders of hedgers and investors. Hedgers orders are exogenous with  $\mathbb{E}[H_t|\Omega_{D,t}^{\text{F}}] = 0$ , but investors' orders are chosen optimally given their expectations concerning the post-Fix change in forex prices,  $\mathbb{E}[S_t^{\text{III}} - S_t^{\text{F}}|\Omega_{D,t}^{\text{II}}]$ . So, from a risk-sharing perspective, dealers need to choose  $S_t^{\text{F}}$  so that in aggregate investors have no incentive to place Fix orders. Under these circumstances,  $F_t = H_t$  so  $\mathbb{E}[F_t|\Omega_{D,t}^{\text{F}}] = \mathbb{E}[H_t|\Omega_{D,t}^{\text{F}}] = 0$ , as desired.

To achieve this outcome, dealers quote a price equal to

$$\begin{aligned} S_t^{\text{F}} &= \mathbb{E}[S_t^{\text{III}}|\Omega_{D,t}^{\text{F}}] - \lambda_A^{\text{F}}A_{t-1} \\ &= S_t^I + \lambda_A^{\text{II}}A_{t-1} + (\mathbb{E}[S_t^{\text{III}}|\Omega_{D,t}^{\text{F}}] - \mathbb{E}[S_t^{\text{III}}|\Omega_{D,t}^I]) \end{aligned} \quad (8)$$

The first line shows that the quote embeds the part of the intraday risk premium  $(\lambda_A^{\text{F}}A_{t-1})$  necessary to dissuade investors from submitting Fix orders. The second line rewrites  $S_t^{\text{F}}$  in terms of the prior price level ( $S_t^I = S_t^{\text{II}}$ ) using (7). The first two terms in this expression are known to dealers from round I. The third term identifies the revision in dealers' expectations concerning  $S_t^{\text{III}}$  based on the new information contained in order flow from round II,  $X_t^{\text{II}} - \mathbb{E}[X_t^{\text{II}}|\Omega_{D,t}^{\text{II}}]$ , as shown in (3c). In particular, dealers optimal trading strategies in round II (discussed below) imply that  $X_t^{\text{II}} - \mathbb{E}[X_t^{\text{II}}|\Omega_{D,t}^{\text{II}}] = \alpha_Z^{\text{II}}\beta Y_t$ , so dealers can infer the value of aggregate foreign income, and revise their expectations accordingly. The Fix benchmark also differs from the round-III price. In particular, (6) and (8) imply that

$$S_t^{\text{III}} = S_t^{\text{F}} + \lambda_A^{\text{F}}A_{t-1} + (S_t^{\text{III}} - \mathbb{E}[S_t^{\text{III}}|\Omega_{D,t}^{\text{F}}]). \quad (9)$$

The round III price includes part of the intraday risk premium  $(\lambda_A^{\text{F}}A_{t-1})$  and the new information needed to share risk efficiently at the end of the day,  $S_t^{\text{III}} - \mathbb{E}[S_t^{\text{III}}|\Omega_{D,t}^{\text{F}}]$ . Equation (3d) shows that this information is conveyed by unexpected order flow,  $X_t^{\text{F}} - \mathbb{E}[X_t^{\text{F}}|\Omega_{D,t}^{\text{F}}]$ . Because dealers' optimal



trading strategy in round F depends on their individual Fix orders,  $F_{d,t}$  (discussed below), their observation of  $X_t^F$  reveals the imbalance in aggregate Fix orders  $F_t (= H_t$  in equilibrium).

Equations (6) - (9) have two important implications for the behavior of pre- and post-Fix price changes:  $S_t^F - S_t^I$  and  $S_t^{III} - S_t^F$ . First, the intraday risk premium is the only source of serial correlation. Because news concerning  $S_t^{III}$  must be serially independent, (8) and (9) imply that  $Cov(S_t^{III} - S_t^F, S_t^F - S_t^I) = \lambda_A^I \lambda_A^F Var(A_{t-1})$ . In this model  $\lambda_A^I$  and  $\lambda_A^F$  are positive, so day-by-day variations in the intraday risk premium produce positive serial correlation in price changes around the Fix. Second, the aggregate imbalance in Fix orders  $F_t (= H_t)$  only contributes to the volatility of the post-Fix price change. While information concerning the value of  $F_t$  is price-relevant from a risk-sharing perspective, it remains dispersed across dealers until they use their individual Fix orders to trade in round F. Consequently, the aggregate imbalance in Fix orders makes no contribution to the serial correlation in price changes around the Fix.

This discussion makes clear that dealers' round II trades have important implications for the behavior of prices around the Fix. In principle, dealers *could* use their individual Fix orders,  $F_{d,t}$ , in their round II trading decisions. If they did, the Fix benchmark  $S_t^F$  would incorporate information about  $F_t$  conveyed by order flow  $X_t^{II}$ . However, as equation (4) shows, this is not the optimal trading strategy. As in the PS model, individual dealers trade to establish optimal speculative positions based on their own private forecasts about future price changes. They have two pieces of private information available for this purpose: the investors' orders they filled in round I,  $Z_{d,t}^I$ , and their Fix orders,  $F_{d,t}$ . In equilibrium only  $Z_{d,t}^I$  has forecasting power for  $S_t^F - S_t^{II}$ , so it is not optimal for dealers to base their round II trades on  $F_{d,t}$ .<sup>16</sup>

The equilibrium in this model provides an interesting perspective on the empirical results in Melvin and Prins (2015). They use a simple model to estimate the end-of-month Fix orders of international equity managers wanting to hedge their forex risk and then show that the estimates are positively correlated with pre-Fix price changes, and negatively correlated with post-Fix price changes. These findings are hard to reconcile with the model. Much of the information Melvin and Prins use to estimate managers' end-of-month Fix orders was known to many market participants before the start of the last trading day each month (Suisse, 2009). So, *if* aggregate Fix orders were largely determined by their estimates, the aggregate imbalance  $F_t$  should have been predicted by market participants well before the Fix. This is inconsistent with efficient risk-sharing. Alterna-

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<sup>16</sup>Equation (4) shows that dealers' round F trades depend on many more factors, including  $F_{d,t}$ . This arises because dealers need to fill their Fix orders and  $F_{d,t}$  has forecasting power for post-Fix price changes,  $S_t^{III} - S_t^F$ . Note, also, that the equilibrium in the Proposition is the only one that shares risk efficiently. If dealers quoted an alternate Fix benchmark, post-Fix price changes would be correlated with foreign income,  $Y_t$ . Under these circumstances, investors would have an incentive to submit Fix orders so that  $\mathbb{E}[F_t | \Omega_{d,t}^F] \neq 0$ , which is inconsistent with efficient risk-sharing.

tively, the Fix orders by investors could have largely offset Melvin and Prins' estimated hedging orders, so that the aggregate imbalance  $F_t$  was largely unpredictable. While this possibility is consistent with efficient risk-sharing, it does not explain why the portion of Fix orders estimated by Melvin and Prins should contribute to pre-Fix price changes. After all, pre-fix price changes in the model are unaffected by the aggregate imbalance in Fix orders.

To summarize, the competitive trading model examined above makes strong predictions concerning the behavior of forex prices and trading patterns around the Fix. First, any correlation between pre- and post-Fix price changes reflects variations in the intraday risk premium. The correlations are not related to the aggregate imbalance in Fix orders. Second, the aggregate imbalance in Fix orders only contributes to the volatility of price changes once the information becomes aggregated and disseminated across the market via dealers' trading decisions. This trade-based information aggregation process does *not* take place *until* dealers fill their Fix orders.

### 3 Empirical Analysis

My empirical analysis examines the behavior of spot rates around the Fix across 21 currency pairs between the start of 2004 and end of 2013. In this section I report findings for representative currencies. A complete set of empirical results are contained in the on-line appendix.

#### 3.1 Data and Methods

I use data from three sources. The daily 4:00 pm Fixes are taken from Datastream. The intraday price data comes from Gain Capital, the parent company of Forex.com. Their data archive includes tick-by-tick bid and offer prices for a wide range of currencies. I focus on 21 currency pairs: the four majors involving the U.S. Dollar (USD/EUR, CHF/USD, USD/GBP and JPY/USD) and 17 further pairs that use either the Euro, Pound or Dollar as the base currency. I also use three month samples of EBS data from 2007, 2010 and 2013.<sup>17</sup>

Gain Capital aggregates data from more than 20 banks and brokerages to construct the bid and offer prices. To gauge how accurately these data represent prices across the forex market, Gain provides a comparison of the mid-point between its bid and ask prices with the mid-point for the best tradable bid and ask prices aggregated from 150 market participants by Interactive Data Corporation GTIS. These comparisons (available at <http://www.forex.com/pricing-comparison.html>) show very small differences between the two mid-point series in current data. As a further check on the

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<sup>17</sup>I am grateful to a market participant for allowing me limited access to these data.

accuracy of the Gain data, I compared the mid-points from the tick-by-tick data with the 4:00 pm Fix benchmarks on each trading day in the sample. This comparison showed that the tick-by-tick prices around 4:00 pm very closely match the prices used in computing the actual Fixes.

My statistical analysis uses a set of observation windows that define market events in clock time around 4:00 pm to accommodate the irregularly spacing of the tick-by-tick data. The observation windows range in durations from one to 60 minutes, covering the period between 3:00 and 5:00 pm. I compute statistics that summarize the behavior of the mid-point price (i.e., the average of the bid and offer prices) within the windows; including the first, last, maximum and minimum prices.

It is informative to compare the behavior of forex prices around the Fix with their behavior under typical market conditions summarized by a bootstrap distribution. To build this distribution, I first pick a random starting time between 7:00 am and 5:00 pm on any day. I then use this time as the starting time for a set of observation windows with durations from one to 60 minutes. If any of the randomly selected windows cover the 4:00 pm Fix, the ECB Fix, or the scheduled release of U.S. macro data, I discard the starting time. If not, I compute and record the statistics for the mid-point prices in the windows. This process is repeated 10,000 times to build up the bootstrap distribution summarizing the typical behavior of prices away from the Fix.

### 3.2 Pre-Fix Prices

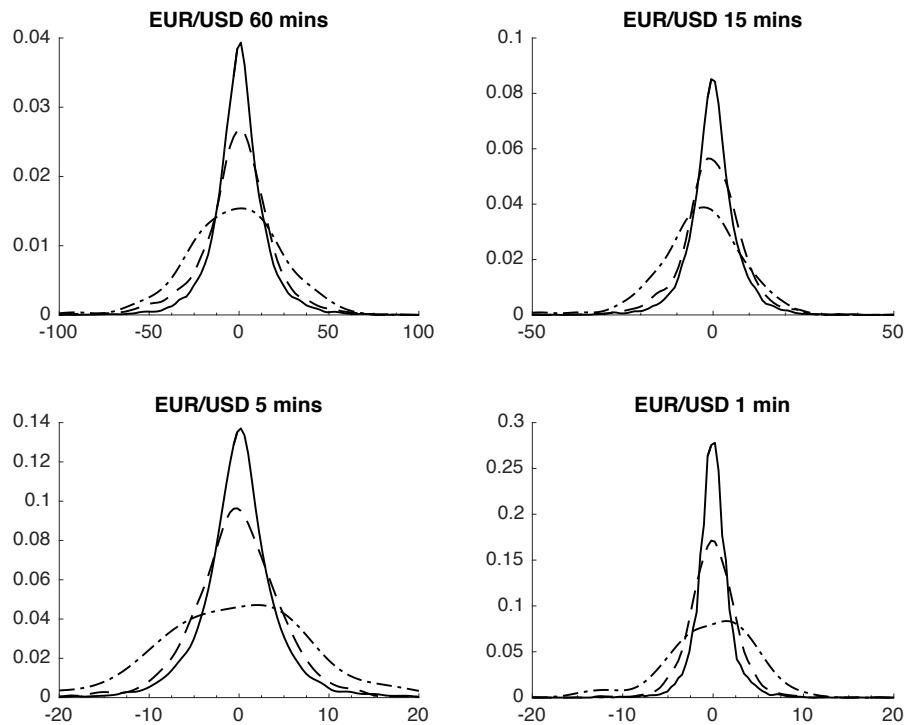
To begin my empirical analysis, I examine forex price-changes in the hour before the Fix. Figure 2 shows the densities for changes in the EUR/USD rate over horizons of 60, 15, 5, and one minute before 4:00 pm on intra month and end-of-month days, and the price-change density for the same horizons from the bootstrap. The plots display two features that are common across all the 21 currency pairs. First, the behavior of pre-Fix rate changes are quite unlike the changes associated with normal trading activity. The estimated densities for the pre-Fix changes are quite different from the bootstrap densities. This visual evidence is confirmed by Kolmogorov-Smirnov (KS) tests for the equality of the two distributions; they give very small p-values for all currency pairs and horizons.<sup>18</sup> Second, the behavior of pre-Fix rate changes at the end of the month appear more

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<sup>18</sup>Two versions of the KS test can be found in the statistics literature. The one-sample KS test is a nonparametric test of the null hypothesis that the population cdf of the data is equal to the hypothesized cdf. The two-sample KS test is a nonparametric hypothesis test of the null that the data in two samples are from the same continuous distribution. Here I compute the two-sample KS test which uses the maximum absolute difference between the cdfs of the distributions of the two data samples. The test statistic is computed as  $D = \max_x (|\hat{F}_1(x) - \hat{F}_2(x)|)$  where  $\hat{F}_1(x)$  is the proportion of the first data sample less than or equal to  $x$ , and  $\hat{F}_2(x)$  is the proportion of the second data sample less than or equal to  $x$ . The KS test and its asymptotic p-value are computed with the Matlab “kstest2” function.

atypical than those on other days. More specifically, the dispersion of pre-Fix rate changes at the end of the month is significantly larger than the dispersion in the bootstrap distribution and the dispersion of pre-Fix changes during the month. These differences are more pronounced at shorter horizons (particularly below 15 minutes). Recall from Section 1 that there is a strong hedging incentive for fund managers to submit Fix orders at the end of the month. The density plots indicate that this institutional factor affects the behavior of forex prices before the Fix.

Figure 2: Pre-Fix Price Change Densities



Notes: Distribution for price changes (in basis points) away from Fixes (solid), intra-month pre-Fix (dashed), and end-of-month pre-Fix (dashed-dot).

How atypical are the forex price movements before 4:00 pm? To answer this question, I compare the pre-Fix price-changes to the tail probabilities in the bootstrap distribution. Table 2 reports the percentage of end-of-month and intra-month days where the absolute pre-Fix change is larger than

the 95th. percentile of the bootstrap distribution. If pre-Fix changes are consistent with typical trading patterns, they should be above the 95th. percentile on approximately one day in twenty. The table shows a much higher incidence of unusually large pre-Fix rate changes, particularly at the end of the month. This pattern holds across all the currency pairs and over all the horizons. It reinforces the visual evidence in Figure 2. Notice, also, that the incidence of unusually large pre-Fix changes rises as the horizon shortens. This means that if we compare the level of the Fix with the level of rates in the prior 30 minutes on a randomly chosen day, we are likely to see an unusually large jump in rates shortly before 4:00 pm.

Examples of large forex price movements immediately before 4:00 pm on particular days for specific currencies have been reported by Vaughan and Finch (2013), Melvin and Prins (2015) and others. The statistics in Table 2 show that unusually large pre-Fix price changes are almost commonplace. For example, atypically large changes in the minute before the Fix on intra-month days occur at more than three times the rate that would be consistent with normal trading activity across the four major currency pairs, and at higher rates across the other currency pairs. The incidence of atypically large price changes immediately before the Fix is even higher at the end of the month. At the one-minute horizon atypical changes occur between four and twelve times the rate consistent with normal trading activity. These are remarkably high numbers. For two of the major currency pairs (JPY/USD and USD/GBP) atypically large price changes in the minute before 4:00 pm occur at more than ten times the rate consistent with normal trading activity.

It is also informative to examine the incidence of atypically large pre-Fix price changes through time. For this purpose Table 3 reports the number of atypical changes (again using the 95th. percentile threshold) over a one-minute horizon at the end of the month during each year covered by the dataset. P-values for the null hypothesis that the number of atypical end-of-month changes occurs by chance (based on the bootstrap distribution) are reported in parenthesis. As the table clearly shows, the high incidence of atypically large pre-Fix price changes is not concentrated in a few years or currency pairs. On the contrary, it is pervasive. For example, in the USD/GBP there have been a high number of atypically large changes in every year between 2004 and 2013. In fact, the numbers are so high in nine of the years that the probability of this representing price movements from normal forex trading in USD/GBP in any year is less than 0.001 (i.e., less than one in one thousand). This repeated high incidence of atypically large pre-Fix price changes is also evident in the JPY/USD, JPY/EUR, CHF/GBP, EUR/GBP, JPY/GBP, USD/USD and CAD/USD. The results in Table 3 also show that the peak incidence of atypically large rate changes did not occur around the world financial crisis. Aggregating across all 21 currency pairs, the peak year was 2010

with a total of 148.

Table 2: Tail Probabilities for pre-Fix Price Changes

		I: End-of-month				II: Intra-Month			
		30 (ii)	15 (iii)	5 (v)	1 (vi)	30 (ii)	15 (iii)	5 (v)	1 (vi)
A:	EUR/USD	22.222	18.803	22.222	33.333	11.653	9.380	7.107	10.496
	CHF/USD	21.698	21.698	25.472	37.736	13.242	10.939	9.433	14.969
	JPY/USD	28.846	38.462	47.115	61.539	12.114	11.071	10.799	22.051
	USD/GBP	27.586	29.310	33.621	51.724	10.822	9.665	11.276	20.446
B:	CHF/EUR	23.276	25.862	29.310	33.621	9.987	9.819	11.589	15.086
	JPY/EUR	28.205	29.915	42.735	52.137	10.574	8.013	10.905	15.572
	NOK/EUR	29.032	24.194	35.484	58.065	14.330	14.562	19.597	29.202
	NZD/EUR	30.882	29.412	41.177	48.529	16.549	15.559	20.368	27.581
	SEK/EUR	25.424	30.509	45.763	45.763	13.975	16.149	15.450	29.115
C:	AUS/GBP	30.435	34.783	34.783	56.522	12.940	13.008	14.160	26.423
	CAD/GBP	28.169	30.986	38.028	39.437	14.614	16.238	22.463	30.176
	CHF/GBP	30.172	37.069	31.035	50.000	10.923	11.378	12.743	21.804
	EUR/GBP	31.304	40.000	37.391	50.435	10.603	12.399	12.185	22.488
	JPY/GBP	27.586	32.759	43.966	56.035	10.132	10.008	11.373	21.464
	NZD/GBP	25.373	25.373	26.866	47.761	13.272	12.420	21.221	30.518
D:	AUS/USD	28.448	23.276	32.759	46.552	12.427	11.259	13.136	19.516
	CAD/USD	31.897	29.310	34.483	43.966	16.722	15.183	16.889	26.040
	DKK/USD	15.254	10.170	18.644	30.509	10.881	6.820	7.126	10.575
	NOK/USD	22.581	19.355	29.032	46.774	12.481	9.954	12.864	24.043
	SEK/USD	20.339	23.729	33.898	40.678	12.336	11.334	11.411	22.282
	SGD/USD	11.475	9.836	16.393	19.672	9.667	8.917	10.083	18.667

Notes: Each cell reports the percentage of days in which the absolute basis point change in prices in the window before the Fix is larger than the 95th. percentile from the bootstrap distribution of absolute basis point price changes away from the Fix. Panel I reports the percentage for end-of-month price changes, panel II the percentage for intra-month price changes.

Table 3: Pre-Fix Tail Events By Year (1 minute window)

	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
A: EUR/USD	2 (0.165)	5 (0.000)	1 (0.600)	6 (0.000)	5 (0.000)	6 (0.000)	6 (0.000)	3 (0.028)	4 (0.003)	2 (0.138)
CHF/USD	1 (0.450)	4 (0.001)	0 (0.569)	5 (0.000)	3 (0.007)	4 (0.002)	5 (0.000)	7 (0.000)	7 (0.000)	4 (0.002)
JPY/USD	3 (0.011)	4 (0.001)	7 (0.000)	11 (0.000)	5 (0.000)	6 (0.000)	9 (0.000)	8 (0.000)	4 (0.003)	7 (0.000)
USD/GBP	6 (0.000)	5 (0.000)	6 (0.000)	3 (0.028)	5 (0.000)	9 (0.000)	7 (0.000)	8 (0.000)	5 (0.000)	7 (0.000)
B: CHF/EUR	4 (0.003)	1 (0.550)	1 (0.550)	3 (0.028)	4 (0.003)	4 (0.003)	9 (0.000)	7 (0.000)	0 (0.540)	6 (0.000)
JPY/EUR	6 (0.000)	4 (0.002)	4 (0.002)	7 (0.000)	8 (0.000)	8 (0.000)	9 (0.000)	5 (0.000)	5 (0.000)	5 (0.000)
NOK/EUR					1 (0.200)	8 (0.000)	8 (0.000)	10 (0.000)	6 (0.000)	4 (0.002)
NZD/EUR					8 (0.000)	7 (0.000)	5 (0.000)	4 (0.003)	5 (0.000)	4 (0.002)
SEK/EUR					1 (0.200)	4 (0.003)	7 (0.000)	6 (0.000)	5 (0.000)	6 (0.000)
C: AUS/GBP					10 (0.000)	9 (0.000)	8 (0.000)	6 (0.000)	5 (0.000)	2 (0.138)
CAD/GBP					6 (0.000)	5 (0.000)	6 (0.000)	4 (0.003)	4 (0.003)	3 (0.021)
CHF/GBP	4 (0.003)	3 (0.021)	4 (0.002)	5 (0.000)	7 (0.000)	7 (0.000)	8 (0.000)	7 (0.000)	7 (0.000)	7 (0.000)
EUR/GBP	3 (0.028)	3 (0.021)	4 (0.002)	4 (0.003)	7 (0.000)	8 (0.000)	9 (0.000)	7 (0.000)	8 (0.000)	6 (0.000)
JPY/GBP	4 (0.003)	3 (0.021)	4 (0.002)	8 (0.000)	7 (0.000)	9 (0.000)	10 (0.000)	6 (0.000)	6 (0.000)	9 (0.000)
NZD/GBP					6 (0.000)	8 (0.000)	7 (0.000)	6 (0.000)	4 (0.003)	2 (0.138)
D: AUS/USD	4 (0.002)	3 (0.021)	5 (0.000)	4 (0.003)	9 (0.000)	9 (0.000)	9 (0.000)	4 (0.003)	3 (0.028)	4 (0.002)
CAD/USD	4 (0.003)	3 (0.021)	5 (0.000)	7 (0.000)	6 (0.000)	5 (0.000)	4 (0.003)	3 (0.028)	7 (0.000)	8 (0.000)
DKK/USD					3 (0.001)	5 (0.000)	6 (0.000)	2 (0.165)	1 (0.600)	2 (0.138)
NOK/USD					2 (0.015)	8 (0.000)	6 (0.000)	6 (0.000)	4 (0.003)	4 (0.002)
SEK/USD					3 (0.001)	3 (0.021)	7 (0.000)	5 (0.000)	3 (0.028)	5 (0.000)
SGD/USD					2 (0.015)	3 (0.028)	3 (0.028)	1 (0.600)	1 (0.600)	2 (0.138)

Notes: Each cell reports the number of months in each year where the absolute change in prices in the one minute before the Fix falls in the 95th percentile of the bootstrap distribution of price changes away from the Fix. P-values for the null that the number of months occurs purely by chance are reported in parentheses. Empty cells signify the absence of data for the currency pair in that year.

These findings extend the volatility results in Melvin and Prins (2015). They showed that on average across currencies, volatility rises in the hour before the Fix at the end of the month. Here we see that changes in forex prices observed immediately before the 4:00 pm Fix are extraordinarily unusual when compared to their behavior in normal trading away from the Fix: prices regularly jump by an amount that is very rarely seen elsewhere. Moreover, the incidence of these atypically large pre-Fix price changes is particularly high at the end of each month, appears pervasive across all the currency pairs and throughout the sample period.

### 3.3 Post-Fix Prices

The high incidence of unusually large changes in prices immediately before the Fixes carries over into the behavior of prices after 4:00 pm. Table 4 reports the incidence of large post-Fix price changes (starting at 4:00 pm) over horizons of one to 30 minutes. As above, I use the 95th. percentile threshold from the bootstrap distribution to identify atypically large price changes, and report their incidence for individual currency pairs at the end of each month and on other intra-month days. As the table shows, the incidence of atypically large changes on intra-month days is almost twice the rate we would expect to see in trading away from the Fix for many of the currency pairs. The incidence of unusually large price changes is much higher at the end of the month. For most currency pairs, the incidence at the one-minute horizon is at least four times higher than we would expect to see in normal trading, declining to between two and three times normal at the 30-minute horizon. While high, these incidence rates are well below those reported for pre-Fix price changes in Table 2 over comparable horizons.

The statistics in Tables 2 and 4 clearly establish that forex prices are unusually volatile immediately before and after the Fix, particularly at the end of the month. I now consider how the pre- and post-Fix behavior of prices are linked. For this purpose, I estimate the bivariate density for pre- and post-Fix price changes  $g(\ln(S_{t+h}/S_t^{fix}), \ln(S_t^{fix}/S_{t-h}))$  at different horizons,  $h$ .<sup>19</sup> Figure 3 shows a contour plot of estimated density for the EUR/USD in end-of-month data at different horizons. The solid line shows the projection (i.e. regression) of  $\ln(S_{t+h}/S_t^{fix})$  on  $\ln(S_t^{fix}/S_{t-h})$ . This splits the post-Fix price change into a portion that is perfectly correlated with the pre-Fix change, the projection  $\mathcal{P}(\ln(S_t^{fix}/S_{t-h}))$ ; and a projection error,  $\eta_{t+h}$ , that is uncorrelated with the pre-Fix change:

$$\ln(S_{t+h}/S_t^{fix}) = \mathcal{P}(\ln(S_t^{fix}/S_{t-h})) + \eta_{t+h}.$$

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<sup>19</sup>Estimation uses a Gaussian Kernel with the bandwidth determined as in Bowman and Azzalini (1997).



Table 4: Tail Probabilities for Post-Fix Price Changes

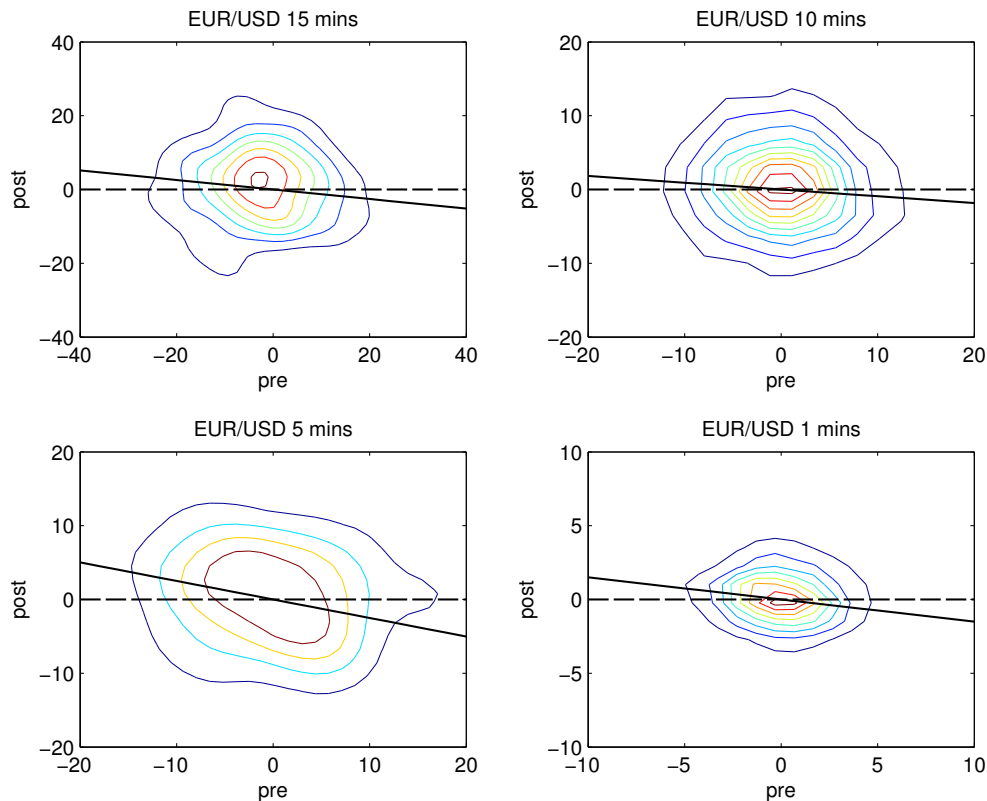
horizon		I: End-of-Month				II: Intra-Month			
		30 (ii)	15 (iii)	5 (v)	1 (vi)	30 (ii)	15 (iii)	5 (v)	1 (vi)
A:	EUR/USD	14.530	15.385	17.094	20.513	9.711	9.298	8.554	6.157
	CHF/USD	15.094	18.868	18.868	26.415	9.965	9.699	8.193	6.997
	JPY/USD	18.269	16.346	21.154	21.154	8.439	8.893	8.394	9.483
	USD/GBP	15.517	14.655	13.793	18.103	8.137	7.228	8.922	6.939
B:	CHF/EUR	10.345	16.379	19.828	16.379	8.681	7.965	8.681	8.260
	JPY/EUR	12.821	16.239	18.803	25.641	8.468	7.600	8.798	7.435
	NOK/EUR	8.065	4.839	12.903	20.968	7.591	6.739	10.380	18.048
	NZD/EUR	22.059	16.177	26.471	41.177	8.911	7.638	10.113	11.245
	SEK/EUR	11.864	13.559	16.949	40.678	7.531	7.609	8.385	16.537
C:	AUS/GBP	20.290	23.188	28.986	26.087	7.859	6.911	7.114	8.537
	CAD/GBP	19.718	19.718	33.803	23.944	7.375	7.510	8.660	8.187
	CHF/GBP	11.207	14.655	20.690	21.552	7.199	7.613	8.440	9.102
	EUR/GBP	14.783	19.130	19.130	26.087	6.156	6.841	7.738	10.389
	JPY/GBP	11.207	15.517	15.517	14.655	7.568	8.189	8.519	7.610
	NZD/GBP	20.896	13.433	32.836	31.343	7.239	6.529	9.226	11.001
D:	AUS/USD	10.345	18.103	27.586	24.138	9.425	8.674	9.008	7.381
	CAD/USD	18.103	17.241	30.172	30.172	9.942	9.318	10.399	9.193
	DKK/USD	11.864	10.170	15.254	18.644	8.736	9.042	8.429	5.900
	NOK/USD	14.516	14.516	22.581	24.194	8.959	8.499	8.116	11.792
	SEK/USD	16.949	11.864	18.644	28.814	8.790	8.867	7.941	11.103
	SGD/USD	4.918	3.279	8.197	27.869	7.167	7.917	7.667	14.667

Notes: Each cell reports the percentage of days in which the absolute basis point change in prices in the window after the Fix is larger than the 95 percentile from the bootstrap distribution. Panel I reports the percentage for end-of-month price changes, panel II the percentage for intra-month price changes.

Several features of the EUR/USD plots in Figure 3 appear across all the currency pairs. First, the maximum width of each contour exceeds its maximum height because prices are more volatile immediately before than after the Fix. Second, the contours generally appear as ellipses that are rotated clockwise around the point (0,0). This pattern implies that positive post-Fix price changes

are more likely than negative changes if they were preceded by a negative pre-Fix change, and vice-verse. Third, the projection lines slope downwards (from left to right) at all horizons and across all currency pairs.

Figure 3: Bivariate Pre- and Post- Fix Price Change Densities



Notes: Each plot shows the contours of the estimated bivariate density for pre- and post-Fix price changes (in basis points) over horizons of 1 to 15 minutes. The solid line in each plot is the estimated regression line from the regression on the post-Fix change in the pre-Fix change. All estimates are based on end-of-month data.

Table 5 reports the estimated projection coefficients, their (heteroskedastic-consistent) standard errors, and the uncentered  $R^2$  statistics for the projections over the horizons of  $\{1, 5, \text{ and } 15\}$  minutes. The estimated coefficients are uniformly negative, ranging in value from -0.07 to -0.61. They are statistically significant at the five percent level for all but three currencies (EUR/USD, CHF/USD and CAD/GBP) for at least one horizon. The  $R^2$  statistics measure the variance

contribution of the projections to the post-Fix price changes. As the table shows, these statistics are generally small (i.e. below 0.2). This indicates that most of the variation in post-Fix changes over time is attributable to projection errors that are uncorrelated with the pre-Fix changes. Notable exceptions to this pattern include the NZD/GBP, AUD/GBP, NZD/EUR and JPY/EUR, where the  $R^2$  statistics are a good deal larger. In these currencies, price reversion accounts for a significant fraction of the time series variation in post-Fix price changes.

Table 5: Post-Fix Projection Estimates

		15 Minutes			5 Minutes			1 Minute		
		Coeff	Std Error	$R^2$	Coeff	Std Error	$R^2$	Coeff	Std Error	$R^2$
A:	EUR/USD	-0.129	(0.077)	0.018	-0.251	(0.165)	0.060	-0.150	(0.082)	0.048
	CHF/USD	-0.107	(0.150)	0.009	-0.112	(0.209)	0.015	-0.160	(0.138)	0.035
	JPY/USD	-0.081	(0.090)	0.011	-0.126	(0.068)	0.051	-0.164*	(0.045)	0.173
	USD/GBP	-0.201	(0.118)	0.115	-0.357	(0.255)	0.243	-0.105*	(0.046)	0.066
B:	CHF/EUR	-0.235*	(0.078)	0.113	-0.199	(0.107)	0.104	-0.096	(0.129)	0.020
	JPY/EUR	-0.375*	(0.154)	0.257	-0.467*	(0.168)	0.408	-0.605*	(0.200)	0.633
	NOK/EUR	-0.167*	(0.073)	0.089	-0.211*	(0.049)	0.162	-0.075	(0.110)	0.009
	NZD/EUR	-0.309*	(0.077)	0.307	-0.439*	(0.126)	0.447	-0.141	(0.118)	0.061
	SEK/EUR	-0.233*	(0.061)	0.209	-0.410*	(0.107)	0.307	-0.199*	(0.070)	0.068
C:	AUD/GBP	-0.303*	(0.042)	0.377	-0.431*	(0.050)	0.464	-0.031	(0.050)	0.008
	CAD/GBP	-0.038	(0.130)	0.002	-0.344	(0.260)	0.079	-0.040	(0.103)	0.003
	CHF/GBP	-0.267*	(0.108)	0.161	-0.410*	(0.180)	0.298	-0.150	(0.085)	0.079
	EUR/GBP	-0.228*	(0.097)	0.134	-0.473*	(0.185)	0.365	-0.209*	(0.047)	0.168
	JPY/GBP	-0.147	(0.145)	0.066	-0.256	(0.223)	0.149	-0.155*	(0.039)	0.179
	NZD/GBP	-0.397*	(0.049)	0.536	-0.505*	(0.053)	0.633	-0.246*	(0.075)	0.239
D:	AUD/USD	-0.247*	(0.056)	0.170	-0.256*	(0.106)	0.144	-0.124	(0.080)	0.061
	CAD/USD	-0.189*	(0.074)	0.069	-0.315*	(0.052)	0.140	-0.178*	(0.064)	0.071
	DKK/USD	-0.259*	(0.108)	0.054	-0.312	(0.255)	0.079	-0.164	(0.102)	0.065
	NOK/USD	-0.135	(0.085)	0.029	-0.169	(0.089)	0.043	-0.079	(0.086)	0.014
	SEK/USD	-0.237*	(0.102)	0.111	-0.396*	(0.159)	0.161	-0.234*	(0.068)	0.126
	SGD/USD	-0.443	(0.238)	0.212	-0.313	(0.161)	0.156	-0.154	(0.309)	0.015

Notes: The table reports the estimated projection coefficient, its (heteroskedastic consistent) standard error, and the  $R^2$  statistic from the projection of the post-Fix price change on the pre-Fix change over the horizons shown at the top of each panel. The “\*” indicates statistical significance at the 5 percent level.

In summary, forex prices display an unusually high level of volatility in the minutes immediately following 4:00 pm. They also appear to be influenced by the pre-Fix behavior of prices: Over a wide range of currencies and horizons, there is a statistically significant negative correlation between pre- and post-Fix price changes.

## 4 Economic Perspective

This section provides economic perspective on my empirical results. First, I examine the implications of the negative correlation between pre- and post-Fix price changes for average price paths around the Fix. I then investigate whether this correlation could have supported the presence of attractive and exploitable trading opportunity to market participants at the time. Finally, I use the predictions from the trading model in Section 2 to interpret my empirical findings and other results in the literature.

### 4.1 Average Price Paths

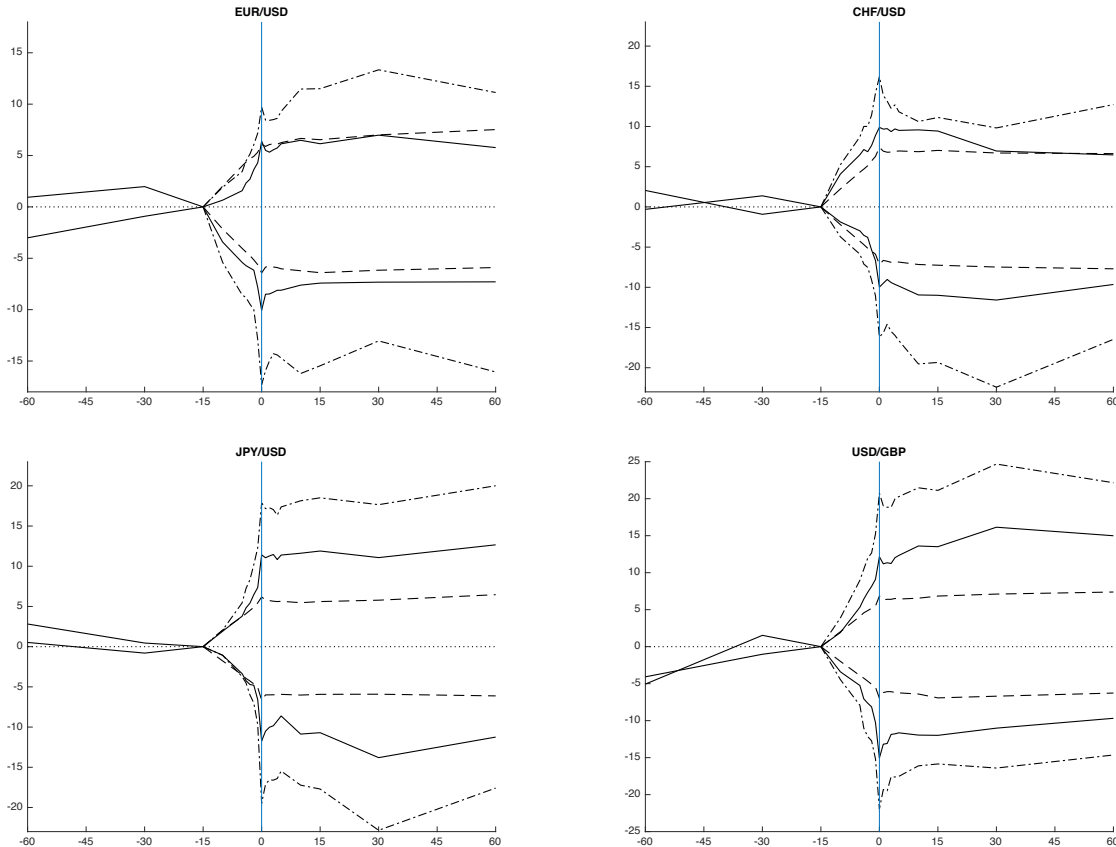
The projection results in Table 5 show the existence of a strong statistical link between pre- and post-Fix price changes. Figure 4 provides another perspective on the temporal dependence in forex prices. Here I plot the average price paths for the major currency pairs around the Fix conditioned on the pre-Fix price change. The vertical axis shows basis points relative to the price at 3:45 pm while the horizontal axis shows minutes relative to 4:00 pm. Each panel shows six average paths that are conditioned on the change in prices between 3:45 and 4:00 pm. I condition on this horizon because 3:45 pm is the cut-off time for dealer-banks to accept Fix orders. The solid black lines in each plot depicts the average path across all end-of-month trading days. Average paths for intra-month days are shown by dashed lines. The remaining upper and lower lines (drawn with dashes and dots) identify the average paths on end-of-the-month trading days where the pre-Fix price change is in the 75th. and 25th. percentiles of the pre-Fix price-change distribution, respectively.

There are several noteworthy features in Figure 4 that are present in the plots for the other currencies (see Appendix). First, consider the paths on intra-month days. These paths identify very small reversals during the first minute after the Fix (on the order of one basis point). Thereafter, the paths are almost flat for all the currency pairs. These patterns imply that all the relevant trade-based information is fully assimilated into prices by the end of the Fix window, so there is no systematic tendency for rates to rise or fall after that. In this sense, it appears that post-Fix equilibrium prices are quickly established on intra-month trading days.

The price paths from end-of-month trading days are quite different. Consistent with the statistics on pre-Fix rate volatility, changes in prices between 3:45 and 4:00 pm are larger (in absolute value). Prices also tend to move in a systematic pattern after 4:00 pm. The plots for many of the currencies show that both positive and negative pre-Fix price changes are followed by a sizable reversal in prices in the first few minutes (see, e.g., AUD/GBP, AUD/USD and NZD/EUR). Figure 4 shows that for other currencies (see, e.g., JYP/USD and USD/GDP) the reversals are larger

following pre-Fix rate changes in one direction. These asymmetric effects were not captured by the projection results in Table 5. Figure 4 also shows that large pre-Fix price changes are followed by bigger price reversals than on average across all end-of-month trading days for some currency pairs (see, e.g., CHF/USD).

Figure 4: Rate Paths Around the Fix



Notes: Average rate path in basis points around 3:45 pm level conditioned on: (i) pre-Fix changes (over 15 mins) at end of month (solid black); (ii) pre-Fix changes above the 75th. percentile of end-of-month distribution (dashed dot); (iii) pre-Fix changes in the 25th. percentile of end-of-month distribution (dashed dot-dot); (iv) positive and negative pre-Fix changes on intra-month days (dashed). For the sake of clarity, both the dotted and dash-dotted lines are hidden to the left of -15.

One further feature of these plots deserves note. All the plotted price paths are conditioned on the change in prices between 3:45 and 4:00 pm without regard to when prices changed during that 15-minute window. Thus, if most of the movement in prices occurred immediately before or at the start of the Fix window, the paths would be flat until a point just to the left of the vertical line. Instead, the paths for all the currencies show that on average prices start “drifting” upwards or

downwards soon after 3:45 pm. In other words, the actions of market participants start a process that moves prices towards the Fix benchmark well before 4:00 pm on both intra- and end-of-month trading days. I discuss this feature of the data further below.

## 4.2 Trading Around the Fix

The projection results in Table 5 and price paths in Figure 4 suggest that a simple end-of-month trading strategy of taking a long (short) position at 4:00 pm if prices fell (rose) towards the Fix should generate positive returns on average. Would such a strategy be attractive to a sophisticated trader who has access to the best bid and ask prices in the market?

To address this question, I computed the realized returns  $R$  on trading strategies that initiated long and short positions at the end-of-month Fixes with durations of  $h = \{1, 5, 15\}$  minutes. The long and short positions are selected according to the price changes over the  $h$  minutes before 4:00 pm.<sup>20</sup> I assume that the benchmark well-approximates the transaction price that sophisticated traders actually face when initiating a position at 4:00 pm because spreads fall during the Fix window.<sup>21</sup> Thereafter, for the next hour or so, spreads return to their normal level. This pattern suggests that a representative price facing a trader closing out a position from one to fifteen minutes after the Fix would be equal to the mid-point price  $\pm$  one half the normal spread between the best offer and bid rates. I compute these spreads from my 2013 sample of EBS data.<sup>22</sup>

I compute three performance measures to assess the attractiveness of the strategies: (i) the average return, (ii) the Sharpe Ratio and (iii) the Maximum Drawdown. The Sharpe Ratio is calculated as  $SR = \frac{1}{\sqrt{252}} (\mathbb{E}_T[R_i] - 1) / \sqrt{\mathbb{V}_T[R_i]}$ , where  $R_i$  is the (gross) return on day  $i$ .  $\mathbb{E}_T[.]$  and  $\mathbb{V}_T[.]$  are the sample mean and variance from the  $T$  returns computed over the span of the data. Because returns are generated at the daily frequency, I include the  $1/\sqrt{252}$  scale factor to “annualize” the ratio (using the convention that a year equals 252 trading days). Sharpe Ratios are

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<sup>20</sup>Notice that this selection method does not require any estimation, so the returns I construct are from a strategy that could be executed in real time.

<sup>21</sup>Recall that the benchmark is computed from the median bid and ask prices sampled each second during the Fix window. Because second-by-second variations in prices within the Fix window are much larger than the spreads, there are many opportunities during the window to trade at more advantageous (bid or ask) prices than the benchmark. Insofar as sophisticated traders are able to take advantage of these opportunities, using the benchmark price as the transaction price will produce lower returns on the strategy.

<sup>22</sup>Recall that Gain Capital aggregates data from banks and brokerages to construct bid and offer prices. Although the average of these bid and ask prices is an accurate measure of the mid-point between the best tradable prices available to retail trading platforms, the spread between these prices are roughly twice as large as the spreads between the best bid and ask prices on interbank trading venues run by EBS and Reuters. Sophisticated traders, such as hedge fund managers, can trade on these interbank venues via prime brokerage accounts, so I use spreads from EBS to estimate the transaction costs these traders face. The cost is estimated as half the average spread each minute (excluding the Fix window) between 7:00 am and 5:00 pm.

widely used by financial market participants to judge the attractiveness of trading strategies. The Maximum Drawdown statistic is another widely-used measure. It is computed as the maximum percentage drop (i.e. from peak to trough) in the cumulated return from following the trading strategy over the span of data. As such, it provides a measure of downside risk.

Table 6 reports the performance measures for the end-of-month trading strategies across all the currency pairs. Columns (i) - (iii) show that average returns are positive for the majority of currencies and horizons. In fact, the returns are positive for at least one horizon in all but the JPY/USD and USD/GBP. Furthermore, the average returns are well over five percent (on an annualized basis) for nine currency pairs at some horizons. The strategies for many currency pairs also appear attractive when judged by the Sharpe Ratios and Drawdown Statistics. The ratios are above one for at least one horizon in 15 of the currency pairs, and over two in eight pairs. The downside risk associated with the strategies is also generally low with most of the Drawdown statistics below two percent.

Overall, the performance statistics in Table 6 show that there was an economic incentive for sophisticated traders to exploit the temporal dependence in forex prices around the Fix at the end of each month. While these incentives were not equally strong for all the currency pairs, in many cases the Sharpe ratios are well above the minimum threshold most financial institutions require before allocating capital to a trading strategy. For example, Lyons (2001) reports that a reasonable range for these thresholds is between 0.5 and 1. He also notes that the Sharpe ratios for carry trading strategies that exploit forward bias are typically around 0.4 (see, also Burnside, 2012). With this perspective, the results in Table 6 indicate that for many currency pairs there is a stronger economic incentive to exploit the systematic behavior in end-of-month prices around the Fix than there is to exploit the better-known forward bias anomaly.

It is also instructive to compare these results with Melvin and Prins (2015). Their regression estimates (discussed in Section 2) imply that traders who accurately estimated the forex flows of equity managers could have designed trading strategies to exploit forex price movements before and after the Fix. Suisse (2009) is an example of an industry news letter that discusses such a

Table 6: Trading Around the Fix with Transaction Costs

Horizon	Average Return			Sharpe Ratio			Drawdown		
	15 (i)	5 (ii)	1 (iii)	15 (iv)	5 (v)	1 (vi)	15 (vii)	5 (viii)	1 (ix)
A: EUR/USD	3.965	0.486	0.068	1.881	0.222	0.042	1.791	1.780	1.732
CHF/USD	0.878	0.721	2.539	0.320	0.335	1.223	1.809	1.167	0.743
JPY/USD	-1.646	-0.667	-1.234	-0.748	-0.336	-0.750	1.348	1.556	0.780
USD/GBP	-3.054	-2.091	-5.804	-1.182	-0.759	-2.793	2.827	1.736	3.092
B: CHF/EUR	2.105	2.327	1.757	1.679	2.392	2.460	0.612	0.437	0.294
JPY/EUR	3.800	4.813	1.651	1.330	1.687	0.693	1.445	1.047	0.814
NOK/EUR	1.396	4.739	2.050	0.688	2.802	1.174	0.699	0.405	0.526
NZD/EUR	10.698	15.098	2.450	3.691	4.954	0.971	0.844	0.573	1.121
SEK/EUR	5.553	0.384	2.300	1.838	0.133	0.868	0.787	1.188	0.559
C: AUD/GBP	5.182	3.719	0.194	1.602	1.098	0.100	1.568	0.999	1.171
CAD/GBP	-4.725	2.710	-1.480	-1.353	0.862	-0.532	2.594	1.601	1.461
CHF/GBP	2.818	2.815	-0.858	1.236	1.201	-0.472	1.137	0.816	1.121
EUR/GBP	8.325	8.614	6.657	2.865	3.162	2.460	0.893	0.652	0.633
JPY/GBP	-0.505	0.368	-1.971	-0.143	0.125	-0.770	2.443	1.927	2.315
NZD/GBP	0.483	5.451	4.735	0.144	1.448	1.946	2.103	1.422	0.996
D: AUD/USD	9.187	12.292	8.353	3.375	4.353	3.124	1.531	1.197	1.176
CAD/USD	2.486	9.538	8.429	0.852	3.450	3.176	1.956	1.041	0.864
DKK/USD	8.214	2.805	0.882	3.177	1.445	0.538	0.932	0.997	0.713
NOK/USD	-0.409	3.240	7.714	-0.098	1.052	3.722	2.032	1.251	0.368
SEK/USD	2.699	-4.673	1.090	0.640	-1.088	0.308	1.636	3.283	1.343
SGD/USD	0.309	0.352	-1.386	0.272	0.364	-1.607	0.801	0.514	0.608

Notes: Columns (i) - (iii) report the average return (in annual percent) from a trading strategy of holding a long (short) position for horizon  $h = \{1, 5, 15\}$  minutes following the end-of-month Fix if the Fix is below (above) the price level  $h$  minutes earlier. Columns (iv) - (vi) report the associated Sharpe ratios (annualized), while columns (vii) - (ix) show the maximum drawdown in percent from following the strategy on every end-of-month trading day. Returns are inclusive of trading costs, computed to be zero at the Fix and one half the average bid-ask spread when the position is closed.

strategy. The strategies studied here require a lot less sophistication. There is no need to estimate forex flows of equity managers. Instead, one only needs to track changes in forex prices before the Fix. Moreover, Melvin and Prins (2015) report a Sharpe ratio of 0.4 for a strategy (without trading costs) that exploits the effects of managers' flows on prices before the Fix and argue that this is



below the threshold institutions require before allocating capital. By this metric, the economic significance of their regression results appears less than the temporal patterns in prices around the Fix examined here.

### 4.3 Competition and Collusion

Banks view their trading data as highly proprietary, and as such have not made this data available to researchers. Consequently, it is impossible to *directly* examine whether the collusive trading activity described by regulators can account for the behavior of forex prices around the Fix presented above.<sup>23</sup> Nevertheless, there are several strands of indirect evidence that shed light on this question.

The first strand comes from the model of competitive forex trading in Section 2. Recall that Fix orders only contribute to the volatility in equilibrium prices when they are filled by dealers during round F. This theoretical prediction is consistent with the empirical evidence on post-Fix price changes in Table 4. By contrast, the model cannot account for the volatility in pre-Fix prices shown in Tables 2 and 3. Nor can it explain the negative correlation between pre- and post-Fix changes documented in Table 5 and depicted in Figure 3. The most likely source of these inconsistencies concerns the information available to dealers in the model. If, contrary to the model's assumptions, dealers had the opportunity to share information about their individual Fix orders before quoting prices in round II, they would find it beneficial to do so. Moreover, the aggregate imbalance in Fix orders would become known to all dealers as a result of this information sharing, and would be incorporated into their price quotes to efficiently share risk. In this alternate collusive equilibrium the aggregate imbalance in Fix orders would impact forex prices before the Fix; which would be more consistent with the empirical results in Tables 2 and 3. This is indirect evidence suggesting that the sharing of information about Fix orders plays a role in determining the empirical behavior of forex prices before the Fix.

The second strand of evidence comes from the empirical microstructure literature on the behavior of prices around the close of equity trading. From their examination of equity prices for Russell 1000 stocks in the 1990s, Cushing and Madhavan (2000) document exceptionally high volatility immediately prior to the close and a negative correlation between pre- and post-close price changes. These empirical characteristics are similar to those documented here, but the market setting is very different. Explanations for the behavior of equity prices that rely on the lack of liquidity in after-hours trading (e.g., via price pressure a la Hendershott and Menkveld, 2014) rather than the

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<sup>23</sup>Regulators' reports provide examples of collusive activity by individual bank dealers, but they do not contain any systematic analysis of how these activities affected forex prices across the market. The on-line appendix gives further details concerning these reports and the settlements reached by the banks.

presence of collusion between market-makers, do not offer a reasonable explanation for the behavior of forex prices examined here because the 4:00 pm Fix occurs *during* rather than at the end of the forex trading day.

Regulators' investigation reports contain the third strand of evidence. According to the U.K. Financial Conduct Authority and the U.S. Department of Justice, the dealers shared information about their Fix orders around 3:45 pm and colluded to front run their joint imbalance in orders prior to the start of the Fix window. That is to say, the dealers would purchase forex before the Fix window when they collectively had net orders to buy; and sell forex before the Fix window when they collectively had net orders to sell. The dealers also traded among themselves to concentrate the net order imbalance at one or two trading desks. Dealers at these desks would then attempt to manipulate the Fix by aggressively trading in the interbank market (on EBS or Reuters) once the Fix window opened. This typically involved placing a large number of market purchase orders when they had net orders to buy forex, and market sale orders when they had net orders to sell forex (during the first 30 seconds of the window). Trading in this manner increased the likelihood that the Fix would move in the desired direction because the WMR methodology took no account of trading volume. After successfully manipulating the Fix by this means, the colluding dealers would close out the speculative positions they established before the Fix window at a profit.

The market-wide effects of these activities depend critically on whether the imbalance in Fix orders across all the colluding dealers was in the same direction as the imbalance across the entire market. In cases where the imbalances are in the same direction we would expect to see patterns like those in Figure 4. In particular, the anticipatory movements in forex prices soon after 3:45 pm is consistent with colluding dealers establishing speculative positions by front running, while the reversal of prices after 4:00 pm is consistent with colluding dealers closing these positions after the Fix. In addition, aggressive trading within the Fix window by some colluding dealers would add to volatility in spot rates due to the market-wide imbalance in Fix orders, consistent with the volatility results in Tables 2-4.<sup>24</sup>

The presence of collusion also provides a simple explanation for the existence of economically attractive trading strategies exploiting the negative serial correlation in price changes around the Fix. Normally, we would expect these temporal patterns in prices to disappear soon after they are discovered as traders attempted to exploit them. However, the results in Table 6 suggest that here the actions of some market participants impeded this process. The regulators' descriptions of the collusive front running by dealers could have played this role.

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<sup>24</sup>Collusive front running could also account for the results in Melvin and Prins (2015) if their estimated end-of-month flows of forex orders were positive correlated with the imbalance in Fix orders among colluding dealers.

## 5 Conclusion

This paper has examined the behavior of forex prices around the WRM Fix from both a theoretical and empirical perspective. The theoretical perspective was provided by a new microstructure model of competitive trading that incorporated the key institutional features of the Fix. The model showed that Fix orders have a limited affect on the behavior of forex prices. In particular, prices adjust to the aggregate imbalance in Fix orders across the market when dealers trade to fill their individual Fix orders. They do not contribute to the behavior of prices before the Fix nor are they are source of serial correlation in price changes around the Fix.

My empirical results provide a very different perspective. They show that across all time periods and currency pairs changes in prices before and after the Fix are regularly of a size rarely seen in normal trading activity. This atypical behavior is particularly strong at the end of each month. Furthermore, the temporal dependence in forex prices around the Fix (i.e., the negative correlation between pre- and post-Fix price changes) are sufficiently strong to support economically attractive end-of-month trading strategies for 15 of the 21 currency pairs studied.

When the many differences between these theoretical and empirical perspectives are combined with the material contained in regulators' reports, the balance of the evidence suggests that the admitted collusive activities by the banks' dealers had a more significant and far-reaching market impact (across time and currency pairs) than has been hitherto realized.

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## Online Appendix (Not for Publication)

This appendix provides information on the investigations into banks' collusive activities, a description of the WMR methodology, mathematical details of the microstructure model, and additional empirical results.

### Investigations

Law enforcement and regulatory authorities in the United States, United Kingdom, European Union, Switzerland, Germany, Asia, Australia, New Zealand, and the international Financial Stability Board have been investigating the forex trading activities of the world's largest banks since 2013. The first penalties arising from the investigation were announced in the U.K. by the Financial Conduct Authority (FCA). On November 11, 2014, the FCA imposed fines totaling \$1.7 billion on Citibank, HSBC, JPMorgan Chase & Co., The Royal Bank of Scotland, and UBS for failing to control their forex trading in G10 currencies, specifically with respect to trading around the Fix.<sup>25</sup> The FCA also released transcripts detailing examples of misconduct by traders attempting to manipulate the Fix.<sup>26</sup> Further penalties were imposed in the U.S. in 2015. On May 20, the U.S. Department of Justice (DoJ) announced plea agreements with Citicorp, JPMorgan Chase & Co., Barclays, and The Royal Bank of Scotland in which the banks admitted to manipulating and rigging the Fixes and agreed to pay criminal fines totaling more than \$2.5 billion.<sup>27</sup> Additional penalties

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<sup>25</sup>See, FCA fines five banks £1.1 billion for FX failings and announces industry-wide remediation programme (available at <http://www.fca.org.uk/news/fca-fines-five-banks-for-forex-failings>). On May 20, 2015, the FCA also fined Barclays £284,432,000 (\$441,000,000): See, FCA fines Barclays £284,432,000 for forex failings (available at <http://fca.org.uk/news/fca-fines-barclays-for-forex-failings>).

<sup>26</sup>See, e.g., FCA Final Notice to Citibank N.A., No. 124704, Nov. 11, 2014 (available at <http://www.fca.org.uk/your-fca/documents/final-notice/2014/citibank-na>).

<sup>27</sup>Citicorp agreed to pay a fine of \$925 million. Barclays agreed to pay a fine of \$650 million. JPMorgan agreed to pay a fine of \$550 million. RBS agreed to pay a fine of \$395 million. See, DOJ Citigroup Plea Agreement, May 20, 2015 (available at: <http://www.justice.gov/file/440486/download>), DOJ Barclays Plea Agreement, May 20, 2015 (available at: <http://www.justice.gov/file/440481/download>), DOJ JPMorgan Plea Agreement, May 20, 2015 (available at: <http://www.justice.gov/file/440491/download>) and DOJ RBS Plea Agreement, May 20, 2015 (available at: <http://www.justice.gov/file/440496/download>). On May 20, 2015, UBS AG pleaded guilty to manipulating LIBOR and other benchmark interest rates and paid a \$230 million criminal penalty, after the DOJ determined UBS breached its earlier Non-Prosecution Agreement resolving the LIBOR investigation. UBS admitted to coordinating the trading of the EUR/USD currency pair in connection with ECB and WMR benchmark currency 'fixes'. See, DOJ UBS Plea Agreement, May 20, 2015 (available at: <http://www.justice.gov/file/440521/download>).

have been imposed by the Federal Reserve,<sup>28</sup> the Commodity Futures Trading Commission<sup>29</sup>, the Office of the Comptroller of the Currency<sup>30</sup> and the New York Department of Financial Services.<sup>31</sup> Criminal and regulatory investigations into forex trading are on-going in many countries.

## WMR Methodology

The following description of the original WMR methodology was taken from The WM Company website (<http://www.wmcompany.com>) in June 2014:

“Over a one-minute Fix period, bid and offer order rates from the order matching systems and actual trades executed are captured every second from 30 seconds before to 30 seconds after the time of the Fix. Trading occurs in milliseconds on the trading platforms and therefore not every trade or order is captured, just a sample. Trades are identified as a bid or offer and a spread is applied to calculate the opposite bid or offer.

Using valid rates over the Fix period, the median bid and offer are calculated independently and then the mid rate is calculated from these median bid and offer rates, resulting in a mid trade rate and a mid order rate. A spread is then applied to calculate a new trade rate bid and offer and a new order rate bid and offer. Subject to a minimum number of valid trades being captured over the Fix period, these new trade rates are used for the Fix; if there are insufficient trade rates, the new order rates are used for the Fix.”

## Model Details

This Appendix provides mathematical details of the microstructure model, and shows that the equilibrium takes the form shown in the Proposition. Evans (2011) provides a detailed description of how this model can be solved when round F is missing, so below I emphasize the new elements in the solution.

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<sup>28</sup>On May 20, 2015, the Federal Reserve announced the following fines: \$342 million each for UBS, Barclays, Citigroup, and JPMorgan; \$274 million for the Royal Bank of Scotland; and \$205 million for Bank of America. See, Press Release, Board of Governors of the Federal Reserve System (available at <http://www.federalreserve.gov/newsevents/press/enforcement/20150520a.htm>).

<sup>29</sup>Barclays was fined \$400 million by the CFTC on May 20, 2015. The CFTC had fined Citibank and JPMorgan \$310 million, and issued fines of \$290 million each for RBS and UBS and \$275 million for HSBC in November 2014.

<sup>30</sup>On November 12, 2014, the Office of the Comptroller of the Currency assessed penalties of \$250 million against Bank of America, \$350 million against Citibank, and \$350 million against JPMorgan.

<sup>31</sup>On May 20, 2015, the NYDFS fined Barclays \$485 million and ordered the termination of eight employees.

## Market Participants

**Investors** The forex orders from investor  $n$  during day  $t$  are determined by their desire to maximize expected utility defined over wealth on day  $t + 1$  :

$$\mathcal{U}_{n,t}^i = \mathbb{E} \left[ -\theta \exp(-\theta W_{n,t+1}^i) | \Omega_{n,t}^i \right], \quad (10)$$

with  $\theta > 0$  where  $W_{n,t+1}^i$  is the wealth of investor  $n$  at the start of round  $i$  on day  $t + 1$ . The information available to investor  $n$  at the start of round  $i$  on day  $t$  is denoted by  $\Omega_{n,t}^i$ .

At the start of day  $t$ , investors receive two pieces of information. First, everyone learns the dividend paid by each unit of forex,  $D_t$ . Second, each investor  $n$  receives foreign income  $Y_{n,t}$ , that comprises an aggregate component,  $Y_t$ , and an idiosyncratic component  $\xi_{n,t}$ . The value of  $Y_{n,t}$  represents private information to each investor, but they do not initially observe either component. In equilibrium each investor learns the value of  $Y_t$  by the end of day  $t$ . In the interim, the conditional distribution of  $Y_t$  is given by  $Y_t | Y_{n,t} \sim N(\kappa_n Y_{n,t}, (1 - \kappa_n) \sigma_Y^2)$ , where  $\kappa_n \equiv \sigma_Y^2 / (\sigma_Y^2 + \sigma_\xi^2)$ .

Investors face makes three decisions each day. In rounds I and III they choose their trades with dealers, while in round II they choose their Fix order which is filled in round F. Let  $A_{n,t}^i$  denote investor  $n$ 's holding of forex at the end of round  $i$ . Since dealers quote common prices in equilibrium (i.e.  $S_{d,t}^i = S_t^i$ ), the budget constraints facing the investor are

$$W_{n,t}^{\text{III}} = A_{n,t}^{\text{I}}(S_t^{\text{III}} - S_t^{\text{I}}) + (A_{n,t}^{\text{F}} - A_{n,t}^{\text{I}})(S_t^{\text{III}} - S_t^{\text{F}}) + W_{n,t}^{\text{I}} + S_t^{\text{I}} Y_{n,t}, \quad \text{and} \quad (11a)$$

$$W_{n,t+1}^{\text{I}} = A_{n,t}^{\text{III}}(S_{t+1}^{\text{I}} + D_{t+1} - (1+r)S_t^{\text{III}}) + (1+r)W_{n,t}^{\text{III}}. \quad (11b)$$

In round I,  $A_{n,t}^{\text{I}}$  is chosen to maximize  $\mathcal{U}_{n,t}^{\text{I}}$  subject to (11) with private information  $\Omega_{n,t}^{\text{I}} = \left\{ \{S_{d,t}^{\text{I}}\}_{d=1}^{\text{D}}, Y_{n,t}, D_t, \Omega_{n,t-1}^{\text{III}} \right\}$ . The forex orders of investor  $n$  are  $A_{n,t}^{\text{I}} - A_{n,t-1}^{\text{III}} - Y_{n,t}$ . In round II, the investor chooses  $A_{n,t}^{\text{F}}$  to maximize  $\mathcal{U}_{n,t}^{\text{II}}$  subject to (11) with private information  $\Omega_{n,t}^{\text{II}} = \left\{ \{S_{d,t}^{\text{II}}\}_{d=1}^{\text{D}}, \Omega_{n,t}^{\text{I}} \right\}$ . The investors' Fix order is  $A_{n,t}^{\text{F}} - A_{n,t}^{\text{I}}$ . In round III the investor chooses  $A_{n,t}^{\text{III}}$  to maximize  $\mathcal{U}_{n,t}^{\text{III}}$  subject to (11b) with  $\Omega_{n,t}^{\text{III}} = \left\{ \{S_{d,t}^{\text{III}}\}_{d=1}^{\text{D}}, S_t^{\text{F}}, \Omega_{n,t}^{\text{II}} \right\}$ . Their round III forex orders are  $A_{n,t}^{\text{III}} - A_{n,t}^{\text{F}}$ .

**Dealers** Each dealer  $d$  makes decisions during day  $t$  to maximize expected utility

$$\mathcal{U}_{d,t}^i = \mathbb{E} \left[ -\theta \exp(-\theta W_{d,t+1}^i) | \Omega_{d,t}^i \right],$$



where  $W_{d,t}^i$  and  $\Omega_{d,t}^i$  denote the wealth and information of dealer  $d$  at the start of round  $i$  on day  $t$ . The problem for each dealer is to choose the price quotes  $S_{d,t}^i$  in rounds  $i = \{I, II, F, III\}$  and inter-dealer trades,  $T_{d,t}^i$ , in rounds II, F and III to maximize expected utility given the following sequence of budget constraints:

$$W_{d,t}^{II} = W_{d,t}^I + (A_{d,t}^I - Z_{d,t}^I)(S_t^{II} - S_t^I) + Z_{d,t}^I(S_{d,t}^I - S_t^I), \quad (12a)$$

$$W_{d,t}^F = W_{d,t}^{II} + (A_{d,t}^{II} + T_{d,t}^{II} - Z_{d,t}^{II})(S_t^F - S_t^{II}) + Z_{d,t}^{II}(S_{d,t}^{II} - S_t^{II}), \quad (12b)$$

$$W_{d,t}^{III} = W_{d,t}^F + (A_{d,t}^F + T_{d,t}^F - Z_{d,t}^F - F_{d,t})(S_t^{III} - S_t^F) \quad (12c)$$

$$W_{d,t+1}^I = (1+r)W_{d,t}^{III} + (A_{d,t}^{III} + T_{d,t}^{III} - Z_{d,t}^{III})(S_{t+1}^I + D_{t+1} - (1+r)S_t^{III}) + Z_{d,t}^{III}(S_{d,t}^{III} - S_t^{III}). \quad (12d)$$

where  $A_{d,t}^i$  denotes the dealer's forex holding at the start of round  $i$ .

Dealers choose their quotes in each round, and their trades in rounds II, F and III to maximize expected utility  $\mathcal{U}_{d,t}^i$  subject to the budget constraints in (12) with their available information. The information available to dealer  $d$  at the start of round I is  $\Omega_{d,t}^I = \{D_t, \Omega_{d,t-1}^{III}\}$ . At the start of round II the dealer knows his Fix orders as well as the quotes and his trades from round I:  $\Omega_{d,t}^{II} = \{\{S_{d,t}^I\}_{d=1}^D, Z_{d,t}^I, F_{d,t}, \Omega_{d,t}^I\}$ . Importantly individual dealers do not know aggregate imbalance in trades from round I or the aggregate imbalance in Fix orders. By the start of the Fix round individual dealers have seen round II quotes and aggregate order flow, so  $\Omega_{d,t}^F = \{X_t^II, \{S_{d,t}^{II}\}_{d=1}^D, S_{B,t}^{II}, Z_{d,t}^{II}, \Omega_{d,t}^{II}\}$ . Finally, at the start of round III individual dealers know the Fix benchmark and the aggregate order from round F, so  $\Omega_{d,t}^{III} = \{X_t^F, \{S_{d,t}^F\}_{d=1}^D, Z_{d,t}^F, \Omega_{d,t}^F\}$ .

**The Broker** The foreign exchange broker chooses quotes in rounds II, F and III,  $S_{B,t}^{II}$ ,  $S_{B,t}^F$  and  $S_{B,t}^{III}$ , to maximize expected utility defined over wealth on day  $t+1$ :  $\mathcal{U}_{B,t}^i = \mathbb{E}[-\theta \exp(-\theta W_{B,t+1}^I) | \Omega_{B,t}^i]$ , where  $W_{B,t}^i$  and  $\Omega_{B,t}^i$  denote the wealth and information of the broker at the start of round  $i$  on day  $t$ . The broker's wealth follows the dynamics of dealer  $d$ 's wealth in (12) except that  $Z_{d,t}^I = 0$ ,  $F_{d,t} = 0$  and  $T_{d,t}^{II} = T_{d,t}^F = T_{d,t}^{III} = 0$  because brokers do not receive customer orders in round I, Fix orders, nor can they initiate trades in rounds II, F and III. The information available to brokers evolves in the same way as that of dealer  $d$  with  $Z_{d,t}^I = 0$  and  $F_{d,t} = 0$

## Solving for the Equilibrium

The steps here closely follow those on pages 279-287 in Evans (2011), so I concentrate on the new elements arising from the introduction of the Fix.

**Information** Consider the common information of dealers at the start of round  $i$ :  $\Omega_{D,t}^i = \bigcap_d \Omega_{d,t}^i$ . In round I common information is  $\Omega_{D,t}^I = \{D_t, \Omega_{D,t-1}^{III}\}$ . Trading between dealers and investors in round I does not change dealer' common information so  $\Omega_{D,t}^{II} = \Omega_{D,t}^I$ .

Next we turn to the common information revealed by trading in rounds II and F. Equations (4) and (5a) imply that aggregate order flows in rounds II and F are

$$X_t^{II} = \sum_{d=1}^D \alpha_Z^{II} Z_{d,t}^I + \alpha_A^{II} A_{t-1} = \alpha_Z^{II} \beta Y_t + \alpha_A^{II} A_{t-1}, \quad (13)$$

$$X_t^F = \sum_{d=1}^D \alpha_Z^F Z_{d,t}^I + \sum_{d=1}^D \alpha_F^F F_{d,t} + \alpha_A^F A_{t-1} + \alpha_X^F X_t^{II} = \alpha_F^F \beta Y_t + \alpha_F^F F_t + \alpha_A^F A_{t-1} + \alpha_X^F X_t^{II}, \quad (14)$$

In equilibrium (shown below),  $F_t = H_t$  and  $A_t = A_{t-1} + Y_t - H_t$ . Following the steps on page 279 we can use this expression with the equations above to show that order flow from round II reveals the value of  $Y_t$ , so  $\Omega_{D,t}^F = \{Y_t, \Omega_{D,t}^{II}\}$ , and order flow from round F reveals  $H_t$ , so  $\Omega_{D,t}^{III} = \{H_t, \Omega_{D,t}^F\}$ . From these results it follows that (i)  $\mathbb{E}[Y_t | \Omega_{D,t}^I] = 0$ , (ii)  $\mathbb{E}[H_t | \Omega_{D,t}^{II}] = 0$ , and (iii)  $\mathbb{E}[A_{t-1} | \Omega_{D,t}^I] = A_{t-1}$  because  $A_{t-1} = \sum_{i=1}^{\infty} (Y_{t-i} - H_{t-i})$  and  $Y_{t-i} - H_{t-i} \in \Omega_{D,t}^I$  for  $i \geq 1$ . Consequently,  $X_t^{II} - \mathbb{E}[X_t^{II} | \Omega_{D,t}^I] = \alpha_Z^{II} \beta Y_t$  and  $X_t^F - \mathbb{E}[X_t^F | \Omega_{D,t}^I] = \alpha_F^F H_t$ . We can therefore rewrite (3b) and (3c) as

$$S_t^F = S_t^{II} + \lambda_A^{II} A_{t-1} + \lambda_X^{II} \alpha_Z^{II} \beta Y_t, \quad (15a)$$

$$S_t^{III} = S_t^F + \lambda_A^F A_{t-1} + \lambda_X^F \alpha_F^F H_t, \quad (15b)$$

Now consider the common information of investors,  $\Omega_t^i = \bigcap_n \Omega_{n,t}^i$ . In round I all investors observe the dividend shock from which they compute the value for  $D_t$ . They also observe the common equilibrium quote from all the dealers, so  $\Omega_t^I = \{S_t^I, D_t, \Omega_{t-1}^{III}\}$ . Equation (15) implies that all investors know the values of  $Y_t$  and  $H_t$  by the end of each days so  $A_{t-1} \in \Omega_{t-1}^I$ . It therefore follows from (15) that  $\Omega_t^{III} = \{Y_t, H_t, \Omega_{t-1}^I\}$ . In sum, therefore, dealers and investors share the same common information set in rounds I and III:

$$\Omega_{D,t}^I = \Omega_t^I = \{D_t, \Omega_{t-1}^{III}\} \quad \text{and} \quad \Omega_{D,t}^{III} = \Omega_t^{III} = \{Y_t, H_t, \Omega_t^I\}.$$

**Investors Trades** Consider investor  $n$ 's choice of FX holdings in round III,  $A_{n,t}^{\text{III}}$ . As in the PS model, in equilibrium the distribution of excess returns  $\mathcal{R}_{t+1} \equiv S_{t+1}^{\text{I}} + D_{t+1} - (1+r)S_t^{\text{III}}$  is normal conditioned on information,  $\Omega_{n,t}^{\text{III}}$ . Maximizing expected utility  $\mathcal{U}_{n,t}^{\text{III}}$  subject to (11b) gives

$$A_{n,t}^{\text{III}} = \frac{1}{\gamma} \mathbb{E}[S_{t+1}^{\text{I}} + D_{t+1} - (1+r)S_t^{\text{III}} | \Omega_{n,t}^{\text{III}}], \quad (16)$$

where  $\gamma \equiv \theta \mathbb{V}[\mathcal{R}_{t+1} | \Omega_{n,t}^{\text{III}}]$ . In equilibrium, all investors have the same conditional expectations concerning  $S_{t+1}^{\text{I}}$  and  $D_{t+1}$ , so their overnight FX holdings are the same, i.e.,  $A_{n,t}^{\text{III}} = A_t^{\text{III}}$  for all  $n \in [0, 1]$ .

In round I, investor  $n$  chooses  $A_{n,t}^{\text{I}}$  to maximize  $\mathcal{U}_{n,t}^{\text{I}}$  subject to the sequence of budget constraints in (11). In equilibrium, these constraints take the same form as in the PS model (see below) so, like there, optimal round I holdings are given by

$$A_{n,t}^{\text{I}} = \eta_{\text{A}}^{\text{I}} A_{t-1}^{\text{III}} + \eta_{\text{S}}^{\text{I}} \mathbb{E}[S_t^{\text{III}} - S_t^{\text{I}} | \Omega_{n,t}^{\text{I}}]. \quad (17)$$

(The coefficients  $\eta_{\text{A}}^{\text{I}}$  and  $\eta_{\text{S}}^{\text{I}}$  are given by the formulae on pages 299 and 230 in Evans (2011) with  $Y_t - H_t$  replacing  $Y_t$ .) From (15) and (3b) we find that

$$\begin{aligned} \mathbb{E}[S_t^{\text{III}} - S_t^{\text{I}} | \Omega_{n,t}^{\text{I}}] &= (\lambda_{\text{A}}^{\text{II}} + \lambda_{\text{A}}^{\text{F}}) A_{t-1} + \lambda_{\text{X}}^{\text{II}} \alpha_{\text{Z}}^{\text{II}} \beta \mathbb{E}[Y_t | \Omega_{n,t}^{\text{I}}] + \lambda_{\text{X}}^{\text{F}} \alpha_{\text{F}}^{\text{F}} \mathbb{E}[H_t | \Omega_{n,t}^{\text{I}}]. \\ &= (\lambda_{\text{A}}^{\text{II}} + \lambda_{\text{A}}^{\text{F}}) A_{t-1} + \lambda_{\text{X}}^{\text{II}} \alpha_{\text{Z}}^{\text{II}} \beta \kappa_n Y_{n,t} \end{aligned}$$

so

$$A_{n,t}^{\text{I}} = \eta_{\text{S}}^{\text{I}} \lambda_{\text{X}}^{\text{II}} \alpha_{\text{Z}}^{\text{II}} \beta \kappa_n Y_{n,t} + (\eta_{\text{S}}^{\text{I}} (\lambda_{\text{A}}^{\text{II}} + \lambda_{\text{A}}^{\text{F}}) + \eta_{\text{A}}^{\text{I}}) A_{t-1}^{\text{III}} = \eta_{\text{S}}^{\text{I}} \lambda_{\text{X}}^{\text{II}} \alpha_{\text{Z}}^{\text{II}} \beta \kappa_n Y_{n,t} + A_{t-1}^{\text{III}}$$

because  $\lambda_{\text{A}}^{\text{II}} + \lambda_{\text{A}}^{\text{F}} = (1 - \eta_{\text{A}}^{\text{I}}) / \eta_{\text{S}}^{\text{I}}$  (see below). By definition, the FX order from investor  $n$  in round I is  $A_{n,t}^{\text{I}} - A_{n,t-1}^{\text{III}} - Y_{n,t} = (\eta_{\text{S}}^{\text{I}} \lambda_{\text{X}}^{\text{II}} \alpha_{\text{Z}}^{\text{II}} \beta \kappa_n - 1) Y_{n,t}$ . Since all dealers quote a common round I price in equilibrium, each dealer receives an equal share of investors' FX orders,  $Z_{d,t}^{\text{I}} = (\beta/D) Y_t + \varepsilon_{d,t}$  with  $\beta = 1 / (\eta_{\text{S}}^{\text{I}} \lambda_{\text{X}}^{\text{II}} \alpha_{\text{Z}}^{\text{II}} \kappa_n - 1)$ , as shown in (5a).

At the start of round II investors optimally choose their Fix orders,  $A_{n,t}^{\text{F}} - A_{n,t}^{\text{I}}$  to maximize  $\mathcal{U}_{n,t}^{\text{II}}$  subject to the sequence of budget constraints in (11). Once again we can follow Evans (2011) to show that the optimal choice is

$$A_{n,t}^{\text{F}} - A_{n,t}^{\text{I}} = \eta_{\text{A}}^{\text{F}} A_{n,t-1}^{\text{III}} + \eta_{\text{S}}^{\text{F}} \mathbb{E}[S_t^{\text{III}} - S_t^{\text{F}} | \Omega_{n,t}^{\text{II}}]. \quad (18)$$

Substituting for  $S_t^{\text{III}} - S_t^{\text{F}}$  from (15) gives  $\mathbb{E}[S_t^{\text{III}} - S_t^{\text{F}}|\Omega_{n,t}^{\text{II}}] = \lambda_A^{\text{F}}A_{t-1}$ , so traders' Fix orders are  $A_{n,t}^{\text{F}} - A_{n,t}^{\text{I}} = (\eta_A^{\text{F}} + \eta_S^{\text{F}}\lambda_A^{\text{F}})A_{t-1}$ . We will see below that when dealers quote prices in round F to efficiently share risk,  $\eta_A^{\text{F}} + \eta_S^{\text{F}}\lambda_A^{\text{F}} = 0$ . Consequently,  $A_{n,t}^{\text{F}} = A_{n,t}^{\text{I}}$ , so traders find it optimal not to submit Fix orders in equilibrium and  $F_t = \int_n (A_{n,t}^{\text{F}} - A_{n,t}^{\text{I}}) dn + H_t = H_t$  (noted above). Furthermore, when  $A_{n,t}^{\text{F}} = A_{n,t}^{\text{I}}$  the budget constraints in (11b) simplify to those in the PS model, which is why (16) and (17) take the same form.

**Quotes** As in the PS model, dealers and the broker quote prices to support an efficient risk-sharing allocation of forex holdings. This means that the round III quote is chosen so that investors are willing to hold the entire stock of forex,  $A_t$ . Recall that the distribution of  $\mathcal{R}_{t+1}$  conditioned on individual investors information,  $\Omega_{n,t}^{\text{III}}$ , is the same across all investors, and is equal to the distribution conditioned on common information,  $\Omega_t^{\text{III}}$ . Under these circumstances, (16) implies that the aggregate demand for forex is  $A_t = \frac{1}{\gamma}\mathbb{E}[\mathcal{R}_{t+1}|\Omega_t^{\text{III}}]$ . Combining this expression with the definition of  $\mathcal{R}_{t+1}$  gives,

$$S_t^{\text{III}} = \frac{1}{1+r}\mathbb{E}[S_{t+1}^{\text{III}} + D_{t+1}|\Omega_t^{\text{III}}] - \frac{1}{1+r}\mathbb{E}[S_{t+1}^{\text{III}} - S_{t+1}^{\text{I}}|\Omega_t^{\text{III}}] - \frac{\gamma}{(1+r)}A_t.$$

Equation (3) of Proposition 1 implies that  $\mathbb{E}[S_{t+1}^{\text{III}} - S_{t+1}^{\text{I}}|\Omega_t^{\text{III}}] = (\lambda_A^{\text{II}} + \lambda_A^{\text{F}})A_t$ . Making this substitution in the expression above and solving forward gives

$$S_t^{\text{III}} = \sum_{i=1}^{\infty} \left(\frac{1}{1+r}\right)^i \mathbb{E}[D_{t+i} - (\gamma + \lambda_A^{\text{II}} + \lambda_A^{\text{F}})A_{t+i-1}|\Omega_t^{\text{III}}]. \quad (19)$$

This equation identifies the value for the round III quote needed to induce investors to hold  $A_t$  given their expectations concerning future dividends and holdings,  $D_{t+1+i}$ , and  $A_{t+i}$ .

Efficient risk-sharing and market clearing imply that the aggregate investor demand for forex,  $A_t$ , be equal to aggregate holdings at the end of day  $t-1$ ,  $A_{t-1}$ , plus foreign income received during round I,  $Y_t = \int_0^1 Y_{n,t}dn$ , minus the the forex needed to fill the hedger Fix orders,  $H_t$ . Hence, the equilibrium dynamics of investors forex holdings follow  $A_t = A_{t-1} + Y_t - H_t$  (as noted above). Using this equation to forecast investor demand, and the facts that  $\mathbb{E}[D_{t+i}|\Omega_t^{\text{III}}] = D_t$ ,  $\mathbb{E}[Y_{t+i}|\Omega_t^{\text{III}}] = 0$  and  $\mathbb{E}[H_{t+i}|\Omega_t^{\text{III}}] = 0$  for all  $i > 0$ , we can rewrite (19) as

$$S_t^{\text{III}} = \frac{1}{r}D_t - \frac{1}{r}(\gamma + \lambda_A^{\text{II}} + \lambda_A^{\text{F}})A_t. \quad (20)$$

We have thus found the value of the round III quote that achieves an efficient risk-sharing allocation.

Dealers' quotes in round I and II are determined in exactly the same way as in the PS model. Dealers choose  $S_t^I$  such that the allocation of holdings is ex ante efficient conditioned on common information  $\Omega_{D,t}^I$ , i.e.  $\mathbb{E}[Z_{d,t}^I|\Omega_{D,t}^I] = 0$ . To find this value for  $S_t^I$ , we take expectations with respect to dealers' common information on both sides of the market clearing condition in (2a):

$$\sum_{d=1}^D \mathbb{E}[Z_{d,t}^I|\Omega_{D,t}^I] = \int_0^1 E[A_{n,t}^I - A_{n,t-1}^{III} - Y_{n,t}|\Omega_{D,t}^I] dn.$$

Substituting for  $A_{n,t}^I$  with (17) and noting that  $\mathbb{E}[Y_{n,t}|\Omega_{D,t}^I] = 0$  for all  $n$  gives

$$\sum_{d=1}^D \mathbb{E}[Z_{d,t}^I|\Omega_{D,t}^I] = \eta_S^I \mathbb{E}[S_t^{III} - S_t^I|\Omega_{D,t}^I] + (\eta_A^I - 1) A_{t-1}^{III}.$$

Finally, we impose the risk-sharing restriction  $\mathbb{E}[Z_{d,t}^I|\Omega_{D,t}^I] = 0$ , and solve for  $S_t^I$  as

$$S_t^I = \mathbb{E}[S_t^{III}|\Omega_{D,t}^I] - (\lambda_A^H + \lambda_A^F) A_{t-1}. \quad (21)$$

where  $\lambda_A^H + \lambda_A^F \equiv (1 - \eta_A^I)/\eta_S^I$ .

In rounds II and F dealers choose  $S_t^{II}$  and  $S_t^F$  so that  $\mathbb{E}[F_{d,t}|\Omega_{D,t}^{II}] = 0$  and  $\mathbb{E}[F_{d,t}|\Omega_{D,t}^F] = 0$ . In words, dealers choose quotes so there is no expected imbalance in Fix orders conditioned on common dealer information. Recall that Fix orders comprise orders from hedgers and investors,  $F_t = H_t + \int_n (A_{n,t}^F - A_{n,t}^I) dn$ . Since  $\mathbb{E}[H_t|\Omega_{D,t}^{II}] = 0$ , dealers will achieve an ex ante efficient allocation with their choice for  $S_t^{II}$  provided  $\mathbb{E}[A_{n,t}^F - A_{n,t}^I|\Omega_{D,t}^{II}] = 0$ . We showed above that the optimal choice for  $A_{n,t}^F - A_{n,t}^I$  depends on  $A_{t-1}$  and  $\mathbb{E}[S_t^{III} - S_t^F|\Omega_{D,t}^{II}]$ . Neither of these terms depends on the value of  $S_t^{II}$ . Dealers can therefore set

$$S_t^{II} = S_t^I \quad (22)$$

to eliminate the risk of unexpected round I trades (just as in the PS model) and use their choice for  $S_t^F$  to ensure that  $\mathbb{E}[F_{d,t}|\Omega_{D,t}^{II}] = 0$  and  $\mathbb{E}[F_{d,t}|\Omega_{D,t}^F] = 0$ . This is achieved by setting

$$S_t^F = S_t^I + \lambda_A^H A_{t-1} + \lambda_X^H \alpha_Z^H \beta Y_t, \quad (23)$$

with  $\lambda_A^H = (1 - \eta_A^I)/\eta_S^I - (\eta_A^F/\eta_S^F)$  and  $\lambda_X^H = -\frac{1}{r}(\gamma + \lambda_A^H + \lambda_A^F)/\alpha_Z^H \beta$ . (Recall that both  $A_{t-1}$  and  $Y_t$  are in  $\Omega_{D,t}^F$ ) This choice for  $S_t^F$  implies that  $S_t^{III} - S_t^F = \frac{1}{r}(\gamma + \lambda_A^H + \lambda_A^F)H_t + \lambda_A^F A_{t-1}$ , so traders'

optimal fix orders are

$$A_{n,t}^F - A_{n,t}^I = \eta_A^F A_{n,t-1}^{III} + \eta_S^F \mathbb{E}[S_t^{III} - S_t^F | \Omega_{n,t}^I] = (\eta_A^F + \eta_S^F \lambda_A^F) A_{t-1} = 0.$$

This means that  $F_t = H_t + \int_n (A_{n,t}^F - A_{n,t}^I) dn = H_t$ , so  $\mathbb{E}[F_{d,t} | \Omega_{D,t}^F] = \frac{1}{D} \mathbb{E}[H_t | \Omega_{D,t}^F] + \mathbb{E}[\xi_t | \Omega_{D,t}^F] = 0$  and  $\mathbb{E}[F_{d,t} | \Omega_{D,t}^I] = 0$  (by iterated expectations) as required. Finally, it is straightforward to check that (20) - (23) can be rewritten in the form of (3) with  $\lambda_X = -(\gamma + \lambda_A^I + \lambda_A^F) / (\beta r \alpha_Z)$  and  $\lambda_X^F = \frac{1}{\alpha_F^F r} (\gamma + \lambda_A^I + \lambda_A^F)$ .

**Dealer Trades** All that now remains is to confirm that inter-dealer trades take the form of (4). Dealers trades in round III take the same form as in the PS model and allow each dealer to eliminate their overnight FX holdings. Hence, in equilibrium, the budget constraints in (12b) and (12d) of dealer  $d$  become

$$W_{d,t+1}^I = (1+r)[W_{d,t}^F + \hat{A}_{d,t}^F (S_t^{III} - S_t^F) - (Z_{d,t}^F - \mathbb{E}[Z_{d,t}^F | \Omega_{d,t}^F]) (S_t^{III} - S_t^F)], \quad (24)$$

where  $\hat{A}_{d,t}^F \equiv A_{d,t}^F + T_{d,t}^F - \mathbb{E}[Z_{d,t}^F | \Omega_{d,t}^F] - F_{d,t}$  is dealer's desired FX position. In round F dealers choose  $T_{d,t}^F$  given the incoming orders from other dealers following BNE strategies,  $Z_{d,t}^F$ , and the fix orders he took at the start of round II,  $F_{d,t}$ , such that  $\hat{A}_{d,t}^F$  maximizes expected utility given (24).<sup>32</sup> As in the PS model, the dealer's desired position is given by

$$\hat{A}_{d,t}^F = \frac{1}{1+D} \varphi_A^F A_{t-1} + \frac{1}{1+D} \varphi_Y^F \mathbb{E}[F_t | \Omega_{d,t}^I], \quad (25)$$

where  $\varphi_Y^F$  and  $\varphi_A^F$  are coefficients again given in the appendix of Evans (2011). We showed above that  $S_t^{III} - S_t^F = \lambda_A^F A_{t-1} + \lambda_X^F \alpha_F^F H_t$ , (with  $H_t = F_t$ ) so dealer's use their private forecast of aggregate imbalance in Fix orders to determine their desired position. Dealers' trades in round II are derived from their desired position in an analogous fashion:

$$\hat{A}_{d,t}^I = \frac{1}{1+D} \varphi_A^I A_{t-1} + \frac{1}{1+D} \varphi_Y^I \mathbb{E}[Y_t | \Omega_{d,t}^I], \quad (26)$$

because  $S_t^F - S_t^{II} = \lambda_A^I A_{t-1} + \lambda_X^I \alpha_Z^I \beta Y_t$ . Dealers' desired position in round II depend on the private forecasts of aggregate income. As in the PS model, I assume that dealers view the shocks that distribute traders orders in round  $i$  and Fix orders in round II as i.i.d. normal variables so their

<sup>32</sup>Notice that dealer  $d$  cannot condition his choice for  $T_{d,t}^F$  on actual incoming orders,  $Z_{d,t}^F$ , because all dealers must act simultaneously. Instead, each dealer must choose  $T_{d,t}^F$  based on his expectations regarding incoming orders  $\mathbb{E}[Z_{d,t}^F | \Omega_{d,t}^I]$ .

private forecasts are given by  $\mathbb{E}[Y_t|\Omega_{d,t}^{\text{II}}] = \kappa_d Z_{d,t}$  and  $\mathbb{E}[F_t|\Omega_{d,t}^{\text{II}}] = \kappa_{\text{F}} F_{d,t}$ . (Recall from (5a) that  $Z_{d,t}$  is a noisy signal of aggregate income.)

We can now compute the BNE trading strategies for each dealer in rounds II and F. If all other dealers trade according to (4), and orders are equally split between the broker and the dealers because they quote the same price, incoming order flow from other dealers is

$$Z_{d,t}^{\text{II}} = \frac{1}{1+\text{D}} \alpha_{\text{Z}}^{\text{II}} \beta Y_t + \frac{1}{1+\text{D}} \alpha_{\text{A}}^{\text{II}} \alpha_{\text{A}} A_{t-1}.$$

Dealer  $d$ 's forecast of this order flow is therefore

$$\mathbb{E}[Z_{d,t}^{\text{II}}|\Omega_{d,t}^{\text{II}}] = \frac{1}{1+\text{D}} \alpha_{\text{Z}}^{\text{II}} \beta E[Y_t|\Omega_{d,t}^{\text{II}}] + \frac{1}{1+\text{D}} \alpha_{\text{A}}^{\text{II}} A_{t-1}.$$

By definition, dealer  $d$ 's trade is given by  $T_{d,t}^{\text{II}} = \hat{A}_{d,t}^{\text{II}} - A_{d,t}^{\text{II}} + \mathbb{E}[Z_{d,t}^{\text{II}}|\Omega_{d,t}^{\text{II}}]$ . Since dealers hold no overnight positions, their FX holdings at the start of round II simply reflect the customer orders they filled in round I, i.e.,  $A_{d,t}^{\text{II}} = -Z_{d,t}^{\text{I}}$ . Combining this fact with the definition above gives

$$\begin{aligned} T_{d,t}^{\text{II}} &= \hat{A}_{d,t}^{\text{II}} + Z_{d,t}^{\text{I}} + \mathbb{E}[Z_{d,t}^{\text{II}}|\Omega_{d,t}^{\text{II}}], \\ &= \frac{1}{1+\text{D}} (\varphi_{\text{Y}}^{\text{II}} + \alpha_{\text{Z}}^{\text{II}} \beta) \mathbb{E}[Y_t|\Omega_{d,t}^{\text{II}}] + Z_{d,t}^{\text{I}} + \frac{1}{1+\text{D}} (\varphi_{\text{A}}^{\text{II}} + \alpha_{\text{A}}^{\text{II}}) A_{t-1}, \\ &= \left(1 + \frac{1}{1+\text{D}} (\varphi_{\text{Y}}^{\text{II}} + \alpha_{\text{Z}}^{\text{II}} \beta) \kappa_d\right) Z_{d,t}^{\text{I}} + \frac{1}{1+\text{D}} (\varphi_{\text{A}}^{\text{II}} + \alpha_{\text{A}}^{\text{II}}) A_{t-1}. \end{aligned} \quad (27)$$

Thus, the BNE strategy for each dealer is to initiate an inter-dealer trade that is a linear function of his own customer orders,  $Z_{d,t}^{\text{I}}$ , and the outstanding stock of FX,  $A_{t-1}$ , as shown in equation (4a) of the Proposition. Equating coefficients gives the formulas for the  $\alpha^{\text{II}}$  coefficients.

In the Fix round incoming orders are

$$\begin{aligned} Z_{d,t}^{\text{F}} &= \frac{1}{1+\text{D}} \alpha_{\text{Z}}^{\text{F}} \beta Y_t + \frac{1}{1+\text{D}} \alpha_{\text{A}}^{\text{F}} A_{t-1} + \frac{1}{1+\text{D}} \alpha_{\text{F}}^{\text{F}} F_t + \frac{1}{1+\text{D}} \alpha_{\text{X}}^{\text{F}} X_t^{\text{II}}, \\ &= \frac{1}{1+\text{D}} (\alpha_{\text{Z}}^{\text{F}} + \alpha_{\text{X}}^{\text{F}} \alpha_{\text{Z}}^{\text{II}}) \beta Y_t + \frac{1}{1+\text{D}} (\alpha_{\text{A}}^{\text{F}} + \alpha_{\text{X}}^{\text{F}} \alpha_{\text{A}}^{\text{II}}) A_{t-1} + \frac{1}{1+\text{D}} \alpha_{\text{F}}^{\text{F}} F_t \end{aligned}$$

so

$$\mathbb{E}[Z_{d,t}^{\text{F}}|\Omega_{d,t}^{\text{F}}] = \frac{1}{1+\text{D}} (\alpha_{\text{Z}}^{\text{F}} + \alpha_{\text{X}}^{\text{F}} \alpha_{\text{Z}}^{\text{II}}) \beta E[Y_t|\Omega_{d,t}^{\text{II}}] + \frac{1}{1+\text{D}} (\alpha_{\text{A}}^{\text{F}} + \alpha_{\text{X}}^{\text{F}} \alpha_{\text{A}}^{\text{II}}) A_{t-1} + \frac{1}{1+\text{D}} \alpha_{\text{F}}^{\text{F}} E[F_t|\Omega_{d,t}^{\text{F}}].$$

Substituting these terms in the definition,  $\hat{A}_{d,t}^{\text{F}} - A_{d,t}^{\text{F}} = \hat{A}_{d,t}^{\text{F}} - \hat{A}_{d,t}^{\text{II}} + (Z_{d,t}^{\text{II}} - \mathbb{E}[Z_{d,t}^{\text{II}}|\Omega_{d,t}^{\text{II}}])$ , and

simplifying with (26) and (25) gives

$$\hat{A}_{d,t}^F - A_{d,t}^F = \frac{1}{1+D} (\varphi_A^F - \varphi_A^H) A_{t-1} + \frac{1}{1+D} \varphi_Y^F \kappa_F F_{n,t} - \frac{1}{1+D} \varphi_Y^H \kappa_d Z_{d,t} + \frac{1}{1+D} \alpha_Z^H \beta (Y_t - \kappa_d Z_{d,t}).$$

Finally, by definition  $T_{d,t}^F = \hat{A}_{d,t}^F - A_{d,t}^F + \mathbb{E}[Z_{d,t}^F | \Omega_{d,t}^F] + F_{d,t}$ . So substituting from above we find that

$$\begin{aligned} T_{d,t}^F &= \frac{1}{1+D} (\varphi_A^F - \varphi_A^H) A_{t-1} + \frac{1}{1+D} \varphi_Y^F \kappa_F F_{n,t} - \frac{1}{1+D} \varphi_Y^H \kappa_d Z_{d,t} + \frac{1}{1+D} \alpha_Z^H \beta (Y_t - \kappa_d Z_{d,t}) \\ &\quad + \frac{1}{1+D} \alpha_F^F \kappa_F F_{d,t} + \frac{1}{1+D} (\alpha_A^F + \alpha_X^F \alpha_A^H) A_{t-1} + \frac{1}{1+D} (\alpha_Z^F + \alpha_X^F \alpha_Z^H) \beta \kappa_d Z_{d,t} + F_{d,t} \\ &= \frac{1}{1+D} (\varphi_A^F - \varphi_A^H + \alpha_A^F + \alpha_X^F \alpha_A^H) A_{t-1} + \frac{1}{1+D} (\varphi_Y^F \kappa_F + \alpha_F^F \kappa_F + 1) F_{d,t} \\ &\quad + \frac{1}{1+D} (X_t^H - \alpha_A^H A_{t-1}) + \frac{1}{1+D} ((\alpha_Z^F + \alpha_X^F \alpha_Z^H - \alpha_Z^H) \beta - \varphi_Y^H) \kappa_d Z_{d,t} \\ &= (\alpha_A^F/D) A_{t-1} + \alpha_F^F F_{d,t} + (\alpha_X^F/D) X_t^H + \alpha_Z^F Z_{d,t} \end{aligned}$$

as shown in the Proposition. Equating coefficients gives the formulas for the  $\alpha^F$  coefficients.

## Additional Empirical Analysis

### Data

Table A.1 provides information on the Gain Capital used in the empirical analysis. The rates are listed in column (i). Columns (ii) and (iii) report the span and scope of the tick-by-tick data for each rate. For 11 currency pairs, I use a decade of tick-by-tick bid and offer rates starting at midnight on December 31 st. 2003. Continuous data is not available for the other currency pairs in 2004 – 2007 so I use tick-by-tick rates starting after midnight on December 31 st. 2007, when continuous data becomes available. The data samples for all the currency pairs end at midnight on December 31 st. 2013. As column (iii) shows, the time series for each currency pair contains tens of millions of data points. Each series contains a date and time stamp, where time is recorded to the nearest 1/100 of a second, and a bid and offer rate. Unlike standard time series, the time between observations is irregular, ranging from a few minutes to a hundredth of a second.

As noted in the text, I checked the accuracy of the Gain data by comparing the mid-points from the tick-by-tick data with the 4:00 pm Fix benchmarks on each trading day in the sample. Fixes are computed as the mid point of the median bid and ask rates across multiple transactions in a one minute window that starts 30 seconds before 4:00 pm. For comparison, I computed an analogous mid-point from the median of the bid and ask rate data on every trading day covered by each currency pair. Differences between this mid-point and the Fixes represent the tracking error of the



Gain data relative to the rates used to determine the Fixes. Table A.1 reports the percentiles of the tracking-error distribution, measured in basis points relative to the Fix benchmark, for each of the currency pairs I study. I separate the tracking errors on end-of-month trading days from the errors on other trading days and report percentiles for both the intra- and end-of-month distributions.

### Bootstrap Distribution

I summarize the behavior of forex prices away from the Fix with a bootstrap distribution of price changes computed from 10,000 randomly chosen times (excluding the times of scheduled releases U.S. macro data, the 4:00 WMR Fix and ECB Fix). Table A.2 reports statistics for this distribution of spot rate changes over horizons of five, fifteen, and thirty minutes. Columns (iii) - (vii) report statistics for the distribution of changes in the log rates expressed in basis points per minute, i.e.,  $\Delta^h s_t \equiv (\ln(S_{t+h}) - \ln(S_t)) * 10000/h$  for horizons  $h = \{5, 15, 60\}$  minutes, where  $S_t$  denotes the mid-point rate a time  $t$ . Columns (viii) and (ix) report the first-order autocorrelation in  $\Delta^h s_{t+h}$  and the p-value for the null of a zero autocorrelation, respectively. Column (x) reports the Kolmogorov-Smirnov (KS) test for the null that the two conditional distributions  $f(\Delta^h s_{t+h} | \Delta^h s_t > 0)$  and  $f(\Delta^h s_{t+h} | \Delta^h s_t \leq 0)$  are the same. The p-value for the test is shown in column (xi).

As Table A.2 shows, the rate-change distributions have several common characteristics across all the currency pairs. First, the dispersion in the rate-change distributions declines as the horizon rises. Columns (iii) and (iv) show that the absolute values for the 5th. and 95th. percentiles of the distributions fall as the horizon rise from five to 30 minutes. The change in dispersion is also reflected by the standard deviations shown in column (v), which fall as the horizon rises. Second, all the rate-change distributions are strongly leptokurtic. As column (vii) shows, the kurtosis statistics across all the currency pairs are large; much larger than the value of three implied by the normal distribution. These statistics indicate that atypically large changes in rates occur quite frequently away from the Fixes and scheduled macro news releases.

The third feature concerns temporal dependence between rate changes. Column (viii) shows that rate changes display some small degree of autocorrelation. Across currency pairs, the autocorrelation is generally negative.<sup>33</sup> This fact accounts for the declining dispersion of the rate-change distributions as the horizon rises, noted above. Although small in (absolute) value, the statistics in column (ix) indicate that many of the estimated autocorrelation coefficients are statistical signifi-

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<sup>33</sup>While the estimated autocorrelations imply that future rate changes are forecastable using past rates, these correlations are computed from the mid-points of the bid and ask rates. As such, the estimated autocorrelations are not a reflection of so-called bid-ask bounce. Nor do they imply that the future returns available to traders (i.e. changes in log rates that account for the bid/offer spread) can be forecast.

Table A.1: Data Characteristics

FX Rate	Data Span	Prices (millions)	Intra Month Trading Days				End-of-Month Trading Days			
			Number	Tracking Error Distribution Percentiles (basis points)			Number	Tracking Error Distribution Percentiles (basis points)		
				(iv)	(v)	(vi)		(vii)	(viii)	(ix)
(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)
A:	EUR/USD	55.370	2420	-1.113	0.055	1.209	117	-1.232	0.109	1.771
	CHF/USD	51.966	2258	-1.510	0.060	1.761	106	-2.462	0.058	2.345
	JPY/USD	38.931	2204	-1.268	0.140	1.790	104	-3.241	0.207	2.608
	USD/GBP	60.859	2421	-1.087	0.083	1.200	116	-1.258	0.051	2.566
B:	CHF/EUR	37.858	2373	-1.144	0.000	1.145	116	-2.135	0.000	1.243
	JPY/EUR	78.813	2421	-1.538	0.021	1.542	117	-4.939	0.090	2.822
	NOK/EUR	15.780	1291	-1.710	0.008	2.015	62	-2.624	0.252	3.829
	NZD/EUR	56.633	1414	-2.211	0.079	2.381	68	-2.927	0.284	3.844
	SEK/EUR	17.424	1288	-1.643	-0.010	1.551	59	-2.336	-0.089	1.980
C:	AUS/GBP	68.169	1476	-1.209	0.225	1.730	69	-2.695	0.544	4.016
	CAD/GBP	57.455	1478	-1.180	0.293	1.777	71	-1.676	0.371	2.578
	CHF/GBP	83.686	2417	-1.476	0.087	1.634	116	-3.412	0.020	2.013
	EUR/GBP	41.643	2339	-1.651	0.114	2.176	115	-2.302	0.157	2.523
	JPY/GBP	88.578	2418	-1.564	0.020	1.582	116	-2.612	0.137	2.378
	NZD/GBP	58.216	1409	-2.197	0.089	2.350	67	-2.529	0.397	5.046
D:	AUS/USD	49.016	2398	-1.601	0.144	2.283	116	-2.373	-0.117	2.053
	CAD/USD	36.163	2404	-1.461	0.138	1.864	116	-1.909	0.256	2.821
	DKK/USD	66.719	1305	-0.696	0.081	0.825	59	-0.779	0.115	0.996
	NOK/USD	55.350	1306	-1.696	0.071	2.152	62	-3.983	0.602	3.999
	SEK/USD	58.296	1297	-1.811	0.053	1.792	59	-2.783	0.101	2.067
	SGD/USD	10.567	1200	-1.440	0.000	1.517	61	-1.980	0.168	1.982

Notes: Columns (i) - (iii) show the data span and the number of quotes (in millions) for each of the currency pairs in the data set. Columns (iv) and (viii) report the number of intra-month and end-of-month trading days for which there are intraday quotes, respectively. Quote errors on each day are defined as the difference between the mid-point of the average bid and ask quotes computed over a one minute window centered on 4:00 pm and the Fix benchmark. Quote errors are expressed in basis points. Columns (v) - (vii) and (ix) - (xi) show the 5th., 50th. and 95th. percentiles of the quote error distribution computed on all intra-month and end-of-month trading days.

cant at standard levels. There is also evidence of temporal dependence from the KS tests reported in column (ix). Under the null of temporal independence, future changes in rates should not depend on the sign of past changes, i.e.,  $f(\Delta^h s_{t+h} | \Delta^h s_t > 0) = f(\Delta^h s_{t+h} | \Delta^h s_t \leq 0)$ . As column (x) shows, this null can easily be rejected at standard levels of significance for most currency pairs and horizons  $h$ .

Table A.2: Bootstrap Distribution

	Price Changes (bps per minute)						Temporal Dependence					
	(i)	horizon (ii)	5% (iii)	95% (iv)	std (v)	skew (vi)	kurtosis (vii)	Autocorrelation (viii)	p-value (ix)	Independence (x)	p-value (xi)	
A:	EUR/USD	5	-1.345	1.378	0.886	-0.033	8.777	-0.018	(0.137)	0.055	(0.000)	
		15	-0.730	0.729	0.466	-0.122	7.694	-0.007	(0.587)	0.047	(0.002)	
		30	-0.468	0.468	0.302	0.057	9.717	0.025	(0.037)	0.047	(0.001)	
	CHF/USD	5	-1.481	1.532	0.968	-0.166	11.873	-0.021	(0.097)	0.051	(0.001)	
		15	-0.774	0.787	0.511	-0.090	8.259	-0.036	(0.005)	0.046	(0.003)	
		30	-0.510	0.492	0.318	-0.235	8.301	0.045	(0.000)	0.051	(0.001)	
	JPY/USD	5	-1.259	1.265	0.818	-0.009	8.457	-0.044	(0.001)	0.049	(0.002)	
		15	-0.657	0.672	0.429	0.310	8.110	-0.047	(0.000)	0.055	(0.000)	
		30	-0.421	0.413	0.276	0.198	9.298	0.033	(0.007)	0.050	(0.001)	
	USD/GBP	5	-1.317	1.338	0.915	0.285	12.967	-0.041	(0.001)	0.043	(0.006)	
		15	-0.717	0.711	0.501	-0.421	20.581	0.028	(0.024)	0.026	(0.251)	
		30	-0.460	0.473	0.329	-0.633	28.025	-0.049	(0.000)	0.047	(0.001)	
	B:	CHF/EUR	5	-0.818	0.889	0.630	0.213	33.326	-0.046	(0.000)	0.072	(0.000)
			15	-0.464	0.463	0.335	0.429	26.405	-0.004	(0.718)	0.057	(0.000)
			30	-0.301	0.282	0.212	0.465	23.065	-0.010	(0.416)	0.047	(0.002)
JPY/EUR		5	-1.607	1.633	1.089	0.234	12.711	-0.007	(0.545)	0.039	(0.016)	
		15	-0.895	0.885	0.585	0.397	11.241	-0.033	(0.007)	0.048	(0.002)	
		30	-0.570	0.567	0.379	0.411	11.997	-0.008	(0.495)	0.034	(0.039)	
NOK/EUR		5	-1.232	1.402	0.854	0.251	9.228	0.035	(0.036)	0.036	(0.209)	
		15	-0.697	0.747	0.487	0.162	9.704	0.005	(0.761)	0.017	(0.958)	
		30	-0.446	0.484	0.319	-0.036	12.685	-0.068	(0.000)	0.083	(0.000)	
NZD/EUR		5	-1.695	1.699	1.170	0.349	15.685	-0.044	(0.006)	0.040	(0.104)	
		15	-0.932	0.904	0.610	-0.188	9.959	-0.059	(0.000)	0.073	(0.000)	
		30	-0.582	0.571	0.383	-0.806	17.827	-0.061	(0.000)	0.066	(0.000)	
SEK/EUR		5	-1.365	1.389	0.885	-0.148	8.334	0.046	(0.007)	0.036	(0.221)	
		15	-0.730	0.778	0.488	0.087	8.384	0.017	(0.314)	0.048	(0.035)	
		30	-0.503	0.484	0.321	-0.092	8.763	-0.039	(0.017)	0.072	(0.000)	

Notes: see below.

Table A.2: Bootstrap Distribution (cont.)

		Price Changes (bps. per minute)					Temporal Dependence				
(i)	horizon (ii)	5% (iii)	95% (iv)	std (v)	skew (vi)	kurtosis (vii)	Autocorrelation (viii)	p-value (ix)	Independence (x)	p-value (xi)	
C:	AUS/GBP	5	-1.683	1.821	1.230	-0.229	17.100	-0.110	(0.000)	0.047	(0.029)
		15	-0.918	0.929	0.639	-0.211	13.506	-0.022	(0.157)	0.017	(0.944)
		30	-0.581	0.591	0.420	-1.893	44.503	-0.097	(0.000)	0.043	(0.045)
	CAD/GBP	5	-1.709	1.722	1.152	-0.064	12.740	-0.085	(0.000)	0.040	(0.084)
		15	-0.931	0.913	0.604	0.080	8.627	0.010	(0.540)	0.029	(0.375)
		30	-0.602	0.580	0.392	-0.080	9.988	-0.129	(0.000)	0.051	(0.010)
	CHF/GBP	5	-1.388	1.390	0.943	0.051	13.442	-0.037	(0.003)	0.067	(0.000)
		15	-0.766	0.726	0.520	0.226	16.612	0.037	(0.003)	0.032	(0.074)
		30	-0.479	0.464	0.342	-0.877	28.940	-0.059	(0.000)	0.048	(0.001)
	EUR/GBP	5	-1.165	1.162	0.764	-0.193	9.183	-0.041	(0.001)	0.054	(0.001)
		15	-0.598	0.629	0.421	-0.147	15.589	0.035	(0.004)	0.019	(0.662)
		30	-0.401	0.418	0.282	0.324	21.871	-0.053	(0.000)	0.068	(0.000)
	JPY/GBP	5	-1.692	1.757	1.181	0.516	14.281	-0.039	(0.001)	0.045	(0.003)
		15	-0.913	0.952	0.640	0.338	16.520	0.013	(0.294)	0.048	(0.001)
		30	-0.578	0.612	0.419	-0.038	23.768	-0.048	(0.000)	0.053	(0.000)
	NZD/GBP	5	-1.877	1.938	1.314	0.264	15.240	-0.053	(0.001)	0.027	(0.491)
		15	-1.032	1.045	0.691	-0.605	16.852	0.022	(0.178)	0.051	(0.014)
		30	-0.648	0.633	0.456	-2.661	62.103	-0.159	(0.000)	0.083	(0.000)
D:	AUS/USD	5	-1.693	1.687	1.160	0.086	18.120	-0.087	(0.000)	0.054	(0.000)
		15	-0.905	0.883	0.610	-0.088	12.623	-0.030	(0.015)	0.033	(0.075)
		30	-0.591	0.562	0.399	0.411	13.210	-0.041	(0.001)	0.034	(0.044)
	CAD/USD	5	-1.467	1.435	0.921	-0.085	8.762	-0.003	(0.789)	0.023	(0.428)
		15	-0.776	0.778	0.510	0.290	10.587	-0.025	(0.039)	0.053	(0.000)
		30	-0.505	0.488	0.329	-0.103	13.586	-0.044	(0.000)	0.043	(0.004)
	DKK/USD	5	-1.578	1.548	1.014	0.095	7.817	-0.015	(0.358)	0.050	(0.024)
		15	-0.822	0.831	0.536	0.094	6.901	0.012	(0.480)	0.048	(0.036)
		30	-0.567	0.549	0.351	-0.103	8.885	0.022	(0.187)	0.048	(0.025)
	NOK/USD	5	-2.089	2.184	1.352	0.094	6.047	0.011	(0.523)	0.032	(0.325)
		15	-1.176	1.184	0.747	0.168	6.938	0.000	(0.995)	0.031	(0.379)
		30	-0.730	0.784	0.490	-0.049	8.592	-0.048	(0.004)	0.031	(0.320)
	SEK/USD	5	-2.304	2.276	1.436	-0.076	6.168	0.012	(0.477)	0.023	(0.710)
		15	-1.215	1.204	0.783	0.211	8.700	0.012	(0.487)	0.047	(0.039)
		30	-0.810	0.784	0.511	-0.057	8.471	-0.012	(0.468)	0.025	(0.587)
	SGD/USD	5	-0.736	0.813	0.523	0.094	9.615	-0.027	(0.121)	0.059	(0.016)
		15	-0.434	0.432	0.278	-0.046	9.321	-0.036	(0.033)	0.043	(0.105)
		30	-0.284	0.285	0.181	0.128	8.823	-0.059	(0.000)	0.062	(0.003)

Notes: Columns (iii) - (vii) report statistics on the distribution of changes in the log prices (spot rates) over horizons  $h$  of 5, 15, and 30 minutes. The change in rates are expressed in basis points per minutes, i.e.,  $\Delta^h_{S_{t+h}} \equiv (\ln(S_{t+h}) - \ln(S_t)) * 10000/h$  for  $h = \{5, 15, 60\}$ , where  $S_t$  is the mid-point price at time  $t$ . All statistics are computed from 10000 starting times  $t$  sampled at random from the span of the available time series for each currency pair. Columns (viii) and (ix) report the first-order autocorrelation in  $\Delta^h_{S_{t+h}}$  and the p-value for the null of a zero autocorrelation, respectively. Column (x) reports the KS test for the null that the two conditional distributions  $f(\Delta^h_{S_{t+h}}|\Delta^h_{S_t} > 0)$  and  $f(\Delta^h_{S_{t+h}}|\Delta^h_{S_t} \leq 0)$  are the same. The asymptotic p-value for the null is shown in column (xi).

Table A.3 examines the stability of forex price dynamics away from the Fix the the 14 currency pairs with data spanning a decade. Columns (iii) - (vii) and (viii) - (xii) report statistics on the distribution of price changes (basis points per minute) at random times between Jan 1st, 2004 and Dec 31st. 2007, and between Jan 1st. 2010 and Dec. 31st. 2013. Both of these subsamples cover periods that are far removed from the height of the 2008-9 crisis. To examine the stability of the rate-change distribution across the two subsamples, I again use the KS test and report its asymptotic p-value in the right-hand column of the table.

The statistics in Table A.3 show that there has been change in the price-change distributions over the past decade. Formally, this can be seen from the very small p-values for the KS tests reported in column (xiv). A comparison of the statistics in columns (iii) - (vii) with those in (viii) - (xii) reveals that the tails of the distributions, measured by the percentiles and kurtosis, generally exhibit the largest differences across the two subsamples. In other words, the incidence and size of atypical rate changes appear to have evolved over the decade. That said, the majority of the statistics from the two subsamples are very similar. In particular, the standard deviations are similar in size and decline with the rise in the horizon in the same manner as their counterparts in Table A.2. As above, this pattern is symptomatic of the generally negative autocorrelation in price changes that is present in both subsamples. Estimated autocorrelations (unreported) are generally negative, and statistically significantly different from zero in the two subsamples, but the estimates are uniformly small (in absolute value), like those in Table A.3.

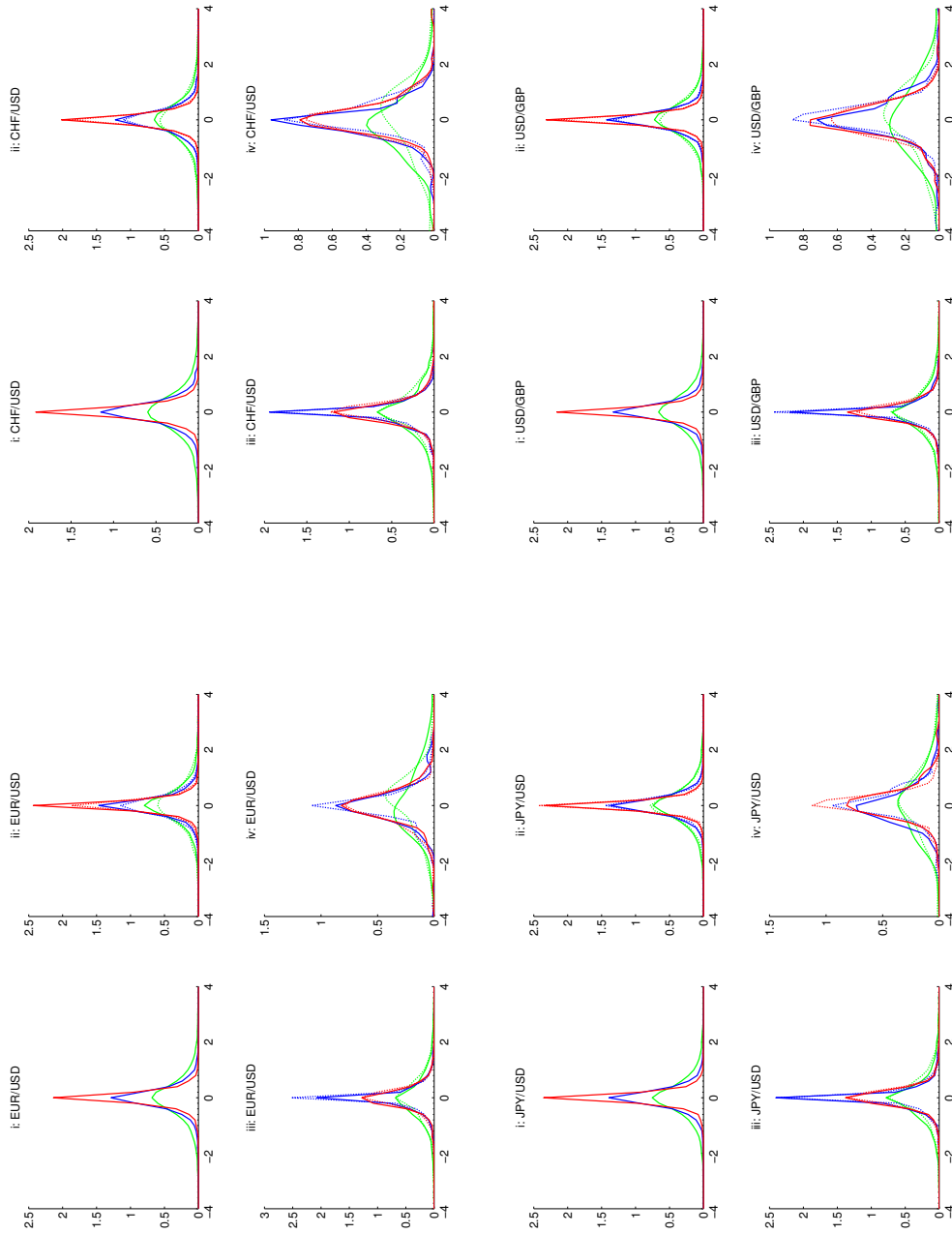
Figures A.1-A.6 provide visual evidence that compliments the statistics reported in Tables A.2 and A.3. The figure plots the price-change densities for all the currency pairs. Plot (i) in each panel shows density functions for  $\Delta^h s_t$  for  $h = \{5, 15, 30\}$  minutes in green, blue, and red, respectively. Here we can clearly see how that dispersion of the densities increases as the horizon shortens from 30 to five minutes. Plot (ii) in each panel shows the distributions from the pre-2008 and post-2009 subsamples. On close inspection, it is possible to see differences between the densities, but they are extremely small. Moreover, the densities from the subsamples do not look dissimilar to the densities in plot (i). Thus, while the differences between the subsample price-change distributions are statistically significant, the differences in the estimated densities do not appear economically important. In sum, despite the large institutional changes in forex trading over the sample period, the intraday dynamics of prices away from Fixes (and other scheduled announcements) appears to have been stable.

Table A.3: Stability of Price-Change Dynamics

		2004-2007					2010-2013					KS Test p-value (xiv)	
(i)	horizon (ii)	5% (iii)	95% (iv)	std (v)	skew (vi)	kurtosis (vii)	5% (viii)	95% (ix)	std (x)	skew (xi)	kurtosis (xii)		
A:	EUR/USD	5	-1.236	1.234	0.833	0.029	11.279	-1.369	1.462	0.890	0.193	6.333	0.000
		15	-0.671	0.636	0.436	-0.466	9.937	-0.727	0.737	0.463	0.135	6.094	0.001
		30	-0.671	0.636	0.282	-0.208	10.384	-0.478	0.487	0.299	0.267	5.886	0.000
	CHF/USD	5	-1.324	1.404	0.889	0.436	9.868	-1.580	1.528	1.012	-0.652	14.041	0.000
		15	-0.725	0.753	0.474	0.268	7.398	-0.805	0.767	0.522	-0.416	9.485	0.001
		30	-0.725	0.753	0.294	0.018	6.575	-0.530	0.489	0.325	-0.559	9.684	0.201
	JPY/USD	5	-1.235	1.312	0.829	0.151	8.780	-1.173	1.099	0.757	-0.174	9.364	0.000
		15	-0.658	0.673	0.428	0.007	7.115	-0.603	0.610	0.392	0.568	9.330	0.001
		30	-0.658	0.673	0.272	-0.291	9.113	-0.415	0.386	0.261	0.337	8.359	0.021
USD/GBP	5	-1.261	1.216	0.887	0.506	18.506	-1.226	1.232	0.782	0.141	8.286	0.000	
	15	-0.648	0.653	0.487	-0.854	35.074	-0.677	0.647	0.440	0.415	9.644	0.000	
	30	-0.648	0.653	0.322	-1.450	49.030	-0.408	0.452	0.282	0.489	8.096	0.003	
B:	CHF/EUR	5	-0.644	0.695	0.488	0.871	22.664	-1.059	1.082	0.764	0.014	31.187	0.000
		15	-0.360	0.382	0.267	1.205	32.256	-0.617	0.542	0.405	0.101	22.086	0.000
		30	-0.360	0.382	0.171	0.614	34.213	-0.371	0.371	0.253	0.433	17.937	0.000
	JPY/EUR	5	-1.360	1.331	1.000	0.562	23.980	-1.711	1.743	1.104	0.085	6.712	0.000
		15	-0.742	0.728	0.532	0.679	17.937	-0.943	0.967	0.595	0.339	8.313	0.000
		30	-0.742	0.728	0.352	0.562	20.992	-0.608	0.600	0.383	0.270	7.020	0.000
C:	CHF/GBP	5	-1.146	1.216	0.815	0.447	14.632	-1.496	1.392	0.987	-0.404	15.134	0.000
		15	-0.646	0.612	0.459	0.909	28.974	-0.803	0.738	0.532	-0.001	12.160	0.000
		30	-0.646	0.612	0.308	-1.634	62.633	-0.482	0.502	0.334	-0.101	9.787	0.001
	EUR/GBP	5	-0.895	0.903	0.667	-0.108	12.598	-1.147	1.215	0.761	-0.185	7.534	0.000
		15	-0.495	0.503	0.365	-0.516	25.800	-0.613	0.646	0.422	-0.228	11.588	0.000
		30	-0.495	0.503	0.244	0.968	46.873	-0.431	0.418	0.274	-0.210	7.546	0.000
	JPY/GBP	5	-1.533	1.547	1.144	0.952	22.271	-1.619	1.614	1.045	0.177	7.440	0.001
		15	-0.814	0.832	0.608	0.481	27.077	-0.880	0.919	0.573	0.466	9.680	0.007
		30	-0.814	0.832	0.405	-0.597	41.379	-0.538	0.598	0.372	0.391	9.238	0.038
D:	AUS/USD	5	-1.658	1.562	1.221	0.350	24.528	-1.557	1.559	0.968	-0.052	7.257	0.000
		15	-0.897	0.849	0.639	-0.161	16.309	-0.827	0.757	0.500	0.162	6.025	0.002
		30	-0.897	0.849	0.420	0.418	16.003	-0.511	0.500	0.323	0.365	7.591	0.001
	CAD/USD	5	-1.496	1.467	0.946	0.024	10.811	-1.207	1.217	0.782	-0.211	6.614	0.000
		15	-0.787	0.787	0.528	0.575	12.847	-0.700	0.650	0.416	-0.084	6.713	0.008
		30	-0.787	0.787	0.342	0.020	16.472	-0.419	0.415	0.264	0.004	7.356	0.004

Notes: Columns (iii) - (vii) and (viii) - (xii) report statistics on the distribution of changes in the log prices over horizons  $h$  of 5, 15, and 30 minutes from quotes made between Jan 1st 2004 and Dec 31st. 2007, and between Jan 1st. 2010 and Dec. 31st. 2013. The change in quotes are expressed in basis points per minutes, i.e.,  $\Delta^h s_t \equiv (\ln(S_{t+h}) - \ln(S_t))10000/h$  for  $h = \{5, 15, 60\}$ . All statistics are computed from 10000 starting times  $t$  sampled at random. Column (xiv) reports the asymptotic p-value from the KS test of the null that the distributions from the two subsamples are the same.

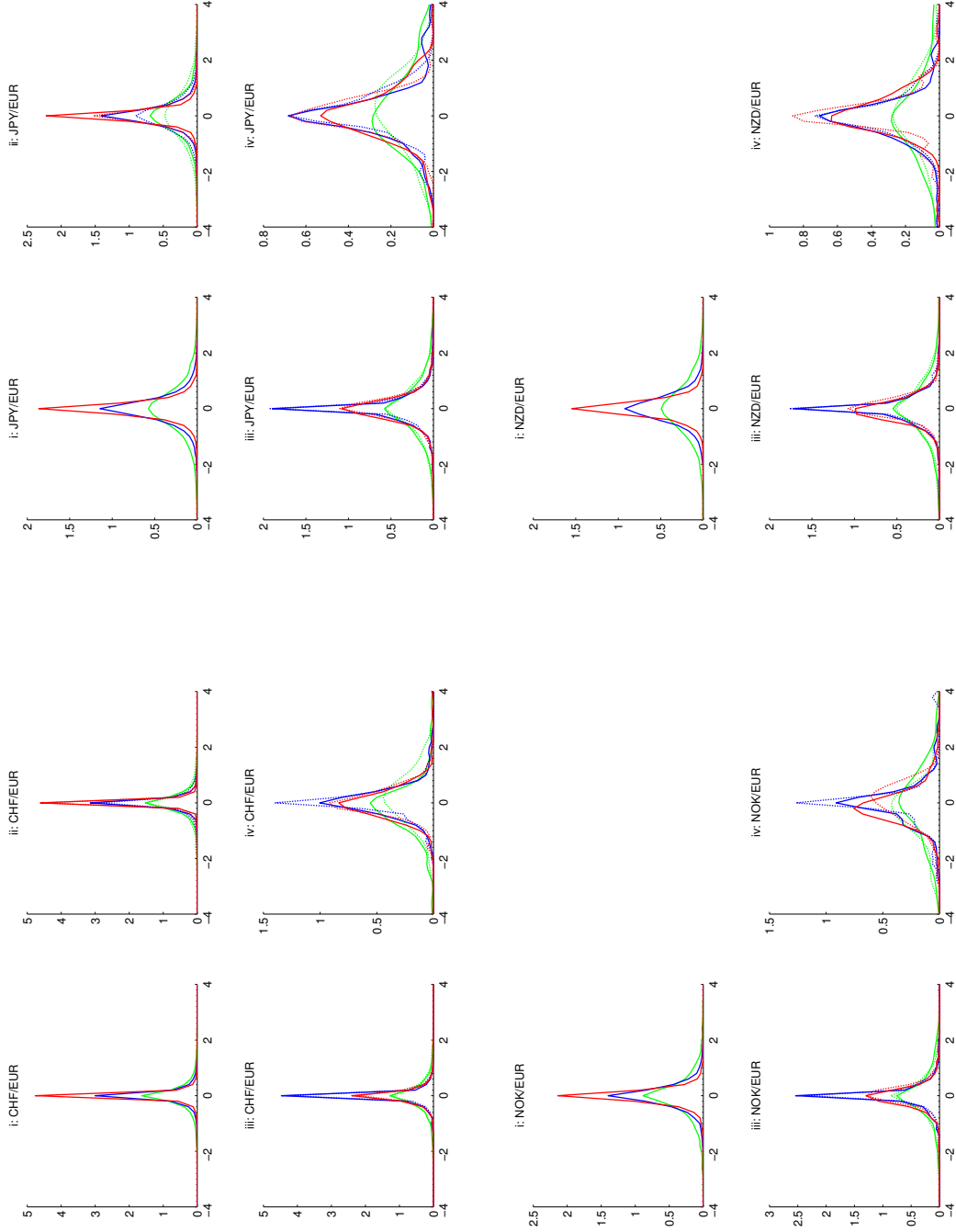
Figure A.1: Price Change Densities



Notes: Plots (i) shows the density functions for  $\Delta^h s_t$  for  $h = \{5, 15, 30\}$  minutes in green, blue, and red, respectively. Plot (ii) shows the density functions  $\Delta^h s_t$  from pre-2008 and post 2009 data with solid and dotted lines, respectively. Plots (iii) and (iv) show the conditional densities for  $f(\Delta^h s_t | \Delta^h s_{t-h} > \kappa^+)$  (solid) and  $f(\Delta^h s_t | \Delta^h s_{t-h} < \kappa^-)$  (dotted), where  $\kappa^+$  and  $\kappa^-$  denote the upper and lower percentiles of the price-change distribution, respectively: equal to  $\{75\%, 25\%$  in plot (iii) and  $\{97.5\%, 2.5\%\}$  in plot (iv).

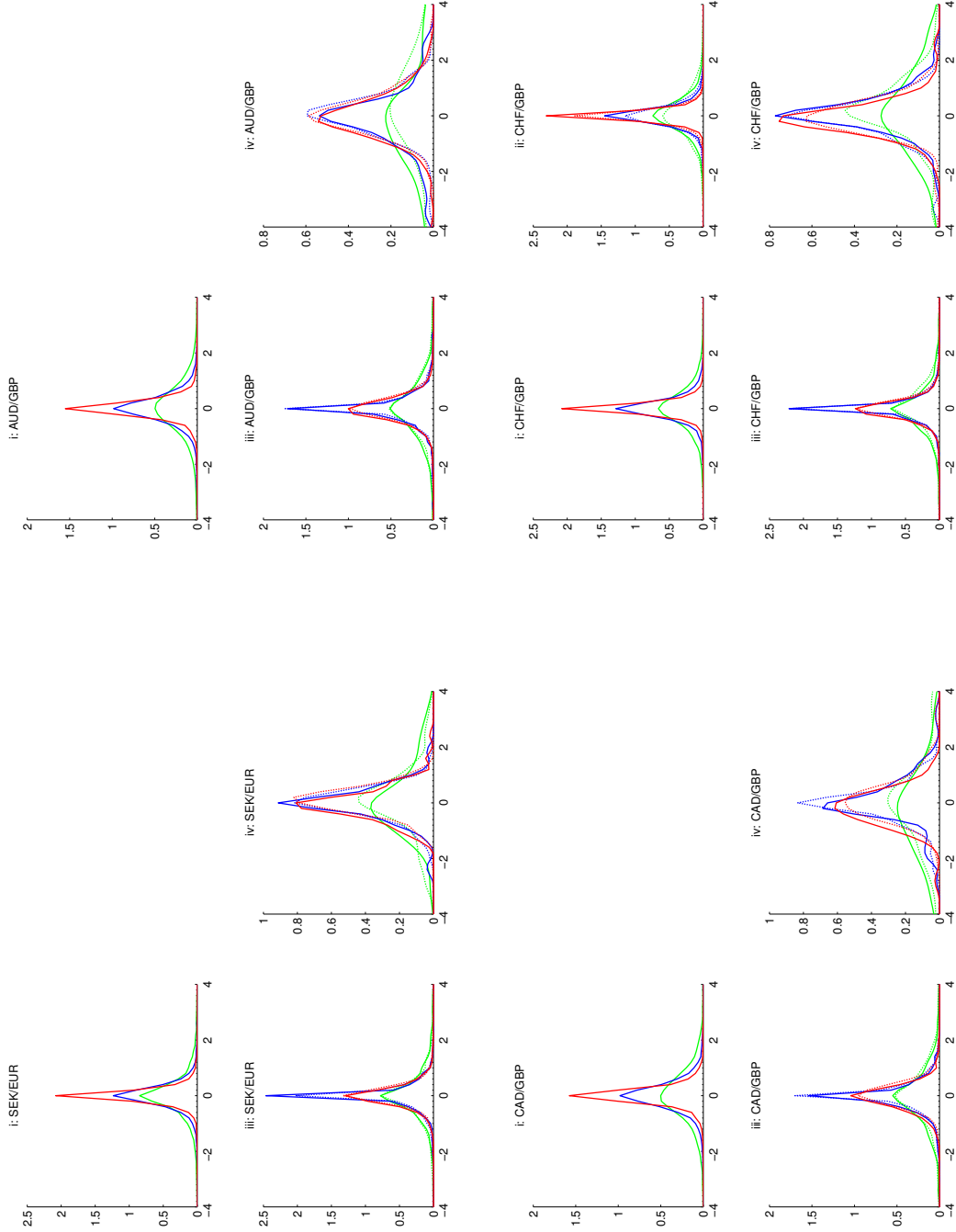


Figure A.2: Price Change Densities



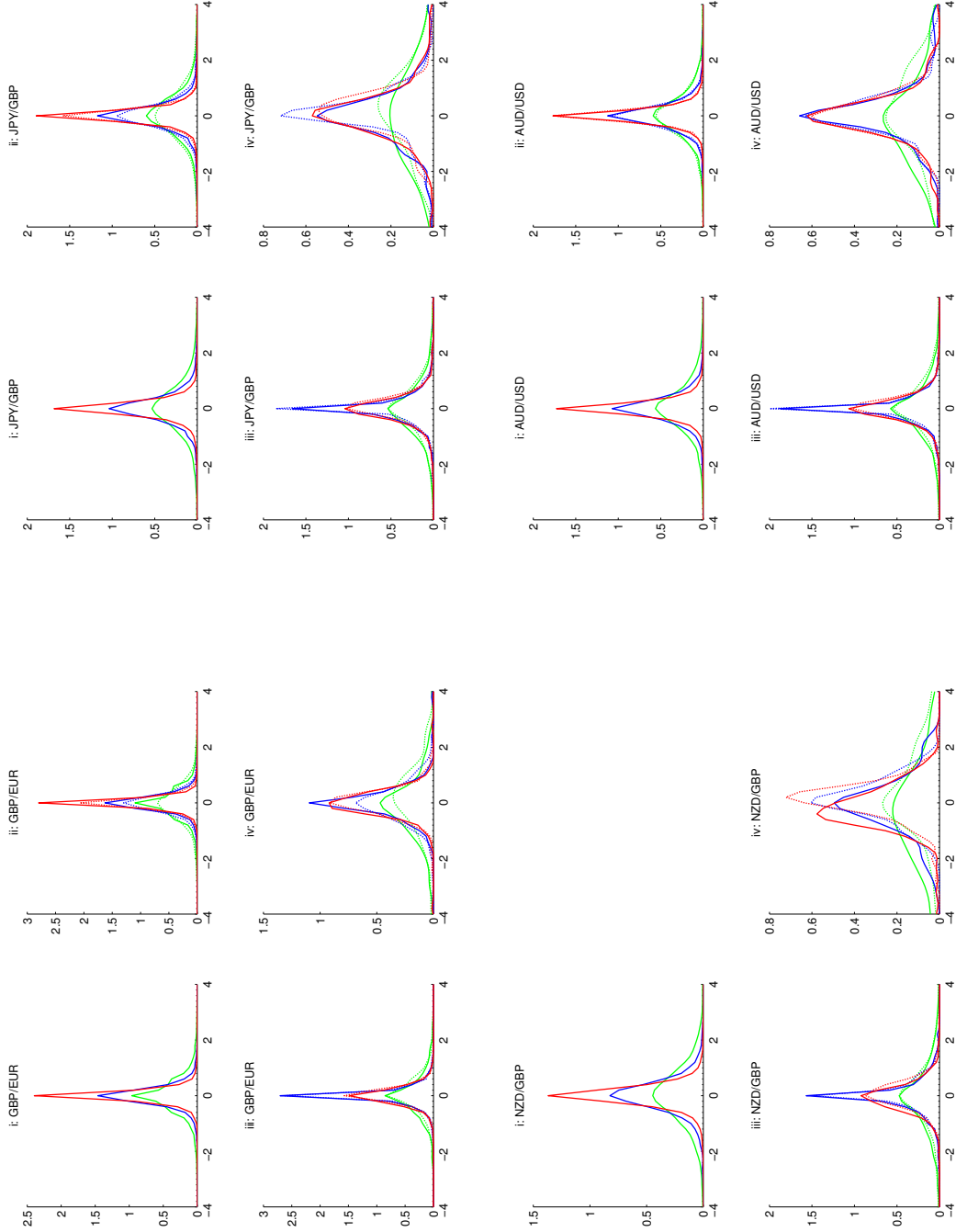
Notes: Panel i plots the density functions for  $\Delta^h s_t$  for  $h = \{5, 15, 30\}$  minutes in green, blue, and red, respectively. Panel ii plots the density functions  $\Delta^h s_t$  from pre-2008 and post-2008 data with solid and dotted lines, respectively. Panels iii and iv plot the conditional densities for  $f(\Delta^h s_t | \Delta^h s_{t-h} > \kappa^+)$  (solid) and  $f(\Delta^h s_t | \Delta^h s_{t-h} < \kappa^-)$  (dotted) for  $\{\kappa^+, \kappa^-\} = \{75\%, 25\%\}$  (panel iii) and  $\{97.5\%, 2.5\%\}$  (panel iv).

Figure A.3: Price Change Densities



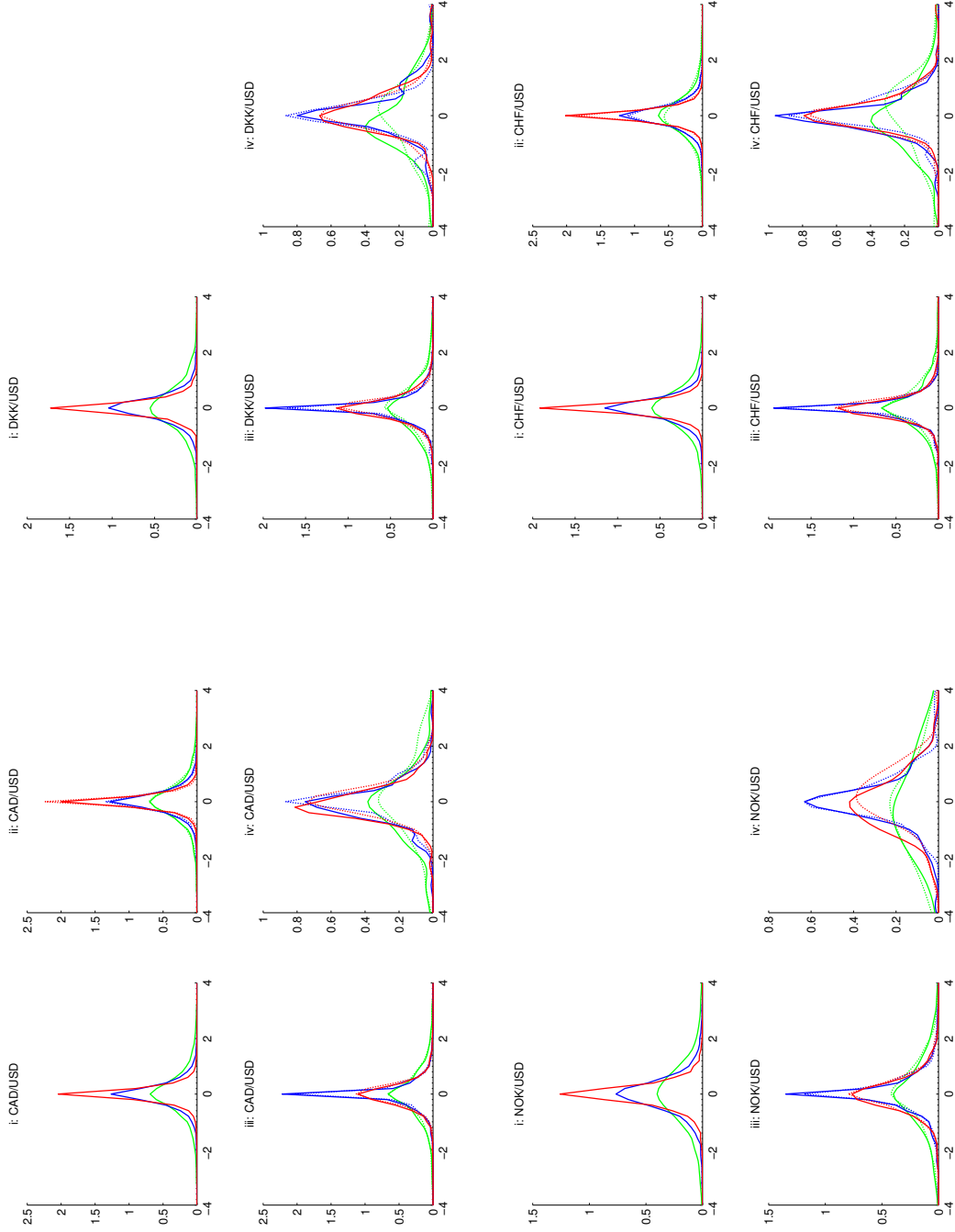
Notes: Panel i plots the density functions for  $\Delta^h s_t$  for  $h = \{5, 15, 30\}$  minutes in green, blue, and red, respectively. Panel ii plots the density functions  $\Delta^h s_t$  from pre-2008 and post-2008 data with solid and dotted lines, respectively. Panels iii and iv plot the conditional densities for  $f(\Delta^h s_t | \Delta^h s_{t-h} > \kappa^+)$  (solid) and  $f(\Delta^h s_t | \Delta^h s_{t-h} < \kappa^-)$  (dotted) for  $\{\kappa^+, \kappa^-\} = \{75\%, 25\%\}$  (panel iii) and  $\{97.5\%, 2.5\%\}$  (panel iv).

Figure A.4: Price Change Densities



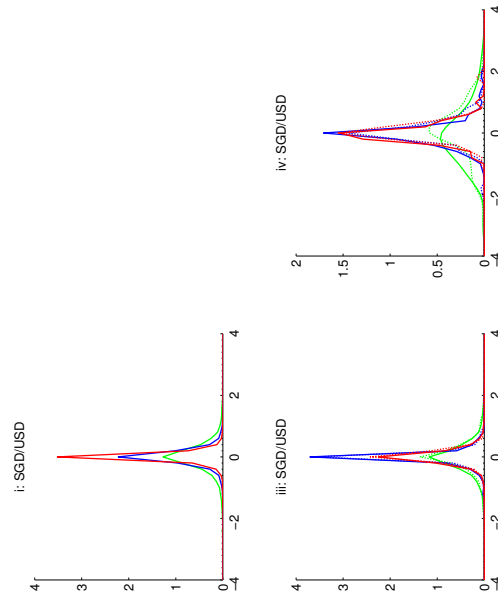
Notes: Panel i plots the density functions for  $\Delta^h s_t$  for  $h = \{5, 15, 30\}$  minutes in green, blue, and red, respectively. Panel ii plots the density functions  $\Delta^h s_t$  from pre-2008 and post-2009 data with solid and dotted lines, respectively. Panels iii and iv plot the conditional densities for  $f(\Delta^h s_t | \Delta^h s_{t-h} > \kappa^+)$  (solid) and  $f(\Delta^h s_t | \Delta^h s_{t-h} < \kappa^-)$  (dotted) for  $\{\kappa^+, \kappa^-\} = \{75\%, 25\%\}$  (panel iii) and  $\{97.5\%, 2.5\%\}$  (panel iv).

Figure A.5: Price Change Densities



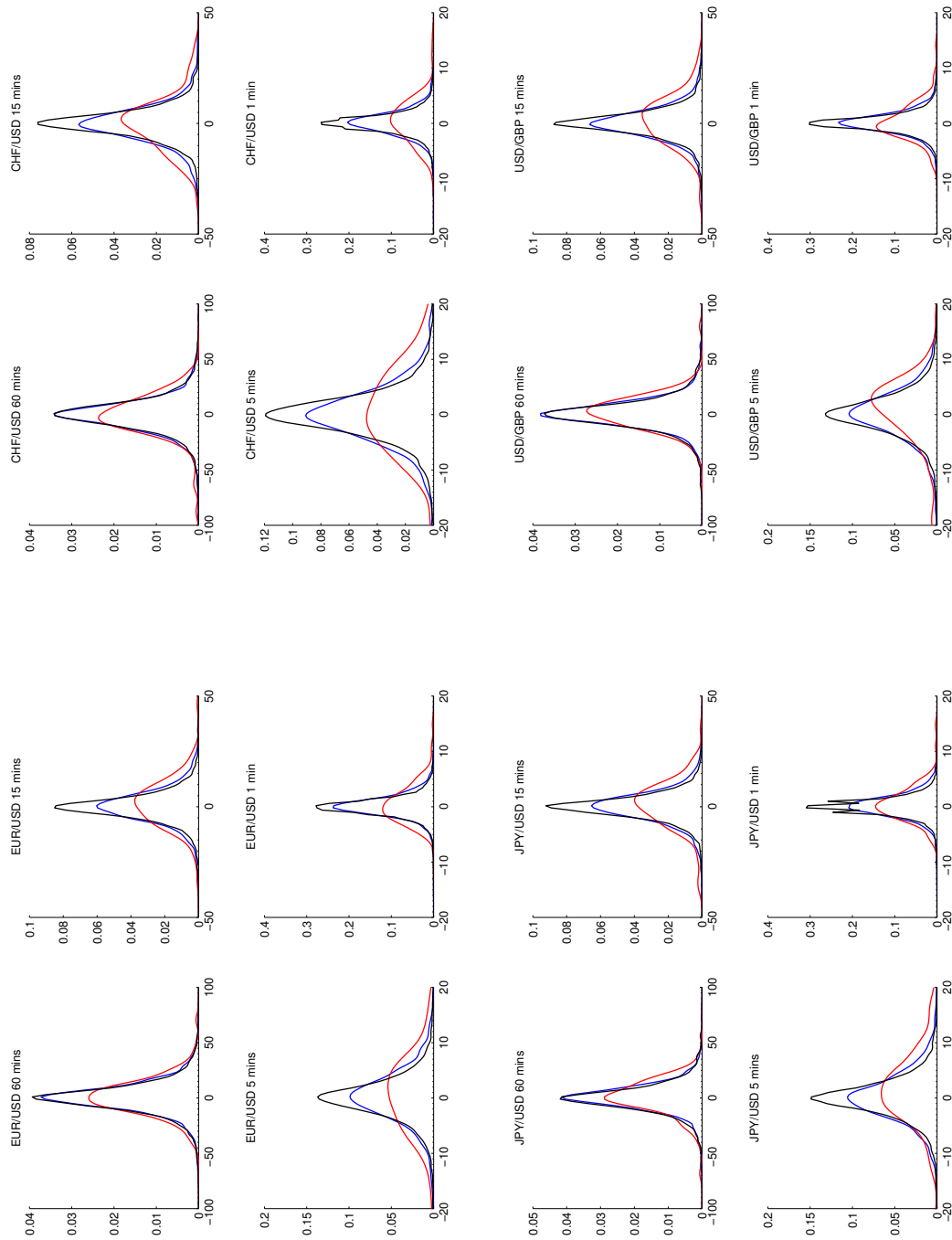
Notes: Panel i plots the density functions for  $\Delta^h s_t$  for  $h = \{5, 15, 30\}$  minutes in green, blue, and red, respectively. Panel ii plots the density functions  $\Delta^h s_t$  from pre-2008 and post-2008 data with solid and dotted lines, respectively. Panels iii and iv plot the conditional densities for  $f(\Delta^h s_t | \Delta^h s_{t-h} > \kappa^+)$  (solid) and  $f(\Delta^h s_t | \Delta^h s_{t-h} < \kappa^-)$  (dotted) for  $\{\kappa^+, \kappa^-\} = \{75\%, 25\%\}$  (panel iii) and  $\{97.5\%, 2.5\%\}$  (panel iv).

Figure A.6: Price Change Densities



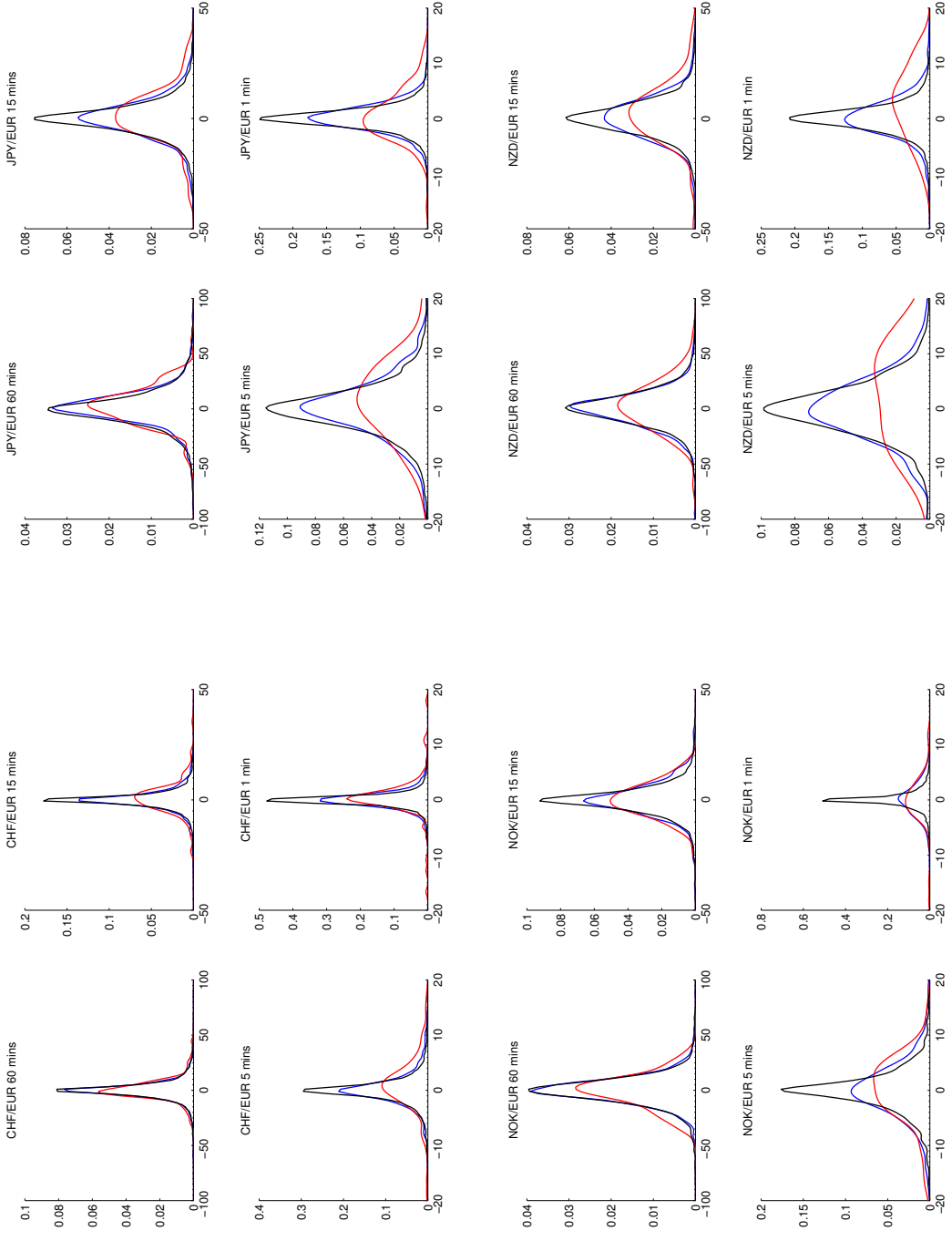
Notes: Panel i plots the density functions for  $\Delta^h s_t$  for  $h = \{5, 15, 30\}$  minutes in green, blue, and red, respectively. Panel ii plots the density functions  $\Delta^h s_t$  from pre-2008 and post 2009 data with solid and dotted lines, respectively. Panels iii and iv plot the conditional densities for  $f(\Delta^h s_t | \Delta^h s_{t-h} > \kappa^+)$  (solid) and  $f(\Delta^h s_t | \Delta^h s_{t-h} < \kappa^-)$  (dotted) for  $\{\kappa^+, \kappa^-\} = \{75\%, 25\%\}$  (panel iii) and  $\{97.5\%, 2.5\%\}$  (panel iv).

Figure A.7: Pre-Fix Price Change Densities



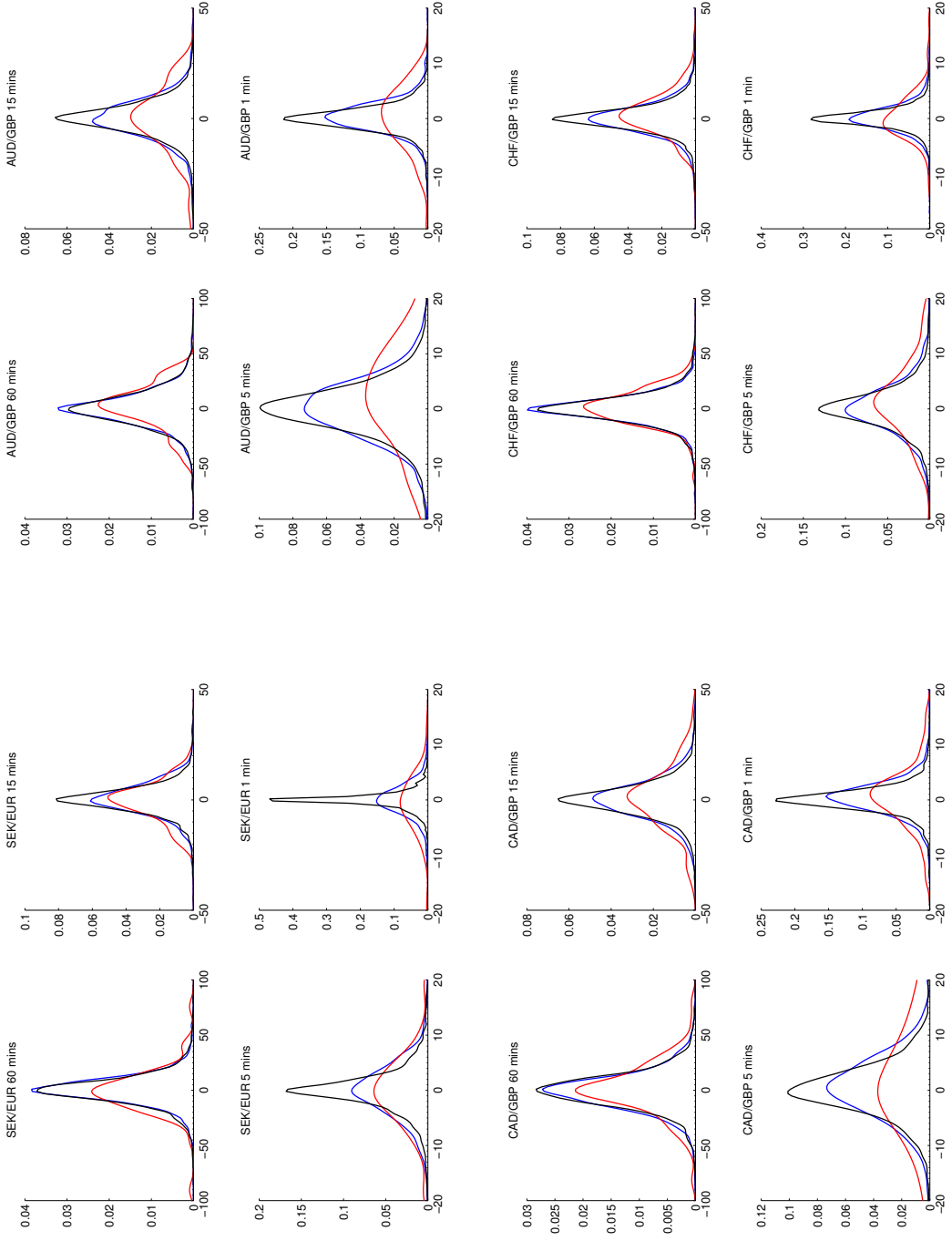
Notes: Distribution for rate changes (in basis points) away from Fixes (black), intra-month pre-Fix (blue), and end-of-month pre-Fix (red).

Figure A.8: Pre-Fix Price Change Densities



Notes: Densities of price changes (in basis points) away from Fix (black) intra-month pre-Fix (blue) and end-of-month pre-Fix (red).

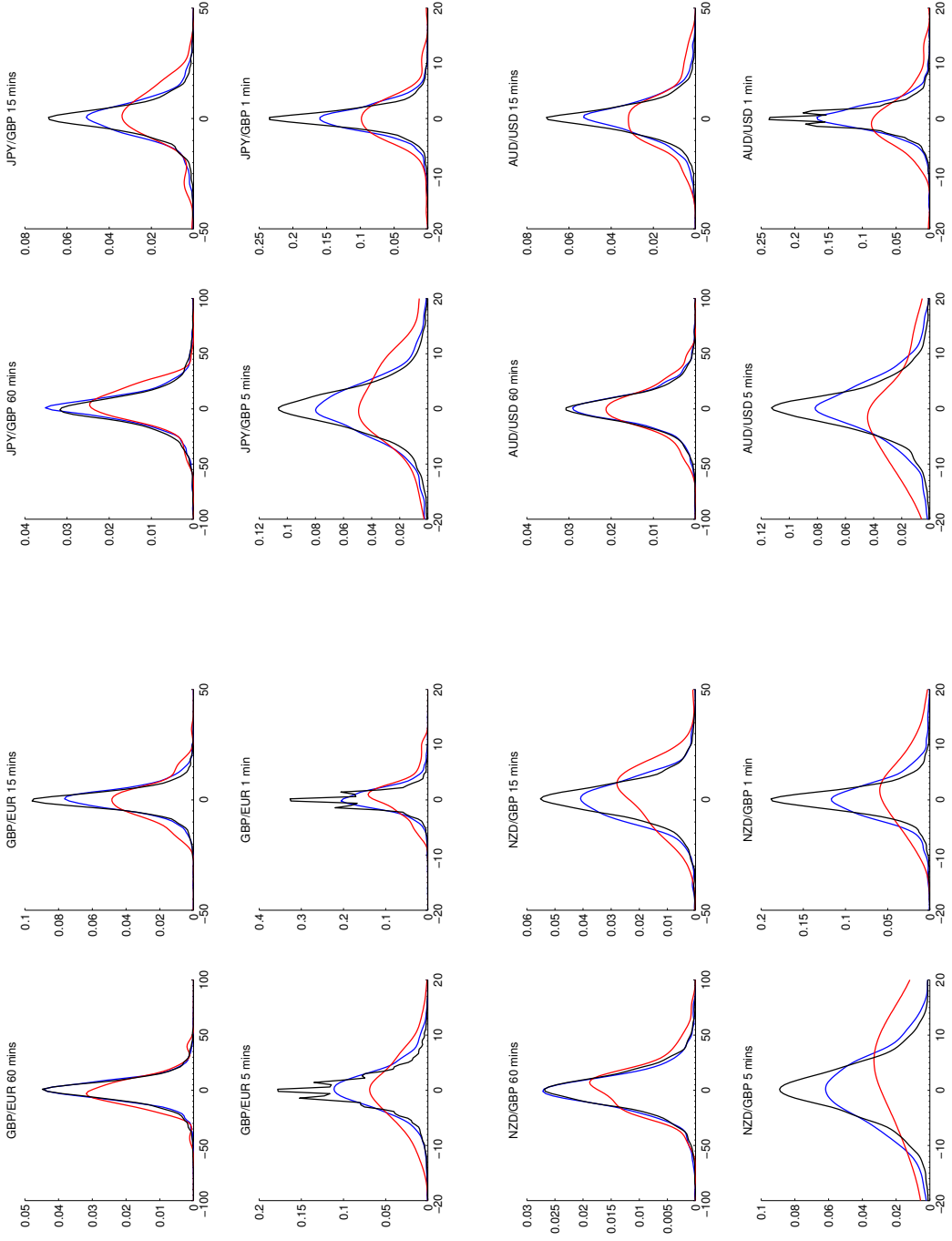
Figure A.9: Pre-Fix Price Change Densities



Notes: Densities of price changes (in basis points) away from Fix (black) intra-month pre-Fix (blue) and end-of-month pre-Fix (red).

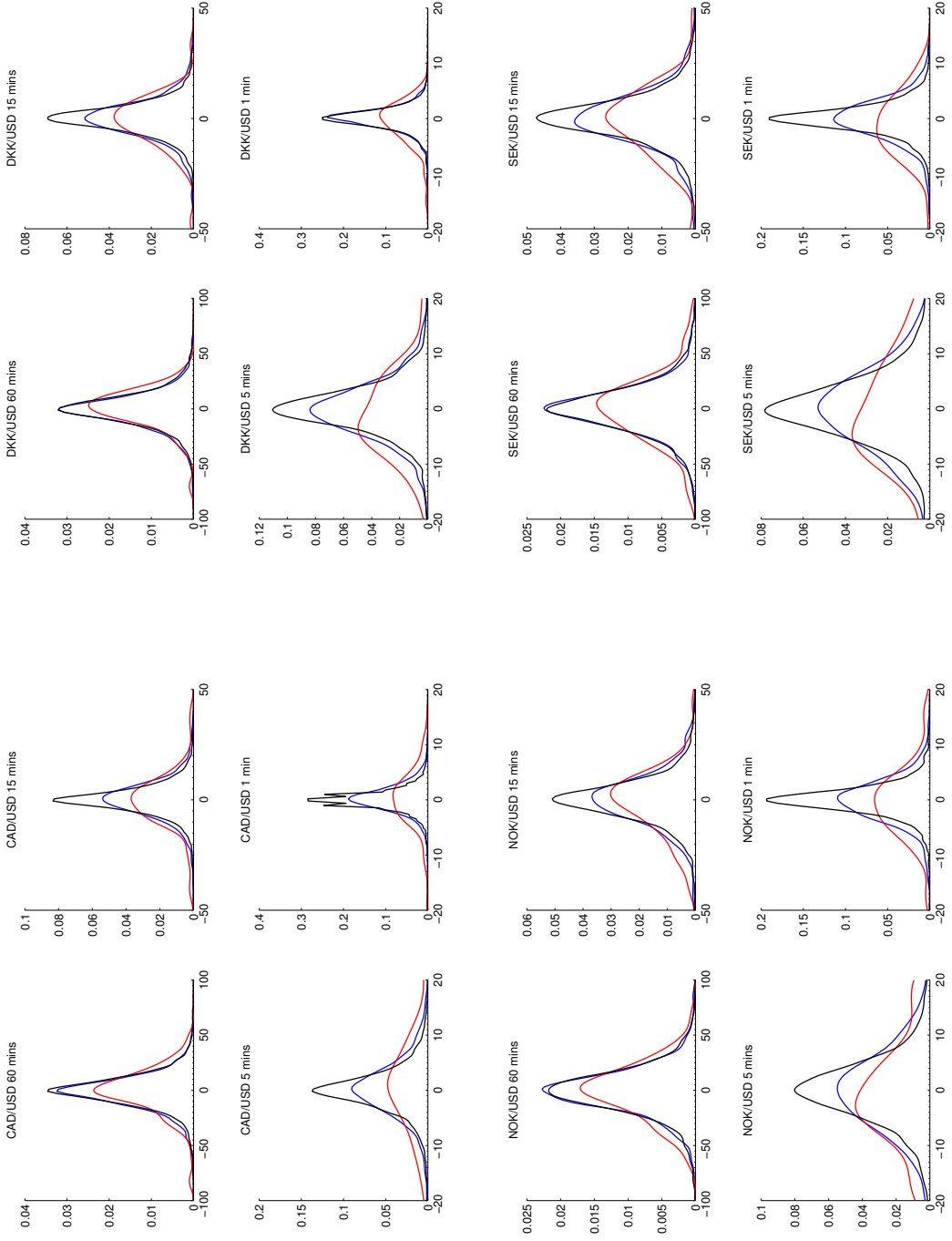


Figure A.10: Pre-Fix Price Change Densities



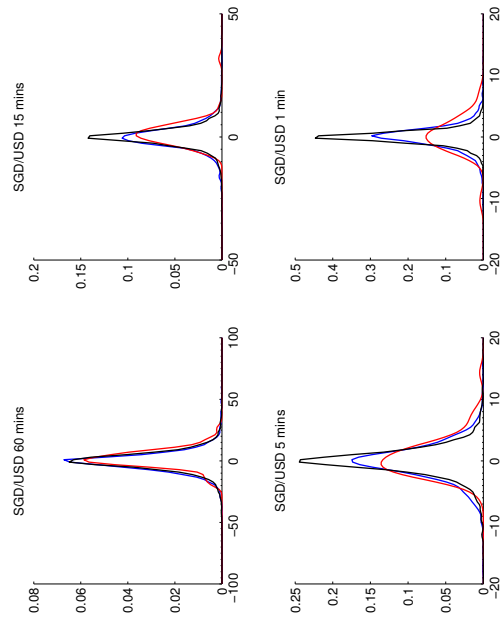
Notes: Densities of price changes (in basis points) away from Fix (black) intra-month pre-Fix (blue) and end-of-month pre-Fix (red).

Figure A.11: Pre-Fix Price Change Densities



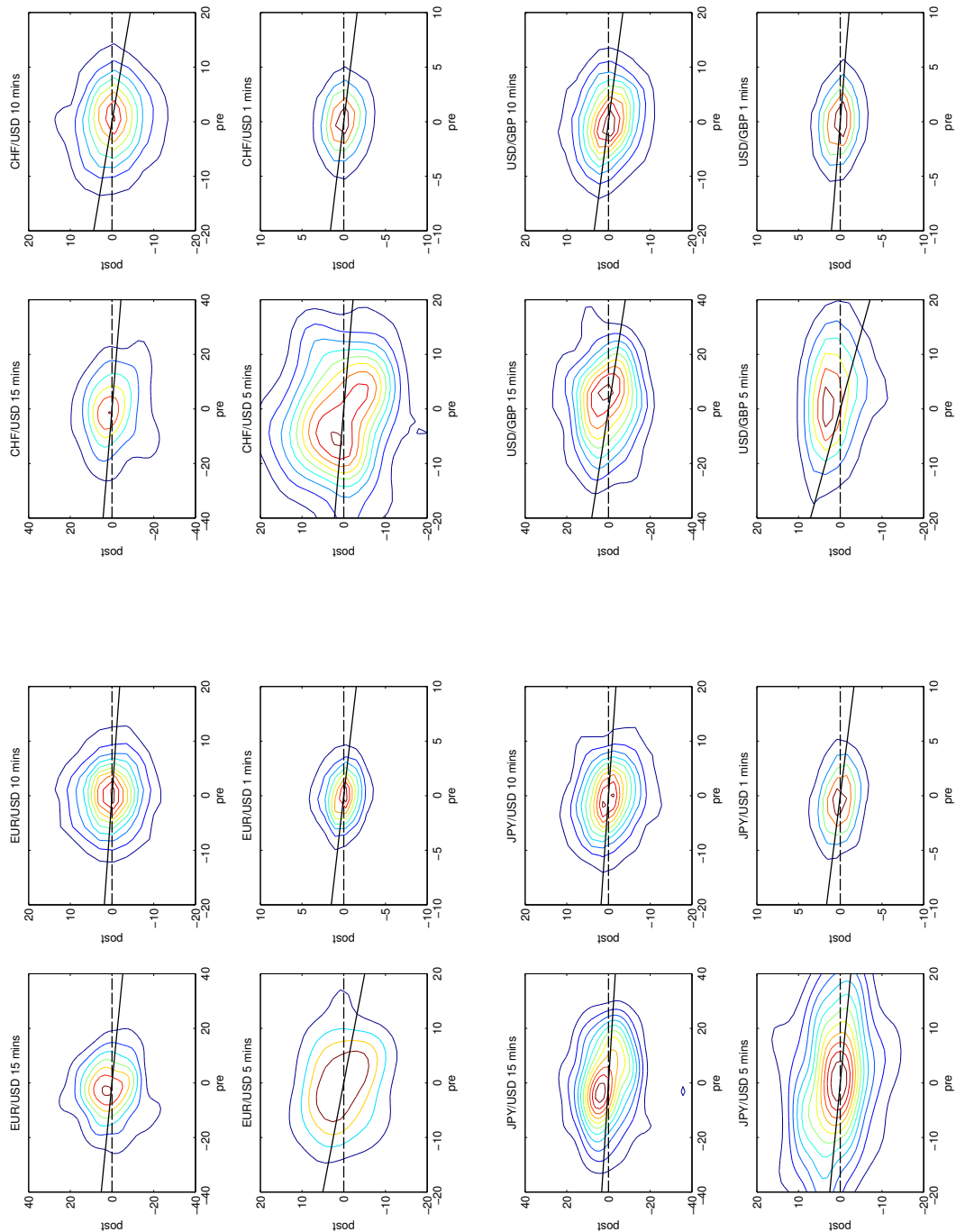
Notes: Densities of price changes (in basis points) away from Fix (black) intra-month pre-Fix (blue) and end-of-month pre-Fix (red).

Figure A.12: Pre-Fix Price Change Densities



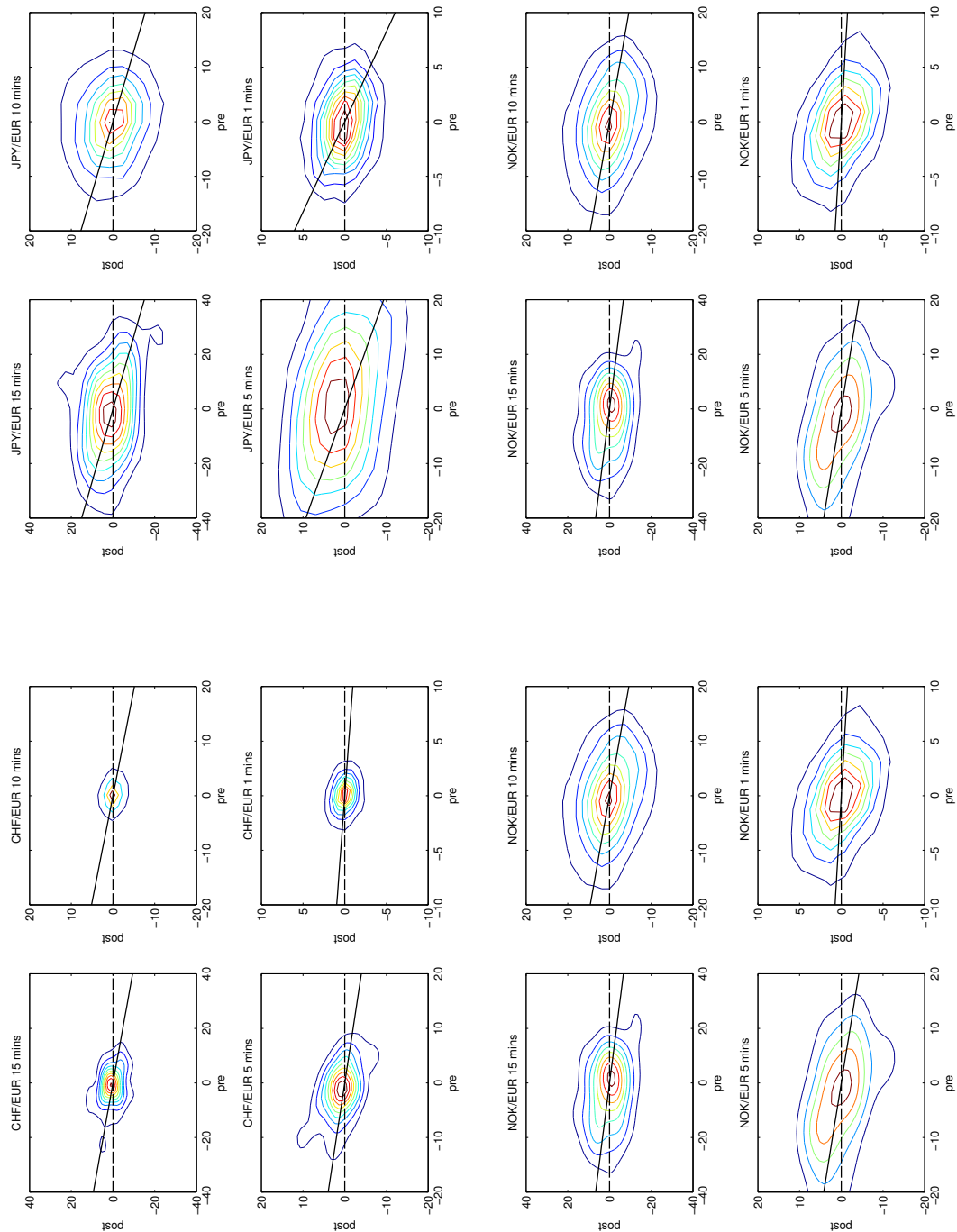
Notes: Densities of price changes (in basis points) away from Fix (black) intra-month pre-Fix (blue) and end-of-month pre-Fix (red).

Figure A.13: Bivariate Pre- and Post-Fix Price Change Densities



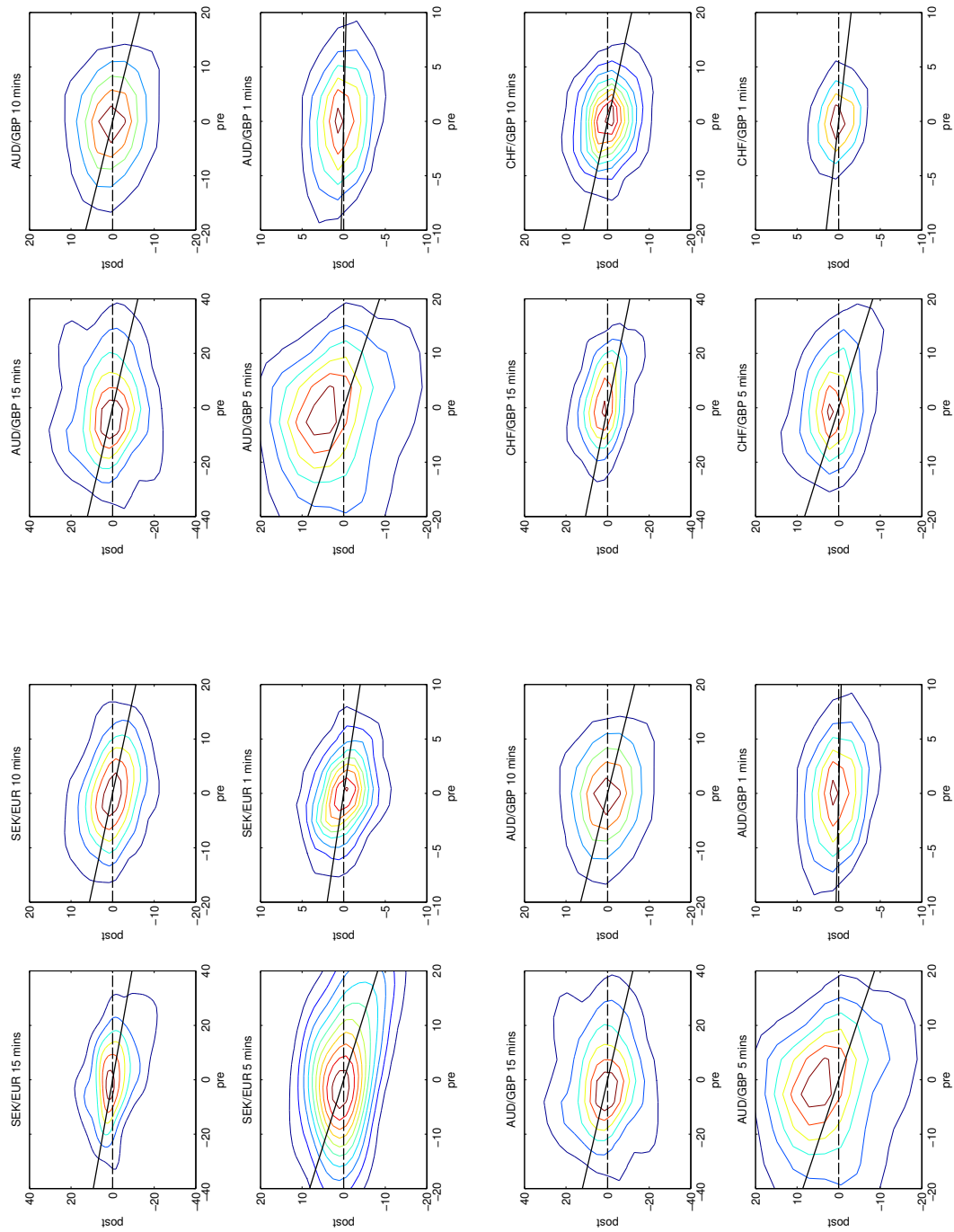
Notes: Each plot shows the contours of the estimated bivariate density for pre- and post-Fix price changes (in basis points) over horizons of 1 to 15 minutes. The solid line in each plot is the estimated regression line from the regression of the post-Fix price change on the pre-Fix change. All estimates are based on end-of-month data.

Figure A.14: Bivariate Pre- and Post-Fix Price Change Density



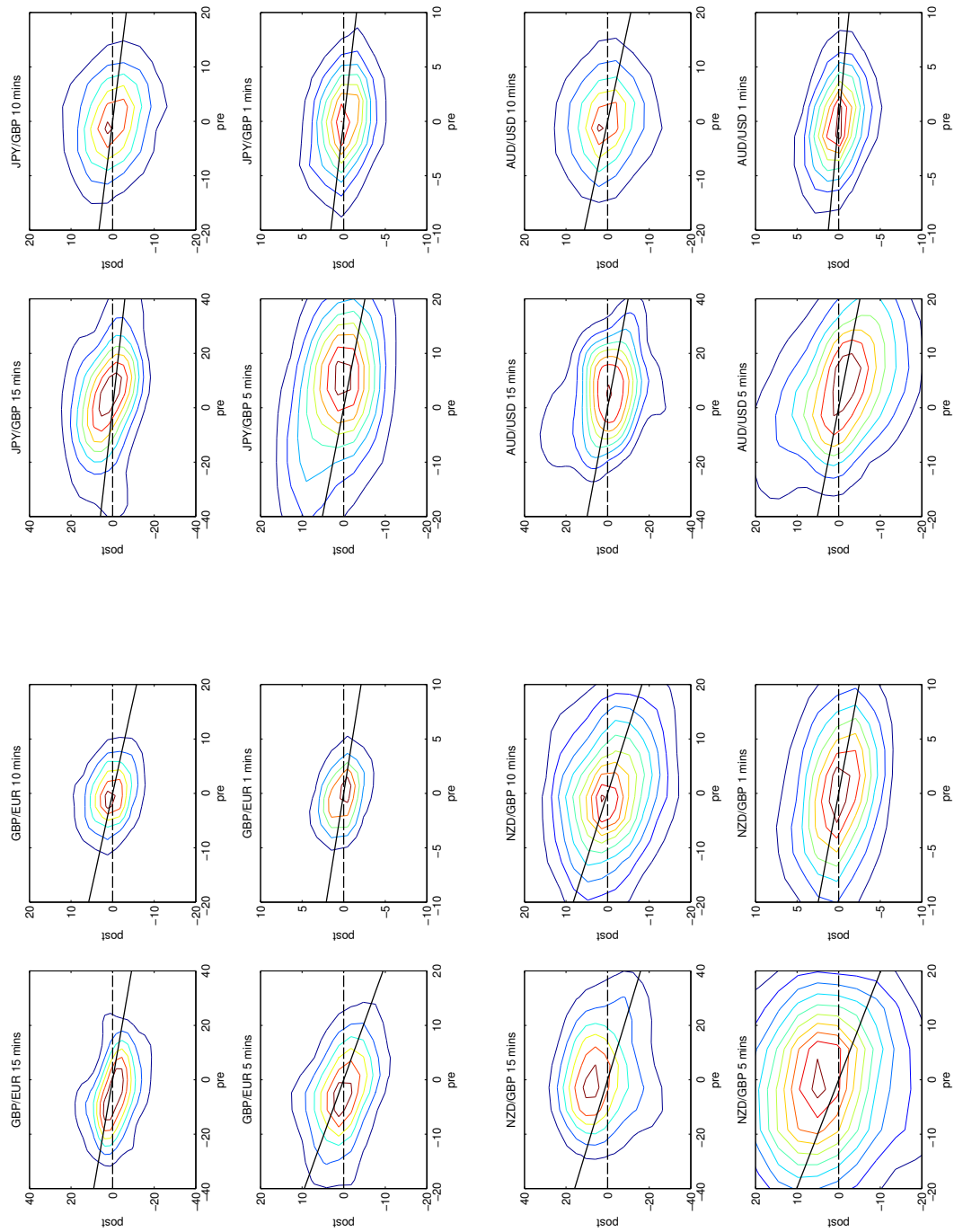
Notes: Each plot shows the contours of the estimated bivariate density for pre- and post-Fix price changes (in basis points) over horizons of 1 to 15 minutes. The solid line in each plot is the estimated regression line from the regression of the post-Fix price change on the pre-Fix change. All estimates are based on end-of-month data.

Figure A.15: Bivariate Pre- and Post- Fix Price Change Density



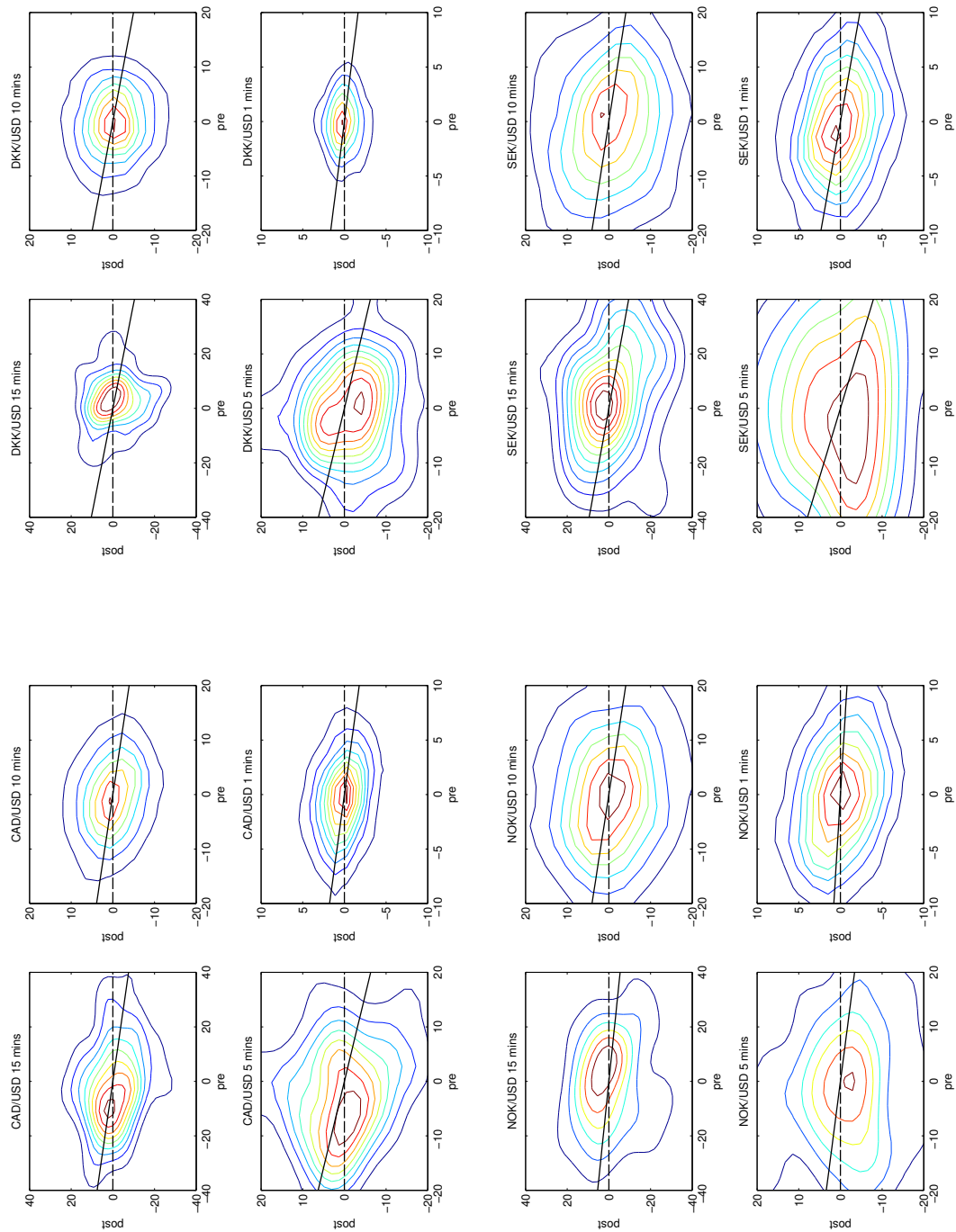
Notes: Each plot shows the contours of the estimated bivariate density for pre- and post-Fix price changes (in basis points) over horizons of 1 to 15 minutes. The solid line in each plot is the estimated regression line from the regression of the post-Fix price change on the pre-Fix change. All estimates are based on end-of-month data.

Figure A.16: Bivariate Pre- and Post- Fix Price Change Density



Notes: Each plot shows the contours of the estimated bivariate density for pre- and post-Fix price changes (in basis points) over horizons of 1 to 15 minutes. The solid line in each plot is the estimated regression line from the regression of the post-Fix price change on the pre-Fix change. All estimates are based on end-of-month data.

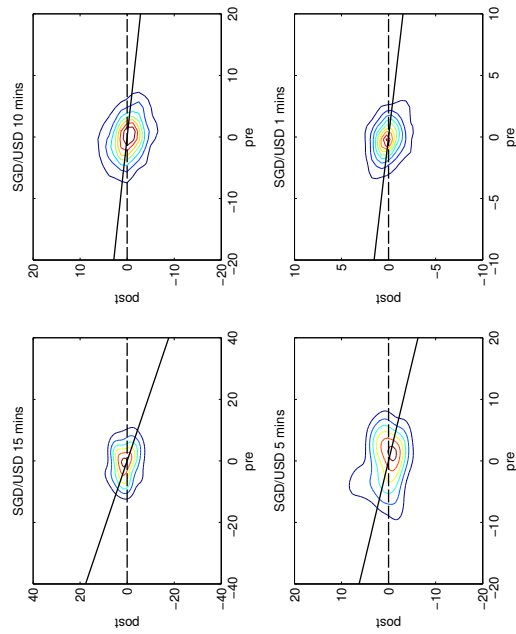
Figure A.17: Bivariate Pre- and Post-Fix Price Change Density



Notes: Each plot shows the contours of the estimated bivariate density for pre- and post-Fix price changes (in basis points) over horizons of 1 to 15 minutes. The solid line in each plot is the estimated regression line from the regression of the post-Fix price change on the pre-Fix change. All estimates are based on end-of-month data.

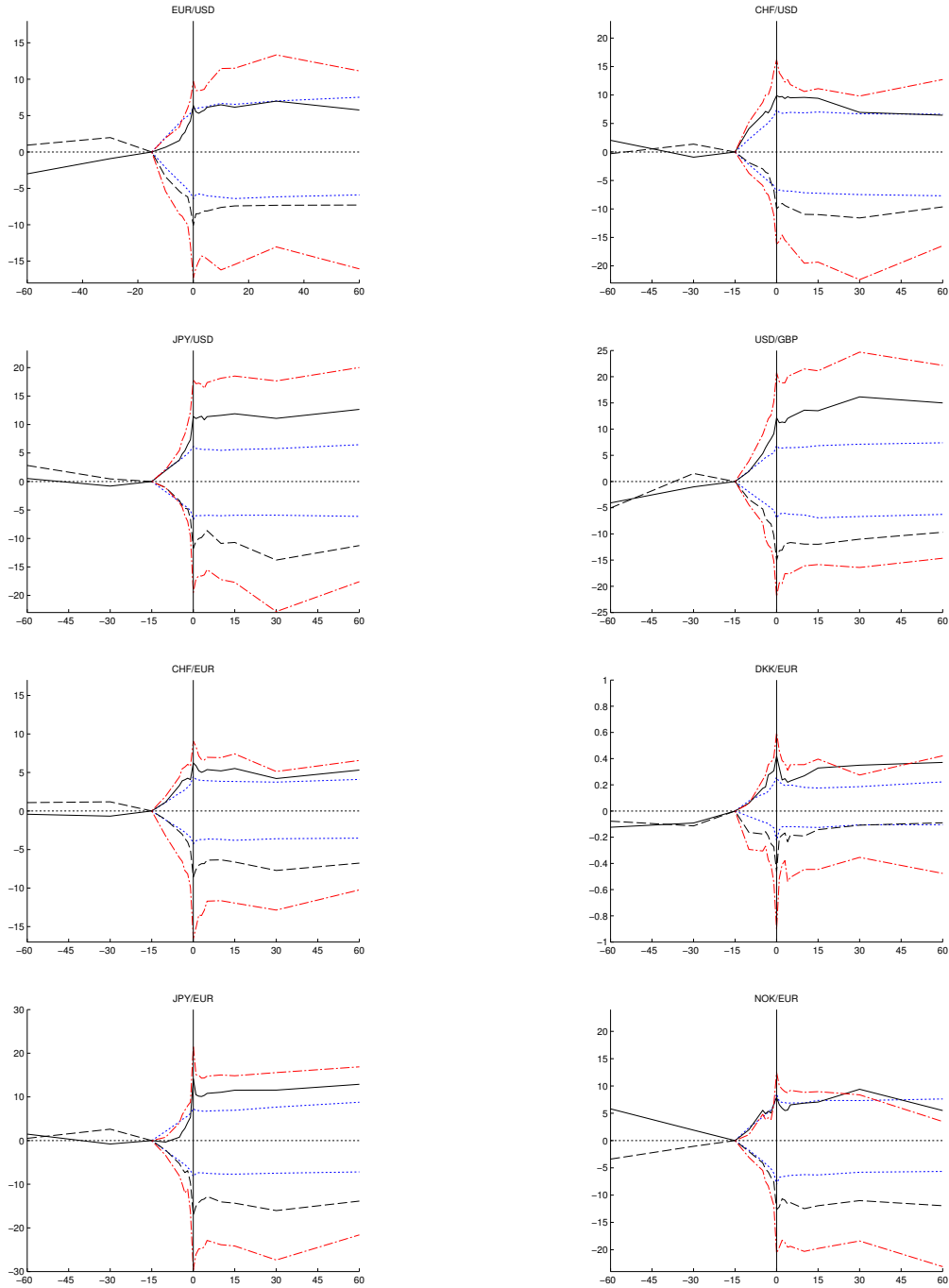


Figure A.18: Bivariate Pre- and Post- Fix Price Change Density



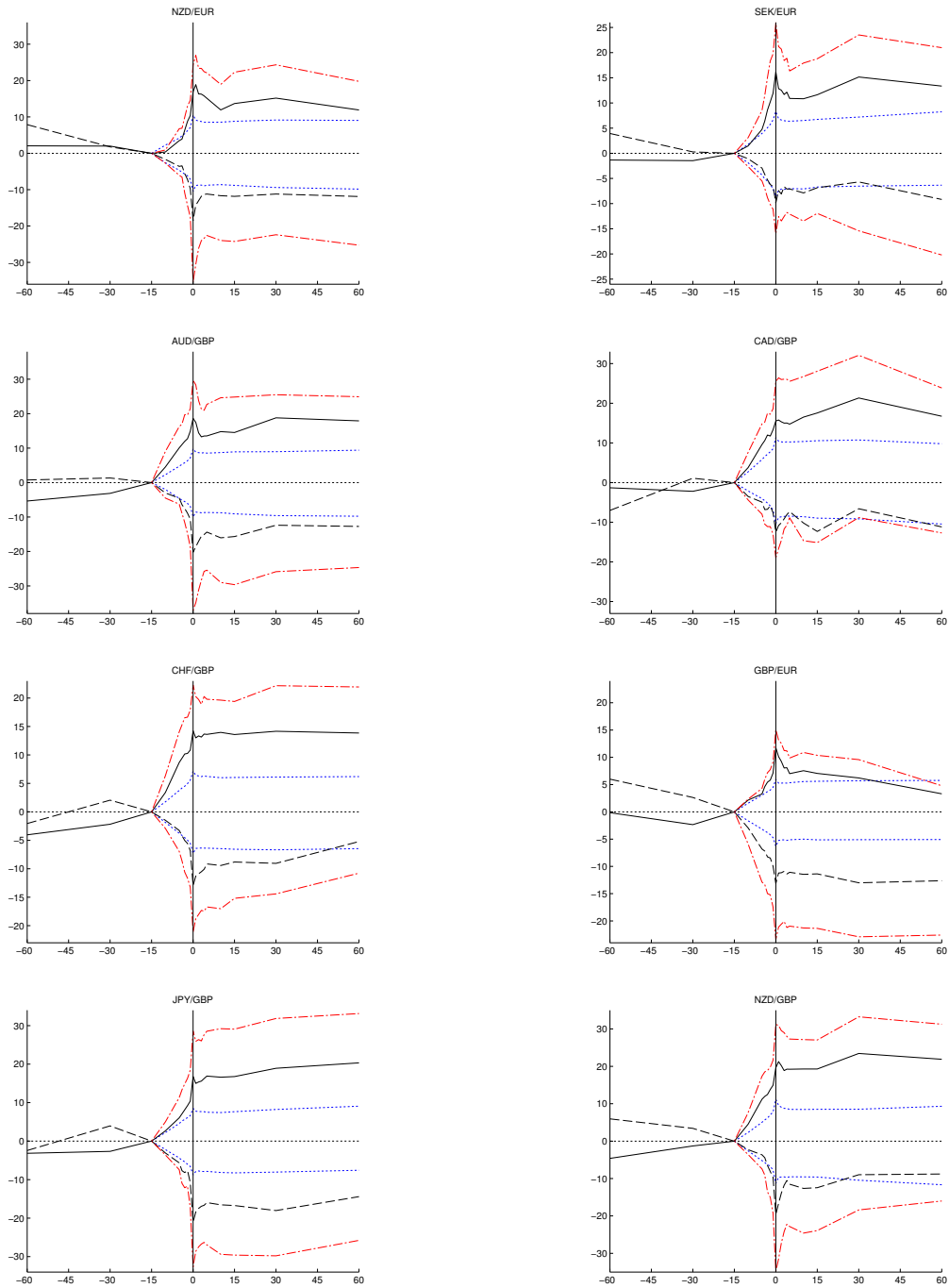
Notes: Each plot shows the contours of the estimated bivariate density for pre- and post-Fix price changes (in basis points) over horizons of 1 to 15 minutes. The solid line in each plot is the estimated regression line from the regression of the post-Fix price change on the pre-Fix change. All estimates are based on end-of-month data.

Figure A.19: Price Paths Around the Fix



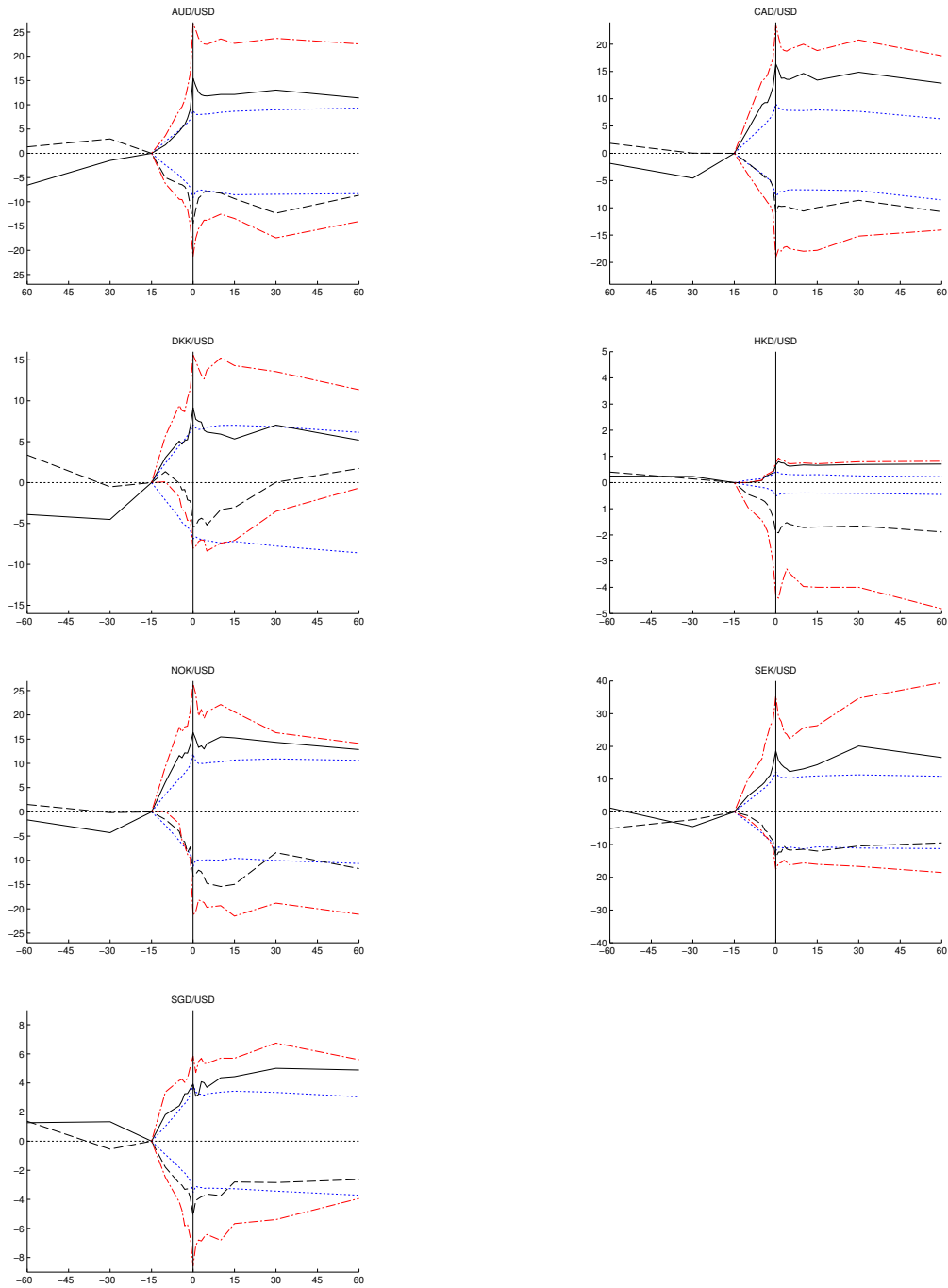
Notes: See below

Table A.19: Price Paths Around the Fix (cont.)



Notes: See below

Table A.19: Price Paths Around the Fix (cont.)



Notes: Average rate path in basis points around 3:45 pm level conditioned on: (i) positive pre-Fix changes (over 15 mins) at end of month (solid black); (ii) negative pre-Fix changes (over 15 mins) at end of month (dashed black); (iii) pre-Fix changes above the 75th. percentile of end-of-month distribution (upper red dashed dot); (iv) pre-Fix changes in the 25th. percentile of end-of-month distribution (lower red dashed dot); (v) positive and negative pre-Fix changes on intra-month days (upper and lower blue dots).