This paper studies the process of external adjustment. I develop an open economy model with endowment and preference shocks that can account for the empirical behavior of real exchange rates, interest rates and consumption in the U.S. and Europe. The model includes cross border holdings of bonds and equity, and financial frictions that impede international risk-sharing. I find that external adjustment following endowment shocks predominantly takes place via trade flows, consistent with the intertemporal approach to the current account. In contrast, preference shocks that change investors’ risk aversion induce adjustment via the trade and valuation channels; where the latter includes the effects of unexpected capital gains and loss on existing cross border holdings and changes in the expected future return differentials between foreign assets and liabilities. The model estimates imply that the valuation channel of external adjustment is more important for the U.S. than the trade channel. Consistent with this implication, I show that forecasts of future return differentials contributed most to the volatility of the U.S. net foreign asset position in the post Bretton-Woods era.
Introduction

A central challenge in international macroeconomics is to understand how unexpected changes in economic conditions in one country are transmitted internationally via international trade and financial markets. In principle, the process of external adjustment can take place via changes in consumption and investment decisions that generate international trade flows, via unexpected variations in asset prices, and through changes in risk premia that produce portfolio re-allocations driving international capital flows. In practice, our understanding of the process is very incomplete. What factors determine the importance of these different adjustment channels? Does greater integration of world financial markets facilitate adjustment or simply increase an economy’s susceptibility to adverse foreign shocks? Are the persistent and large imbalances in current accounts and net foreign asset positions we observe across the world consistent with a well-functioning adjustment process or are they a symptom of a fragile international financial system?

Unfortunately, standard open economy macro models give limited guidance on these questions. In canonical models where all international borrowing and lending take place via a single risk free bond, external adjustment takes place entirely via the current account reflecting revisions in forward-looking consumption and investment decisions. This perspective on adjustment, referred to as the intertemporal approach to the current account (see, e.g., Obstfeld and Rogoff, 1995), ignores the fact that there are large cross border holdings of many financial assets in different currencies, not just holdings of a single risk free bond. As such, it is silent on the possible adjustment roles played by unexpected capital gains and losses on existing holdings and expected capital gains and losses on future holdings that compensate investors for risk. These valuation effects may be empirically important. Gourinchas and Rey (2013) observe that changes in the net foreign asset positions of many counties appear to be increasingly influenced by capital gains and losses. Are these gains and losses unanticipated; or do they represent, in part, compensation for risk? More generally, are these financial effects benign, reflecting efficient risk sharing, or do they impede adjustment? The aim of this paper is to make progress on these questions. For this purpose I develop a new open economy model that allows for multiple channels of external adjustment and use it to assess their empirical importance.

The model is built around a standard core. There are two countries, each populated by a continuum of infinitely-lived households with preferences exhibiting home bias defined over the consumption of two perishable traded goods. Households have access to a wide array of financial assets: domestic equity and risk free bonds as well as foreign equity and bonds. Their optimal portfolio decisions are the driver of international capital flows in bonds and equity. On the production side I assume that the world supply of each traded good follows an exogenous endowment process and that there are no impediments to international goods trade. I add three key elements to this simple structure: collateral constraints that limit international borrowing, a portfolio friction that limits the flexibility households have in re-allocating their wealth among foreign bonds and equity, and habits in households’ preferences (adapted from Campbell and Cochrane, 1999). The first two elements make markets incomplete so households’ portfolio decisions affect equilibrium consumption, savings and international trade flows. The third element allows me to compare external adjustment following temporary output/endowment shocks with the adjustment following heteroskedastic preference shocks that change households’ risk aversion. The financial and macro effects of these risk shocks are quite different from
output/endowment shocks and play a central role in my analysis.

The model identifies two distinct adjustment processes; one triggered by output shocks and one triggered by risk shocks. Adjustment following output shocks occurs via international trade and unexpected capital gains. For example, a shock that temporarily raises the output of the domestic good induces a deterioration in the terms of trade (i.e., a fall in the relative price of domestic to foreign traded goods), and a rise (fall) in domestic (foreign) aggregate consumption. These adjustments ensure goods market clearing in the presence of home consumption bias. They also imply a jump depreciation in the real exchange rate followed by an expected future appreciation, and a fall in the domestic real interest rate relative to the foreign rate as households smooth consumption intertemporally - consistent with uncovered interest parity (UIP). The shock also produces higher domestic dividends together with unexpected capital gains on domestic equity and foreign asset holdings that finance domestic trade deficits until it dissipates. In sum, output shocks produce adjustments via international trade and unexpected gains/loses on existing net foreign asset positions. Quantitatively, most adjustment takes place via trade, referred to as “the trade channel” by Gourinchas and Rey (2007).

Adjustments triggered by risk shocks follow a different pattern because they change the risk premia embedded in expected future returns on domestic and foreign assets. For example, a shock that temporarily increases the risk aversion of domestic households induces an immediate appreciation in the real exchange rate. This improves international risk sharing (i.e., it reduces the difference between marginal utility across countries) but it also produces unexpected capital losses on domestic equity and foreign asset positions when domestic marginal utility is high. Households view these adverse valuation effects as more likely in the future because the current risk shock increases the likelihood of future shocks (via heteroskedasticity), so in equilibrium the risk premia on domestic equity and foreign assets (equity and bonds) rise to compensate. As a result, the risk shock produces an initial unexpected capital loss on the domestic net foreign asset position and higher expected future returns on foreign assets relative to liabilities until its effects dissipate. Beyond these external valuation adjustments, the risk shock increases domestic precautionary saving which lowers the real interest rate. As a consequence, there is a fall in real interest differential and a rise in the foreign exchange risk premium (the risk premium embedded in the expected excess return on foreign bonds) that together match the expected real depreciation rate following the shock. Unlike the pattern following output shocks, these adjustments are inconsistent with UIP. The risk shock also produces a trade surplus because the appreciation in the exchange rate improves the terms of trade. However, external adjustment via trade is quantitatively much less important than through the capital gains/loses and risk premia variations, collectively referred to as “the valuation channel” by Gourinchas and Rey (2007).

Clearly, these adjustment processes are quite different: temporary output shocks primarily trigger adjustment via the trade channel, while risk shocks mainly produce adjustment via the valuation channel. The contributions of these channels to actual adjustment thus depends on the relative importance of (temporary) output and risk shocks as drivers of international business cycles. To address this issue, I estimate the parameters of endowment and habit processes (driven by output and risk shocks respectively) so that the second moments for equilibrium real depreciation rates, interest rates and consumption growth rates in the model match their counterparts in quarterly U.S. and E.U. data. This estimation procedure reveals that
risk shocks are a much more important driver of exchange rate, interest rate and consumption dynamics in U.S./E.U. data than temporary output shocks. The key reason for this is that risk shocks produce a strong negative correlation between future depreciation rates and the current real interest differential via variations in the foreign exchange risk premium, whereas endowment shocks produce a positive correlation consistent with UIP. In U.S./E.U. data, future depreciation rates are strongly negatively correlated the current interest differential, so the incidence of risk shocks needs to be high for the model to replicate this feature of the data. The high incidence of risk shocks also allows the model to match the empirical volatilities of real depreciation rates, interest differentials and consumption growth rates.

My estimation results imply that valuation effects (driven by risk shocks) should play an important role in the U.S. external adjustment process. To investigate this implication of the model, I estimate the role of valuation effects in the U.S. process. Gourinchas and Rey (2007) first studied the empirical importance of the trade and valuation channels of external adjustment in the U.S. using an approximation that linked a country's external position to forecasts of future net export growth (identifying the trade channel) and forecast of future returns on foreign assets and liabilities (identifying the valuation channel). They found that the valuation channel accounted for approximately 30 percent of the cyclical variations in U.S. external position between 1952 and 2004. I use a similar approximation to estimate the contribution of forecast revisions for net export growth and returns to the variance of the U.S. net foreign asset position between 1973 and 2007. My estimates indicate that the valuation channel was more important than the trade channel during this period, contributing between 60 and 88 percent of variance. When I perform the same calculations with data from the estimated model, I find that the valuation channel accounts for 86 percent of the variance in the net foreign asset position.

Finally, I examine the implications of the estimated model for the behavior of international capital flows. Since households can hold domestic and foreign bonds and equity, the model allows for complex patterns of bond and equity inflows and outflows as part of the external adjustment process. Once again, these patterns depend on the shock triggering adjustment. Output shocks produce very small capital flows because they have negligible effects on households portfolio decisions. The international trade flows induced by the shocks are finance by capital gains/losses on existing positions and changes in dividends. In contrast, risk shocks produce debt and equity flows as households re-allocate their portfolios in the face of changing risk premia and risk aversion. It is here that the effects of financial frictions in the model appear most clearly. Intuitively, the portfolio friction stops the equilibrium risk premia on foreign equity and bonds from adjusting to a point where all households are willing to maintain their pre-existing holdings, so flows arise as household establish new optimal positions. These flows are also affected by the presence of collateral constraints. Unexpected capital losses that push households closer to the point where the constraint binds induce portfolio re-allocations that amplify the capital flows produced by the portfolio friction. According to the model estimates, the presence of collateral constraints doubles the size of the debt and equity capital flows induced by risk shocks.

The remainder of the paper is divided into six sections. The model I develop draws on several lines of research from the asset-pricing and open-economy macro literatures. Section 1 discusses this research and explains where I extend existing work. Section 2 presents the model. Section 3 explains how I solve and
calibrate the model to match the U.S. and E.U. data. In Section 4 I identify the elements that contribute to the external adjustment process. Section 5 contains my quantitative analysis of the model, including my empirical examination of the U.S. external adjustment process. Section 6 concludes. Information concerning the data used in my empirical analysis, the solution method and other mathematical details can be found in the Web Appendix.

1 Related Literature

My analysis is related to three main areas of research: The first studies the asset-pricing implications of non-standard preferences, the second examines portfolio choice in open economy general equilibrium models, and the third focuses on the process of international external adjustment.

This paper adds to a large literature that uses habit formation to explain asset prices in closed and open economies (see, e.g., Sundaresan, 1989, Abel 1990, Constantinides 1990, Detemple and Zapatero 1991, Ferson and Constantinides 1991, Heaton 1995, Jermann 1998, Boldrin, Christiano, and Fisher 2001, Chan and Kogan 2002, Menzly, Santos, and Veronesi 2004, Santos and Veronesi 2010 and Buraschi and Jiltsov 2007). The model I develop adapts the Campbell and Cochrane (1999) habit specification to an open-economy setting. In their model habits are a function of past consumption shocks which are completely determined by an exogenous endowment process. As in Bekaert, Engstrom, and Xing (2009), I allow habits to depend on domestic consumption and exogenous preference shocks. These shocks have the natural interpretation of risk shocks because they change the local curvature of the households’ utility. They are correlated with equilibrium consumption via variations in the terms of trade so that habits remain tightly linked to current and past consumption, as in Campbell and Cochrane (hereafter C&C).

In the C&C model consumption shocks can have differing effects on the marginal utility of investors (depending the proximity of current consumption to the level of habit) producing variations in the price of risk. Several papers in the international asset-pricing literature exploit this feature. Verdelhan (2010) uses the C&C habit specification to account for deviations from UIP in a two-country model with complete markets. In his model shocks to domestic consumption produce pro-cyclical movements in the real interest rates and counter-cyclical variations in the foreign exchange risk premia. Moore and Roche (2010) and Heyerdahl-Larsen (2012) also use models with habits and complete markets to account for UIP deviations, but here habits are relate to the consumption of individual goods (so called deep-habits). The model I present extends this line of research in two directions: First, I estimate parameters of the model so that it replicates the UIP deviations in U.S. and E.U. data in an equilibrium with incomplete risk-sharing and financial frictions. Second, I specify households’ preferences over multiple traded goods to allow for consumption home bias, international trade and realistic co-movements between real exchange rates and the terms of trade (see, e.g., Engel, 1999). These extensions narrow the gap between international asset-pricing and open economy

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1 Under conventional rational expectations assumptions, UIP implies that the slope coefficient from a regression of future exchange rate changes on the current differential between domestic and foreign risk-free rates should equal one. Instead, a very large literature finds estimates less than one and often negative; see, Lewis (1995) and Engel (1996, 2013) for surveys. The literature refers to these findings as deviations from UIP, the UIP puzzle or the forward premium anomaly.

2 In contrast, Verdelhan (2010) assumes that households only consume domestically produced goods. This extreme form of home bias allows him to circumvent the problems induced by variations in the terms of trade but at the cost of eliminating
macro literatures (see, e.g., Engel, 2013 and Lewis, 2011).

The model I develop falls into the class of dynamic general equilibrium models with portfolio choice. Papers by Coeurdacier and Gourinchas (2008), Engel and Matsumoto (2009), Coeurdacier, Kollmann, and Martin (2010), Hnatkovska (2010), Devereux and Sutherland (2011), Coeurdacier and Rey (2012) and others use these models to examine the reasons for home bias in equity portfolio holdings. This literature considers home bias arising from exchange-rate risk and non-tradable income risk, primarily in a complete markets setting. In contrast, home equity bias arises in my model because financial frictions stop the equilibrium risk premia from providing sufficient compensation to households for the poor exchange-rate hedging properties of foreign equity. These frictions also impede international risk sharing. In this respect the my analysis adds to a growing literature studying open economy models with portfolio choice and incomplete markets (see, e.g., Evans and Hnatkovska, 2013 and 2007, Hnatkovska, 2010, Devereux and Sutherland, 2011, and Tille and van Wincoop, 2010). One distinctive feature of my analysis relative to this literature concerns the solution method used to compute the equilibrium of the model. Both Devereux and Sutherland (2011) and Tille and van Wincoop (2010) use a method that approximates the financial side of the model in the neighborhood of deterministic steady state where differences between expected returns on risky assets are very small.\(^3\) This paper adapts the solution method developed by Evans and Hnatkovska (2012) to allow for the presence of collateral constraints. Importantly, it approximates the equilibrium of the model around a stochastic steady state where there are significant differences in the risk characteristics of different financial assets (e.g., domestic vs. foreign equity). These differences are reflected in the steady state risk premia, and in how the premia react to risk shocks - a central focus of my analysis.

My analysis also relates to research on the joint determination of capital flows and equity returns. Representative papers in this area include Bohn and Tesar (1996), Froot and Teo (2004) and Froot, O’Connell, and Seasholes (2001). Hau and Rey (2006, 2004) extend the analysis of the equity return-capital flow interaction to include the real exchange rate. Evans and Hnatkovska (2013), Devereux and Sutherland (2011), Tille and van Wincoop (2010) and Coeurdacier, Kollmann, and Martin (2010) all model capital flows as the result of optimal portfolio reallocations in a similar manner to this paper. Noteeworthy aspects of my analysis relative to these papers include: (i) the contrast between the effects of risk shocks on equity returns and capital flows and the effects of output shocks; and (ii) the role of collateral constraints.

Finally, the paper adds to a growing empirical and theoretical literature on international external adjustment. My empirical analysis of the U.S. external position is most closely related to the work of Gourinchas and Rey (2007) (noted above), but I examine variations in a measure of the total external position rather

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\(^3\) Technically speaking, the approximate solution to the model is computed by a perturbation method where the standard deviation of the exogenous shocks is one of the perturbation variables. Approximations to the expected returns on risky assets derived from this approach differ from zero to second-order (i.e. they depend on the variance of the shocks), so they are very similar across assets near the approximation point used to solve the model.
than the cyclical measure they study. This difference in focus accounts for the larger estimated contribution of the valuation channel I estimate. Corsetti and Constantinou (2012) also study the cyclical dynamics of the U.S. external position. They find that transitory shocks drive changes in the net foreign asset position while variations in aggregate consumption and gross positions are dominated by permanent shocks. In my model, risk shocks are the dominant source of transitory shocks that drive the net foreign asset position while permanent endowment shocks (common to all goods) change gross asset positions. On the theoretical side, Pavlova and Rigobon (2008), Tille and van Wincoop (2010) and Devereux and Sutherland (2011) all study external adjustment in open economy models with incomplete markets. In these models valuation adjustment takes place via unexpected capital gains and losses rather than through changes in the expected return differentials between foreign assets and liabilities.4

2 The Model

The model comprises two symmetric countries, which I refer to as the U.S. and Europe. Each country is populated by infinitely-lived households with preferences defined over the consumption of two perishable traded goods. For simplicity, there are no nontraded goods or impediments to trade in goods between the two countries. I also assume that the world supply of each traded good follows an exogenous endowment process, one located in each country.5 The model includes three key features beyond this simple structure: habits in household preferences, a collateral constrain that limits international borrowing, and a restriction that limits portfolio reallocations among foreign assets.

2.1 Household Preferences

Each country is populated by a continuum of identical households distributed on the interval [0,1]. Households’ preferences are defined over a basket of traded consumption goods. In particular, the expected utility of a representative U.S. household $i \in [0,1]$ in period $t$ is given by

$$U_{i,t} = E_t \sum_{j=0}^{\infty} \beta^j U(C_{i,t+j}, H_{t+j}), \quad \text{with} \quad U(C_{i,t}, H_t) = \frac{1}{1-\gamma} \{(C_{i,t} - H_t)^{1-\gamma} - 1\}, \quad (1)$$

where $1 > \beta > 0$, $\gamma > 0$ and $E_t$ denotes expectations conditioned on period-$t$ information, which is common to all households. Each household’s sub-utility $U(C_{i,t}, H_t)$ depends on current consumption, $C_{i,t}$, and the subsistence level of consumption, or habit level, $H_t$. This habit level is treated as exogenous by individual U.S. households but varies with aggregate U.S. consumption, $C_t$.6 Specifically, let $S_t = (C_t - H_t)/C_t$ denote

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4In Pavlova and Rigobon (2008) households have log preferences so there is no intertemporal hedging and risk premia are constant. Shocks produce negligible changes in expected return differentials between foreign assets and liabilities in Devereux and Sutherland (2011) and Tille and van Wincoop (2010) for the reasons discussed above.

5This assumption rules out the possibility that domestic production could be impaired by the effects of capital outflows on domestic credit markets, as in the literature on sudden stops. Clearly, this is a concern in economies where the domestic banking system is heavily dependent on foreign funding and domestic firms have limited access to world capital markets. My focus in this paper is on large economies where these issues are much less of a concern. I leave the task of extending the model to include production and domestic banking for future work.

6In this specification the level of habit is determined externally by the consumption decisions of all U.S. households. Alternatively, the level of habit could be determined internally by the consumption decisions of individual households. C&C find
the aggregate surplus consumption ratio. Following C\&C, I assume that the log surplus ratio \( s_t = \ln(S_t) \) follows a heteroskedastic AR(1) process

\[
s_{t+1} = (1 - \phi)\bar{s} + \phi s_t + \omega(s_t)v_{t+1},
\]

where \( 1 > \phi > 0 \) and \( \bar{s} \) is the steady state value of \( s_t \). (Hereafter I use lowercase letters to denote the natural log of their uppercase counterpart.) The i.i.d. mean-zero, unit variance \( v_t \) shocks affect the surplus ratio via a non-negative sensitivity function \( \omega(s_t) \), which I discuss below. Notice that the U.S. habit level is given by \( H_t = [1 - \exp(s_t)]C_t \) so negative \( v_t \) shocks raise habit relative to current aggregate consumption, but they cannot push habit above current consumption. The surplus ratio also determines the local curvature of households’ utility functions. For a representative household \( -C_{i,t}\mathcal{U}_i(C_{i,t}, H_t)/\mathcal{U}_{cc}(C_{i,t}, H_t) = \gamma/S_t \), so their risk aversion tends toward infinity as the surplus ratio goes to zero.

The U.S. consumption basket comprises U.S. and E.U. traded goods:

\[
C_{i,t} = \mathcal{C}(C_{i,t}^{US}, C_{i,t}^{EU}) = \left( \eta \frac{1}{\theta} C_{i,t}^{US \theta - 1} + (1 - \eta) \frac{1}{\theta} C_{i,t}^{EU \theta - 1} \right)^{\frac{\theta}{\theta - 1}},
\]

where \( C_{i,t}^{US} \) and \( C_{i,t}^{EU} \) identify the consumption of U.S. and E.U. goods by U.S. household \( i \). The parameter \( \eta \in (0, 1) \) governs the desired share of each good in the basket and \( \theta \) is the elasticity of substitution between goods. I follow standard practice in the literature and focus on the case where \( \eta > 1/2 \), so that households’ preferences exhibit consumption home-bias.

E.U. household \( i \in [0, 1] \) has analogous preferences:

\[
\hat{\mathcal{U}}_{i,t} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \mathcal{U}(!\hat{C}_{i,t+j}, \hat{H}_{t+j}),
\]

where the E.U. consumption basket is

\[
\hat{C}_{i,t} = \mathcal{C}(\hat{C}_{i,t}^{US}, \hat{C}_{i,t}^{EU}) = \left( \eta \frac{1}{\theta} \hat{C}_{i,t}^{US \theta - 1} + (1 - \eta) \frac{1}{\theta} \hat{C}_{i,t}^{EU \theta - 1} \right)^{\frac{\theta}{\theta - 1}}.
\]

The external E.U. habit level \( \hat{H}_t = [1 - \exp(\hat{s}_t)]\hat{C}_t \) depends on aggregate E.U. consumption, \( \hat{C}_t \), and the log surplus ratio, \( \hat{s}_t = \ln \hat{S}_t \), which follows

\[
\hat{s}_{t+1} = (1 - \phi)\bar{s} + \phi \hat{s}_t + \omega(\hat{s}_t)\hat{v}_{t+1},
\]

where \( \hat{v}_{t+1} \) are i.i.d. mean-zero, unit variance shocks. As above, the surplus ratio \( \hat{S}_t \) determines the local curvature of E.U. household utility: \( -\hat{C}_{i,t}\mathcal{U}_i(\hat{C}_{i,t}, \hat{H}_t)/\mathcal{U}_{cc}(\hat{C}_{i,t}, \hat{H}_t) = \gamma/\hat{S}_t \).

In the C\&C model shocks to the log surplus ratio are perfectly correlated with unexpected changes in aggregate consumption. In their closed economy equilibrium consumption is determined by an endowment process so unexpected consumption depends on the exogenous endowment shock. This specification also that most asset-pricing implications of their model are robust to the presence of internal rather than external habits. Since models with internal habits are inherently more complex, I utilize the external habit specification here.
implies that the level of external habit is a nonlinear function of current and past aggregate consumption. Here, in contrast, each country’s aggregate consumption comprises a basket of traded goods so shocks affecting the endowments of either good and/or relative prices contribute to unexpected consumption in both countries. Thus, in this setting the C&C specification would produce a cross-country correlation between shocks to the log surplus ratios that greatly complicates the analysis of how changes in risk-aversion are transmitted internationally. To facilitate this analysis, I assume instead that shocks to the log surplus processes in (2) and (6) are independent and exogenous. They thus have the natural interpretation as risk shocks because they change the local curvature of households’ utility. This specification enables us to study how changes in risk aversion originating in one country are transmitted internationally by tracing their effects on equilibrium asset prices and capital flows.

My specification for the $s_t$ and $\hat{s}_t$ processes has two other noteworthy implications. First, it does not rule out a correlation between consumption growth and the log surplus ratio in each country. On the contrary, risk shocks affect consumption growth because they alter the real exchange rate and the terms of trade, which in turn induce households to adjust the composition of their consumption baskets. Thus, consumption growth is correlated with the log surplus ratios, but the correlation is imperfect and is determined as part of the model’s equilibrium. This feature of the model is consistent with the results in Bekaert, Engstrom, and Xing (2009). They find evidence in U.S. data of an imperfect correlation between the log surplus ratio and consumption growth in a model that adds exogenous shocks to the C&C specification.

The second implication concerns the dynamics of habits. By definition the level of U.S. habit is given by $H_t = [1 - \exp(s_t)]C_t$, so domestic risk shocks can in principle affect the level of habit independently of their effect on aggregate consumption (i.e., via their impact on $s_t$). This is not an important effect in the calibrated equilibrium of the model. I show below that almost all the variations in the level of habit are related to current and past innovations in domestic consumption.

The processes for the log surplus ratios in (2) and (6) also contain a sensitivity function that governs the reaction of the ratios to risk shocks. I assume that the function is decreasing in the log surplus:

$$
\omega(s) = \begin{cases} 
\sqrt{\omega(s_{\text{max}} - s)} & s \leq s_{\text{max}} \\
0 & s > s_{\text{max}} 
\end{cases},
$$

where $\omega$ is a positive parameter. In the continuous time limit, $s_t$ and $\hat{s}_t$ never exceed the upper bound of $s_{\text{max}}$, so there is a corresponding lower limit on households’ risk-aversion. The sensitivity function in the C&C specification is also decreasing in the log surplus ratio but is parameterized to keep risk free interest rates constant. The specification here allows for variations in the risk free rates depending on the values for $\omega$ and $s_{\text{max}}$. I estimate these parameters from data on real exchange rates, real interest rates and consumption below.

**Prices and Exchange Rates**

The model contains two international relative prices: the terms of trade and the real exchange rate. These prices are linked through the consumption price indices in each country. Let $P_t^{US}$ and $P_t^{EU}$ denote the prices
of the U.S. and E.U. goods in dollars, while \( \hat{P}_t^{US} \) and \( \hat{P}_t^{EU} \) denote their prices in euros, respectively. The U.S. and E.U. price indices corresponding to the consumption baskets in (3) and (5) are

\[
P_t = \left( \eta (P_{US}^t)^{1-\theta} + (1 - \eta) (P_{EU}^t)^{1-\theta} \right)^{\frac{1}{1-\theta}} \quad \text{and} \quad \hat{P}_t = \left( \eta (\hat{P}_{EU}^t)^{1-\theta} + (1 - \eta) (\hat{P}_{US}^t)^{1-\theta} \right)^{\frac{1}{1-\theta}}.
\]

The real exchange rate is defined as the relative price of the E.U. consumption basket in terms of the U.S. basket, \( E_t = S_t \hat{P}_t / P_t \), where \( S_t \) denotes the dollar price of euros. There are no impediments to international trade between the U.S. and the E.U. so the law of one price applies to both the U.S. and E.U. goods: i.e., \( P_{US}^t = S_t \hat{P}_{US}^t \) and \( P_{EU}^t = S_t \hat{P}_{EU}^t \). Combining these expressions with the definitions of the real exchange rate and the price indices gives

\[
E_t = \left( \frac{\eta \hat{T}_t^{1-\theta} + (1 - \eta) \hat{T}_t^{1-\theta}}{\eta + (1 - \eta) \hat{T}_t^{1-\theta}} \right)^{-\frac{1}{1-\theta}},
\]

(7)

where \( \hat{T}_t \) denotes the U.S. terms of trade, defined as the relative price of imports in terms of exports, \( \hat{P}_{EU}^t / \hat{P}_{US}^t \). When there is home bias in consumption (\( \eta > 1/2 \)), a deterioration in the U.S. terms of trade (i.e., a rise in \( \hat{T}_t \)) is associated with a real depreciation of the dollar (i.e., a rise in \( E_t \)).

Equation (7) makes clear that the model attributes all variations in real exchange rates to changes in the relative prices of traded goods via the terms of trade. This feature of the model is broadly consistent with existing empirical evidence on the source of real exchange rate variations. For example, Engel (1999) finds that little of the variation in real depreciations rates over horizons of five years or less originate from differences between the relative prices of traded and nontraded goods across countries. The omission of non-traded goods from the model doesn’t significantly impair its ability to replicate the short- and medium-term dynamics of real exchange rates. Equation (7) also implies that the real depreciation rate is strongly correlated with changes in the terms of trade when there is home bias in consumption. This implication is also consistent with the data. Real depreciation rates are very strongly correlated with changes in terms of trade over short horizons (see, e.g., Evans, 2011).

**Assets and Goods Markets**

There are four financial markets: a market for U.S. equities, E.U. equities, U.S. bonds and E.U. bonds. U.S. equity represents a claim on the stream of U.S. good endowments. In particular, at the start of period \( t \), the holder of one share of U.S. equity receives a dividend of \( D_t = (P_{US}^t / P_t) Y_t \), measured in terms of the U.S. consumption basket, where \( Y_t \) is the endowment of the U.S. good. The ex-dividend price of U.S. equity in period \( t \) is \( Q_t \), again measured in terms of U.S. consumption. A share of E.U. equity pays a period-\( t \) dividend of \( \hat{D}_t = (\hat{P}_{EU}^t / \hat{P}_t) \hat{Y}_t \) and has an ex-dividend price of \( \hat{Q}_t \), where \( \hat{Y}_t \) is the period-\( t \) endowment of the E.U. good. Notice that \( \hat{Q}_t \) and \( \hat{D}_t \) are measured relative to the E.U. consumption basket. The gross real returns on U.S. and E.U. equity are given by

\[
R_{t+1}^{EQ} = (Q_{t+1} + D_{t+1}) / Q_t \quad \text{and} \quad \hat{R}_{t+1}^{EQ} = (\hat{Q}_{t+1} + \hat{D}_{t+1}) / \hat{Q}_t.
\]
Households can also hold one-period real U.S. and E.U. bonds. The gross return on holding U.S. bonds between periods $t$ and $t+1$ is $R_t$, measured in terms of U.S. consumption; the analogous return on E.U. bonds is $\hat{R}_t$, measured in terms of E.U. consumption.

Households have access to both domestic and foreign bond and equity markets subject to two financial frictions (discussed below). Let $B_{i,t}^{US}$, $B_{i,t}^{EU}$, $A_{i,t}^{US}$ and $A_{i,t}^{EU}$ respectively denote U.S. household $i$'s holdings of U.S. and E.U. bonds, and shares of U.S. and E.U. equity in period $t$. The budget constraint facing the household is

$$E_i \hat{Q}_t A_{i,t}^{EU} + E_i B_{i,t}^{EU} + Q_t A_{i,t}^{US} + B_{i,t}^{US} + C_{i,t} = R_{t-1} B_{i,t-1}^{US} + A_{i,t-1}^{US} (Q_t + D_t) + E_i \hat{R}_{t-1} B_{i,t-1}^{EU} + E_i A_{i,t-1}^{EU} (\hat{Q}_t + \hat{D}_t).$$

(8)

Notice that the first two terms on the left-hand-side represent the value of U.S. foreign asset holdings in period $t$. The budget constraint facing E.U. household $i$ is

$$Q_t \hat{A}_{i,t}^{US}/E_t + \hat{B}_{i,t}^{US}/E_t + \hat{B}_{i,t}^{EU} + \hat{Q}_t \hat{A}_{i,t}^{EU} + \hat{C}_{i,t} = \hat{A}_{i,t-1}^{EU} (\hat{Q}_t + \hat{D}_t) + \hat{R}_{t-1} \hat{B}_{i,t-1}^{EU} + R_{t-1} \hat{B}_{i,t-1}^{US}/E_t + \hat{A}_{i,t-1}^{US} (Q_t + D_t)/E_t,$$

(9)

where $\hat{B}_{i,t}^{US}$, $\hat{B}_{i,t}^{EU}$, $\hat{A}_{i,t}^{US}$ and $\hat{A}_{i,t}^{EU}$ respectively denote the number of U.S. bonds, E.U. bonds and the number of shares of U.S. and E.U. equity held by E.U. household $i$ in period $t$.

The market clearing conditions are straightforward. Both U.S. and E.U bonds are in zero net supply so the bond market clearing conditions are

$$0 = \int_0^1 B_{i,t}^{US} di + \int_0^1 \hat{B}_{i,t}^{US} di \quad \text{and} \quad 0 = \int_0^1 B_{i,t}^{EU} di + \int_0^1 \hat{B}_{i,t}^{EU} di.$$

(10)

The supplies of the U.S. and E.U. equities are normalized to one, so market clearing requires that

$$1 = \int_0^1 A_{i,t}^{US} di + \int_0^1 \hat{A}_{i,t}^{US} di \quad \text{and} \quad 1 = \int_0^1 A_{i,t}^{EU} di + \int_0^1 \hat{A}_{i,t}^{EU} di.$$

(11)

Market clearing in the goods markets requires that aggregate demand from U.S. and E.U. households matches the world endowment of each traded good:

$$Y_t = \int_0^1 C_{i,t}^{US} di + \int_0^1 \hat{C}_{i,t}^{US} di \quad \text{and} \quad \hat{Y}_t = \int_0^1 C_{i,t}^{EU} di + \int_0^1 \hat{C}_{i,t}^{EU} di.$$

(12)

Endowments of the two traded goods are driven by exogenous non-stationary processes. Specifically, I assume that the log endowments $y_t = \ln Y_t$ and $\hat{y}_t = \ln \hat{Y}_t$ follow:

$$y_t = \bar{z}_t + z_t \quad \text{and} \quad \hat{y}_t = \bar{\hat{z}}_t + \hat{z}_t,$$

(13a)

$$\bar{z}_t = \bar{z}_{t-1} + g + u_t, \quad z_t = \rho z_{t-1} + e_t \quad \text{and} \quad \bar{\hat{z}}_t = \rho \bar{\hat{z}}_{t-1} + \hat{e}_t.$$  

(13b)
Here $z_t$ identifies the stochastic trend (unit root process), while $z_t$ and $\hat{z}_t$ denote the cyclical components that follow AR(1) processes with $1 > \rho > 0$. The three endowment shocks, $u_t$, $e_t$ and $\hat{e}_t$ are mutually uncorrelated mean-zero random variables, with variances $\sigma_u^2$, $\sigma_e^2$ and $\sigma_{\hat{e}}^2$, respectively. In the absence of any shocks, both endowments grow at rate $g$.

**Financial Frictions and Household Decisions**

Households face two financial frictions when making their optimal consumption and portfolio decisions. The first limits their ability to reallocate their foreign asset holdings between bonds and equities. The second friction takes the form of a collateral constraint that prevents rolling over international debt through the use of Ponzi schemes.

I assume that households cannot hold foreign equity and bonds directly. Rather, they can hold them indirectly in a foreign asset mutual fund with fixed proportions. Specifically let $FA_{i,t} = E_t\hat{Q}_tA^EU_{i,t} + E_tB^EU_{i,t}$ denote the value of the foreign mutual fund held by U.S. household $i$ in period $t$. The portfolio weights for equity and bonds in the fund are fixed at $\{\}$ and $1\}$, so that the value of E.U. equity and bond holdings held indirectly by U.S. household $i$ are

$$E_t\hat{Q}_tA^EU_{i,t} = \varphi FA_{i,t} \quad \text{and} \quad E_tB^EU_{i,t} = (1 - \varphi)FA_{i,t}. \quad (14)$$

Similarly, E.U. households hold U.S. bonds and equities indirectly as part of their foreign mutual fund, $\hat{FA}_{i,t} = Q_t\hat{A}^{US}_{i,t}/E_t + \hat{B}^{US}_{i,t}/E_t$, such that

$$Q_t\hat{A}^{US}_{i,t}/E_t = \varphi \hat{FA}_{i,t} \quad \text{and} \quad \hat{B}^{US}_{i,t}/E_t = (1 - \varphi)\hat{FA}_{i,t}. \quad (15)$$

The return on U.S. and E.U foreign asset mutual funds are given by

$$R^PA_{i+1} = \left(\frac{E_{i+1}}{E_i}\right)(\varphi R^PQ_{i+1} + (1 - \varphi)\hat{R}_i) \quad \text{and} \quad \hat{R}^PA_{i+1} = \left(\frac{E_{i+1}}{E_i}\right)\left(\varphi R^PQ_{i+1} + (1 - \varphi)\hat{R}_i\right).$$

These assumptions limit households’ ability to change the composition of their foreign asset portfolios but not the value of foreign assets in their total wealth. Obviously, they are not an accurate description of the actual re-allocation costs households face, but they can be interpreted as arising from the presence of fixed transaction costs.\footnote{An alternative would be to assume that households could hold foreign assets directly and incur a transaction cost whenever they change their holdings (see e.g., Meier, 2013). This approach would also give households’ more flexibility in reallocating their wealth among domestic than foreign assets, but the explicit introduction of transaction costs would add significantly to an already complex model.} Suppose, for example, that each period households could choose from a continuum of foreign asset mutual funds, each characterized by a different value for $\varphi$. Further, assume that establishing an account with a fund incurs a fixed cost, while changing the amount invested in an existing account does not. In the steady state all households would establish an account in the fund with the value for $\varphi$, say $\varphi^*$, that maximized expected utility. Away from the steady state, they would compare the utility from moving to a new fund net of the fixed cost with the utility of keeping the $\varphi^*$ fund. If the fixed costs were sufficiently large, households would find it optimal to vary the amount of wealth invested in the $\varphi^*$ fund,
so the assumption above would hold with \( \varphi = \varphi^* \). This fixed cost interpretation is broadly consistent with U.S. data. In Section 3.3 I show that the value for \( \varphi^* \) implied by the model is close the value for \( \varphi \) implied by the average equity-to-debt ratio in U.S. foreign asset positions.

The effects of greater financial integration in world equity markets on external adjustment and capital flows can be easily studied by comparing solutions of the model with different values \( \varphi \). For example, all foreign assets are held in the form of bonds when \( \varphi = 0 \), so international bond transactions exclusively drive capital flows. My analysis of the external adjustment process below is based on a calibration of the model where the value of \( \varphi \) matches the average equity-to-debt ratio in U.S. foreign asset positions. These data show that the shares of equity and debt in foreign assets and liabilities are quite stable on a quarter-by-quarter basis and only make a small contribution to the variability of asset and liability returns.

The second financial friction takes the form of a collateral constraint. For the case of U.S. household \( i \), the constraint takes the form of a lower bound on U.S. bond holdings:

\[
B_{i,t}^{US} \geq -(1 + \varepsilon)FA_{i,t},
\]  

(16)

where \( \varepsilon > 0 \). The constraint implies that U.S. households can borrow by issuing domestic bonds up to the point where the real value of their debt is \((1 + \varepsilon)\) times the value of their foreign mutual fund holdings. The constraint facing E.U. household \( i \) takes an analogous form:

\[
\hat{B}_{i,t}^{EU} \geq -(1 + \varepsilon)\hat{FA}_{i,t}.
\]  

(17)

The constraints in (16) and (17) have several noteworthy implications. First, they do not limit the total amount of borrowing by any household. Second, variations in both exchange rates and equity prices affect the collateral value of foreign fund holdings. For example, since \( FA_{i,t} = E_t \hat{Q}_t A_{i,t}^{EU} + \mathcal{E}_t B_{i,t}^{EU} \), a shock inducing a fall in foreign equity prices \( \hat{Q}_t \) and/or a real appreciation of the dollar (i.e. a fall in \( E_t \)) will push U.S. households towards the point where the constraint binds. This, in turn, can lead to financial amplification. If households react in a way that induces a further fall in foreign equity prices and/or an appreciation of the dollar, the effects of the shock will be amplified by the presence of the constraint (see Jeanne and Korinek, 2010 and Brunnermeier, Eisenbach, and Sannikov, 2012). Third, the constraints in (16) and (17) assume that foreign equity and bonds (held indirectly in mutual funds) have collateral value but not domestic equity holdings. I omit domestic equity from the constraints because it cannot be held directly by the lender, even in the event of a default. For example, any U.S. equity seized in a default would have to be held by the E.U. foreign asset mutual fund, and so could only compensate E.U. households (indirectly) if the fund lent more to U.S. households by purchasing additional U.S. bonds. By contrast, E.U. equity and bonds seized in a default can be held directly by E.U. households and so provide compensation without the need for further international lending.\(^8\)

\(^8\)In this model all borrowing and lending takes place internationally because households within each country are identical. If the model were extended to include different agents types (e.g., investors and savers) within each country, domestic equity would also have collateral value insofar as it compensated domestic savers in a default. Devereux and Yetman (2010) analyze a model where borrowing and lending takes place between investors and savers within countries and internationally subject to collateral constraints where both domestic and foreign equity has collateral value.
I now describe the consumption and portfolio allocations decisions facing households. Let $W_{i,t}$ denote the real wealth of U.S. household $i$ at the start of period $t$ and let $\alpha_{i,t}^{EQ}$ and $\alpha_{i,t}^{FA}$ identify the fraction of wealth held in domestic equity and foreign assets at the end of period $t$:

$$\alpha_{i,t}^{EQ} = Q_{t}A_{i,t}^{US}/(W_{i,t} - C_{i,t}) \quad \text{and} \quad \alpha_{i,t}^{FA} = FA_{i,t}/(W_{i,t} - C_{i,t}).$$

The problem facing U.S. household $i$ may now be written as

$$\text{Max}_{\{C_{i,t},C_{i,t}^{EU},\alpha_{i,t}^{EQ},\alpha_{i,t}^{FA}\}} \mathbb{E}_{t} \sum_{j=0}^{\infty} \beta^{j}U(C(C_{i,t+j},C_{i,t+j}^{EU},H_{t+j})) \quad (18a)$$

s.t. $W_{i,t+1} = R_{i,t+1}^{W}(W_{i,t} - C_{i,t})$ and

$$N_{i,t} = (1 - \alpha_{i,t}^{EQ} + \alpha_{i,t}^{FA})(W_{i,t} - C_{i,t}) \geq 0. \quad (18c)$$

Equation (18c) rewrites the collateral restriction in (16) using the portfolio shares. Equation (18b) rewrites the budget constraint in terms of wealth and the real return on the households’ portfolio:

$$R_{i,t+1}^{W} = R_{t} + \alpha_{i,t}^{EQ}(R_{t+1}^{EQ} - R_{t}) + \alpha_{i,t}^{FA}(R_{t+1}^{FA} - R_{t}). \quad (19)$$

The problem facing E.U. household $i$ is analogous:

$$\text{Max}_{\{\hat{C}_{i,t},\hat{C}_{i,t}^{EU},\hat{\alpha}_{i,t}^{EQ},\hat{\alpha}_{i,t}^{FA}\}} \mathbb{E}_{t} \sum_{j=0}^{\infty} \beta^{j}U(\hat{C}(\hat{C}_{i,t+j},\hat{C}_{i,t+j}^{EU},\hat{H}_{t+j})) \quad (20a)$$

s.t. $\hat{W}_{i,t+1} = \hat{R}_{i,t+1}^{W}(\hat{W}_{i,t} - \hat{C}_{i,t})$ and

$$\hat{N}_{i,t} = (1 - \hat{\alpha}_{i,t}^{EQ} + \hat{\alpha}_{i,t}^{FA})(\hat{W}_{i,t} - \hat{C}_{i,t}) \geq 0. \quad (20c)$$

Here $\hat{\alpha}_{i,t}^{EQ} = Q_{t}A_{i,t}^{EU}/(\hat{W}_{i,t} - \hat{C}_{i,t})$ and $\hat{\alpha}_{i,t}^{FA} = FA_{i,t}(\hat{W}_{i,t} - \hat{C}_{i,t})$ are the fractions of wealth invested in E.U. equity and foreign assets; $\hat{W}_{i,t}$ is real wealth (measured in terms of E.U. consumption) at the start of period $t$, and $\hat{R}_{i,t+1}^{W}$ is the real return on the household’s portfolio:

$$\hat{R}_{i,t+1}^{W} = \hat{R}_{t} + \hat{\alpha}_{i,t}^{EQ}(\hat{R}_{t+1}^{EQ} - \hat{R}_{t}) + \hat{\alpha}_{i,t}^{FA}(\hat{R}_{t+1}^{FA} - \hat{R}_{t}). \quad (21)$$

Since households within each country have the same preferences and face the same constraints, we can focus on the behavior of a representative U.S. and E.U. household without loss of generality. Hereafter, I drop the $i$ subscripts on consumption and the portfolio shares to simplify notation.
3 Equilibrium

3.1 Solution Method

An equilibrium in this model comprises a sequence for the real exchange rate \( \{E_t\} \), real interest rates \( \{R_t\}\) and \( \hat{R}_t \), and equity returns \( \{R_{eq}^t\} \) and \( \hat{R}_{eq}^t \), consistent with market clearing in the goods and asset markets given the optimal consumption and portfolio decisions of households and the exogenous endowments. Finding the equilibrium processes for \( \{E_t, R_t, \hat{R}_t, R_{eq}^t, \hat{R}_{eq}^t\} \) is complicated by the presence of incomplete markets, portfolio choice, and occasionally binding collateral constraints. In principle, models with these features can be solved with existing global methods, but here the method is computationally infeasible because the state space is too large.\(^9\) Local methods, based on approximations around the steady state, often provide a computationally attractive alternative when this curse of dimensionality appears. For example, Evans and Hnatkovska (2012) Tille and van Wincoop (2010) and Devereux and Sutherland (2011) show how models with portfolio choice, incomplete markets and large state spaces can be solved with local methods, but they do not accommodate occasionally binding constraints. In recognition of these problems, I use a new solution method developed Evans (2012). It combines barrier methods and approximations around the model’s stochastic steady state to produce an accurate solution in a very computationally efficient manner. Consequently, I am able to use the solution method as part of an GMM estimation procedure in which the model is solved thousands of times to match moments of U.S. and E.U. data. Below, I provide a brief overview of the method. A detailed description and accuracy assessment can be found in the Web Appendix.

Barrier methods are widely used in the optimal control literature to solve optimization problems involving inequality constraints (see, e.g., Forsgren Anders and Wright, 2002). The basic idea is to modify the objective function so that the optimizing agent is penalized as his actions bring him closer to the barrier described by the inequality constraint. This approach converts the original optimization problem with inequality constraints into one with only equality constraints that can be readily combined with the other equilibrium conditions to derive an approximate solution to the model. Preston and Roca (2007) and Kim, Kollmann, and Kim (2010) use barrier methods in this way to solve incomplete markets’ models with heterogeneous agents.

Following Kim, Kollmann, and Kim (2010), I modify the sub-utility functions for the representative U.S. and E.U. households to

\[
U(C_t, H_t, N_t) = \frac{1}{1 - \gamma} \left\{ (C_t - H_t)^{1-\gamma} - 1 \right\} + \frac{\mu \bar{N}_t}{(\bar{C}_t \bar{S})^{\gamma}} \left\{ \ln \left( \frac{\bar{N}_t}{\bar{N}_t} \right) - \left( \frac{N_t - \bar{N}_t}{N_t} \right) \right\},
\]

and

\[
U(\hat{C}_t, \hat{H}_t, \hat{N}_t) = \frac{1}{1 - \gamma} \left\{ (\hat{C}_t - \hat{H}_t)^{1-\gamma} - 1 \right\} + \frac{\mu \bar{N}_t}{(\bar{C}_t \bar{S})^{\gamma}} \left\{ \ln \left( \frac{\hat{N}_t}{\bar{N}_t} \right) - \left( \frac{\hat{N}_t - \bar{N}_t}{\hat{N}_t} \right) \right\},
\]

where \( \mu > 0 \) and bars denote the values of variables in the steady state. These modifications penalize

---

\(^9\) Rabitsch, Stepanchuk, and Tsyrennikov (2013) use a global solution method to analyze a small model with portfolio choice and incomplete markets, but they are forced for tractability reasons to focus on a wealth-recursive Markov equilibrium in which relative wealth is the only endogenous state variable. Applying their global solution method to this model is impracticable because there are simply too many endogenous state variables.
households as their portfolio choices bring them closer to their respective collateral constraints. For example, as U.S. households’ foreign asset holdings close in on the point where the constraint binds, \( N_t = (1 - \alpha t^{02} + x\alpha t^{03})(W_t - C_t) \) nears zero, and the last term on the right-hand-side of (22a) approaches its limiting value of \(-\infty\). Similarly, the last term in (22b) approaches \(-\infty\) when E.U. households near the point where their collateral constraint binds. As a consequence, households will occasionally choose portfolios that come close to the point where the collateral constraint binds, but never to the point where it actually does. The importance of this distinction depends on the size of the barrier parameter, \( \mu \), that governs the rate at which the utility cost rises as the household approaches the constraint. If we consider a sequence of solutions to the modified households’ problems as \( \mu \) takes smaller and smaller values, the sequence will converge to the solutions of their original problem in the limit as \( \mu \to 0 \) (see, e.g., Forsgren Anders and Wright, 2002). The solutions of the model I examine below are robust to alternative choices for \( \mu \) close to zero. Notice, also, that the last terms on the right-hand-side on (22) disappear when \( N_t = \hat{N}_t = \tilde{N}_t \). This implies that household decisions are unaffected by collateral constraints in the steady state.

I use standard log-linear approximations to the households’ first-order conditions from the modified optimization problems and the market clearing conditions to find the equilibrium process for the real exchange rate, real interest rates, and other endogenous variables. These approximations are computed around the model’s stochastic steady state.\(^{10}\) This is the point at which the exogenous surplus ratios, \( s_t \) and \( \hat{s}_t \), equal their long run value of \( \bar{s} \), and the cyclical components in the endowment processes, \( z_i \) and \( \hat{z}_i \), equal zero so the endowments of U.S. and E.U. goods are equal and follow the stochastic trend (i.e., \( y_t = \hat{y}_t = \bar{z}_t \)).\(^{11}\) In the stochastic steady state households expect both endowments to grow at rate \( g \), (i.e., \( E_t \Delta y_{t+i} = E_t \Delta \hat{y}_{t+i} = g \), for all \( i > 0 \) ), but they do not expect any future changes in the surplus ratios (i.e., \( E_t s_{t+i} = E_t \hat{s}_{t+i} = \bar{s} \) for all \( i > 0 \) ). Households also recognize that future endowments and surplus ratios are subject to shocks. It is this recognition that distinguishes the stochastic steady state from its conventional deterministic counterpart. It implies that households’ steady state portfolios are uniquely identified from the joint conditional distribution of future returns and marginal utility.\(^{12}\)

### 3.2 Parameterization

The model contains 15 parameters: the preference parameters, \( \beta, \gamma, \eta \) and \( \theta \); the parameters governing the surplus ratios, \( \phi, \bar{s}, s_{\text{max}} \) and \( \omega \); the parameters of the endowment processes, \( \rho, g, \sigma_e^2 \) and \( \sigma_i^2 \); the collateral constraint, \( \kappa \), the barrier parameter, \( \mu \), and foreign asset weight, \( \varphi \). I set the values for some parameters to be consistent with the values that appear elsewhere in the literature. The values for other parameters are estimated by the Generalized Method of Moments (GMM) using U.S. and E.U. data.

---

\(^{10}\)In some models, incomplete risk-sharing induces non-stationary dynamics in household wealth so there is no unique deterministic steady state around which to approximate equilibrium dynamics. Schnitt-Grohe and Uribe (2003) discuss how the introduction of endogenous discounting in households’ preferences, asset-holding costs and (ad hoc) debt-elastic interest rates can induce stationarity. In this model stationarity is induced endogenously via the collateral constraints that make interest rates sensitive to a country’s net foreign asset position.

\(^{11}\)Coeurdacier, Rey, and Winant (2011) use a similar steady state concept, but their definition includes all the state variables not just the exogenous state variables as I do here.

\(^{12}\)All assets have the same riskless return in the deterministic steady state so portfolios are not uniquely identified. One approach to this identification problem is to consider the portfolio choices in the limit as the variance of exogenous shocks goes to zero; see, e.g., Judd and Guu (2001).
Table 1: **Parameterization**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: Assigned Values</td>
<td></td>
<td></td>
</tr>
<tr>
<td>β</td>
<td>discount function</td>
<td>0.990</td>
</tr>
<tr>
<td>γ</td>
<td>utility curvature</td>
<td>2.000</td>
</tr>
<tr>
<td>η</td>
<td>home good share</td>
<td>0.850</td>
</tr>
<tr>
<td>θ</td>
<td>elasticity of substitution</td>
<td>0.110</td>
</tr>
<tr>
<td>g</td>
<td>steady state growth rate</td>
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</tr>
<tr>
<td>$\bar{S}$</td>
<td>average log consumption-surplus</td>
<td>0.050</td>
</tr>
<tr>
<td>φ</td>
<td>equity share in foreign asset portfolios</td>
<td>0.420</td>
</tr>
<tr>
<td>χ</td>
<td>collateral constraint</td>
<td>0.500</td>
</tr>
<tr>
<td>μ</td>
<td>barrier parameter</td>
<td>0.030</td>
</tr>
<tr>
<td>B: GMM Estimates</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>autocorrelation in log surplus</td>
<td>0.826</td>
</tr>
<tr>
<td>$S_{\text{max}}$</td>
<td>upper bound on log surplus</td>
<td>0.060</td>
</tr>
<tr>
<td>$\omega$</td>
<td>variance sensitivity</td>
<td>0.198</td>
</tr>
<tr>
<td>$\rho$</td>
<td>autocorrelation in endowments</td>
<td>0.877</td>
</tr>
<tr>
<td>$\sigma_{\epsilon}$</td>
<td>standard deviation of endowment shocks*</td>
<td>0.777</td>
</tr>
</tbody>
</table>

Notes: * expressed in percent per quarter.

The values assigned to the model’s parameters are shown in Table 1. The model is parameterized so that one period corresponds to one quarter. β is set equal to 0.99, while γ and η are assigned standard values of 2 and 0.85, respectively. Thus, households are risk-averse and have a strong bias towards the consumption of domestic goods. In actual economies the local prices of domestic- and foreign-produced consumer goods are relatively unresponsive to quarterly variations in spot exchange rates because the effects are absorbed by the production and distribution sectors. These sectors are absent in the model. Consequently, variations in the real exchange rate are directly reflected in the relative prices that drive households’s consumption decisions. To compensate for this feature, I treat θ as a composite parameter, $\theta^* (1 - \varsigma)$, where $\varsigma$ denotes the fraction of exchange rate variations absorbed by the un-modeled production and distribution sectors, and $\theta^*$ is the “true” elasticity of substitution. Setting $\theta^*$ equal to 0.72, as in Hnatkovska (2010) and Corsetti, Dedola, and Leduc (2008), and $\varsigma$ equal to 0.85, gives a value for $\theta$ of 0.11.\(^{13}\)

The parameters φ, χ and μ determine how financial frictions affect the model’s equilibrium. In the benchmark parameterization I set the share of equity in foreign asset portfolios φ equal to 0.42. This is one half the average share of equity and FDI in U.S. foreign assets and liability portfolios between 1973 and 2007. The values for χ and μ are chosen to imply reasonable restrictions on the degree of international borrowing.

\(^{13}\)Obviously, this is a very reduced-form approach of capturing the low rate of exchange-rate pass-through we observe empirically (see, e.g., Campa and Goldberg, 2008).
I set the value of $\kappa$ equal to 0.5 so that households can issue debt up to 150 percent of the value of their foreign asset holdings. This limit implies an upper bound on the ratio of net foreign debt (i.e., debt minus foreign assets) to trend GDP of approximately 190 percent. I set $\mu$ equal to 0.03 and check that the solution is robust to using alternative small values for the barrier parameter.

The remaining parameters govern the endowment and log surplus processes. I assign values to two of these parameters. The first is the value for the long run growth rate, $g$, which I set equal to 0.028. The values of $\beta$, $\gamma$ and $g$ together imply that the steady state real interest rate in both countries equals 1.5 percent per year. Following Campbell and Cochrane (1999) I also assign a value to the steady state surplus consumption ratio, $\bar{S}$, of 0.057, so the steady state level of habit is 94 percent of consumption. The remaining parameters are estimated by GMM so that the model’s equilibrium matches five key moments of quarterly U.S. and E.U. data (described below): (i) the variance of the real depreciation rate for the USD/EUR, (ii) the variance in the per capita consumption growth differential between the U.S. and E.U., (iii) the variance of the real interest differential between the U.S. and E.U., (iv) the first-order autocorrelation in the real interest differential, and (v) the slope coefficient from a regression of the future real depreciation rate on the current real interest differential. I find the GMM estimates for $\phi$, $s_{max}$, $\omega$, $\rho$, and $\sigma_{r}^2$ that match the unconditional moments computed from the equilibrium of the model with statistics computed from quarterly data spanning 1990:I to 2007:IV. The results are reported in the panel B of Table 1.14

Three aspects of GMM estimates deserve comment. First, matching the moments in this model requires less persistence in the log surplus ratios that is assumed in other models. For example, C&C and Verdelhan (2010) use values for $\phi$ very close to one, well above the GMM value of 0.826. Second, the GMM value for the upper bound on the surplus ratio $S_{max}$ is close to $\bar{S}$ so the unconditional distribution for $s_t$ is skewed further to the left of $\bar{s}$ than in other habit models. The third feature concerns the relative importance of endowment shocks and risk shocks in the stochastic steady state. The GMM values imply that in the steady state the standard deviation of the log surplus ratios ($s_t$ and $\hat{s}_t$) is 11 times that of the cyclical endowments ($\hat{z}_t$ and $\hat{z}_t$). This means that time series variations in the log surplus ratios are the dominant driver of equilibrium exchange rates, real rate and consumption growth differentials.

3.3 Equilibrium Dynamics

Exchange Rates, Interest Rates and Consumption

Table 2 compares the unconditional moments of the real exchange rate, real interest rates, and consumption growth produced from the equilibrium of the model with sample moments computed in U.S. and E.U. data. These data come from Datastream and span the period 1990:I to 2007:IV. The real exchange rate at the start of month $t$, $E_t \equiv \exp(\tilde{e}_t)$, is computed as $S_t \hat{P}_t / P_t$, where $S_t$ is the spot rate (USD/EUR) at the end of trading (i.e. 12:00 noon E.S.T.) on the last trading day (Monday - Friday) in quarter $t - 1$. $P_t$ and $\hat{P}_t$ are the last reported levels for the U.S. and E.U. consumer price indices before the start of quarter $t$. The real interest

---

14The endowment process also depends on the variance of common growth shocks, $\sigma_u^2$. I choose the value for $\sigma_u^2$ so that the long run correlation between consumption growth in each country matches the unconditional sample correlation in US and Euro-area data. Because growth shocks affect both countries equally, the value of this parameter does not affect my examination of the external adjustment process below.
differential is computed from inflation and the three month nominal rates on Eurodeposits. Specifically, I estimate the U.S. real rate at the start of quarter \( t \) as the fitted value from an AR(2) regression for the ex post real return on Eurodeposits, \( i_t - (p_{t+1} - p_t) \), where \( i_t \) is the midpoint of the bid and offer rates on the last trading day of quarter \( t - 1 \). The E.U. real rate is similarly computed as the fitted value from an AR(2) for the ex post real return, \( \hat{i}_t - (\hat{p}_{t+1} - \hat{p}_t) \). The consumption growth rates are computed from real consumption expenditures on nondurables.

Table 2: Exchange Rates, Interest Rates and Consumption

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: Variances</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta \hat{e}_t ) real depreciation rate ( ^\dagger )</td>
<td>19.787</td>
<td>19.787</td>
</tr>
<tr>
<td>( \Delta c_t - \Delta \hat{c}_t ) consumption growth differential ( ^\dagger )</td>
<td>2.448</td>
<td>2.448</td>
</tr>
<tr>
<td>( r_t - \hat{r}_t ) real interest rate differential ( ^\dagger )</td>
<td>1.739</td>
<td>1.739</td>
</tr>
<tr>
<td>B: First Order Autocorrelation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r_t - \hat{r}_t ) real interest rate differential ( ^\dagger )</td>
<td>0.833</td>
<td>0.833</td>
</tr>
<tr>
<td>( \Delta \hat{e}_t ) real depreciation</td>
<td>-0.116</td>
<td>-0.043</td>
</tr>
<tr>
<td>( \Delta c_t - \Delta \hat{c}_t ) consumption growth differential</td>
<td>-0.233</td>
<td>-0.090</td>
</tr>
</tbody>
</table>

Notes: Moments included in GMM estimation are indicated by \( ^\dagger \).

Panel A of Table 2 shows that the variance of the real depreciation rate is approximately four times larger than the variance of the consumption growth differential, which in turn is 30 percent larger than the variance of the interest differential. Panel B reports the autocorrelation properties of the real interest differential, depreciation rate and consumption growth differential. The GMM estimation procedure did not use the latter two autocorrelations, so a comparison of these moments is informative about the model’s ability to represent this aspect of the data. As the table shows, this calibration of the model has no trouble replicating the lack of serial correlation in the real USD/EUR depreciation rate. The model also produces little serial correlation in the consumption growth differential, but in this case the correlation is somewhat smaller in absolute value than the correlation estimated from the data.

The GMM procedure also ensures that the model can match a key co-movement in real depreciations rates and interest differentials. In U.S. and E.U. data a regression of the future real depreciation rate, \( \Delta \hat{e}_{t+1} \), on the current real interest differential, \( r_t - \hat{r}_t \), produces a slope coefficient of -2.95. This estimate is significantly different from the value of one implied by UIP. The GMM estimation procedure ensures that the calibration of the model exactly reproduces this feature of the data. As in Verdelhan (2010), the negative coefficient reflects the fact that equilibrium interest differentials and foreign exchange risk premium move in different directions, but here these movement arise in a world of incomplete risk sharing and financial frictions (discussed in detail below).

The model is less successful in replicating one other feature of the U.S. and E.U. data; the correlation
between the real depreciation rate, $\Delta \epsilon_t$, and the consumption growth differential, $\Delta c_t - \Delta \hat{c}_t$. The sample correlation is -0.103, while in the model the unconditional correlation is -0.754. Producing a negative correlation has long been a challenge for standard models using isoelastic time-separable utility with complete markets (see, e.g., Backus and Smith, 1993), but here the combination of external habits and incomplete risk-sharing produces too large a negative correlation. Unfortunately there does not appear to be an alternative (reasonable) calibration of the model that produces a correlation between the real depreciation rate and consumption growth differential closer to -0.1, while replicating the size of the UIP deviation and the moments in Table 2.\(^{15}\)

**Habits**

Habits are tightly linked to consumption in the calibrated equilibrium of the model. Recall that the level of U.S. habit, $H_t$, is related to the surplus ratio, $S_t$, and aggregate consumption, $C_t$, by the identity, $H_t = (1 - S_t)C_t$. In equilibrium intertemporal smoothing by households ensures that aggregate consumption responds immediately and one-for-one with shocks to the endowment trend. The log of equilibrium U.S. consumption can therefore be represented as $c_t = \bar{c}_t + c_t^{CVL}$, where $\bar{c}_t (= \bar{z}_t)$ is the trend in consumption and $c_t^{CVL}$ is the remaining cyclical component driven by the temporary endowment shocks and the risk shocks. Combining this decomposition with the definition of $H_t$ and using the fact that $S_t$ is stationary, we can represent the dynamics of log U.S. habit by

$$h_t = \bar{h}_t + h_t^{CVC},$$

where

$$\bar{h}_t = \bar{c}_t \quad \text{and} \quad h_t^{CVC} = \ln(1 - \exp(s_t)) + c_t^{CVL}.$$ 

Permanent shocks to consumption (i.e., shocks to $\bar{c}_t$) are fully reflected in the level of habit via the trend component $\bar{h}_t$. Other shocks affect the level of habit via the cyclical component $h_t^{CVC}$. In the C&C specification, $c_t^{CVL} = 0$ and the log surplus ratio $s_t$ is driven by shocks to $\bar{c}_t$. As I discuss below, in this model $c_t^{CVL}$ varies in response to temporary endowment shocks and risk shocks because both shocks produce changes in the terms of trade that affect households’ consumption decisions. As a result, even though risk shocks directly affect the level of habit via their impact on the log surplus ratio, $s_t$, they do not produce significant variations in habit that are unrelated to consumption.

This feature can be seen from the time series representation of $h_t^{CVC}$. To derive the representation, I first generate time series over 100,000 quarters for $h_t^{CVC}$ and the innovations in cyclical consumption, $c_t^{CVL} - E_{t-1}c_t^{CVL}$, from the model’s equilibrium. I then use these series to estimate the following ARMA

\(^{15}\)The problem of simultaneously replicating the size of UIP deviation, the volatility of real depreciation rates, and their correlation with consumption growth differentials is long-standing in the literature. For example, Verdelhan (2010) replicates the anomaly but not the volatility of depreciation rates or their correlation with consumption. On the other hand, Colacito and Croce (2011) have more success accounting for the volatility of depreciation rates and correlation with consumption, but their model cannot replicate the UIP deviation because it generates a constant market price of risk.
As the $R^2$ statistic indicates, cyclical variation in habits are tightly tied to the history of consumption innovations in the model’s equilibrium. Of course actual consumption innovations come from four different shocks in the model which have different implications for the dynamics of habits and other variables. This means that the estimated ARMA coefficients have no structural interpretation. They simply summarize the fact that positive shocks to cyclical consumption are, on average, associated with higher levels of cyclical habit that persist far into the future.

Returns on Foreign Assets and Liabilities

Table 3 compares the characteristics of U.S. foreign asset and liability returns with the foreign asset returns generated by the model. The statistics on U.S. returns are based on the dataset from Evans (2012b) that contains U.S. foreign asset and liability positions at market value and their associated returns at the quarterly frequency. The data is constructed following procedures described in Gourinchas and Rey (2005) that combine information on the market value for four categories of U.S. foreign asset and liabilities: Equity, Foreign Direct Investment (FDI), Debt and Other; with information on the U.S. International Investment Position reported by the Bureau of Economic Analysis. To facilitate comparisons with the model, the table reports returns on two categories: “equity” that combines Equity and FDI, and “debt” that combines Debt and Other. I also compute statistics for two sample periods: 1973:I-2007:IV and 1990:I-2007:IV. The former period covers the entire post Bretton-Woods era prior to 2008 financial crisis, while the latter covers the period after the adoption of the Euro used to compute the statistics in Table 2.

Columns (i) and (ii) of panels A and B report the sample means and standard deviations for log excess returns on U.S. foreign assets, liabilities and their equity and debt components computed as $r_{j,t+1} - r_t$, where $r_{j,t+1}$ for $z = \{A, L\}$ denotes the log real return on asset/liability $j$. Column (iii) reports Sharpe ratios, computed as the sample average of gross excess returns, $R_{j,t+1}^z - R_t$, divided by their sample standard deviation, while columns (iv) and (v) show the sample means and standard deviations of the portfolio shares, $\alpha_{j,t}^z$. Panel C reports analogous statistics computed from simulating the model over 100,000 quarters. Since the model is symmetric, simulations of the U.S. foreign liability returns produce identical unconditional moments.

---

16. The estimated coefficients have extremely small standard errors, on the order of 0.003, so sampling error is not a concern here. The estimates are also robust to adding further lags of $h_{t-1}^{Cyc}$ and $c_{t-1}^{Cyc} - E_{t-1}c_{t-1}^{Cyc}$.
17. The Web Appendix provides details concerning the construction of the U.S. foreign asset and liability position data and the associated returns.
18. To be clear, the expected log excess return on the U.S. foreign asset portfolio is equal to the expected log excess return on the U.S. foreign liability portfolio in the stochastic steady state of the model. So, long simulations produce average log excess returns on foreign asset and liability portfolios that are identical (as are their sample variances). In contrast, my examination of the external adjustment process below focuses on how endowment and risk shocks affect conditional expectations of future log returns, specifically conditional expectations concerning the differential between the log return on U.S. foreign assets and liabilities. Table 3 does not provide information on the dynamics of these conditional expectations in either the model or the U.S./E.U. data.
Table 3: **Foreign Asset and Liability Returns**

<table>
<thead>
<tr>
<th>Returns</th>
<th>Mean (i)</th>
<th>Std. (ii)</th>
<th>Sharpe Ratio (iii)</th>
<th>Mean Share (iv)</th>
<th>Std.Share (v)</th>
<th>Variance Contributions (vi)</th>
<th>(vii)</th>
<th>(viii)</th>
</tr>
</thead>
</table>


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<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>All Assets</td>
<td>1.347</td>
<td>13.743</td>
<td>0.115</td>
<td>1.000</td>
<td>0.000</td>
<td>1.022</td>
<td>0.932</td>
<td>0.932</td>
</tr>
<tr>
<td>Equity</td>
<td>1.960</td>
<td>26.927</td>
<td>0.108</td>
<td>0.483</td>
<td>0.105</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Debt</td>
<td>-2.373</td>
<td>14.095</td>
<td>-0.149</td>
<td>0.517</td>
<td>0.105</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Liabilities</td>
<td>0.943</td>
<td>10.648</td>
<td>0.103</td>
<td>1.000</td>
<td>0.000</td>
<td>0.988</td>
<td>1.049</td>
<td>1.064</td>
</tr>
<tr>
<td>Equity</td>
<td>1.805</td>
<td>27.806</td>
<td>0.103</td>
<td>0.361</td>
<td>0.073</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Debt</td>
<td>-2.623</td>
<td>18.831</td>
<td>-0.113</td>
<td>0.639</td>
<td>0.073</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

**B: 1990:I-2007:IV**

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<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>All Assets</td>
<td>2.285</td>
<td>14.878</td>
<td>0.172</td>
<td>1.000</td>
<td>0.000</td>
<td>0.861</td>
<td>0.882</td>
<td>0.749</td>
</tr>
<tr>
<td>Equity</td>
<td>3.187</td>
<td>25.566</td>
<td>0.158</td>
<td>0.568</td>
<td>0.061</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Debt</td>
<td>-2.600</td>
<td>10.912</td>
<td>-0.221</td>
<td>0.432</td>
<td>0.061</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Liabilities</td>
<td>1.816</td>
<td>10.374</td>
<td>0.188</td>
<td>1.000</td>
<td>0.000</td>
<td>1.134</td>
<td>0.957</td>
<td>0.890</td>
</tr>
<tr>
<td>Equity</td>
<td>3.261</td>
<td>24.046</td>
<td>0.168</td>
<td>0.418</td>
<td>0.052</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Debt</td>
<td>-3.142</td>
<td>14.588</td>
<td>-0.194</td>
<td>0.582</td>
<td>0.052</td>
<td></td>
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</tr>
</tbody>
</table>

**C: Model**

<p>| | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>All Assets</td>
<td>3.471</td>
<td>19.764</td>
<td>0.176</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equity</td>
<td>4.470</td>
<td>19.129</td>
<td>0.420</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Debt</td>
<td>0.000</td>
<td>17.127</td>
<td>0.580</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The upper panels report statistics computed from U.S. data over the sample periods indicated. The lower panel reports statistics computed from the stochastic steady state of the model. Columns (i) and (ii) show the means and standard deviations for log excess returns for all assets listed in the left hand column. All log returns are multiplied by 400. Column (iii) reports the Sharpe ratios. Columns (iv) and (v) show the average and standard deviation of the share of each asset and liability category in total assets and liabilities, respectively. Variance decompositions for the returns on U.S. foreign assets and liabilities are in the three right-hand columns. The variance contribution of returns using constant asset or liability shares, the average of asset and liability shares, and the constant average of asset and liability shares are shown in column (vi)–(viii), respectively.

An inspection of the table reveals that the sample statistics computed from the U.S. data are generally comparable with those implied by the model. Log excess foreign asset returns in the model have a somewhat higher mean and standard deviation than in the two data samples, but the Sharpe ratio implied by the model closely matches the ratio computed in the second data sample. It is worth emphasizing that the model’s equilibrium is calibrated to match the moments of exchange rates, interest rates and consumption in Table 2 rather than the behavior of these returns, so some differences between the sample statistics in Panels A and B and the model statistics are to be expected. Moreover, the U.S. data on returns comes from
portfolios with different time-varying shares that are not present in the model. In particular, columns (iv) and (v) show that on average equity comprises a larger share of U.S. foreign asset holdings than liability holdings, and that equity shares have been rising through time and becoming more stable. These portfolio composition effects account for some of the differences between the U.S. data and the model’s implications.

To assess how composition effects affect portfolio returns in the U.S. data, I compute the log returns on three sets of synthetic portfolios for assets and liabilities. The log portfolio returns are constructed as

\[ r^a_{j,t}(\alpha^z_{j,t}) = \ln \left( \sum_l \alpha^z_{j,l,t-1} R^a_{j,l,t} \right) \quad \text{and} \quad r^l_{t}(\alpha^z_{t}) = \ln \left( \sum_j \alpha^z_{j,t-1} R^l_{j,t} \right), \]

for difference portfolio shares, \( \alpha^z_{j,t} \). The first set of portfolios use the average share for each asset and liability class (i.e., \( \alpha^z_{j,t-1} = \alpha^z_{j} \) for \( z = \{A,L\} \) and all \( j \)), giving log portfolio returns of \( r^a_{j,t}(\alpha^z) \) and \( r^l_{t}(\alpha^z) \). Comparing \( r^a_{j,t}(\alpha^z) \) and \( r^l_{t}(\alpha^z) \) with the actual U.S. portfolio returns provides information on how time-varying changes in the composition of asset and liability portfolios contributes to returns. The second set of portfolios use the average of the asset and liability share (i.e., \( \bar{\alpha}_{j,t} = \frac{1}{2} \alpha^a_{j,t} + \frac{1}{2} \alpha^l_{j,t} \)) for each class of assets. This gives log portfolio returns of \( r^a_{j,t}(\bar{\alpha}) \) and \( r^l_{t}(\bar{\alpha}) \). Here any difference between \( r^a_{j,t}(\bar{\alpha}) \) and \( r^l_{t}(\bar{\alpha}) \) and actual returns reflects the effects of asymmetries between asset and liability holdings. The third set of portfolios use the mean of the average asset and liability share (i.e., the mean value of \( \bar{\alpha}_{j,t} \) for each \( j \)) to give log portfolio returns of \( r^a_{j,t}(\bar{\alpha}) \) and \( r^l_{t}(\bar{\alpha}) \). These portfolio returns correspond to the returns in the model because the asset and liability portfolios have the same constant shares.

Columns (vi) - (vii) report the contributions of the synthetic portfolio returns to the variance of actual returns.\(^{19}\) In panel A all the estimated contributions are close to one. Time-variation in the portfolio shares, and differences between composition of U.S. foreign asset and liability portfolios did not materially affect the behavior of U.S. asset and liability returns over this longer sample period. Portfolio composition effects appear to have played a larger role since 1990. In particular, the estimated contributions shown in column (viii) of panel B indicate that time-varying and asymmetric shares contributed between 10 and 25 percent of the sample variability in actual portfolio returns.

Overall, Table 3 shows that the equilibrium of the model produces behavior for foreign asset returns that is broadly consistent with actual U.S. asset and liability returns. Although the actual composition of U.S. foreign portfolios varies between assets and liabilities and through time, these composition effects make a relatively minor contribution to the variability of actual portfolio returns (particularly over the 1973.I-2007.IV sample period).\(^{20}\) Consequently, the absence of composition effects in the model doesn’t greatly impair its ability to replicate the behavior of returns in U.S. data.

\(^{19}\)For example, in the case of the first set of synthetic portfolios, we can decompose the variance of actual returns, \( r^2_t \), as \( \text{Var}(r^2_t) = \text{Cov}(r^2_a, r^2_l) + \text{Cov}(r^2_a - r^2_l, r^2_l) \) for \( z = \{A,L\} \). The variance contribution of the synthetic portfolio is \( \text{Cov}(r^2_{j,t}(\alpha^z), r^2_l)/\text{Var}(r^2_l) \), which can be estimated by the slope coefficient from a regression of \( r^2_{j,t}(\alpha^z) \) on \( r^2_l \).

\(^{20}\)Notice that the standard deviations for U.S. liability returns are only slightly smaller than those for U.S. foreign assets. Even though U.S. liabilities are more heavily weighted towards debt than U.S. assets, because much of this debt is long-term, capital gains and loss on existing dollar liabilities contribute significantly to the volatility of U.S. liability returns. The idea that the U.S. borrows primarily at the risk free short-term dollar rate is thus a poor approximation to reality.
Portfolios Shares

Table 4 reports the shares of domestic and foreign assets in households’ portfolios in the stochastic steady state of the model. Column (i) reports the portfolio shares for the benchmark specification where $\varphi = 0.420$, the value calibrated from U.S. data. Column (ii) shows the steady state portfolio shares in an equilibrium where $\varphi = 0.491$. The shares of foreign equity and bonds in this equilibrium match the steady state shares in an equilibrium where households freely choose the composition of their foreign asset portfolios (i.e., $\varphi = \varphi^*$ as discussed in Section 2.1).

Table 4: Steady State Portfolio Shares

<table>
<thead>
<tr>
<th>Share</th>
<th>Benchmark $\varphi = 0.420$</th>
<th>Optimal $\varphi = 0.491$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic Equity</td>
<td>0.937</td>
<td>0.926</td>
</tr>
<tr>
<td>Domestic Bonds</td>
<td>-0.087</td>
<td>-0.076</td>
</tr>
<tr>
<td>Foreign Equity</td>
<td>0.063</td>
<td>0.074</td>
</tr>
<tr>
<td>Foreign Bonds</td>
<td>0.087</td>
<td>0.076</td>
</tr>
</tbody>
</table>

Notes: The table reports the shares of equity and bonds in household wealth in the stochastic steady state of the model under the benchmark calibration in column (i) and when $\varphi$ is chosen to maximize household utility in column (ii).

As Table 4 shows, the steady state of the model is characterized by a very high degree of equity home bias. This feature of the model arises for two reasons. First, households receive all their income in the form of dividends from domestic equity and their foreign asset holdings. As such, they are not concerned with hedging non-tradable income risk. Following the logic in Baxter, Jermann, and King (1998), extending the model to include non-tradable (wage) income in households’ budgets could lower the degree of equity home bias. The second reason arises from the presence of the portfolio constraints. As I show below, holdings in the foreign asset mutual fund provide domestic households with a poor hedged against risk shocks because they suffer capital losses (via exchange rate appreciation) when marginal utility is unexpected high. In a world without frictions (complete markets), households would hold equal long positions in domestic and foreign equity to benefit from international diversification, and a short position in foreign bonds to hedge against the adverse effects of domestic risk shocks. In the model, by contrast, the presence of the portfolio constraint restricts households to either long or short positions in both foreign equity and bonds (as part of their foreign fund holdings). As a consequence, the expected return on the foreign asset fund needs to compensate for this lack of flexibility if households are to take large equilibrium positions. This level of

---

21 The portfolio shares in column (ii) are computed from the steady state in which the parameters in Panel B of Table 1 are re-estimated by GMM to insure that the moments implied by the equilibrium also match the data as in Table 2.
compensation is absence in the stochastic steady state, so households only take small positions in the foreign asset fund, which in turn imply a high degree of home equity bias.

4 The Elements of External Adjustment

I now use the model to study the process of external adjustment following endowment and risk shocks. This process involves numerous elements that interact in a complex manner so it is useful to identify them individually before studying the quantitative implications of the model’s equilibrium in Section 5.

4.1 Consumption, Trade Flows and Dividends

The model’s implications for the behavior of consumption and trade flows are quite standard. The C.E.S specification for the U.S. and E.U consumption baskets implies that the domestic and foreign demand for U.S. goods are given by

\[ C_u(t) = \eta \left( \frac{P_u(t)}{P(t)} \right) \hat{C}_u(t) \]

and

\[ C_e(t) = \left( 1 - \eta \right) \left( \frac{P_e(t)}{P(t)} \right) \hat{C}_e(t) \]

respectively. Combining these demands with definition of the U.S. terms of trade,

\[ T_t = \frac{P_e(t)}{P_u(t)} \]

goods market clearing in (12) gives

\[ Y(t) = \eta \left( \frac{1}{2} \right) T_t^{1-\theta} C(t) + \left( 1 - \eta \right) \left( \frac{1}{2} \right) T_t^{1-\theta} \hat{C}(t) \]

and

\[ \hat{Y}(t) = \left( 1 - \eta \right) \left( \frac{1}{2} \right) T_t^{\theta-1} C(t) + \eta \left( \frac{1}{2} \right) T_t^{\theta-1} \hat{C}(t) \]

These market clearing conditions pin down the equilibrium levels of aggregate U.S. and E.U. consumption given the real exchange rate and endowments in the presence of home consumption bias. In particular, when \( \eta > 1/2 \) with can use identity linking the terms of trade with the real exchange rate in (7) to rewrite (23) as

\[ C(t) = F_{c,g}(\xi_t)Y(t) - F_{c,g}(\hat{\xi}_t)\hat{Y}(t) \]

and

\[ \hat{C}(t) = F_{c,g}(\hat{\xi}_t)\hat{Y}(t) - F_{c,g}(\xi_t)Y(t) \]

where \( F_{i,j}(\cdot) \) are positive nonlinear functions of the real exchange rate.

Equation (24) has two important implications for the international transmission of endowment and risk shocks. First it implies that shocks to the endowment of either good must produce changes in aggregate consumption across countries at the pre-existing real exchange rate (terms of trade). An increase in the endowment of the U.S. good, for example, must be accompanied by a rise in \( C(t) \) and a fall in \( \hat{C}(t) \) because with home bias these changes in aggregate consumption produce a matching increase in the demand for the U.S. good and no change in the demand for E.U. good at pre-existing relative prices. Equation (24) also makes clear that the consumption effects of both endowment and risk shocks depend critically on how they affect the real exchange rate. For example, the consumption effects of the U.S. endowment shock will be ameliorated by a real depreciation because it implies a deterioration in the terms of trade that lowers the relative price of U.S. goods. The behavior of the real exchange rate is even more critical in determining the
response of aggregate consumption to risk shocks. As (24) clearly shows, all the effects of risk shocks on equilibrium consumption are transmitted via changes in the real exchange rate.

Aggregate consumption and the terms of trade govern international trade flows. By definition U.S. exports and imports are determined by E.U. households’ demand for U.S. goods, and U.S. demand for E.U. goods, respectively. So, when measured in terms of the U.S. consumption basket, U.S. exports and imports are given by

\[ X_t = (1 - \eta) \left( \eta T_t^{1-\theta} + (1 - \eta) \right)^{-1} E_t \hat{C}_t \quad \text{and} \quad M_t = (1 - \eta) \left( \eta T_t^{\theta-1} + (1 - \eta) \right)^{-1} C_t. \] (25)

In the stochastic steady state, \( T_t = E_t = 1 \) and \( C_t = \hat{C}_t \) so these expressions simplify to \( X_t = (1 - \eta) \hat{C}_t \) and \( M_t = (1 - \eta) C_t \) and trade is balanced between the countries.

Recall that equities are claims to the endowments of U.S. and E.U. goods. In particular, one share of U.S. equity entitles the holder to receive a dividend of \( D_t = \left( \frac{P_{tU}}{P_t} \right) Y_t \) while one share of E.U. equity receives a dividend of \( \hat{D}_t = \left( \frac{\hat{P}_{tU}}{P_t} \right) \hat{Y}_t \). Substituting for relative prices with the terms of trade produces

\[ D_t = \left( \eta + (1 - \eta) T_t^{1-\theta} \right)^{1-\theta} \hat{Y}_t \quad \text{and} \quad \hat{D}_t = \left( \eta + (1 - \eta) T_t^{\theta-1} \right)^{\theta} Y_t. \] (26)

Clearly, endowment shocks have a direct impact on domestic dividends. They also have an indirect impact via the terms of trade which also affects foreign dividends. In contrast, risk shocks only affect dividends via the terms of trade. Dividends are also linked to domestic consumption and the trade balance. In particular, equations (23) - (26) imply that

\[ D_t = C_t + X_t - M_t, \quad \text{and} \quad \hat{D}_t = \hat{C}_t + (M_t - X_t)/E_t. \] (27)

Thus U.S. trade surpluses \( (X_t > M_t) \) push the dividends on U.S. equity above consumption and the dividends on E.U. equity below consumption.

### 4.2 Asset Prices

Asset prices play three important roles in the external adjustment process. First, they determine household wealth at pre-existing asset holdings. Shocks that produce unexpected changes in asset prices induce revaluations of household wealth that affect their consumption and saving decisions. Second, asset price variations produce international capital flows as households re-balance their portfolio holdings of foreign assets and liabilities. Changes in asset prices can also affect the risk premia and risk free rates by pushing households towards their collateral constraints.

The prices of domestic equity are readily determined from the first-order conditions of households’ optimizing problems. Let \( D_t/Q_t = 1/\Lambda_t^{eq} \) denote the dividend-price ratio for U.S. equity. The first-order condition governing optimal share of domestic equity in U.S. households’ portfolios implies that

\[ \Lambda_t^{eq} = \mathbb{E}_t \left[ M_{t+1} (1 + \Lambda_t^{eq}) \frac{D_{t+1}}{D_t} \right] \quad \text{with} \quad M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{S_{t+1}}{S_t} \right)^{-\gamma}. \] (28)
Analogously, the first-order condition for E.U. households implies that

\[
\hat{\lambda}_t^{eq} = E_t \left[ \hat{\mathcal{M}}_{t+1} (1 + \hat{\lambda}_t^{eq}) \frac{\hat{D}_{t+1}}{\hat{D}_t} \right] \text{ with } \hat{\mathcal{M}}_{t+1} = \beta \left( \frac{\hat{C}_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{\hat{S}_{t+1}}{\hat{S}_t} \right)^{-\gamma},
\]  

(29)

where \( \hat{D}_t/\hat{Q}_t = 1/\hat{\lambda}_t^{eq} \) is the dividend-price ratio for E.U. equity. The prices of U.S. and E.U. equity, measured in terms of domestic consumption, are therefore

\[
Q_t = \lambda_t^{eq} D_t \quad \text{and} \quad \hat{Q}_t = \hat{\lambda}_t^{eq} \hat{D}_t.
\]

(30)

Endowment shocks and risk shocks affect equity prices in several different ways. First, risk shocks directly affect the intertemporal marginal rates of substitution, \( \mathcal{M}_{t+1} \) and \( \hat{\mathcal{M}}_{t+1} \), that govern dividend-price ratios via their impact on the surplus ratios, \( S_t \) and \( \hat{S}_t \). Intuitively, equity prices must change in response to risk shocks to compensate for the uncertainty associated with future dividend flows. Risk shocks also affect equity prices indirectly via their impact on the terms of trade, aggregate consumption and dividends in both countries as discussed above. Similarly, endowment shocks to either good have a direct effect on equity prices via dividends, and indirect effects via the terms of trade.

Shocks to the common trend in the endowments have a different effect on equity prices. They have a one-to-one impact on the level of U.S. and E.U. consumption because households prefer smooth consumption paths, so goods markets clear without any change in the terms of trade. This has two consequences. First, current U.S. and E.U. dividends rise one-for-one with the endowment shock. Second, the shock has no effect on dividend-price ratios because the conditional distributions of future consumption, dividend and surplus growth remain unchanged. As a result, shocks to the common trend in the endowments have the same proportional effect on U.S. and E.U. equity prices.

The real exchange rate also acts as an asset price in the model because it determines the domestic value of the foreign asset mutual funds: i.e., \( FA_t = E_t (B_{eu} + \hat{Q}_t A_{eu}^{eq}) \) and \( \hat{FA}_t = (Q_t \hat{A}_{us}^{eq} \hat{B}_{us}) / E_t \). The equilibrium real exchange rate is determined by the real interest differentials and the foreign exchange risk premium. Let \( \delta_t = E_t \varepsilon_{t+1} - \varepsilon_t + \hat{r}_t - r_t \) define the foreign exchange risk premium (hereafter simply the “FX premium”).\(^{22}\) Rewriting this definition as a difference equation in \( \varepsilon_t \), solving forward, and applying the Law of Iterated expectations produces

\[
E_t = \exp \left( -E_t \sum_{i=0}^{\infty} \{r_{t+i} - \hat{r}_{t+i} + \delta_{t+i} \} + E_t \bar{\varepsilon} \right),
\]

(31)

where \( \bar{\varepsilon} = \lim_{t \to \infty} \varepsilon_t \). In the steady state of the model the real exchange rate equals unity so \( E_t \bar{\varepsilon} = 0 \). Equilibrium variations in the real exchange rate must reflect changes in current and expected future real interest differentials, \( \hat{r}_{t+i} - r_{t+i} \); and/or changes in current and expected future FX premia, \( \delta_{t+i} \).

The equilibrium interest differential is determined by the first-order conditions governing the optimal

\(^{22}\)Some definitions of the risk premium also include one half the variance of the real depreciation rate to account for the fact that we are dealing with log rather than gross returns. I use this simpler definition for clarity but fully account for the presence of log returns when solving the model.
holdings of domestic bonds

\[ 1 = E_t [\mathcal{M}_{t+1}] R_t + B_t \quad \text{and} \quad 1 = E_t [\mathcal{M}_{t+1}] \hat{R}_t + \hat{B}_t, \tag{32} \]

with \(B_t = C_t S_t^2 L_t\) and \(\hat{B}_t = \hat{C}_t S_t^2 \hat{L}_t\), where \(L_t\) and \(\hat{L}_t\) are the Lagrange multipliers on the collateral constraints in (18c) and (20c), respectively. Like equity prices, endowment and risk shocks affect real interest rates via their impact on the intertemporal marginal rates of substitution. They can also affect interest rates via the collateral constraints. Shocks that change the value of foreign assets so that the constraints bind produce positive values for \(B_t\) and \(\hat{B}_t\) and push real rates down relative to \(E_t [\mathcal{M}_{t+1}]\) and \(E_t [\mathcal{M}_{t+1}]\). Intuitively, households’ expected utility from additional borrowing exceeds the real interest rate when the collateral constraint is binding. Notice, also, that portfolio constraints have no direct effect on real interest rates. The conditions in (32) would still apply if households were able to freely choose their foreign equity and debt holdings.

The equilibrium FX premium is similarly determined by combining the first-order conditions governing households’ choice of domestic bonds and foreign asset holdings:

\[
0 = E_t \left[ \mathcal{M}_{t+1} \left( \frac{E_{t+1} \hat{R}_t}{E_t} - R_t \right) \right] + \varphi E_t \left[ \mathcal{M}_{t+1} \frac{E_{t+1}}{E_t} \left( \hat{R}_t^{eq} - \hat{R}_t \right) \right] + \zeta B_t, \tag{33a} \]

\[
0 = E_t \left[ \mathcal{M}_{t+1} \left( \frac{E_t R_t}{E_{t+1}} - \hat{R}_t \right) \right] + \varphi E_t \left[ \mathcal{M}_{t+1} \frac{E_t}{E_{t+1}} \left( R_t^{eq} - R_t \right) \right] + \zeta \hat{B}_t. \tag{33b} \]

These equations identify how financial frictions play a role in the determination of the equilibrium FX premium. The first term on the right-hand-side of each equation jointly determine the premium in an equilibrium where households are free to choose their holdings of foreign bonds and equity directly, and there are no collateral constraints. Financial frictions can push the FX premium away from this benchmark. When collateral constraints are absent but households can only hold foreign assets indirectly via the foreign asset fund, optimal portfolio choice requires that (33) holds with \(\zeta = 0\). In this case the equilibrium FX premium will also depend on the riskiness of foreign equity returns, identified by the second terms on the right-hand-side of (33). The size of these terms depends upon \(\varphi\) and the degree of international risk sharing.

In an equilibrium where risk sharing is complete, the expectations terms equal zero, and the presence of the portfolio constraint does not affect the equilibrium FX premium. When risk sharing is incomplete, the FX premium must adjust so that households are compensated for the risk of holding foreign bonds and equities so its size will depend on \(\varphi\). Things are more complex when both portfolio and collateral constraints are present. First, the presence of collateral constraints impair international risk sharing, so the FX premium adjusts to reflect the risk of holding foreign equity when \(\varphi > 0\). Second, the collateral constraints directly affect the equilibrium expected excess return on the foreign asset fund via the last terms on the right-hand

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23The first-order conditions governing the choice of domestic and bonds by U.S. and E.U. households imply that \(E_t [\mathcal{M}_{t+1} (R_t^{eq} - R_t)] = 0\) and \(E_t \left[ \mathcal{M}_{t+1} (R_t^{eq} - R_t) \right] = 0\), respectively. Combining these expressions with the complete risk-sharing condition, \(\hat{M}_{t+1} = (E_{t+1} / E_t) M_{t+1}\), gives \(E_t \left[ \hat{M}_{t+1} \frac{E_{t+1}}{E_t} (R_t^{eq} - R_t) \right] = 0\) and \(E_t \left[ \mathcal{M}_{t+1} \frac{E_{t+1}}{E_t} (R_t^{eq} - R_t) \right] = 0\), so the conditions in (33) simplify back to the case without financial frictions.
side of (33). These terms are positive when the constraints bind. Ceteris paribus, this lowers the equilibrium expected excess return on foreign assets and the FX premium. I examine the quantitative significance of both financial frictions on the behavior of the equilibrium FX premium in Section 5.

4.3 Saving, Portfolio Holdings and Capital Flows

Households’ savings and portfolio decisions play a central role in the international transmission mechanism. U.S. and E.U. households enter period $t$ with real wealth $W_t$ and $\hat{W}_t$ that comprises pre-existing holdings of domestic equity, bond and foreign assets (chosen in period $t-1$) valued at current asset prices, and the period $t$ dividend payments from their equity holdings.\(^{24}\) Households then choose how much of this wealth to save, and how to allocate their savings across domestic bonds, equity and foreign assets. Together these decisions determine households desired changes in asset holdings that drive international capital flows.

Endowment and risk shocks affect households’ consumption and savings decisions via the consumption-saving ratios: $C_t/(W_t - C_t) = 1/\Lambda^C_t$ and $\hat{C}_t/(\hat{W}_t - \hat{C}_t) = 1/\hat{\Lambda}^C_t$. Combining the households’ first-order conditions with these definitions and the budget constraints in (18b) and (20b) produces

\[
\Lambda^C_t = \mathbb{E}_t \left[ \mathcal{M}_{t+1}(1 + \Lambda^C_{t+1}) \frac{C_{t+1}}{C_t} \right] \quad \text{and} \quad \hat{\Lambda}^C_t = \mathbb{E}_t \left[ \hat{\mathcal{M}}_{t+1}(1 + \hat{\Lambda}^C_{t+1}) \frac{\hat{C}_{t+1}}{\hat{C}_t} \right].
\]

(34)

The level of saving and consumption in each country relative to beginning-of-period wealth are therefore

\[
\text{U.S.:} \quad W_t - C_t = \frac{\Lambda^C_t}{1 + \Lambda^C_t} W_t, \quad C_t = \frac{1}{1 + \Lambda^C_t} W_t, \quad \text{and}
\]

\[
\hat{W}_t - \hat{C}_t = \frac{\hat{\Lambda}^C_t}{1 + \hat{\Lambda}^C_t} \hat{W}_t, \quad \hat{C}_t = \frac{1}{1 + \hat{\Lambda}^C_t} \hat{W}_t.
\]

(35a)

(35b)

The equations display two noteworthy features: First, ceteris paribus, $\Lambda^C_t$ and $\hat{\Lambda}^C_t$ rise when households anticipate higher growth in their future consumption. As a result, household save a larger fraction of their wealth, consistent with standard intertemporal consumption smoothing. Second, the Lagrange multipliers on the collateral constraints are absent from (34). Financial frictions only affect households consumption/saving decisions indirectly via their impact on the equilibrium intertemporal marginal rate of substitution and prospective future consumption growth.

Savings decisions, portfolio choices and asset price variations all contribute to capital flows. Consider the flows associated with foreign equity. By definition the value of E.U. equity and bonds in U.S. households’ portfolio are given by $\mathcal{E}_t \hat{Q}_t A^\text{EU}_t = \psi \alpha^\text{FA}_t (W_t - C_t)$ and $\mathcal{E}_t B^\text{EU}_{t,t} = (1 - \psi) FA_t$, so the flows of equity and bonds\(^{24}\) the real value of dividend payments also depends upon the period $t$ real exchange rate/terms of trade; see (26) above.
during period $t$ are

$$
\varepsilon_t \hat{Q}_t \Delta A^\text{EU}_t = \alpha^\text{PA}_t \varphi (W_t - C_t) - \alpha^\text{PA}_{t-1} \varphi (W_{t-1} - C_{t-1}) \frac{\varepsilon_t \hat{Q}_t}{\hat{E}_{t-1} \hat{Q}_{t-1}} \\
= \Delta \alpha^\text{PA}_t \varphi (W_t - C_t) + \varphi \alpha^\text{PA}_{t-1} \left[ \Delta (W_t - C_t) - \left( \frac{\varepsilon_t \hat{Q}_t}{\hat{E}_{t-1} \hat{Q}_{t-1}} - 1 \right) (W_{t-1} - C_{t-1}) \right], \quad (36)
$$

and

$$
\varepsilon_t \Delta B^\text{EU}_t = \alpha^\text{PA}_t (1 - \varphi) (W_t - C_t) - \alpha^\text{PA}_{t-1} (1 - \varphi) (W_{t-1} - C_{t-1}) \frac{\varepsilon_t}{\hat{E}_{t-1}} \\
= \Delta \alpha^\text{PA}_t (1 - \varphi) (W_t - C_t) + (1 - \varphi) \alpha^\text{PA}_{t-1} \left[ \Delta (W_t - C_t) - \left( \frac{\varepsilon_t}{\hat{E}_{t-1}} - 1 \right) (W_{t-1} - C_{t-1}) \right]. \quad (37)
$$

The first term in the second line of each equation identifies the flow resulting from U.S. households’ desire to alter the share of foreign assets in their savings. Variations in these flow components are perfectly correlated across equity and bond flows because households cannot adjust the composition of their foreign asset holdings. The second term in each equation identifies the effects of changing savings behavior via $\Delta (W_t - C_t)$ and the effects of capital gains or losses on pre-existing positions. Notice that these portfolio rebalancing effects can have differing affects on equity and bond flows. Real capital gains and losses on pre-existing equity positions depend on the variations in foreign equity prices and the real exchange rate, while the gains and losses on bonds depend only on changes in the real exchange rate.

Equations (36) and (37) identify U.S. gross capital outflows. By convention, negative outflows represent the purchase of foreign assets by U.S. households which correspond to positive values for $\varepsilon_t \hat{Q}_t \Delta A^\text{EU}_t$ and $\varepsilon_t \Delta B^\text{EU}_t$. Purchases of U.S. equity and bonds by E.U. households, $(Q_t / E_t) \Delta A^\text{US}_t$ and $(1 / E_t) \Delta B^\text{US}_t$ represent positive U.S. capital inflows, and are identified from E.U. households’ choice for $\alpha^\text{PA}_t$, and savings, $\hat{W}_t - \hat{C}_t$ in an analogous manner to (36) and (37).

5 Quantitative Analysis

I now use the model to study the external adjustment process following endowment and risk shocks. The goal of this analysis is to quantify the roles played the trade and valuation channels in the model and to compare them against their estimated contributions to external adjustment in the U.S.. To identify how financial frictions affect the adjustment process, I compare the impulse responses to endowment and risk shocks in the benchmark specification with the portfolio and collateral constraints against the responses in an alternative specification where the portfolio constraint is the only financial friction.

5.1 Internal Adjustment

Figure 1 shows the impulse responses of consumption and the terms of trade following a positive shock to the U.S. endowment process, and a negative shock to the U.S. log surplus ratio. These responses identify the effect of a one standard deviation shock when the economy is initially in the stochastic steady state.
The solid plots show the responses of variables in the benchmark equilibrium where both financial frictions are present; responses from the equilibrium where portfolio constraints are the only friction are shown by the dashed plots. Overall, the plots show that the presence of collateral constraints have little quantitative impact on the reaction of consumption and the terms of trade to either endowment or risk shocks. In the upper panel, the consumption and terms of trade responses are essentially identical. In the lower panel the responses are smaller (in absolute value) when collateral constraints are present, but the differences are not economically significant.

To develop the intuition behind the responses in Figure 1, it proves useful to consider how the financial frictions affect the degree of international risk sharing. Recall from (24) that equilibrium consumption depends on the endowments and the real exchange rate: \( C_t = F_{c,y}(\ddot{e}_t)Y_t - F_{c,y}(\ddot{e}_t)\dot{Y}_t \) and \( \dot{C}_t = F_{c,y}(\ddot{e}_t)\dot{Y}_t - F_{c,y}(\ddot{e}_t)\dot{Y}_t \). In the absence of any financial frictions, households could trade the two equities and bonds to completely share the idiosyncratic risks they face (i.e., markets would be dynamically complete) so the real exchange rate would equalize marginal utilities internationally: \( E_t = (C_t/\dot{C}_t)^\gamma (S_t/\dot{S}_t)^\gamma \). This risk-sharing condition and (24) pin down equilibrium aggregate consumption and the real exchange rate in terms of the exogenous endowments and surplus ratios: i.e., \( C_t^{CM} = F(S_t, \dot{S}_t, Y_t, \dot{Y}_t) \), \( \dot{C}_t^{CM} = F(S_t, \dot{S}_t, Y_t, \dot{Y}_t) \), and \( E_t^{CM} = (F(S_t, \dot{S}_t, Y_t, \dot{Y}_t))/F(S_t, \dot{S}_t, Y_t, \dot{Y}_t)\)^\gamma (S_t/\dot{S}_t)\)^\gamma \).\(^{25}\)

Now consider how the introduction of the portfolio restriction affects the equilibrium degree of international risk sharing. If the processes for \( C_t^{CM}, \dot{C}_t^{CM} \) and \( E_t^{CM} \) fail to satisfy the market clearing conditions and the first-order conditions governing household decision-making with the portfolio constraints, their presence must impair international risk sharing. Alternatively, if the conditions are satisfied, \( C_t^{CM}, \dot{C}_t^{CM} \) and \( E_t^{CM} \) identify the consumption and real exchange rate processes in an equilibrium where the portfolio constraint is present and risk sharing is complete. Assessing these alternatives is straightforward. Our examination of the first-order conditions (32) and (33) in Section 4.2 showed that the presence of the portfolio constraints does not affect real interest differentials and the FX premium if aggregate consumption in each country follow the \( C_t^{CM} \) and \( \dot{C}_t^{CM} \) processes. Consequently, the process for the real exchange rate implied by the present value relation (31) matches \( E_t^{CM} \), so the \( C_t^{CM} \) and \( \dot{C}_t^{CM} \) processes satisfy the goods market clearing conditions in (24). Thus, the \( C_t^{CM}, \dot{C}_t^{CM} \) and \( E_t^{CM} \) processes satisfy the model’s equilibrium conditions when the portfolio constraints are present, and allow for complete international risk sharing: i.e., \( E_t^{CM} = (C_t^{CM}/\dot{C}_t^{CM})^\gamma (S_t/\dot{S}_t)^\gamma \).

In contrast, international risk sharing is incomplete in the equilibrium where both the portfolio and collateral constraints are present (i.e. the benchmark specification). In this case the collateral constraints affect the behavior of real interest rates and the foreign exchange risk premium even if consumption follow the \( C_t^{CM} \) and \( \dot{C}_t^{CM} \) processes, so the present value relation in (31) gives values for the real exchange rate that differ from \( E_t^{CM} \). Consequently, the \( C_t^{CM}, \dot{C}_t^{CM} \) and \( E_t^{CM} \) process do not satisfy the model’s equilibrium conditions when both financial frictions are present, so the actual equilibrium processes imply incomplete risk sharing.

Understanding the economics behind the plots in Figure 1 is straightforward with this risk-sharing per-

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\(^{25}\)More formally, from (24) aggregate consumption must satisfy \( C_t = F_{c,y}((C_t/\dot{C}_t)^\gamma (S_t/\dot{S}_t)^\gamma )Y_t - F_{c,y}((C_t/\dot{C}_t)^\gamma (S_t/\dot{S}_t)^\gamma )\dot{Y}_t \) and \( \dot{C}_t = F_{c,y}((C_t/\dot{C}_t)^\gamma (S_t/\dot{S}_t)^\gamma )\dot{Y}_t - F_{c,y}((C_t/\dot{C}_t)^\gamma (S_t/\dot{S}_t)^\gamma )Y_t \). Solving these equations gives \( C_t^{CM} = F(S_t, \dot{S}_t, Y_t, \dot{Y}_t) \) and \( \dot{C}_t^{CM} = F(S_t, \dot{S}_t, Y_t, \dot{Y}_t) \).
Figure 1: Impulse Responses of Consumption and the Terms of Trade

Notes: The upper and low rows show the effects of a positive shock to the U.S. endowment and negative shock to the U.S. surplus ratio, respectively. Both shocks are one standard deviation in magnitude, and their effects on the endowment and surplus ratio are shown in panels A and C. Solid plots identify the responses in the benchmark equilibrium with both financial frictions, while dashed plots identify the equilibrium response when portfolio constraints are the only friction. In panels B and E the plots with bullets identify the paths of E.U. consumption, those without identify U.S. consumption. The horizontal axis measures the time elapsed since the shock in years.
spective. For example, a positive shock to the U.S. endowment (see Panel A) requires a deterioration in the U.S. terms of trade (i.e., a rise in $P_{EU}^{t}/P_{US}^{t}$, see Panel C) to maintain goods market clearing at pre-existing consumption levels. This, in turn, implies a real depreciation of the dollar. So, from risk-sharing perspective, there needs to be a rise in relative consumption, $C_t/\hat{C}_t$. Panel B of Figure 1 shows that this comes about via a larger rise in U.S. consumption than the fall in E.U. consumption because home bias in consumption ($\eta > 1/2$) produces the higher demand for U.S. goods necessary to clear markets. Notice, also, that the responses of consumption are essentially the same in both equilibria because the presence of the collateral constraint does little to impair the ability of households to fully share risk following the endowment shock.

In principle endowment shocks produce capital gains and losses that drive households towards the collateral constraints thereby pushing equilibrium real interest rates and the foreign exchange risk premium away from adjustment paths that allow for complete risk sharing. In practice, these affects are very small when a typically sized endowment shock hits the economy in the steady state. Consequently, the equilibrium adjustment paths for consumption, the real exchange rate and the terms of trade following such shocks are very similar to paths in the equilibrium with just the portfolio constraint that permit complete risk sharing.

The presence of collateral constraints has a larger impact on the adjustments following risk shocks. The lower panels of Figure 1 show the effects of positive U.S. risk shock that increases local curvature of U.S. households’ utility by approximately 6.5 percent and temporarily lowers their surplus ratio (see Panel D). From risk-sharing perspective, the risk shock must produce immediate appreciation in the real exchange rate to equalize marginal utility across countries. This appreciation also induces an improvement in the U.S. terms of trade in the presence of home bias (see Panel F), so there must be a rise in U.S. consumption and fall in E.U. consumption to clear goods markets (see Panel E). In this case the presence of the collateral constraints have a small effect on consumption and the terms of trade (real exchange rate) because they dampen the response of the real interest differential and the FX premium (see below).

The left hand panels of Figure 2 show the responses of log dividends, $d_t$ and $\hat{d}_t$; and equity prices, $q_t$ and $\hat{q}_t$; to the endowment and risk shocks. Panels A and B show that following the endowment shock there is a rise in both U.S. and E.U. dividends, a large rise in U.S. equity prices and a small fall in E.U. equity prices. Intuitively, the rise in U.S. dividends reflects the positive effect of the shock on U.S. output which is partially offset by the deterioration in the U.S. terms of trade. In addition, the shock increases the expected intertemporal marginal rate of substitution via its effects on U.S. consumption, producing a rise in the equity price-to-dividend ratio $\Lambda^{EQ}_t$ [see (28)], so U.S. equity prices rise more than dividends. The endowment shock also affects E.U. dividends and equity prices via its affect on the terms of trade. In this case the deterioration in the U.S. terms of trade raise E.U. dividends ($\hat{D}_t$ in line with $D_t$) and lowers $\hat{\Lambda}^{EQ}_t$ via its impact on expected future dividend and consumption growth. The net effect is a small fall in E.U. equity prices $\hat{Q}_t = \hat{\Lambda}^{EQ}_t \hat{D}_t$ (see Panel B).

Panels E and F show the effects of the risk shock. Here all the variations in dividends reflect the effect of the improvement in the U.S. terms of trade that produce a temporary rise (fall) in the real value of U.S. (E.U.) dividends. These valuation effects have very different impacts on equity prices. In the U.S., the risk shock produces a large fall in $\Lambda^{EQ}_t$ via expectations of lower intertemporal marginal rate of substitution and dividend growth [see (28)] that overwhelm the effects of higher dividends, so U.S. equity prices fall. In
Notes: The upper and lower rows show the effects of the positive shock to the U.S. endowment and negative shock to the U.S. surplus ratio depicted in Figure 1. The vertical axis in each plot shows the response per one percent of the shock. Solid plots identify the responses in the benchmark equilibrium with both financial frictions, while dashed plots identify the equilibrium response when portfolio constraints are the only friction. In panels A-C and E-G the plots with bullets identify the paths of E.U. variables, those without identify U.S. variables. The horizontal axis measures the time elapsed since the shock in years.
contrast, the fall in E.U. dividends produces a small drop in equity prices because the combined effect of the risk shock on expected E.U. consumption and dividend growth produces a small rise in $A^E_t$.

The right hand panels of Figure 2 show the responses of the log risk free rates, $r_t$ and $\hat{r}_t$, and the FX premium, $\delta_t$. Panels C and D show that the endowment shock affects the risk free rates but not the premium. Specifically, lower (higher) expected consumption growth following the shock (see Panel B in Figure 1) produce a significant fall in the U.S. rate and small rise in the E.U. rate via intertemporal smoothing. As in Figure 1, the responses in the benchmark equilibrium are very similar to those in the equilibrium with only the portfolio constraint where risk sharing is complete. Panels G and H show how real interest rates and the FX premium respond to the risk shock. Here, in contrast to Panels C and D, the interest rate responses are far smaller than the premium response. Although the exact effects of the risk shock on interest rates differ between the two equilibria, their (absolute) size amounts to a few basis points whereas the FX premium rises by approximately 30 basis points. These differences imply quite different adjustment mechanisms for real exchange rates. Recall from (31) that variations in the real exchange rate must reflect changes in current and expected future real interest differentials and/or changes in the current and expected FX premia. The responses shown here imply that risk shocks primarily affect the real exchange rate via changes in the premia, while all the exchange-rate effects of endowment shocks are transmitted via real interest differentials.

We can gain a better understanding of how different shocks affect interest rates and the FX premium by considering the first-order conditions governing households’ portfolio choices. In the equilibrium without collateral constraints, these conditions (approximately) imply that

$$
\begin{align*}
  r_t &= -\ln \beta + \gamma E_t \Delta c_{t+1} + \gamma E_t \Delta s_{t+1} - \frac{1}{2} \gamma^2 V_t(c_{t+1} + s_{t+1}), \\
  \hat{r}_t &= -\ln \beta + \gamma E_t \Delta \hat{c}_{t+1} + \gamma E_t \Delta \hat{s}_{t+1} - \frac{1}{2} \gamma^2 V_t(\hat{c}_{t+1} + \hat{s}_{t+1}), \quad \text{and} \\
  \delta_t &= \frac{1}{2} \gamma CV_t(c_{t+1} + \epsilon_{t+1}, \epsilon_{t+1}) + \frac{1}{2} \gamma CV_t(s_{t+1} + \hat{s}_{t+1}, \epsilon_{t+1}).
\end{align*}
$$

Following the U.S. endowment shock, households lower (raise) their expectations of future U.S. (E.U.) consumption growth but not their forecasts for changes in the log surplus ratios or conditional second moments involving consumption. Thus, the interest rates responses to the endowment shock in Panel B are entirely produced by intertemporal smoothing via the second terms on the right-hand-side of (38a) and (38b), and there is no change in the FX premium as shown by Panel D.

Risk shocks have more complex interest-rate effects. In particular, the U.S. risk shock raises both $\gamma E_t \Delta s_{t+1}$ and $\gamma^2 V_t(c_{t+1} + s_{t+1})$ because U.S. households anticipate a future rise in the log surplus ratio and greater volatility in marginal utility. These effects act on the equilibrium U.S. real interest rate in opposite directions. The rise in $\gamma E_t \Delta s_{t+1}$ puts upward pressure on the real interest rate because households want to borrow more, while in the increase in $\gamma^2 V_t(c_{t+1} + s_{t+1})$ induces greater precautionary saving. The risk shock also lowers expectations of future U.S. consumption growth via its effects on the terms of trade. These effects largely offset each other, so their combined impact on the U.S. real rate is very small (i.e., a few basis points). Risk shocks also produce a small change in E.U. interest rates because the improvement in the terms of trade induce a rise in $\gamma E_t \Delta \hat{c}_{t+1}$. The small size of these interest-rate effects is not surprising. In
the C&C model the parameters of the habit process are chosen so that interest rates are constant. Here the parameters are estimated so that the volatility of interest differentials match the data. This requires that the intertemporal smoothing and precautionary savings effects of risk shocks on real interest rates largely offset each other.

Risk shocks have significant effects of the FX premium because they change the hedging properties of foreign asset portfolios. In particular, the U.S. risk shock produces a real appreciation of the dollar that generates an unanticipated capital loss on the foreign asset holdings when U.S. marginal utility is unexpectedly high. Thus, the expected equilibrium excess return on foreign assets must compensate U.S. households for the poor hedge they provide against the adverse effects of future U.S. risk shocks. When the U.S. risk shock hits, this compensation rises because the fall in \( s_t \) makes marginal utility more sensitive to future risk shocks [see (2)]. As a result, the expected excess return on U.S. foreign assets increases via a rise in the FX risk premium.\(^{26}\) Equation (38c) identifies this hedging effect by an increase in \( \text{CV}_t \left( s_{t+1} + \hat{s}_{t+1}, \varepsilon_{t+1} \right) \).

The right hand panels in Figure 2 show that the presence of collateral constraints only visibly affect the interest-rate and FX risk premium responses to risk shocks. In particular, the fall in U.S. equity prices and appreciation of the dollar reduces the collateral value of U.S. and E.U. households’ foreign asset holdings which in turn lowers interest rates by a few basis points. The fall in the collateral value of foreign asset holdings also dampens the rise in the FX premium (again by a few basis points). Intuitively, equilibrium expected excess returns need not rise as much to compensate for the deterioration in the hedging properties of U.S. foreign assets when they also have collateral value.

One final aspect of Figure 2 deserves special comment. As I noted above, all the variations in the real exchange rate induced by endowments shocks are produced by changes in current and expected future real interest differentials because the shocks have no discernible effect on the FX premium (even when collateral constraints are present). Consequently, the equilibrium dynamics of real depreciation rates and interest differentials driven by endowment shocks are consistent with UIP: A regression of \( \Delta \varepsilon_{t+1} \) on \( r_t - \hat{r}_t \) generated by endowment shocks would produce a slope coefficient of one. In contrast, risk shocks produce variations in the FX premium and interest differentials. Indeed, an inspection of Panels G and H reveals that \( r_t - \hat{r}_t \) falls and \( \delta_t \) rises following the U.S. risk shock. More generally, risk shocks produce a negative correlation between the real interest differential and the FX premium, and greater volatility in the FX premium than the interest differential. These are the statistical properties necessary to produce a negative slope coefficient from a regression of \( \Delta \varepsilon_{t+1} \) on \( r_t - \hat{r}_t \) (see, e.g., Fama, 1984), so risk shocks must play an important role in the GMM calibration of the model to replicate the coefficient of -2.95 estimated from U.S./E.U. data (together with the other moments in Table 2).\(^{27}\)

\(^{26}\)Recall from Section 4.2 that the expected excess return on foreign assets equals the FX premium in the equilibrium without collateral constraints.

\(^{27}\)Although Figure 2 shows that endowment shocks produce no visible variations in the FX premia, this feature arises endogenously in the model rather than directly from a specific assumption. In principle, endowment shocks could induce large enough changes in the value of foreign asset holdings to produce sizable variations in the FX premia in the benchmark equilibrium with collateral constraints - via their effects on \( B_t \) and \( \hat{B}_t \) in the first-order conditions (32) and (33). Calibrations producing such effects are implicitly considered by the GMM estimation procedure, but they are discarded in favor of the benchmark calibration because it produces a match between the model moments and the data. In sum, the model allows endowment shocks to produce variations in the FX premium, but it cannot do so without generating counterfactual behavior for exchange rates, consumption growth and interest rates.
5.2 External Adjustment

I now quantify the roles played by the trade and valuation channels of external adjustment. First I derive a simple approximation that links the U.S. foreign asset position to exports, imports and the returns on foreign assets and liabilities. I then use this approximation to study how endowment and risk shocks affect the U.S. net foreign asset position, and compare these implications of the model with estimates from U.S. data.

Net Foreign Assets, Trade and Returns

In the steady state of the model households hold long positions in domestic equity, and foreign assets, and a short position in domestic bonds. Thus U.S. foreign assets comprise a portfolio of E.U. equity and bonds with a real value of

\[ FA_t = E_t(B_{EU}^t + \hat{Q}_t A_{EU}^t), \]

while U.S. foreign liabilities comprise U.S. equity and bonds held by E.U. households with a real value of

\[ FL_t = \hat{B}_{US}^t + Q_t \hat{A}_{US}^t (= E_t F A_t). \]

Combining these definitions with the bond and equity market clearing conditions in (10) and (11); the aggregated budget constraint for U.S. households in (8); and the equation for dividends in (27) gives

\[ FA_t - FL_t = X_t - M_t - R_{FL}^t FL_{t-1} + R_{FA}^t FA_{t-1}, \]  

(39)

where \( R_{FL}^t = R_{FA}^t (E_{t+1}/E_t) \) is the gross real return on U.S. foreign liabilities.

Equation (39) is the consolidated U.S. budget constraint that links changes in foreign assets and liabilities to exports, imports and returns. These links place restrictions on the process of external adjustment. To see how, let \( NFA_t = FA_t - FL_t \) and \( R_{NFA}^t = (R_{FA}^t FA_{t-1} - R_{FL}^t FL_{t-1})/NFA_{t-1} \) denote the value of and return on the U.S. net foreign asset position. Using these definitions we can rewrite (39) as

\[ NFA_t = X_t - M_t + R_{NFA}^t NFA_{t-1}. \]

Iterating this expression forward, and taking conditional expectations produces

\[ NFA_t = -E_t \left[ \sum_{i=1}^{\infty} \left( \prod_{j=1}^{i} R_{FA}^{t+j} \right)^{-1} (X_{t+i} - M_{t+i}) \right] + \lim_{k \to \infty} E_t \left( \prod_{j=1}^{k} R_{NFA}^{t+j} \right)^{-1} NFA_{t+k}. \]

In the steady state of the model net foreign asset positions are zero, so the last terms on the right hand side disappears leaving

\[ NFA_t = -E_t \left[ \sum_{i=1}^{\infty} \left( \prod_{j=1}^{i} R_{FA}^{t+j} \right)^{-1} (X_{t+i} - M_{t+i}) \right]. \]  

(40)

Thus, at any point in time, the value of the U.S. net foreign asset position must reflect households expectations concerning the future path of net exports, \( X_t - M_t \), and the returns on net foreign assets, \( R_{NFA}^t \).

Equation (40) identifies the international solvency constraint that must hold in any model where international Ponzi-schemes are ruled out. In a world where the only internationally traded asset is a single risk free real bond with a constant return \( R \), (39) simplifies to \( NFA_t = -E_t \sum_{i=1}^{\infty} R^{-i} (X_{t+i} - M_{t+i}) \), so changes in the net foreign asset position only reflect revisions in the expected future path of net exports. Under these circumstances all external adjustments to shocks take place through the “trade channel”: that is via changes in future net exports. Moreover, since the net foreign asset position simply reflects a country’s long or short position in bonds, capital gains and losses on pre-existing net asset positions cannot play any role.
in the adjustment process.

In this model equation (40) places a more complex set of restrictions on the adjustment process because two bonds and two equities are traded internationally. As a consequence, shocks can induce revisions in expected future returns on both foreign assets and liabilities, as well as net exports, and unexpected changes in exchange rates and equity prices can produce capital gains or losses on pre-existing net foreign asset positions. While (40) continues to hold in this more complex environment, examining how these factors interact is considerably facilitated by considering an approximation.

To derive the approximation, I first rewrite (39) as

$$\ln \left( \frac{FA_t}{R_i^{PL}FA_{t-1}} - 1 - \frac{M_t}{R_i^{PL}FA_{t-1}} \right) = \ln \left( \frac{FL_t}{R_i^{PL}FL_{t-1}} - 1 - \frac{X_t}{R_i^{PL}FL_{t-1}} \right) + \ln \left( \frac{R_i^{PL}FL_{t-1}}{R_i^{PL}FA_{t-1}} \right). \tag{41}$$

In the stochastic steady state, exports and imports are constant shares of consumption, $X_t = (1-\eta)\hat{C}_t$ and $M_t = (1-\eta)\hat{C}_t$; the returns on foreign assets and liabilities are equal and constant, $R_i^{PL} = R_i^{PS} = R_i^f$; and both foreign assets and liabilities are constant fractions of households savings, $FA_{t-1} = \alpha^f(W_{t-1} - C_{t-1})$ and $FL_{t-1} = \alpha^f(W_{t-1} - \hat{C}_{t-1})$, where $\alpha^f$ is the steady state value for the portfolio shares, $\alpha_i^{PS}$ and $\alpha_i^{PL}$. The steady state is also characterized by a constant common consumption growth rate $g$ and savings-to-consumption ratios, $\Lambda^c_i = \hat{\Lambda}_i^c = \Lambda^c$. Under these circumstances, the terms in square brackets in (41) are constants equal to $\kappa = 1 - \frac{1}{R_i^f\alpha^f_N}$ and savings-to-savings-to-foreign asset ratio $n_{fa_t} = \ln(FA_t/FL_t)$ and $n_{x_t} = \ln(X_t/M_t)$ are the log ratios of foreign assets to liabilities and exports to imports. Finally, I iterate forward and take conditional exceptions (noting that $\lim_{K \to \infty} \sum_{k=1}^\infty \kappa^{k-1} n_{fa_t+k} = 0$) to obtain

$$n_{fa_t} = n_{fa_t}^{TR} + n_{fa_t}^{VAL} \quad \text{where} \tag{42}$$

$$n_{fa_t}^{TR} = -(1-\kappa)\sum_{i=1}^\infty \kappa^{i-1} n_{x_{t+i}} \quad \text{and} \quad n_{fa_t}^{VAL} = -\sum_{i=1}^\infty \kappa^{i-1} (r_i^{SP} - r_{t+i}^{PL}).$$

The approximation in (42) embodies the central features of the international solvency constraint in (40). For example, (42) implies that a shock producing an upward revision in the expected future path for U.S. net exports (i.e., a rise in $E_t n_{x_{t+i}}$) must also produce either an fall in expected future returns on foreign assets $E_t r_{t+i}^{PS}$, a rise in expected future liability returns $E_t r_{t+i}^{PL}$, and/or a fall in value of net foreign assets (i.e., a fall in $n_{fa_t}$). In the special case were a single riskless bond is the only internationally traded asset, returns on foreign assets and liabilities are equal so (42) simplifies to $n_{fa_t} = n_{fa_t}^{TR} = -(1-\kappa)\sum_{i=1}^\infty \kappa^{i-1} n_{x_{t+i}}$. I refer to $n_{fa_t}^{TR}$ as the trade component of the net foreign asset ratio. The effects of revisions in expected
future returns are identified by the valuation component, \( nfa_t^{VAL} \).

There are two main advantages to using (42) rather than (40) to study external adjustment. First, (42) links trade flows and returns to the net foreign asset position via a linear present value relation rather than through the complex nonlinear relation in (40). This facilitates the examination of how net foreign assets behave in the model and in actual U.S. data. Second, expressions for the log returns on U.S. foreign assets and liabilities are readily determined from the model’s equilibrium conditions. Variations in these returns are empirically important, so it proves useful to examine their theoretical drivers in the model.

**Drivers of the U.S. Net Foreign Asset Position**

We can use the approximate solvency constraint in (42) and a VAR to study the U.S. external adjustment process. The VAR is used to construct the forecasts for net exports, \( nx_t \), and the return differentials, \( r_t^{NFA} = r_t^{PA} - r_t^{PL} \), that appear in the definitions of the trade and valuation components, \( nfa_t^{TR} \) and \( nfa_t^{VAL} \). I then compare the actual movements in \( nfa_t \) against these estimates of \( nfa_t^{TR} \) and \( nfa_t^{VAL} \) derived from the VAR.

I estimate the trade and valuation components with several VAR specifications. In the base specification I estimate a VAR for three variables: the return differential, \( r_t^{NFA} \); the growth differential between exports and imports, \( \Delta nx_t = \Delta x_t - \Delta m_t \); and the sum the log net asset position and exports, \( nx_a_t = nfa_t + nx_t \). I include the latter two variables because the U.S. time series for \( nx_t \) and \( nfa_t \) are quiet persistent in the sample period (1973:I to 2007:IV). The approximation in (42) implies that \( nx_a_t = -E_t \sum_{i=1}^{\infty} \kappa^{i-1} (\Delta nx_{t+i} + r_{t+i}^{NFA}) \), so shocks producing persistent variations in \( nx_a_t \) should have little impact on \( nx_a_t \) because they have small effects on the near-term forecasts for \( \Delta nx_t \). In addition, I estimate VARs that include \( \Delta x_t \) and \( \Delta m_t \) instead of \( \Delta nx_t \), as well as \( r_t^{PA} \) and \( r_t^{PL} \) instead of \( r_t^{NFA} \). In all these specifications the returns on foreign assets and liabilities are constructed from the actual U.S. portfolios with time-varying shares. To facilitate comparisons between the U.S. data and the model, I also estimate VARs where the returns on assets and liabilities are constructed from portfolios with constant shares equal to the sample average, \( r_t^{PA} (\bar{\alpha}) \) and \( r_t^{PL} (\bar{\alpha}) \) (as discussed in Section 3.3).

Let \( Y_t = \tilde{A} Y_{t-1} + \tilde{V}_t \) represent the estimates of a VAR written in first-order form, where the vector \( Y_t \) appropriately stacks the current and lagged values of all the VAR’s variables, and \( \tilde{A} \) is the companion matrix of estimated VAR coefficients. I compute estimates of the trade and valuation components as

\[
\tilde{nfa}_t^{TR} = -i^{TR} \tilde{A} (I - \kappa \tilde{A})^{-1} Y_t - nx_t \quad \text{and} \quad \tilde{nfa}_t^{VAL} = -i^{VAL} \tilde{A} (I - \kappa \tilde{A})^{-1} Y_t,
\]

where \( i^{TR} \) and \( i^{VAL} \) are vectors such that \( \Delta nx_t = i^{TR} Y_t \) and \( r_t^{NFA} = i^{VAL} Y_t \).

By definition, \( nfa_t = \tilde{nfa}_t^{TR} + \tilde{nfa}_t^{VAL} + \xi_t \), where \( \xi_t \) is a composite error comprising the approximation error in (42) and the estimation error associated with forecasting the future path for \( \Delta nx_t \) and \( r_t^{NFA} \).

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28Gourinchas and Rey (2007) derive a similar approximation to the dynamics in (39) around a deterministic trend path where solvency is satisfied. Their approximation links the cyclical components of exports, imports and the foreign asset and liability positions rather than the trade flows and positions themselves.
the VAR. We may therefore decompose the unconditional variance of the log net foreign asset ratio as

\[ \mathbb{V}(nfa_t) = \mathbb{CV}(\tilde{nfa}_t^{TR}, nfa_t) + \mathbb{CV}(\tilde{nfa}_t^{VAL}, nfa_t) + \mathbb{CV}(\xi_t, nfa_t), \]

where \( \mathbb{V}(\cdot) \) and \( \mathbb{CV}(\cdot, \cdot) \) denote the variance and covariance operators, respectively. The first term on the right-hand-side identifies the variance contribution of the estimated trade component, the second identifies the contribution of the valuation component. I estimate these contributions as the slope coefficient from the OLS regression of the estimated component on \( nfa_t \):

\[ \tilde{nfa}_t^{TR} = \beta^{TR} nfa_t + \zeta^{TR} \quad \text{and} \quad \tilde{nfa}_t^{VAL} = \beta^{VAL} nfa_t + \zeta^{VAL}. \quad (43) \]

By least squares, \( \beta^{TR} = \mathbb{CV}(\tilde{nfa}_t^{TR}, nfa_t)/\mathbb{V}(nfa_t) \) and \( \beta^{VAL} = \mathbb{CV}(\tilde{nfa}_t^{VAL}, nfa_t)/\mathbb{V}(nfa_t) \) so the regression coefficients provide estimates of the variance contributions. I also compute confidence bands for these estimates using White (1980) heteroskedastic consistent standard errors.

Panel A of Table 5 reports estimates of \( nfa_t \) variance decomposition for the U.S. between 1973:I and 2007:IV computed from different VAR specifications. In all cases I report results based on second-order VARs, but I obtain similar results using first- and third-order ones as well. The left-hand column lists the variables included in the VAR. The center columns report the estimates of \( \beta^{TR} \), \( \beta^{VAL} \) and \( \beta^{TR} + \beta^{VAL} \) together with their 95 percent confidence bands. The right-hand column of the table reports the value for \( \kappa \) used to compute the \( nfa_t \) components. The values for \( \kappa \) of 0.988 and 0.980 maximize \( \beta^{TR} + \beta^{VAL} \) for the VAR specifications in rows (1) and (2), respectively.

The results in Panel A show a consistent pattern in the estimated variance contributions of the trade and valuation components across the different VAR specifications. In all cases the variations in the estimates of the expected future return differentials contribute more to the sample variance of the log net foreign asset ratio than the estimated revisions future trade flows.\(^{29}\) There are, however, some noteworthy differences across the estimates based on different VAR specifications. Comparing rows (1) and (2) we see that the variance contribution of the trade (valuation) component is larger (smaller) when based on separate VAR forecasts for export and import growth and returns than when based on VAR forecast for net export growth and return differentials. Although both VAR specifications generate estimates of the trade and valuation components that account for almost all the sample variation in the \( nfa_t \) (the estimates of \( \beta^{TR} + \beta^{VAL} \) are close to one), it is hard to precisely pin down how real-time forecasts of future trade flows and returns differentials were changing over the sample period. The estimates in rows (3) and (4) provide information on the importance of changes in the composition of U.S. asset and liabilities portfolios. Here I report results derived from VARs that replace the returns on actual portfolios with the returns on asset and liability portfolios where the share of equity are fixed at the sample average of 0.42. This restriction reduces the estimated variance contributions of the valuation components. It also reduces the explanatory power of the VAR for \( nfa_t \), as measured by the estimates of \( \beta^{TR} + \beta^{VAL} \), compared to their counterparts in rows (1) and

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\(^{29}\) Gourinchas and Rey (2007) estimate a smaller variance contribution (of approximately 30 percent) from the valuation component in their decomposition of the cyclical component of \( nfa_t \). If I replace \( nfa_t \) with a cyclical component estimated with the HP filter in my analysis, I also find a much small variance contribution from revisions in forecasts of future return differentials.
Table 5: Variance Decompositions for $nfa_t$

<table>
<thead>
<tr>
<th>Panel</th>
<th>Description</th>
<th>Trade: $\hat{\beta}_{TR}$</th>
<th>Valuation: $\hat{\beta}_{VAL}$</th>
<th>Total: $\hat{\beta}<em>{TR} + \hat{\beta}</em>{VAL}$</th>
<th>Discount Factor $\kappa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: VAR Estimates</td>
<td></td>
<td>lower bound (i)</td>
<td>estimate (ii)</td>
<td>upper bound (iii)</td>
<td>lower bound (i)</td>
</tr>
<tr>
<td>(1) {(\Delta nx_t, r_t^{nxa}, nxa_t)}</td>
<td>-0.025</td>
<td>0.092</td>
<td>0.210</td>
<td>0.763</td>
<td>0.877</td>
</tr>
<tr>
<td>(2) {(\Delta x_t, \Delta m_t, r_t^{fa}, r_t^{fl}, nxa_t)}</td>
<td>0.312</td>
<td>0.391</td>
<td>0.471</td>
<td>0.530</td>
<td>0.609</td>
</tr>
<tr>
<td>(3) {(\Delta nx_t, r_t^{nxa}(\bar{\alpha}), nxa_t)}</td>
<td>-0.041</td>
<td>0.079</td>
<td>0.198</td>
<td>0.636</td>
<td>0.732</td>
</tr>
<tr>
<td>(4) {(\Delta x_t, \Delta m_t, r_t^{fa}(\bar{\alpha}), r_t^{fl}(\bar{\alpha}), nxa_t)}</td>
<td>0.392</td>
<td>0.463</td>
<td>0.533</td>
<td>0.439</td>
<td>0.504</td>
</tr>
<tr>
<td>B: Model Implications</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) Portfolio Restrictions</td>
<td>0.098</td>
<td></td>
<td>0.902</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2) Collateral and Portfolio Restrictions ($\phi = 0.42$)</td>
<td>0.140</td>
<td></td>
<td>0.860</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2) Collateral and Portfolio Restrictions ($\phi = 0.75$)</td>
<td>0.466</td>
<td></td>
<td>0.534</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Column (ii) in each panel reports the OLS estimate of the slope coefficient from (43) using forecast of net exports and the return differential from the VARs. Columns (i) and (iii) report the lower and upper 95 percent bounds associated with the slope estimates computed as $\hat{\beta} \pm 1.96\hat{\sigma}^2$, where $\hat{\sigma}$ is the coefficient standard error computed from the White (1980) procedure. The variables included in the VAR are listed in the left hand column. The value of the discount factor parameter $\kappa$ used in computing the trade and value components are shown in the right hand column.
This finding is consistent with the idea that real-time forecasts for future return differentials factor in the effects of changing portfolio compositions.

Panel B of Table 5 reports values for $\beta^{TR}$ and $\beta^{VAL}$ implied by different equilibria of the model. These values are computed from unconditional variances and covariances for $nfa_t$, $nfa_t^{TR}$ and $nfa_t^{VAL}$ using $\kappa = 0.993$, which is the value for $\kappa$ implied by the model’s steady state. Rows (1) and (2) show that variations in the valuation components account for high fractions of the variance in $nfa_t$ in the equilibrium with the portfolio restriction (that permits complete risk sharing) and the equilibrium with both collateral and portfolio constraints (that impair risk sharing). Indeed, the contribution of the valuation component in the benchmark equilibrium in row (2) appears at the upper end of the range of estimates based on the U.S. data. Of course these calculations are based on an equilibrium of a model that was calibrated to match the sample moments of real exchange rates, interest rates and consumption growth differentials. It is not too surprising that the values for $\beta^{TR}$ and $\beta^{VAL}$ implied by the external adjustment mechanism in the model do not exactly match the estimates from U.S. data. Moreover, the values for $\beta^{TR}$ and $\beta^{VAL}$ implied by model appear to be quite sensitive to the value for $\phi$, the share of equity in households’ foreign asset portfolios. For example, row (3) shows that when $\phi$ is set to 0.75, and all the other parameters are at their benchmark values, the model’s equilibrium implies that $\beta^{TR} = 0.466$ and $\beta^{VAL} = 0.534$.

Overall, the results in Table 5 show that variations in expected future return differentials between foreign assets and liabilities made a major historic contribution to the variability of the U.S. net foreign asset ratio. They also show that variations in expected future return differentials make a similarly important contribution to the dynamics of the net foreign asset ratio in the model. As I noted in Section 2, this feature of the model is new to the literature. To understand its theoretical foundation, I next consider how endowment and risk shocks affect the value of net foreign assets via their impact on forecasts for future foreign asset returns and trade flows.

**Trade and Valuation Channels**

Figure 3 shows the impulse responses of the log net export ratio, $nx_t$, the expected return differential on net foreign assets, $E_t(r_{t+1}^{NFA})$, the log net foreign asset ratio, $nfa_t$, and its valuation component, $nfa_t^{VAL}$. As above, the solid plots show the responses of variables in the benchmark equilibrium where both financial friction are present; responses from the equilibrium where portfolio constraints are the only friction are shown by the dashed plots. Panels A-C plot the responses to the U.S. endowment shock. Recall that the shock produces a rise in U.S. consumption and a deterioration in the U.S. term of trade. These effects produce a rise in U.S. demand for E.U. goods and a fall in the E.U. demand for U.S. goods so $nx_t$ initially falls before gradually rising back to its steady state level as shown in Panel A. These plots also show that the adjustment path for $nx_t$ is not materially affected by the presence of the collateral constraints. Panel B plots the responses of the expected return differential on U.S. foreign assets, $E_t(r_{t+1}^{NFA}) = E_t(r_{t+1}^{FA} - \Delta \hat{r}_{t+1}^{FA} - \hat{r}_{t+1}^{FA})$. The endowment shock has no effect on $E_t(r_{t+1}^{NFA})$ in the equilibrium without collateral constraints, but when they are present there is a very small rise in $E_t(r_{t+1}^{NFA})$ (i.e., by a fraction of a basis point). Thus, most of the variation in the value of U.S. net foreign asset position following an endowment shock occurs via the trade channel. This is
Figure 3: External Adjustment

Notes: The upper and lower rows show the effects of the positive shock to the U.S. endowment and negative shock to the U.S. surplus ratio depicted in Figure 1. The vertical axis in each plot shows the response per one percent of the shock. Solid plots identify the responses in the benchmark equilibrium with both financial frictions, while dashed plots identify the equilibrium response when portfolio constraints are the only friction. The right hand panels plot the responses of $n_{t}$ without circles, and $n_{t}^{VAL}$ with circles. The horizontal axis measures the time elapsed since the shock in years.
illustrated by the responses for $nfa_t$ and $nfa_t^{\text{val}}$ plotted in Panel C.

These responses are consistent with the standard process of international adjustment that underlies the intertemporal approach to the current account. In this case the transitory endowment shock produces capital gains on U.S. households’ holdings of domestic equity and foreign assets that they use to raise consumption, producing temporary current account deficits that are financed by higher dividends on their equity holdings. As in a standard model, the adjustment of the net foreign asset position following the shock comes primarily via the trade channel, but at the time the shock hits there are capital gains and losses on existing holdings. Thus, unexpected valuation effects play a role in the external adjustment process following (temporary) endowment shocks even though revisions in expected future trade flows account for most of the variations in the net foreign asset position. The adjustment processes examined by Pavlova and Rigobon (2008), Tille and van Wincoop (2010) and Devereux and Sutherland (2011) have these same features.

Panels D-F in Figure 3 show the external responses produced by the U.S. risk shock. Recall that this shock induces a real appreciation of the dollar, a rise in U.S. consumption and a fall in E.U. consumption. As a consequence there is a fall in the ratio of E.U. to U.S. consumption expenditure $\mathcal{E}_t \hat{C}_t/C_t$, and a rise in the share of spending on U.S. goods in each country’s consumption basket. These factors affect net exports in opposite directions, but in both equilibria the latter factor dominates producing the rise in $nx_t$ shown in Panel D. Notice that this trade response is smaller (in absolute value) than the response to the endowment shock. In contrast, the risk shock has a much larger effect on the expected return differential, $\mathbb{E}_t r_{t+1}^{\text{NFA}}$. As Panel E shows, the presence of collateral constraints dampens the rise in $\mathbb{E}_t r_{t+1}^{\text{NFA}}$ by approximately 50 percent, but it is still 25 times larger than the rise produced by the endowment shock. The response of the U.S. net foreign asset ratio are shown in Panel F. Here we see the valuation component, $nfa_t^{\text{val}}$, accounts for the lion’s share of the $nfa_t$ responses in both equilibria. Since $nfa_t^{\text{val}}$ equals the presence value of expected future return differentials, the $\mathbb{E}_t r_{t+1}^{\text{NFA}}$ responses in Panel E translate into sizable $nfa_t^{\text{val}}$ variations. In fact the initial effect of the risk shock on $nfa_t$ in the benchmark equilibrium is roughly twice the (absolute) size of the effect produced by the endowment shock.

The plots in Figure 3 reconcile the results in Table 5 and the analysis of the adjustment process following endowment and risk shocks. Specifically, the small value for $\beta^{\text{TR}}$ and large value $\beta^{\text{VAL}}$ implied by the model’s equilibrium arise because: (i) risk shocks affect the net foreign asset ratio primarily through the valuation channel (i.e., via changes in $\mathbb{E}_t r_{t+1}^{\text{NFA}}$ that drive variations in $nfa_t^{\text{val}}$), and (ii) they are the dominant source of cyclical dynamics in the model. In contrast, (temporary) endowment shocks induce adjustment via the trade channel and through unexpected valuation effects, so they cannot account for the estimates of $\beta^{\text{TR}}$ and $\beta^{\text{VAL}}$ from U.S. data reported in Panel A of Table 5 unless they are unimportant drivers of cyclical U.S./E.U. dynamics; which is, of course, consistent with the GMM estimates of the model.

To understand why the valuation channel is so important in the external adjustment process following risk shocks, it proves useful to consider the links between the FX and equity risk premia and the expected returns on the foreign asset mutual funds. Following Campbell and Viceira (2002), these returns can be
approximated by

\[ r_{t+1}^{FA} = \hat{r}_t + \Delta \varepsilon_{t+1} + \frac{1}{2} \varphi(1 - \varphi) V_t \left( \hat{r}_{t+1}^{EQ} \right) + \varphi \left( r_{t+1}^{EQ} - \hat{r}_t \right) \quad \text{and} \]

\[ \hat{r}_{t+1}^{FA} = r_t - \Delta \varepsilon_{t+1} + \frac{1}{2} \varphi(1 - \varphi) V_t \left( r_{t+1}^{EQ} \right) + \varphi \left( r_{t+1}^{EQ} - r_t \right). \]

Subtracting the log of the domestic real interest rate from each equation, and combining the results with the definition of the FX premium, \( \delta_t \), produces

\[ \mathbb{E}_t r_{t+1}^{FA} - r_t = \delta_t + \varphi \hat{r}_t^{EQ} - \frac{1}{2} \varphi^2 V_t \left( \hat{r}_{t+1}^{EQ} \right) \quad \text{and} \]

\[ \mathbb{E}_t \hat{r}_{t+1}^{FA} - \hat{r}_t = -\delta_t + \varphi r_t^{EQ} - \frac{1}{2} \varphi^2 V_t \left( r_{t+1}^{EQ} \right), \]

where \( r_t^{EQ} = \mathbb{E}_t r_{t+1}^{EQ} - r_t + \frac{1}{2} V_t (\hat{r}_{t+1}^{EQ}) \) and \( \hat{r}_t^{EQ} = \mathbb{E}_t \hat{r}_{t+1}^{EQ} - \hat{r}_t + \frac{1}{2} V_t (\hat{r}_{t+1}^{EQ}) \) are the U.S. and E.U. equity premia. In equilibrium these premia are determined by the first-order conditions governing households' domestic equity and bond holdings [see (28), (29) and (32)], and are well-approximated by

\[ r_t^{EQ} = \gamma CV_t (r_{t+1}^{EQ}, c_{t+1} + s_{t+1}) - \ln(1 - B_t), \quad \text{and} \]

\[ \hat{r}_t^{EQ} = \gamma CV_t (r_{t+1}^{EQ}, \hat{c}_{t+1} + \hat{s}_{t+1}) - \ln(1 - \hat{B}_t). \]

Finally, combining (44) with the definition of the return differential, \( r_{t+1}^{NPA} = r_{t+1}^{PA} - \Delta \varepsilon_{t+1} - \hat{r}_{t+1}^{PA} \), gives

\[ \mathbb{E}_t r_{t+1}^{NPA} = \delta_t + \varphi \left[ r_t^{EQ} - \hat{r}_t^{EQ} \right] + \frac{1}{2} \varphi^2 \left[ V_t (r_{t+1}^{EQ}) - V_t (\hat{r}_{t+1}^{EQ}) \right]. \]  

These equations provide the key to understanding why the valuation channel plays such different roles following endowment and risk shocks. Consider first the effects of the endowment shock. Recall from Figure 2 that the shock has no effect on the FX premia in either equilibrium so the valuation channel will only be operable if the shock changes \( \mathbb{E}_t r_{t+1}^{NPA} \) via the equity risk premia, \( r_t^{EQ} \) and \( \hat{r}_t^{EQ} \), or the variance terms in (46). Equation (45) shows that the equilibrium equity risk premia compensate for the hedging properties of equities identified by the covariance terms, and the effects of the collateral constraints via \( B_t \) and \( \hat{B}_t \). When the constraints are absent, the endowment shock has no affect on the conditional second moments so there is no change in the equity premia in (45), or the variance terms in (46). As a result there is no revision in the expected return differentials (i.e., \( \mathbb{E}_t r_{t+1}^{NPA} \) for \( i > 0 \)), so the valuation channel remains inoperable. When the collateral constraints are present, the capital loss suffered by E.U. households on their foreign asset holdings drives down \( \hat{r}_t \) thereby raising the E.U. equity premium via the last term on the right-hand-side of (45b). This produces a rise in \( \mathbb{E}_t r_{t+1}^{FA} - r_t \) [see (44a)] which is reflected in \( \mathbb{E}_t r_{t+1}^{NPA} \), but the effect is extremely small.

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\( ^{30} \)As is standard, I include the Jensen inequality terms, \( \frac{1}{2} V_t (r_{t+1}^{EQ}) \) and \( \frac{1}{2} V_t (\hat{r}_{t+1}^{EQ}) \), to account for the fact that we are working with log returns.
as was seen in Panel B of Figure 3. Consequently, the valuation channel plays almost no role in the external adjustment process following (temporary) endowment shocks even when collateral constraints are present.

In contrast, the valuation channel plays an important role in the adjustment process following risk shocks for three reasons: First, as discussed above, risk shocks have sizable effects on the FX premium because they change the hedging properties of foreign bonds. Recall from Figure 2 that the U.S. risk shock had a larger effect on $\delta_t$ than on the real interest differential in both equilibria. These effects are directly reflected in $E_t r_{t+1}^{PA}$ via the first term on the right-hand-side of (46). Second, risk shocks change the hedging properties of equities. Recall that the U.S. risk shock produced an unanticipated fall in U.S. equity prices when U.S. marginal utility was unexpectedly high, so the increased likelihood of further risk shocks must produce a rise in the U.S. equity premium to compensate. This hedging effect is identified by an increase in the covariance term on the right-hand side of (45a). U.S. risk shocks have minimal effects on E.U. equity prices and marginal utility, so hedging properties of E.U. equity remain almost unchanged. In the equilibrium without collateral constraints, the rise in $r_{t+1}^E$ dampens the effects of the higher FX premium on the expected return differential so the rise in $E_t r_{t+1}^{PA}$ (see Panel C of Figure 3) is smaller than the rise in $\delta_t$ (see Panel H of Figure 2). In the equilibrium with collateral constraints, the dampened effects of the equity premia on $E_t r_{t+1}^{PA}$ are larger. In this case, the capital losses U.S. households suffer on their foreign asset position drive the U.S. equity premium even higher so that the effect of the risk shock on the expected return differential is reduced by approximately 50 percent (see Panel E of Figure 3).

5.3 Asset Holdings and Capital Flows

I now turn to the remaining aspect of the external adjustment process, the behavior of capital flows. Panel A in Table 6 reports the unconditional standard deviations from simulations of the benchmark equilibrium with both portfolio and collateral constraints and from the equilibrium with just the portfolio constraints for U.S. log savings, $w_t - c_t$; the log value of U.S. foreign assets, $f_{a_t}$; the U.S. portfolio shares for domestic equity and foreign assets, $\alpha_{eq}^t$ and $\alpha_{fa}^t$; and the U.S. capital outflows for bonds and equity, $E_t \Delta B_{\text{EU}}^t$ and $E_t \Delta A_{\text{EU}}^t$. These statistics show that the presence of collateral constraints has little impact on the volatility of savings. This is consistent with the similarity of the impulse responses for consumption, dividends and trade flows across the different equilibria depicted in Figures 1-3. Households find it optimal in the presence of collateral constraints to approximate the real consumption and savings decisions they would make under complete markets. The constraints play a larger role on households’ financial decisions. As the ratios in column (iv) show, the volatility of the U.S. foreign asset position is approximately 20 percent higher when both constraints are present. By definition $f_{a_t} = \ln \alpha_{eq}^t + w_t - c_t$, so the higher volatility in $f_{a_t}$ induced by the financial frictions reflects more variability in households’ choice for $\alpha_{fa}^t$. In fact the standard deviations of both the foreign asset and domestic equity portfolio shares are approximately 15 percent higher in the benchmark equilibrium. The presence of collateral constraints also raises the standard deviations of U.S. capital outflows by approximately 10 percent. Bond flows are more volatile than equity flows in both equilibria because the share of E.U. equities in U.S. foreign assets is fixed at $\varphi = 0.42$. The volatility of equity outflows is higher than bond outflows in equilibria with $\varphi > 0.5$. 
### Table 6: Capital Flows

<table>
<thead>
<tr>
<th>Variables</th>
<th>Equilibria</th>
<th>Ratio</th>
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<tr>
<td></td>
<td>Portfolio and Collateral Constraints</td>
<td>Portfolio Constraints</td>
</tr>
<tr>
<td>(i)</td>
<td>(ii)</td>
<td>(iii)</td>
</tr>
</tbody>
</table>

#### A: Standard Deviations

- $w_t - c_t$: 0.302 0.297 1.016
- $fa_t$: 0.782 0.654 1.195
- $\alpha^E_t$: 0.089 0.077 1.155
- $\alpha^P_t$: 0.212 0.183 1.158
- $\mathcal{E}_t \Delta B_t^{EU}$: 0.167 0.152 1.098
- $\mathcal{E}_t \hat{Q}_t \Delta A_t^{EU}$: 0.120 0.109 1.101

#### B: Return Chasing Contributions

- Bond Flows: $\mathcal{E}_t \Delta B_t^{EU}$: 0.717 0.714
- Equity Flows: $\mathcal{E}_t \hat{Q}_t \Delta A_t^{EU}$: 1.381 1.381

**Notes:** Panel A reports the unconditional standard deviation from simulations of the benchmark equilibrium with portfolio and collateral constraints in column (ii) and the equilibrium with just portfolio constraints in column (iii) for the variables listed in column (i). Column (iv) reports the ratio of the statistics in columns (ii) to (iii). Panel B reports the variance contribution of the return chasing components in U.S. foreign bond flows and equity flows in the benchmark and complete markets equilibria. All statistics are computed from simulations spanning 100,000 quarters.

The statistics in Panel B identify the contribution of U.S. households’ portfolio reallocation decisions to U.S. capital outflows. Recall from equations (36) and (37) that a portion of the equity and bond outflow, $\mathcal{E}_t \hat{Q}_t \Delta A_t^{EU}$ and $\mathcal{E}_t \Delta B_t^{EU}$, can be attributed to the change in the portfolio share $\alpha^P_t$: $\Delta \alpha^P_t \nu(W_t - C_t)$ and $\Delta \alpha^P_t (1 - \nu)(W_t - C_t)$. Panel B reports the variance contributions of these so-called “return chasing contributions” to the bond and equity outflows. Even though these components are perfectly correlated across the flows, their variance contributions are quite different because the effects of capital gains and losses on pre-existing foreign bond and equity positions differs. Notice, also, that the variance decompositions are almost identical across the two equilibria. The presence of collateral constraints raise the volatility of capital outflows but has little affect on the role played by portfolio reallocations.\(^{31}\)

\(^{31}\)These variance decomposition results are similar to the findings reported elsewhere in the literature. For example, Tille and van Wincoop (2010) find that changes in portfolio shares are an important driver of flows following endowment shocks in a model where only equity is traded internationally. Similarly, Evans and Hnatkovska (2013) show that the return chasing component contributes most to the variance of bond and equity flows driven by productively shocks in a model with incomplete
We can study the role of capital flows in the external adjustment process in greater detail with the aid of Figure 4. Here I plot the impulse responses of U.S. and E.U. bond and equity holdings following the endowment and risk shocks. Each panel shows the percent deviation from the steady state holdings. For example, Panel A plots the deviations in U.S. holdings of U.S. bonds, \(100 \times (B^\text{US}_t - B^\text{US}_t)/B^\text{US}_t\), and the deviations in E.U. holdings of E.U. bonds, \(100 \times (B^\text{RU}_t - B^\text{RU}_t)/B^\text{RU}_t\), where bars indicate steady state values. As above, the upper and lower panels show the responses following the endowment and risk shocks. The solid lines indicate the responses in the benchmark equilibrium with portfolio and collateral constraints and the dashed lines show the responses in the equilibrium with only the portfolio constraints.

Two features of the plots in Figure 4 immediately stand out: First, the changes in bond and equity positions following endowment shocks are extremely small and are far smaller than the changes induced by the risk shocks. Since \(\varepsilon_t \hat{Q}_t \Delta A^\text{RU}_t, \varepsilon_t \Delta B^\text{RU}_t, (Q_t/\varepsilon_t) \Delta \hat{A}^\text{US}_t\) and \((1/\varepsilon_t) \Delta \hat{B}^\text{US}_t\) represent U.S. capital outflows and inflows for equities and bonds, the capital flows produced by endowment shocks must be very small. Indeed, it turns out that endowment shocks account for less than one percent of the variance in bond and equity flows produced by simulations of both equilibria. Thus, capital flows appear to play an insignificant role in the adjustment process following endowment shocks. Intuitively, endowment shocks are typically too small to affect equilibrium risk premia so households keep their portfolio shares constant. As a consequence, when the shock hits U.S. households issue more domestic bonds to balance the capital gains on their pre-existing domestic equity and foreign asset positions. The corresponding change in the U.S. and E.U. bond positions are shown in panels A and B of Figure 4. This small bond flow is sufficient to finance the initial U.S. current account deficit that follows the endowment shock. It is then reversed as U.S. households buy back their bonds with the proceeds of higher domestic dividends.

The second striking feature in Figure 4 appears in the lower panels. Here we see that the initial changes in the foreign bond and equity positions produced by risk shocks are economically large, and are approximately twice the size when collateral constraints are present. These position changes are an important driver of bond and equity capital flows. In particular, the plots in panels F and H show that risk shocks induce sharp initial falls in foreign bond and equity positions of both U.S. and E.U. households. They therefore represent large positive bond and equity foreign capital outflows across the world economy.

To understand the source of these capital flows, recall that the U.S. risk shock produces a real appreciation of the dollar and a fall in the domestic value of E.U. equity. Thus, E.U. equity holdings contribute more to the capital loss U.S. households sustain on their foreign asset portfolio than do E.U. bonds. If households were free to choose the composition of their foreign asset portfolio, the risk shock would produce a larger risk premium on foreign equity than bonds to compensate U.S. households for their inferior hedging properties in the face of future risk shocks. However, since the composition of the portfolio is fixed, the risk shock raises the equilibrium expected excess return on the foreign assets instead; i.e., \(\mathbb{E}_t r_{t+1}^{\text{FA}} - r_t\) [see (44a)], but not enough to fully compensate U.S. households for the higher risk associated with holding E.U. equity, so they

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32Of course it is possible that an endowment shock hits when households are already close to their collateral constraints so that the equilibrium risk premia and portfolio shares change. Under these circumstances, the shock would produce a more complex pattern of capital flows, but the simulations indicate that such patterns are very rare.
Figure 4: Foreign Asset Holdings and Capital Flows

Notes: The upper and lower rows show the effects of the positive shock to the U.S. endowment and negative shock to the U.S. surplus ratio depicted in Figure 1. The vertical axis in each plot shows the deviation in percent from the steady state asset holdings. Solid plots identify the responses in the benchmark equilibrium with both financial frictions, while dashed plots identify the equilibrium response when portfolio constraints are the only friction. Plots with bullets identify the paths of E.U. variables, those without identify U.S. variables. The horizontal axis measures the time elapsed since the shock in years.
reduce their exposure to foreign assets by lowering $\alpha_t^{PA}$. When coupled with the fall in U.S. saving, this portfolio reallocation produces an 8 percent fall in U.S. holdings of foreign bonds and equity, as shown by the dashed black plots in panels F and H of Figure 4. Similarly, the rise in $E_t^{PA} - \tilde{r}_t$ under-compensated E.U. households for the higher risk of future capital losses on U.S. equity, so they reduce their exposure to foreign assets by lowering $\tilde{\alpha}_t^{PA}$. Although E.U. households savings remain relatively stable (because the capital loss on U.S. equity is offset by the exchange rate), the fall in $\tilde{\alpha}_t^{PA}$ and appreciation of the dollar produce a fall in E.U. holdings of U.S. bonds of approximately 12 percent (see Panel F).

Collateral constraints amplify the effects of the risk shocks on capital flows. The U.S. risk shock produces capital losses on pre-existing U.S. equity and foreign asset positions as well as a desired portfolio reallocation that lowers $\alpha_t^{PA}$. Together, these effects lower the value of foreign asset holdings that can be used as collateral when issuing U.S. bonds. In response, U.S. households sell a larger fraction of their foreign asset portfolio in order to buy back more outstanding U.S. bonds. These actions produce the larger U.S. capital outflows depicted in panels F and H of Figure 4. The reallocation of E.U. portfolios has a similar collateral effect on E.U. outflows. In this case the fall in $\tilde{\alpha}_t^{PA}$ reduces the value of foreign assets available for E.U. households to use when issuing E.U. bonds. And, as a result, E.U. households sell foreign assets to buy back previously issued domestic bonds. As Figure 4 shows, the size of the E.U. bond and equity outflows produced by these transactions are very similar to their U.S. counterparts.

Table 7: U.S. Asset and Liability Position Changes

<table>
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<tr>
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<th>Equity Assets</th>
<th>Debt Assets</th>
<th>Equity Liabilities</th>
<th>Debt Liabilities</th>
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<td>0.903</td>
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<td>0.383</td>
</tr>
<tr>
<td>Debt Liabilities</td>
<td>-0.015</td>
<td>0.617</td>
<td>-0.062</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes: The table reports correlations between changes in U.S. foreign asset and liability positions at the annual frequency above and quarterly frequency below the leading diagonal. All positions are valued at market prices converted to constant dollars. Correlations are computed from the 1973:I-2007:IV sample period.

Finally, it is instructive to compare the dynamics of foreign asset and liability holdings in the model with actual changes in U.S. foreign asset and liability positions. Table 7 reports the correlations between quarterly and annual changes in gross U.S. asset and liability positions in equity and debt in the post Bretton-Woods era. Here we see that annual changes (reported above the leading diagonal) in gross positions are positively correlated across assets and liabilities and across debt and equity. At the quarterly frequency (reported below the leading diagonal) the correlations are smaller, and slightly negative for equities with debt liabilities. In contrast, changes in foreign equity and bond holdings are perfectly correlated in the

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33 Because the risk shock increases the local curvature of U.S. utility, households are more concerned about the under-compensation for equity risk than over-compensation for bond risk in the foreign portfolio.

34 Coeurdacier, Kollmann, and Martin (2010) examine correlations of annual changes between net foreign equity positions
model because households cannot change the composition of their foreign asset portfolios. The correlations in Table 7 indicate the presence of some “stickiness” in the composition of U.S. foreign asset and liability portfolios, but the portfolio constraint in the model is too restrictive to accurately account for this aspect of the data.

6 Conclusion

In this paper I developed a new open economy macro model to study the process of external adjustment. The model contains a novel combination of features (i.e., external habits, financial frictions, portfolio choice and incomplete risk-sharing) and can replicate key features of international data when risk shocks are the dominant drivers of international business cycles. My analysis of the model reveals two distinct adjustment processes; one triggered by output/endowment shocks and one trigged by risk shocks. External adjustment following (temporary) endowment shocks predominantly takes place via trade flows (consistent with the intertemporal approach to the current account), without large gross capital flows. In contrast, risk shocks primarily induce adjustment through and valuation channel via revisions in the expected future return differential between foreign asset and liability portfolios. In this case adjustment is accompanied by sizable variations in risk premia and large gross capital flows. The model estimates imply that the valuation channel is more important for the U.S. than the trade channel. Consistent with this implication, I find that forecasts of future return differentials contribute most to the volatility of the U.S. net foreign asset position in the post Bretton-Woods era.

References


and bond positions. They report an average correlation across the G-7 countries of -0.27 and in the U.S. -0.09. In these data the net correlations are 0.186 and -0.135 at the quarterly and annual frequency, respectively. Interpreting these net correlations is tricky because, as Table 7 shows, the correlations between equity and debt asset positions differ from the correlation between equity and debt liability positions at the quarterly and annual frequencies.


