Exchange Rates, Equity Prices and Capital Flows

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1 Introduction

The last 25 years have been characterized by a remarkable increase in international capital mobility. While gross cross-border transactions in bond and equity for the U.S. were equivalent to only 4 percent of GDP in 1975, this share increased to 100 percent in the early 1990s and has grown to 245 percent by 2000. Furthermore, a growing proportion of these capital flows consists of equity as opposed to bank loans or government bonds.[1] The increasing size and equity content of current capital flows has not yet inspired a new financial market paradigm for exchange rate theory, in which exchange rates, equity market returns and capital flows are jointly determined.

Recently, positive exchange rate theory has advanced mostly outside the scope of traditional macroeconomic theory, plagued with its notoriously poor empirical performance (Meese and Rogoff (1983a, 1983b)) and with widespread pessimism about the explanatory power of macro variables in general.[2] The empirical microstructure literature has examined the role of foreign exchange (forex) order flow defined as the difference between buy and sell orders. Evans and Lyons (2001a,b), Lyons (2001), Rime (2001), Killeen *et al.* (2001), and Hau *et al.* (2001) show that order flow from electronic brokerage systems have remarkably high correlation with contemporaneous exchange rate changes. These empirical results have been established both for inter-dealer order flow and for customer-dealer order flow. Since customer-dealer order flow in the foreign exchange market is at least partly determined by investors' desires for portfolio shifts, these results suggest an important linkage between exchange rate dynamics and investor behavior. The most comprehensive order flow data is owned by global custodians like State Street, which undertake a large proportion of global equity clearing. Such (proprietary) data has been analyzed by Froot *et al.* (2001) and Froot and Ramadorai (2002). The results show that the impact of investor order flow on the exchange rate is very persistent and peaks at horizons of about a month for major currencies. But the order flow exchange rate linkage has not yet been imbedded in a theoretical framework in which order flow is derived from international investment behavior. The literature therefore still misses a model that bridges the gap between foreign exchange microstructure and macroeconomic fundamentals. To develop such a framework and explore its empirical implications is the main objective of this paper.

Our most important structural assumption concerns incomplete forex risk trading. In complete markets, exchange rate risk hedging is a free lunch (Perold and Schulman (1988), Karolyi and Stulz (2001)). Investors in the home country can simply swap and eliminate forex risk by trading it with foreign investors holding the reciprocal risk.[3] Under full forex risk hedging, the domestic and international investment problems are alike.[4] But the evidence on forex hedging strongly suggests that market completeness represents a highly counterfactual benchmark. We dispose of survey evidence

[1] The London based research firm Cross Border Capital reports that during the period 1975-1984 bank loans accounted on average for 39.5% of total outflows from major industrialized countries (60.3% of inflows), while equities accounted for only 9.5% of outflows (6.1% of inflows). During the 1985-94 period these proportions were reversed. Bank loans accounted only for 8.3% of outflows (16.3% of inflows), while equities jumped to 35.9% of outflows (31.6% of inflows).

[2] Frankel and Rose (1995) summarize the situation by saying that "... no model based on such standard fundamentals like money supplies, real income, interest rates, inflation rates, and current account balances will ever succeed in explaining or predicting a high percentage of the variation in the exchange rate, at least at short- or medium-term frequencies." More recently Devereux and Engel (2002) argue that one cannot match some stylized facts regarding exchange rate volatility and disconnect without adding ingredients such as noise traders to the standard models.

[3] For example, U.S. holdings of foreign long-term securities amounted to \$1.755 trillion and foreign holdings in U.S. long-term securities amounted to \$2.806 trillion as of December 31, 1997 (Board of Governors of the Federal Reserve System (2000)).

[4] Only informational asymmetries may still separate home and foreign investors.

on mutual funds and other institutional investors which manage a large proportion of U.S. foreign equity investments. Their lower transaction costs and higher financial sophistication make them better candidates for forex risk trading compared to individual investors. Do they swap forex risk with their foreign counterparts? Levich *et al.* (1999) surveyed 298 U.S. institutional investors and found that more than 20 percent were not even permitted to hold derivative contracts in their investment portfolio. A further 25 percent of institutional investors were formally unconstrained, but did not trade in derivatives. The remaining 55 percent of institutional investors hedged only a minor proportion of their forex exposure. For the full sample, Levich *et al.* calculated that forex risk hedging concerned only 8 percent of the total foreign equity investment.[5] Portfolio managers cited monitoring problems, lack of knowledge and public and regulatory perceptions as most important reasons for the restricted forex derivative use. The development of the derivative market notwithstanding, only a minor proportion of the total macroeconomic forex return risk seems to be separately traded and eliminated. The typical foreign equity investor holds currency return and foreign equity return risk as a bundle.

Exposure to exchange rate risk implies that the international investor generally cares about both the volatility of the exchange rate and the correlation structure of exchange rates and foreign equity returns. For example, higher exchange rate volatility tends to induce a home equity bias. On the other hand, a negative correlation between foreign exchange rate returns and foreign stock market returns reduces the return volatility in home currency terms and makes foreign investment more attractive. Portfolio choice therefore depends on exchange rate dynamics. But dynamic portfolio choice should simultaneously affect the exchange rate. Differences in stock market performances generate imbalances between the dividend income of home and foreign investors. Dynamic rebalancing of equity portfolios then initiates forex order flow, which in turn induces exchange rate movements.

We capture this interaction between optimal portfolio choice under market incompleteness and exchange rate dynamics in a simple model. Exchange rates, portfolio equity flows and equity returns are jointly and endogenously determined. For simplicity we assume that in each of the two countries of our world economy there is a constant risk-free interest rate and an exogenous stochastic dividend process for the equity market. Domestic and foreign investors are risk averse and maximize a simple myopic return trade-off between instantaneous expected return and its variance. They can invest in both the domestic and foreign equity and bond markets. Dividend payments and equity purchases are undertaken in local currency. The exchange rate is determined under market clearing in the forex market where private investor order flows stemming from portfolio rebalancing and dividend repatriation meet a price-elastic forex supply of liquidity-providing financial institutions. The price-elastic forex supply simply captures the imperfect intertemporal forex arbitrage. It implies that order flow drives the exchange rate in accordance with the empirical findings in the recent microstructure literature.

The model we develop has testable implications regarding the relative volatilities of equity and exchange rate returns; correlations between stock index (excess) returns and exchange rate returns; and correlations between portfolio flows and exchange rate returns. We highlight here the three main empirical implications of our model:

[5] We also consulted market experts in two large U.S. custodians. Independent sources at both State Street Bank and Citibank estimated the notional forex hedge at less than 10 percent. This confirms the survey evidence.

1. Market incompleteness in combination with a low price elasticity of forex liquidity supply generates exchange rates which are almost as volatile as equity prices.

2. Foreign stock index returns in (local currency) in excess of the home country stock index returns correlate negatively with foreign currency returns.

3. Net equity inflows into the foreign market are positively correlated with a foreign currency appreciation.

We confront these model predictions with the data. Ratios of exchange rate volatility to equity return volatility are generally smaller than one and in the range replicated by the model. Return correlations are examined with daily, monthly and quarterly stock index and exchange rate return data for 17 OECD countries. Strong statistical evidence is produced for a negative correlation between excess returns on foreign over U.S. equity and returns of the foreign currency as predicted by the model. Hence, both theory and evidence contradict the journalistic wisdom that a strong equity market comes with a strong currency. We also highlight that these findings are produced at high statistical significance in contrast to the well known failure of uncovered interest parity for the same set of countries. The evidence for the negative correlation between excess equity returns and exchange rate is strongest for the post 1990 period when equity markets became more integrated. Cross sectionally, we find that the negative correlation is more pronounced for countries with the most developed equity markets. Finally, we also use monthly equity flow data on the same OECD countries to verify the portfolio flow implications. In accordance with the model, the pooled regressions reveal a positive correlation between equity inflows into the foreign market and the appreciation of the foreign currency.

The following section discusses the literature before we describe the model in section 3. In section 4, we solve the model for two special cases, namely the case of financial autarky and full integration in a complete market setting. These two polar cases provide two benchmarks for the general case of financial integration under market incompleteness explored in section 5. We summarize the most important testable implications in section 6 before confronting them with the data in section 7. Conclusions follow in section 8.

2 Literature Review

It is useful to situate our analysis in the existing exchange rate literature. What distinguishes our approach from previous studies concern (1) the emphasis on equity flows relative to the new open macroeconomics literature, (2) the financial market incompleteness assumption relative to the real business cycle literature, (3) the endogeneity of the order flows relative to the forex microstructure literature and (4) the explicit modeling of the exchange rate relative to the finance literature.

Macroeconomic theory has recently emphasized better microfoundations together with a more rigorous modelling of the dynamic current account. This approach is exemplified by Obstfeld and Rogoff (1995) and surveyed in Lane (2001). But international equity markets do not play an important role in this framework. While monopolistic profits occur in these models, they typically accrue entirely to domestic residents and therefore do not give rise to any equity flows. In the spirit of the traditional asset market approach to exchange rates (surveyed by Branson and Henderson (1985)), we link exchange rate movements and optimal foreign and domestic asset holdings. We obtain sharper testable implications for the correlation structure of forex returns, equity returns and equity flows.

Our analysis features incomplete forex risk trading as an important structural assumption. To the extent that real business cycle models allow for international asset trade, they typically examine the resulting exchange rate dynamics in a complete market setting.[6] In this idealized setting all benefits from international exchange rate risk trading are realized. We argue that this assumption is at odds with current evidence on very low hedge ratios for foreign equity investment as discussed in the introduction. In our view the market succeeds in trading international equity fairly frictionlessly, but fails to realize the full benefit of trading the associated forex risk. This market incompleteness is not related to the absence of the market (forex derivatives exist), but rather to transaction and agency costs of using them.

This paper is inspired by the new empirical literature on the microstructure of the forex market. Order flow is identified as an important determinant of exchange rate dynamics. We interpret this literature as evidence for a price inelastic forex supply and explore its consequences for optimal international portfolio investment. The microstructure literature has always treated the forex order flows as exogenous model primitives and not itself as the object of equilibrium analysis.[7]. In our model forex order flow is derived endogenously from the optimal dynamic portfolio policy. Also the time horizon for our analysis extends to several months unlike the high frequency focus in many microstructure models. We do not incorporate any informational asymmetries to preserve simplicity and tractability.

Finally, our analysis relates to a recent literature on international equity flows. Some of this work is entirely descriptive (Bekaert and Harvey, (2000); Bekaert *et al.* (2002) and Richards (2002)). Brennan and Cao (1997) and Griffin *et al.* (2002) also provide a theoretical analysis of foreign investment behavior. Paradoxically, both treated foreign investment like domestic investment by modelling only dollar returns. Instead of an exchange rate, home and foreign investors are separated by information asymmetries (Brennan and Cao) or by exogenous differences in return expectations (Griffin *et al.*). Unlike these models, our framework assumes that foreign and home investors are separated by an exchange rate and pursue investment objectives in the currency of their respective residence.

3 The Model

A world with two countries has a home and a foreign investor. Both investors are risk averse and can invest in risky home and foreign equities and in riskless bonds. Equities pay a continuous stochastic dividend flow. Purchase of foreign equity is settled in foreign currency and therefore requires a parallel purchase of foreign currency in the forex market. Increases in foreign equity therefore generate an order flow[8] in the forex market. Investors do not hold money balances, which are dominated by investment in the riskless bond. Foreign dividend income is either reinvested in foreign equity, or

[6] Capital market incompleteness and the short sale constraint for foreign bonds set our model apart from the Lucas (1982) model and much of the stochastic dynamic general equilibrium literature. Our model is not however cast in a general equilibrium set up, since dividend processes and riskless rates are exogenous.

[7] Exceptions are Osler (1998) and Carlson and Osler (2000) who model the exchange rate as the price equating supply and demand on a foreign exchange market where 'current account traders' meet 'rational currency speculators'.

[8] We assume that when an agent purchases the foreign equity she initiates the purchase of foreign exchange, so that our net currency flow coincides with the conventional definition of the order flow (net of buyer over seller initiated trades).

repatriated for home country investment. Home investment can occur in home equity or a riskless bond with a constant interest rate. The supply of home and foreign equity is fixed and its price determined by market clearing. The bond supply is assumed to be infinitely price elastic. Central banks in both countries peg the interest rate.

We do not allow for short selling of foreign bonds. A short position in foreign bonds works as a forex hedge on the foreign equity investment. We believe that incomplete hedging of foreign investment is the more realistic benchmark compared to a world of full international exchange-rate risk sharing. It is important to highlight that the short sale constraint is binding in equilibrium (see proof in Appendix F). Intuitively, the home bond investment always strictly dominates the foreign bond investment under identical foreign and home bond returns and additional exchange rate risk on the foreign bond. Since home investors would like to hold a short position in foreign bonds to hedge the currency risk of their foreign equity position, but are prohibited to do so, they can at best choose a zero position of foreign bonds. To simplify the exposition and reduce notation, we present the model as if investors were prevented from investing in foreign bonds altogether. This does not involve any loss of generality. Given that the short-selling constraint is always binding in equilibrium, we can assume zero foreign bond holdings.

The market structure is summarized as follows:

Assumption 1: Asset Market Structure

A home (h) and a foreign (f) stock market provide exogenous stochastic dividend flows D_t^h and D_t^f in local currency. Home and foreign investors can invest in both stock markets. In addition, each investor can invest in a domestic bond providing a riskless constant return r in the respective local currency.

Investors in our model are risk averse and their objective is to find an optimal trade-off between expected profit flow of their asset position and the instantaneous profit risk. Each investor measures profits in home currency. Formally, we assume:

Assumption 2: Investor Behavior

Home and foreign investors are risk averse and maximize (in local currency terms) a myopic mean-variance objective for the profit flow.[9] Home investors choose a portfolio of home and foreign equity, $K_t = (K_t^h, K_t^f)$, and foreign investors choose a portfolio of foreign and home equity, $K_t^* = (K_t^{f*}, K_t^{h*})$, so as to solve the optimization problem

$$\begin{array}{lll} \max_{K_t^h, K_t^f} & \mathcal{E}_t \int_{s=t}^{\infty} e^{-r(s-t)} & \left[d\Pi_s - \frac{1}{2}\rho d\Pi_s^2\right] ds \\ \max_{K_t^{f*}, K_t^{h*}} & \mathcal{E}_t \int_{s=t}^{\infty} e^{-r(s-t)} & \left[d\Pi_s^* - \frac{1}{2}\rho d\Pi_s^{*2}\right] ds \end{array}$$

where $\mathcal{E}_t$ denotes the rational expectation operator. Let $dR_t = (dR_t^h, dR_t^f)^T$ and $dR_t^* = (dR_t^{f*}, dR_t^{h*})^T$ denote the corresponding excess payoffs (in local currency terms over the local riskless bond) for domestic and foreign investors, respectively. We define the stochastic

[9] For the time horizons relevant for our exercise (1 day to several months), good prices can be considered to be sticky in local currency.

profit flows for the domestic and foreign investors as

$$\begin{aligned} d\Pi_t &= K_t dR_t \\ d\Pi_t^* &= K_t^* dR_t^*, \end{aligned}$$

respectively. The investor risk aversion is given by ρ, and the discount rate is given by r.

The myopic investor behavior simplifies the asset demand equations to linear functions in the fundamentals. Intertemporal hedging demand components are ignored under this utility specification. We highlight that both stock markets have to clear under the optimal asset demand. For simplicity we normalize the quantity of outstanding equity to one. This implies

$$\begin{aligned} K_t^h + K_t^{h*} &= 1 \\ K_t^f + K_t^{f*} &= 1 \end{aligned} \tag{1}$$

as the two asset market clearing conditions.

An additional market clearing condition applies to the foreign exchange market with an exchange rate E_t. Denoting home and foreign equity prices by P_t^h and P_t^f, respectively, we can measure the equity related capital outflows dQ_t of the home country (in foreign currency terms) as

$$dQ_t = E_t K_t^{h*} D_t^h dt - K_t^f D_t^f dt + dK_t^f P_t^f - E_t dK_t^{h*} P_t^h. \tag{2}$$

The first two terms capture the outflow if all dividends are repatriated. But investors can also increase their holdings of foreign equity assets. The net capital outflow due to changes in the foreign holdings, dK_t^f and dK_t^{h*} are captured by the third and fourth term. Let us for example denote the euro area as the foreign and the U.S. as the home country. Then dQ_t represents the net capital outflow induced by equity trade out of the U.S. into the euro area in euro terms. An increase in E_t (denominated in euro per dollar) corresponds to a dollar appreciation against the euro. Any capital outflow in our model is identical to a net demand in foreign currency as all investment is assumed to occur in local currency. We can therefore also identify dQ_t with the equity trade induced order flow for foreign currency in the foreign exchange market[10]. Furthermore, the above investor capital outflow (or forex order flow) can be linearly approximated by

$$dQ_t^D = (E_t - \overline{E})\overline{K}\overline{D}dt + (K_t^{h*} - K_t^f)\overline{D}dt + (D_t^h - D_t^f)\overline{K}dt + (dK_t^f - dK_t^{h*})\overline{P}. \tag{3}$$

where the upper bar variables denote the unconditional means of the stochastic variables. The linearization generates a linear order flow and renders the analysis tractable. We normalize $\overline{E}$ to 1, because both countries are exactly symmetric.

The net forex order flow of investors is absorbed by liquidity-supplying banks which can buffer foreign exchange imbalances.[11] The following assumption characterizes the liquidity supply:

Assumption 3: Price-Elastic Excess Supply of Foreign Exchange

[10] Remember that there is no trade in the foreign riskless bond in equilibrium and therefore the forex order flow results only from equity trade and dividend repatriation.

[11] A generalization of the model consists in allowing for additional current account imbalances given by $CA_t dt = \gamma \left(\overline{E} - E_t\right) dt$. The current account for the U.S. is in deficit when the dollar is strong and vice versa (γ is the exchange rate elasticity of the current account). This generalization is straightforward.

The foreign exchange market clears for a price-elastic excess supply curve with elasticity parameter κ. For an equilibrium exchange rate E_t, the excess supply of foreign exchange is given by

$$Q_t^S = -\kappa(E_t - \overline{E})$$

where $\overline{E}$ denotes the steady state exchange rate level.

An increase in E_t (euro depreciation) decreases the excess supply of euro balances. The exchange rate elastic excess supply captures incomplete intertemporal arbitrage of risk averse forex market makers, who sell dollars for euros when the dollar is high and buy dollars when the dollar is low. Such intertemporal arbitrage involves considerable risk and needs to be compensated by expected trading profit. For example, if the exchange rate follows a mean reverting Ornstein-Uhlenbeck process, then the expected exchange rate change $\mathcal{E}_t(dE_t)$ and the expected instantaneous profit of liquidity supply $\mathcal{E}_t(d\Pi_t) = \mathcal{E}_t(Q_t^S dE_t)$ is proportional to the steady state deviation $\overline{E} - E_t$. The liquidity supply Q_t^S should then increase in the steady state deviation $\overline{E} - E_t$. While it is possible to endogenize the elasticity parameter κ, we prefer the simpler parametric representation.

Market clearing in the forex market then requires $Q_t^S = Q_t^D$ and the foreign exchange rate is subject to the constraint

$$-\kappa dE_t = (E_t - \overline{E})\overline{K}\overline{D}dt + (K_t^{h*} - K_t^f)\overline{D}dt + (D_t^h - D_t^f)\overline{K}dt + (dK_t^f - dK_t^{h*})\overline{P}. \tag{4}$$

The exchange rate dynamics is therefore tied to the relative dividend flows, $D_t^h - D_t^f$, the relative level of foreign asset holdings $K_t^{h*} - K_t^f$, and their relative changes $dK_t^{h*} - dK_t^f$. The relative dividend flows are exogenous, but the optimal relative foreign equity holdings are endogenously determined and depend in turn on the exchange rate dynamics.

It is straightforward to express the excess payoffs (over the riskless asset) on a unit of home equity over the interval dt as dR_t^h. To characterize the foreign excess payoff dR_t^f in home currency we use a linear approximation around the steady state exchange rate $\overline{E} = 1$ and the steady state price $\overline{P}$. Formally, excess payoffs are given as

$$\begin{aligned} dR_t^h &= dP_t^h - rP_t^h dt + D_t^h dt \\ dR_t^f &\approx -dE_t\overline{P} + dP_t^f - dE_t dP_t^f - r\left[P_t^f - \overline{P}(E_t - 1)\right]dt + \left[D_t^f - \overline{D}(E_t - 1)\right]dt \end{aligned}$$

for the home and foreign assets, respectively. Excess returns follow as $dR_t^h/\overline{P}$ and $dR_t^f/\overline{P}$, respectively. The exchange rate component of the foreign payoff is given by $-\overline{P}dE_t$ and the exchange rate return by $-dE_t$. [12]

Finally, we have to specify the stochastic structure of the state variables spelled out in the following assumption:

Assumption 4: Stochastic Structure

[12] The model is "closed" and there is no stock-flow inconsistency. A foreign equity purchase of the home investor is settled in foreign currency. But the foreign equity seller immediately reinvests this liquidity and holds zero money balances. He can either exchange it in the FX market if he reinvests in equity abroad, or bring it to his central bank at a fixed riskless rate. Central banks thus absorb the additional liquidity at the fixed rate r.

The home and foreign dividends follow independent Ornstein-Uhlenbeck processes with identical variance and mean reversion ($\alpha_D > 0$) given by

$$\begin{aligned} dD_t^h &= \alpha_D(\overline{D} - D_t^h)dt + \sigma_D dw_t^h \\ dD_t^f &= \alpha_D(\overline{D} - D_t^f)dt + \sigma_D dw_t^f. \end{aligned}$$

The innovations dw_t^h and dw_t^f are independent.

The mean reversion of all stochastic processes simplify the analysis considerably. We can now introduce variables F_t^h and F_t^f which denote the expected present value of the future discounted dividend flow,

$$\begin{aligned} F_t^h &= \mathcal{E}_t \int_{s=t}^{\infty} D_t^h e^{-r(s-t)} ds = f_0 + f_D D_t^h \\ F_t^f &= \mathcal{E}_t \int_{s=t}^{\infty} D_t^f e^{-r(s-t)} ds = f_0 + f_D D_t^f \end{aligned}$$

with constant terms defined as $f_D = 1/(\alpha_D + r)$ and $f_0 = (r^{-1} - f_D)\overline{D}$. The risk aversion of the investors and the endogenous exchange rate variability imply that the asset price will generally differ from this fundamental value.

4 Two Special Cases

It is instructive to explore two special variations of our model. First we cover the extreme case in which no foreign asset holdings are allowed. We refer to this case as financial autarky. It provides a useful closed economy benchmark for the stock market equilibrium, in which investors do not internationally share their domestic equity risk. The opposite extreme assumption is to allow both the equity risk and the exchange rate risk to be fully and separately traded. This second benchmark characterizes the international financial market equilibrium with complete risk sharing. Formally, it is identical to an economy with two freely tradeable assets. The exchange rate is a redundant price. As empirically most relevant we consider a third case in which equity is freely traded but the exchange rate risk is not. We spare the analysis of the latter case for section 5.

Solving the model always requires three steps. First, we postulate a linear solution for the asset prices and the exchange rate. Second, we derive the optimal asset demand under the conjectured solution. Third, we impose the market clearing conditions, show that the resulting price functions are indeed of the conjectured form and finally solve for the coefficients. To provide for a more coherent exposition, we summarize our results in various propositions. All derivations are relegated to the appendix of the paper.

4.1 Equilibrium without Risk Sharing (Financial Autarky)

Under financial autarky, the home investor's foreign equity position (K_t^f) and the foreign investor's home equity position (K_t^{h*}) are assumed to be zero. All domestic assets are owned by domestic investors, hence

$$\begin{pmatrix} K_t^h & K_t^f \\ K_t^{f*} & K_t^{h*} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}.$$

The financial market equilibrium for the home and foreign equity market can be determined separately. Proposition 1 states the result:

Proposition 1: Equilibrium under Financial Autarchy.

Assume a two-country world in which home investors hold the domestic asset and foreign investors the foreign asset. The home and foreign stock market prices are given by

$$\begin{aligned} P_t^h &= p_0 + p_F F_t^h \\ P_t^f &= p_0 + p_F F_t^f \end{aligned}$$

with $p_0 = -\rho\sigma_R^2/r$ and $p_F = 1$. The (instantaneous) return volatility follows as $\sigma_R^2 = \sigma_D^2/(\alpha_D + r)^2$.

Proof: See Appendix A.

A price parameter $p_F = 1$ implies that the asset prices are proportional to their fundamental values F_t^h and F_t^f, respectively. The fundamental values represent the expected discounted future cash flows. The risk aversion of the investors is reflected in the coefficient $p_0 < 0$, which captures the equity risk premium as a price discount. It is proportional to the investor risk aversion ρ and the instantaneous variance σ_R^2 of the excess return processes. These equilibrium results are standard for a closed economy with a fixed asset supply and myopic mean-variance preferences for the investor.

4.2 Equilibrium with Complete Risk Sharing

A second model variation consists in the full risk sharing benchmark. Forex risk can then be fully traded either through derivative contracts or through short sales of the foreign riskless bond. Perfect and complete risk trading results in the elimination of all exchange rate risk. Intuitively, home and foreign investors hold exactly opposite and off-setting exchange rate risk in their global equity portfolio. They just need to swap the forex risk and thereby eliminate it. The resulting financial market equilibrium is stated in proposition 2:

Proposition 2: Equilibrium with Complete Risk Sharing.

The home and foreign stock market prices and the exchange rate are given by

$$\begin{aligned} P_t^h &= p_0 + p_F F_t^h \\ P_t^f &= p_0 + p_F F_t^f \\ E_t &= 1 \end{aligned}$$

where we define $p_0 = -\rho\sigma_R^2/2r$, and $p_F = 1$. The (instantaneous) return volatility follows as $\sigma_R^2 = \sigma_D^2/(\alpha_D + r)^2$. The domestic and foreign portfolio positions of the two investors are equal and constant with

$$\begin{pmatrix} K_t^h & K_t^f \\ K_t^{f*} & K_t^{h*} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}.$$

Proof: An identical riskless rate in the home and foreign country under complete markets implies a constant exchange rate, $E_t = 1$. The complete solution is derived in Appendix B.

First, we note that the exchange rate is constant. In a world of perfect risk sharing, the two country model is not different from one domestic economy with two asset markets. Home and foreign investors each hold equal and constant shares of the world market portfolio. The asset prices are again proportional to their fundamental values, F_t^h and F_t^f, respectively. The risk sharing across the two investor groups implies that the asset price risk discount $p_0 < 0$ is only half as large as in the autarky case for the same return volatility σ_R^2. This implies lower average asset returns under market integration. Evidence that financial integration indeed reduces market stock returns is provided by Bekaert and Harvey (2000), Henry (2000) and Stulz (1999) among others. These authors show reduced capital costs or excess returns on equity for emerging countries following their capital market liberalization.

We further highlight that complete forex risk trading implies no particular correlation structure between exchange rate and equity returns. The exchange rate is a redundant price and constant. This implication is of course at odds with the high exchange rate volatility observed in practise. But it provides a useful benchmark for the following section which explores the case of equity market integration under incomplete exchange rate risk trading.

5 Foreign Investment under Incomplete Risk-Sharing

We now treat the case in which a foreign exchange market allows investment in the foreign equity but exchange rate risk trading is incomplete. If the exchange rate moves stochastically, home investors with foreign equity holdings incur an additional exchange rate risk in addition to the risk of the stochastic dividend flow. Foreign investors hold the opposite risk due to ownership stakes in foreign equity. If this reciprocal exchange rate risk were tradeable, it could be perfectly eliminated as assumed in the perfect market case discussed in section 4.2. But now we assume that such forex risk trading does not occur.

The non-tradeability of the forex risk not only excludes derivative contracts, but also requires that investors cannot short sell the foreign riskless asset. Short selling of foreign riskless assets effectively amounts to a separate trading of the exchange rate risk. As discussed before, assuming a no short-sale constraint on the riskless foreign asset implies zero foreign bond holdings in equilibrium. If unconstrained, investors should seek a short position in the foreign riskless asset equivalent to their foreign equity stake. But they would not seek a long position which adds exchange rate risk to the portfolio. The short selling constraint is binding. Setting the foreign bond position to zero does not represent an additional restriction.

5.1 Exchange Rate Dynamics

Before we conjecture the exchange rate dynamics under incomplete markets, it is useful to highlight two principle equilibrium forces which shape this dynamics. The first equilibrium tendency is governed by the elastic liquidity supply for forex order flow. Forex order flow dQ_t^D in equation (3) is accommodated by financial institutions which finance these home outflows according to an upward sloping supply curve. The elasticity of forex liquidity supply certainly influences the impact of net order flow on the exchange rate and indirectly the adjustment speed towards the steady state exchange rate, $\overline{E}$. We associate the supply induced mean reversion with a first characteristic root (labeled z). A second important parameter for exchange rate dynamics is the mean reversion of the dividend processes. This mean reversion α_D is exogenous and any feedback effect from the exchange rate dynamics to the dividend process is ruled out by assumption.

An important simplifying feature of our model is its symmetry between the home and foreign country. Symmetry implies that the exchange rate can depend only on differences between home and foreign country variables, but not on a country specific variable itself. Otherwise the symmetry would be broken. The symmetry requirement also implies that exchange rate surprises can only be a function of current and past relative dividend innovations, $dw_s = dw_s^h - dw_s^f$. These relative innovations are the only exogenous source of exchange rate dynamics.

Finally, we highlight the linearity of the model structure. The forex order flow constraint is linearized and the exogenous dividend dynamics is linear by assumption. Moreover, we have assumed a myopic mean-variance utility function which translates linear dividend, price and return processes into linear asset demands. It is therefore justified to restrict our attention to the class of linear exchange rate and price processes. The argument for two fundamental equilibrium forces explains why we focus on two state variables Δ_t and Λ_t, both of which depend for reasons of model symmetry on current and past relative dividend innovations dw_s only.

The following proposition 3 states the conjectured exchange rate process and derives its implications for the order flow constraint (4).

Proposition 3: Exchange Rate Dynamics.

Assume that (i) equity prices $P = (P_t^h, P_t^f)$ depend linearly on the exchange rate E_t and the dividend processes $D_t = (D_t^h, D_t^f)$ and (ii) the exchange rate has the following linear representation

$$E_t = 1 + e_\Delta \Delta_t + e_\Lambda \Lambda_t$$

with

$$\begin{aligned} \Delta_t &= D_t^h - D_t^f = \int_{-\infty}^{t} \exp[-\alpha_D(t-s)]\sigma_D dw_s \\ \Lambda_t &= \int_{-\infty}^{t} \exp[z(t-s)] dw_s \end{aligned}$$

where $z < 0$ and $dw_s = dw_s^h - dw_s^f$. Then it follows that the order flow constraint (4) is of the simple form

$$dE_t = k_1 \Delta_t dt + k_2 (E_t - 1) dt + k_3 dw_t,$$

where k_1, k_2 and k_3 represent undetermined coefficients.

Proof: The derivation is provided in Appendix C. We have to show that for a linear price and a linear exchange rate, investor utility maximization implies optimal foreign equity demands K_t^{h*}, K_t^f such that the expression $(K_t^{h*} - K_t^f)\overline{D}dt + (dK_t^f - dK_t^{h*})\overline{P}$ in equation (4) is linear in $E_t - 1$, Δ_t and dw_t.

Under linearity of the price and exchange rate processes, the order flow constraint simplifies to a differential equation in only two state variables Δ_t and $E_t - 1$. This allows us to characterize the exchange rate dynamics as a system of two first-order differential equations,

$$\begin{pmatrix} d\Delta_t \\ dE_t \end{pmatrix} = \begin{pmatrix} -\alpha_D & 0 \\ k_1 & k_2 \end{pmatrix} \begin{pmatrix} \Delta_t \\ E_t - 1 \end{pmatrix} dt + \begin{pmatrix} \sigma_D \\ k_3 \end{pmatrix} dw_t. \tag{5}$$

The associated characteristic polynomial follows as

$$\begin{vmatrix} -\alpha_D - \lambda & 0 \\ k_1 & k_2 - \lambda \end{vmatrix} = (-\alpha_D - \lambda)(k_2 - \lambda) = 0$$

with characteristic roots $-\alpha_D$ and k_2. A stable solution requires $k_2 < 0$. The exchange rate solution can then be written as a linear combination $e_\Delta \Delta_t + e_\Lambda \Lambda_t$ of the two eigenvectors

$$\Delta_t = \int_{-\infty}^{t} \exp[-\alpha_D(t-s)]\sigma_D dw_s \qquad \text{and} \qquad \Lambda_t = \int_{-\infty}^{t} \exp[k_2(t-s)] dw_s$$

as conjectured in proposition 3.

In order to find the solution parameters, we have to impose the market clearing conditions (1) and determine the steady state levels for the exchange rate, $\overline{E}$, the equity price, $\overline{P}$, and the foreign equity holding, $\overline{K}$. Non-negative (steady state) prices ($\overline{P} > 0$) and positive (steady state) home and foreign equity holdings ($0 < \overline{K} < 1$) imply further restrictions on the parameter domain of our model. In particular we have to impose an upper bound $\overline{\rho}$ on the risk aversion and a lower bound $\underline{\kappa}$ on the elasticity of the forex liquidity supply to obtain plausible steady state values.

Proposition 4 characterizes the equilibrium properties:

Proposition 4: Existence and Uniqueness of the Incomplete Risk-Sharing Equilibrium.

Let the economy be characterized by assumptions 1 to 4. For a sufficiently low risk aversion of the investors $\rho < \overline{\rho}$ and a sufficiently price-elastic forex supply $\kappa > \underline{\kappa}$, there exists a unique stable linear equilibrium

$$\begin{aligned} P_t^h &= p_0 + p_F F_t^h + p_\Delta \Delta_t + p_\Lambda \Lambda_t \\ P_t^f &= p_0 + p_F F_t^f - p_\Delta \Delta_t - p_\Lambda \Lambda_t \\ E_t &= 1 + e_\Delta \Delta_t + e_\Lambda \Lambda_t \end{aligned}$$

where we define F_t^h and F_t^f as the expected present values of the future home and foreign dividend flows, respectively (as in section 3). The variable $\Delta_t = D_t^h - D_t^f$ represents the relative dividend flows for the two countries and Λ_t a weighted average of past relative dividend innovations decaying at an endogeneous rate $z < 0$ as defined in proposition 3. The price parameters can be signed as

$$p_0 < 0,\ p_F = 1,\ p_\Delta > 0,\ e_\Delta < 0,\ e_\Delta \sigma_D + e_\Lambda < 0.$$

Optimal portfolio holdings are given by

$$\begin{pmatrix} K_t^h & K_t^f \\ K_t^{f*} & K_t^{h*} \end{pmatrix} = \begin{pmatrix} 1-\overline{K} & \overline{K} \\ 1-\overline{K} & \overline{K} \end{pmatrix} + \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix} \frac{1}{2\rho} (m_\Delta \Delta_t + m_\Lambda \Lambda_t)$$

for the parameters $m_\Delta < 0$, and $m_\Lambda > 0$ defined in Appendix C.

Proof: For a derivation see Appendix C.

As in the previous full risk-sharing case, we find that investor risk aversion requires an equity risk premium in the form of a price discount $p_0 < 0$. As before, a coefficient $p_F = 1$ implies that the equity price reflects the fundamental value of expected future dividends, F^h and F_t^f, respectively. Moreover, two new stochastic terms Δ_t and Λ_t influence asset prices and the exchange rate. These additional terms reflect changes in the asset prices and exchange rate dynamics induced by the incompleteness of forex risk trading. The exchange rate is no longer constant and exchange rate volatility imply asymmetric holdings of home and foreign equity. In addition, the optimal portfolio positions change proportionally to $m_\Delta \Delta_t + m_\Lambda \Lambda_t$. The dynamic equilibrium is characterized by constant rebalancing of the optimal portfolios. We therefore have endogenous equity inflows and outflows as a result of optimal equity risk trading under constrained forex risk trading. The equity flows and the corresponding forex order flow in turn generate the equilibrium exchange rate dynamics under the price elastic forex liquidity supply.

5.2 Economic Interpretation

Investors in the two countries care about nominal trading profits in their home currency. This does not imply however that they only invest in home assets. Given that foreign asset investment provides an equity risk diversification benefit, foreign equity ownership is desirable for the home investor. But the foreign dividend income is repatriated at a fluctuating exchange rate. The exchange rate dynamics is itself related to the relative performance of the two stock markets. For relatively high home dividend income ($\Delta_t = D_t^h - D_t^f > 0$), the home country faces a capital outflow approximated by the term $(D_t^h - D_t^f)\overline{K}dt$ in the flow constraint (4). This creates an excess demand for foreign currency. The value of the foreign currency should therefore be high (*i.e.* the value of the domestic currency should be low) under the limited supply elasticity in the forex market. The home stock price P_t^h and exchange rate E_t should therefore move in opposite directions. This explains the sign of the coefficients $p_\Delta > 0$ and $e_\Delta < 0$ in proposition 4. The country with a highly productive risky asset sees a decline in its currency terms of trade to assure the equilibrium in the forex market. We can formally summarize this effect as follows:

Corollary 1: Negative Correlation of Foreign Stock and Forex Returns.

Under incomplete forex risk trading, foreign stock returns $(dR_t^f/\overline{P})$ and exchange rate returns $(-dE_t)$ are negatively correlated, hence

$$-\mathcal{E}_t(dE_t dR_t^f/\overline{P})dt = \mathcal{E}_t(dE_t dR_t^h/\overline{P})dt < 0.$$

Proof: Appendix D.

The negative correlation implies that the exchange rate provides a partial, but automatic hedge against foreign equity risk. When foreign stock market returns are high, the foreign currency depreciates and vice versa. This reduces the return risk of foreign investment in home currency terms and increases the (steady state) demand for foreign equity. Furthermore, dividend processes are by assumption mean reverting. When home dividends are high ($\Delta_t > 0$), they are expected to decrease and the home currency is therefore expected to appreciate. This makes the home equity at date t more attractive relative to the foreign equity. It adds a price premium ($p_\Delta \Delta_t > 0$) to the home equity and a price discount ($-p_\Delta \Delta_t < 0$) to the foreign equity.

We highlight that the exchange rate more than adjusts to accommodate imbalances in foreign dividend income ($\Delta_t > 0$). If the exchange rate just counterbalanced high dividends outflows, the flow constraint (4) would only consist of the terms $-\kappa dE_t < 0$, $(E_t - \overline{E})\overline{K}\overline{D}dt < 0$ and $(D_t^h - D_t^f)\overline{K}dt > 0$. But investors adjust their optimal portfolio holdings to the exchange rate dynamics and these equilibrium portfolio shifts influence the exchange rate change through the additional terms $(K_t^{h*} - K_t^f)\overline{D}dt$ and $(dK_t^f - dK_t^{h*})\overline{P}dt$ in equation (4). A low home country exchange rate (E_t low) makes foreign equity holdings relatively more attractive for the home investor since the value of foreign dividends in domestic currency is high, implying $K_t^{h*} - K_t^f < 0$ (or $m_\Delta < 0$); an expected appreciation leads to a net home equity inflow $dK_t^f - dK_t^{h*} < 0$. It follows from the flow constraint (4) that the endogenous portfolio shifts require a larger exchange rate appreciation $-\kappa dE_t \ll 0$ than is needed to eliminate the imbalance in dividend income. In this sense the exchange rate overshoots the dividend income imbalance. Optimal international portfolio allocations in the presence of incomplete forex risk trading therefore tend to re-enforce the exchange rate fluctuations.

Finally, we also note that imperfect intertemporal forex arbitrage is a necessary condition for these results. This can be verified by examining the limit case of a completely price elastic forex liquidity supply. In this case the imperfect risk trading equilibrium converges to the special case of complete equity risk sharing:

Proposition 5 (Convergence to Complete Risk Sharing):

The incomplete risk-sharing equilibrium (characterized in proposition 4) converges to the complete risk-sharing equilibrium (characterized in proposition 2) as the currency supply becomes infinitely price elastic, that is $\kappa \to \infty$.

Proof: Appendix C.

In this limit case, the investors can always exchange foreign dividend income at the constant exchange rate $\overline{E} = 1$. Optimal international equity risk sharing is achieved by equally shared ownership of the world equity portfolio. The infinitely elastic currency supply corresponds to a scenario of perfect intertemporal arbitrage in the forex market. In practice, capital constraints for arbitraging speculators impose limits on the amount of intertemporal arbitrage (Shleifer and Vishny (1997)). A relatively, small supply elasticity of currency may therefore represent the correct benchmark.

6 Model Implications

We summarize the main empirical implications of our model, which concern the volatility of the exchange rate return relative to the equity return in section 6.1, the correlation structure of exchange rate and equity returns in section 6.2, and the correlation structure of exchange rate return and equity flows in section 6.3. We also discuss the role of equity market development on the strength of our results in section 6.4.

6.1 Relative Exchange Rate Volatility

In complete markets all exchange rate risk can be traded and state contingent contracts leave no particular role to the exchange rate itself. Hence, general equilibrium models with complete markets and flexible prices typically do not generate sufficient exchange rate volatility. If, on the other hand, forex risk is not widely traded, then we expect the forex liquidity supply to have a relatively low price elasticity. Forex order flows can induce large exchange rate changes. Our model captures the elasticity of the forex supply in the parameter κ. Portfolio flows in the incomplete risk-sharing setting can generate considerable exchange rate volatility if κ becomes small. We can illustrate this effect in Figure 1 by plotting the volatility ratio of the exchange rate returns and the stock market returns (in local currency),

$$\sqrt{\frac{Var(dE_t)}{Var(dR_t^{f*}/\overline{P})}},$$

as a function of two fundamental model parameters, namely the investor risk aversion, ρ, and the elasticity of the liquidity supply, κ. The riskless rate r and the three parameters governing the dividend processes ($\overline{D}$, α_D, σ_D) are held constant. The parameter range is given by $0.04 < \rho < 0.44$ for the degree of risk aversion and $20 < \kappa < 100$ for the liquidity supply parameter. A high price elasticity of forex liquidity supply (κ large) implies a low forex volatility. A decrease in the elasticity of the liquidity supply (lower κ) comes with substantial forex volatility as illustrated by the parametric plot. We summarize this result as follows:

Implication 1: Relative Exchange Rate Volatility.

Market incompleteness in combination with a low price elasticity of forex liquidity supply can generate exchange rates which are almost as volatile as equity returns.

6.2 Equity Returns and Exchange Rate Returns

Market incompleteness implies a negative correlation structure between foreign equity returns and exchange rate returns as stated in Corollary 1. Because of the symmetry of the model, it is most convenient to state the correlation structure for differences of the foreign and home equity returns in local currency, namely $(dR_t^{f*} - dR_t^h)/\overline{P}$. The following corollary provides the result:

Corollary 2:

Under incomplete forex risk trading, exchange rate returns $-dE_t$ and foreign excess returns in local currency over home market returns have a perfect negative correlation, hence

$$-Corr\left[dE_t, (dR_t^{f*} - dR_t^h)/\overline{P}\right] = -1.$$

Proof: See Appendix E.

For example, a U.S. equity market return shortfall relative to the European equity market ($(dR_t^{f*} - dR_t^h)/\overline{P} > 0$) should ceteris paribus coincide with a dollar appreciation ($dE_t > 0$). The negative correlation is perfect, because we have only two exogenous stochastic processes for the dividends which influence the model dynamics. For reasons of symmetry, return differences and exchange rate returns are driven exclusively by relative dividend innovations, $dw_t = dw_t^h - dw_t^f$. The instantaneous correlation between the local currency excess return can therefore only be either perfectly negative or positive or zero. Our analysis shows that the correlation is perfectly negative. Empirically, we cannot expect to find a perfectly negative correlation. Shocks other than dividend innovations and cross country asymmetries will tend to reduce the absolute value of the correlation. As the empirically relevant implication, we therefore retain only the sign of the correlation:

Implication 2: Differential Equity Returns and Foreign Exchange Rate Return

Foreign stock index returns in (local currency) in excess of the U.S. stock index returns (in dollars) correlate negatively with foreign currency returns.

To our knowledge, this particular correlation structure has not yet been related to financial structure in general and the incompleteness of forex risk trading in particular. We explore its empirical validity in section 7.2.

6.3 Exchange Rate Returns and Portfolio Flows

Exchange rates in our model are determined through a price elastic response to forex order flow, which in turn originates in portfolio equity flows. It therefore seems appropriate to relate exchange rate returns directly to equity portfolio flows. Using the price equilibrium in proposition 4, it is straightforward to show that the equity investment into the foreign market by home investors, $dK_t^f = -dK_t^{f*}$, and the equity investment into the home market by foreign investors, $dK_t^{h*} = -dK_t^h$, both correlate positively with the exchange rate return $-dE_t$ and dE_t, respectively. Formally,

$$-\mathcal{E}(dE_t dK_t^f) = \mathcal{E}(dE_t dK_t^{h*}) = \frac{\kappa}{P}(e_\Delta \sigma_D + e_\Lambda)^2 > 0.$$

The symmetry of the model implies that the exchange rate return has the same absolute covariance with foreign and home country inflows, but with opposite signs. We can express the home investors's net foreign inflow as the difference $dK_t^f - dK_t^{h*}$. The net foreign equity inflow exhibits a perfect positive correlation with the exchange rate return. Hence the following corollary:

Corollary 3:

Under incomplete forex risk trading, exchange rate returns $-dE_t$ and the home investor's net foreign equity inflow have a perfect positive correlation,

$$-Corr\left[dE_t, (dK_t^f - dK_t^{h*})\right] = 1.$$

Proof: See Appendix E.

Again, the correlation is perfect, because all variables for country differences or the exchange rate are governed by stochastic innovations which are proportional to the relative dividend innovations, $dw_t = dw_t^h - dw_t^f$. Country heterogeneity in other dimensions will certainly tend to decrease the correlation to a value below 1. We therefore retain only the sign of the correlation as the empirically relevant model implication and refer to the U.S. as the home country:

Implication 3: Forex Return and Net Foreign Equity Inflows.

Foreign exchange returns in dollar terms correlate positively with net foreign equity inflow of U.S. investors.

6.4 The Role of Equity Markets Development

The correlation structure of equity and exchange rate returns was derived for integrated and frictionless equity markets. But equity market development and integration constitutes a relatively recent phenomenon. Only in the 1990s did international equity trading become a prominent feature in international finance. Hence, we expect the empirical model implications to hold best for OECD country data over the last decade. We therefore examine the correlation structure separately over the entire data collection period and for two subsamples starting in 1990 and 1995. An increasingly negative correlation between foreign excess equity returns and the foreign exchange rate return suggests that the correlation structure is indeed induced by increasing equity market integration.

Moreover, the evidence should be strongest for countries with relatively developed equity markets. Such equity market development can be crudely measured by the ratio of market capitalization to GDP. Alternatively, we can measure the integration of a local equity market into the world equity market by the ratio of gross equity trade to GDP. Both market development measures should be correlated with the magnitude of the predicted correlation structure. Such cross sectional evidence suggests again that the exchange rate dynamics represents a financial market phenomenon.

We can summarize both the time series and cross sectional implications as follows:

> **Implication 4: Negative Correlation and Equity Market Development.**
>
> The magnitude of the negative correlation between foreign equity excess return and the exchange rate return should increase in the 1990s and should be strongest for countries with a high degree of equity market development as measured by the ratio of market capitalization to GDP or gross equity trade to GDP.

7 Evidence

The empirical work focuses on OECD countries vis-à-vis the U.S. OECD countries tend to have the most developed equity markets and are therefore most pertinent for the model. The U.S. represents by far the largest source and recipient of international equity flows. Furthermore, the most comprehensive bilateral asset flow data is available for the U.S. only. Within the OECD sample, we excluded three countries for which daily exchange rate data was not available over a sufficiently long time period: Iceland, Greece and New Zealand. Belgium and Luxemburg are treated as one country because of their common currency. Canada was excluded because of its effective exchange rate fixing with the U.S.[13] The remaining 17 OECD countries maintained flexible exchange rates relative to the U.S. and constitute our sample.

The daily equity index and exchange rate data are obtained from Datastream. We used the MSCI series for the end of the day stock index quote and the corresponding dollar exchange rates. Most daily price data are available since 1980. The data are screened for data outliers and errors and do not show any abnormal entries.

Portfolio flow data is more difficult to obtain. We use the so-called TIC data produced by the U.S. Treasury department. Available on a monthly frequency since 1987, the TIC data record transactions

[13] The same exchange rate consideration would also lead to the exclusion of Hong Kong and Singapour, which have developed equity markets, but are not considered OECD countries.

in portfolio equities between U.S. residents and residents of foreign countries.[14] They allow us to compute net purchases of foreign equities by U.S. residents (dK_t^f) and net purchases of U.S. equities by foreigners (dK_t^{h*}). Cross-border equity flows have been growing sizably in the last decade.[15] Hence, we have to find a suitable normalization of the portfolio flow series. We consider a normalization for capital flows by market capitalization and alternatively by the average flows over the previous 12 months (as in Brennan and Cao (1997)). Both methods produce very similar results and we only report tables with the normalization based on past average flows. The stock market capitalization data comes from the S&P Emerging Markets Database.

7.1 Relative Exchange Rate Volatility

First, we examine the volatility ratio of exchange rate returns to stock index returns. We calculate the standard deviation of the log returns of the dollar exchange rate and the stock index returns in local currency. Table 1 reports the ratio of the standard deviations for the entire 1980 (column (a)), the subsample since 1990 (column (b)) and the most recent period since 1995 (column (c)).

The volatility ratio over the full sample varies between 0.369 for Finland and 0.845 for Switzerland with a mean for all countries of 0.6215. Our theoretical framework can explain such high exchange rate volatility with a low price elasticity of the forex liquidity supply. Comparing volatility ratios for the entire period since 1980 to the more recent subsamples since 1990 and 1995, we find declining volatility ratios for most countries. This can mostly be attributed to a decrease in exchange rate volatility. We can speculate that the elasticity of liquidity supply in the forex market (parameter κ in our model) might have increased over time. This would be consistent with increasing forex markets depth in the more recent period.

7.2 Equity Returns and Exchange Rate Returns

The most important model implication concerns the negative correlation between foreign equity and exchange rate returns. We calculate the return correlations based on daily returns for various data periods. Exchange rate returns are in foreign currency per dollar and stock index returns are measured in local currency. The correlation evidence is produced at the daily, monthly and quarterly frequency in Tables 2, 3 and 4, respectively.

Daily correlations in Table 2 provides strong statistical evidence in favor of the our correlation hypothesis. The model prediction of a negative correlation is validated for most countries at a 1 percent statistical significance level.[16] Moreover, the correlations become more negative in the two more recent periods. The correlation in the pooled data decreases from -0.053 over the entire period to -0.0761 for the period since 1990 and to -0.0735 for the most recent period since 1995. The correlation has grown more negative along with the equity market integration, which has intensified since the 1990s. The only countries for which the correlation is still positive after 1995 are Australia

[14] For a thorough presentation of these data see Warnock *et al.* (2001). We note that TIC data records transactions based on the residency of the seller and of the buyer. For example, a German equity sold in London by a US resident to a UK Bank will be recorded as a sale of a foreign security by a US resident to the UK. In our model, this transaction will therefore be interpreted as a dollar pound transaction on the forex market. This inference can be flawed inso far as the real operation was actually performed in euro and not in Sterling or as it was realized on the behalf of a German equity trader.

[15] See Portes and Rey (1999) for a detailed study of the properties of these flows.

[16] Standards errors are corrected for heteroskedasticity and serial correlation.

and Japan.[17] Overall, our evidence strongly supports the predicted negative correlation. Regression evidence on the pooled data sample shows a strong negative correlation significant at the 1 percent level. The monthly return data in Table 3 provides very similar results. In the sample period since 1995 every OECD country features a negative correlation at the monthly frequency. Again we find that the correlation became more negative in the 1990s. For the entire data collection period from 1973 to 2002, the correlation is roughly half as strong as in the last decade.

Table 4 confirms our results on quarterly data for the period 1990 to 2002. We present regressions of exchange rate changes on return differentials for all the countries of our sample. The correlation is again negative and strongly significant for most countries. Furthermore, the variance of the exchange rate explained by our simple return differential variable is strikingly high for some countries: with a single variable, equity return differential, we explain 30% of quarterly exchange rate movements in Spain, 28% in Sweden, 25% in Germany. For the pooled data the R^2 is 13%. These results offer a sharp contrast with the dismal performance of monetary variables of standard exchange rate models at quarterly horizons.

The negative correlation has previously been noted by other researchers for some particular countries. But they were mostly puzzled for lack of a coherent theoretical explanation. Brooks *et al.* (2001) for example document negative correlations between European equity excess returns over U.S. equity and the euro-dollar exchange rate. Interestingly, they discard their finding as "counter-intuitive" (p. 17), since it contradicts the popular view that a strengthening U.S. equity market should be mirrored by a strengthening of the dollar. Incomplete forex risk trading offers a coherent theoretical explanation for the observed correlation structure. From an empirical perspective, the negative correlation deserves to be highlighted because of its strong statistical significance and increasing magnitude. Moreover, it stands out relative to the empirical failure of uncovered interest parity for the same set of countries.[18]

7.3 Exchange Rate Returns and Portfolio Flows

Data on equity flows allow for another test of our model. Model implication 3 highlights a positive correlation between equity inflow into the foreign market and the foreign currency return. Data on bilateral equity flows relative to the U.S. is unfortunately only available at the monthly frequency. Table 5 summarizes the evidence on the correlation of net U.S. inflows into the same 17 OECD countries as before and the corresponding foreign exchange rate returns.

Only France and Portugal show positive correlation at the 1 percent significance level for the entire data period since 1987. Pooling the entire data for all countries even produces a negative, albeit insignificant, correlation. However, this picture is reversed for the more recent data period since 1990. The correlation is now significantly positive at the 1 percent level for 6 countries. It is positive but insignificant for 4 others. The correlation for the pooled sample increases to 0.114 and is statistically significant at the 1 percent level. Overall, the evidence is supportive of a linkage between

[17] We conjecture that the Australian evidence might be tainted by the role of natural resource prices. Chen and Rogoff (2001) and Cashin, Cespedes and Sahay (2002) show indeed that the Australian exchange rate is strongly related to world commodity price fluctuations. They underline the specificity of this country in this respect. Japan on the other hand is special because international portfolio flows concern mostly bonds as opposed to equity.

[18] One could theoretically offer an alternative hypothesis for this negative correlation. A depreciation of the exchange rate could be associated with higher equity returns via a competitiveness effect for exporting firms. Such a mechanism has however been tested and been rejected by previous empirical studies (see in particular...). Moreover, that alternative explanation could not account for intertemporal increase in the correlation.

net equity portfolio flows and exchange rate returns. Net foreign equity inflows tend to appreciate the local currency.

7.4 Equity Market Development and the Correlation Structure

The evidence in Tables 2, 3 and 4 suggest that foreign equity excess returns became a more important determinant of exchange rate behavior in the 1990s, presumably because of increased equity market development and integration. We can test this hypothesis further by examining the cross sectional variation of equity market development within the OECD sample. Two crude measures of equity market development are given by the quarterly market capitalization of the OECD country relative to its GDP and by the gross equity trade with the U.S. relative to GDP. Both measures of equity market development are highly correlated at 0.84.

Figure 2 plots the average return correlation for the 17 countries as a function of the (log) market capitalization to GDP ratio for the sample period 1995-2001. Countries with higher equity market development tend to show a more negative correlation between foreign equity market excess return and the exchange rate return. Table 6 reports the results of a more formal panel regression analysis. We calculate quarterly realized correlations from daily returns for all 17 OECD countries and regress those alternatively on both measures of market development and a fixed time effect for each quarter. Both market development proxies are statistically significant at a 1 percent level. We conclude that the correlation structure of equity and exchange rate returns is related to the level of equity market development. The correlation is more negative for OECD countries with the most developed equity markets.

8 Conclusion

This paper develops a new integrated analysis of exchange rates, equity prices and equity portfolio flows. Such a framework is warranted by the increasing magnitude of international equity flows over the last decade. We argue that the integration of equity markets does not imply convergence to a financial structure based on full exchange rate risk trading. The available evidence from U.S. global mutual funds suggests to the contrary that forex risk in international equity portfolios is mostly unhedged and therefore not internationally traded. The main theoretical contribution of this paper is to explore the role of incomplete forex risk trading for the correlation structure of exchange rate and equity returns and exchange rate returns and net portfolio flows.

The theoretical analysis incorporates a stylized fact from the recent microstructure research on exchange rates, namely that net forex order flow tends to generate large and relatively persistent exchange rate changes. We simply assume a price elastic forex supply curve to mimic this exchange rate reaction. But the forex order flow itself is tied to the endogenous portfolio flows which emerge under optimal dynamic investment in an incomplete market setting. The entire exchange rate dynamics is therefore based exclusively on the financial market structure as opposed to traditional macroeconomic determinants.

We highlight three dimensions in which this parsimonious approach is successful. First, the model can explain a large degree of exchange rate volatility if the elasticity of forex liquidity supply is sufficiently low. Second, we derive a negative correlation between foreign equity excess returns (in local currency) and the corresponding exchange rate returns. This correlation contradicts the (undocumented) common wisdom that strong equity markets are accompanied by currency appreciation. This correlation structure has not been highlighted in the previous exchange rate literature. Such a negative correlation decreases the risk of foreign investment in home currency terms as negative foreign equity returns tend to be compensated by positive exchange rate returns. This automatic hedge reduces the home bias and facilitates international equity risk sharing. We find very strong empirical support for the predicted return correlation at daily, monthly and quarterly horizons. Stock return differentials explain as much as 30% of the variance of the exchange rate at the quarterly frequency for some countries. Moreover, the negative correlation becomes more pronounced after 1990, presumably because of more developed and integrated international equity markets. The cross sectional evidence also points to the role of financial market development. Countries with a higher equity market capitalization relative to GDP tend to have a more negative return correlation. Third, we explore the correlation between exchange rate returns and net foreign equity inflows. The model predicts a positive correlation since net foreign equity flows are tied to forex order flows. The period after 1990 shows a highly significant positive correlation for the pooled data of 17 OECD countries.

Our analysis can be extended in various directions. We believe that the results are robust to positive correlation between home and foreign dividends. Dividends were so far assumed to be independent. But such internationally correlated equity market risk is *per se* devoid of risk trading benefits and should not alter the allocation problem for the remaining uncorrelated equity return risk. A more interesting extension would take account of asymmetric information or differences in opinion concerning the international equity returns between the home and foreign investors. This would introduce an additional and potentially important new trading motive alongside the risk sharing concerns of the present framework.

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Appendix A: Equilibrium under Financial Autarchy

Proposition 1:

We conjecture a linear price equilibrium of the form

$$\begin{aligned} P_t^h &= p_0 + p_F F_t^h \\ P_t^f &= p_0 + p_F F_t^f. \end{aligned}$$

Let $F_t^h = f_0 + f_D D_t^h$ denote the fundamental equity value in the home country with $f_D = 1/(\alpha_D + r)$ and $f_0 = (r^{-1} - f_D)\overline{D}$. The excess payoffs over the risk-less rate of home country equity follows as

$$dR_t^h = dP_t^h - rP_t^h dt + D_t^h dt = \boldsymbol{\alpha}_\Psi^h \boldsymbol{\Psi}_t^h dt + b_\Psi^h dw_t^h$$

with $\boldsymbol{\Psi}_t^h = (1, D_t^h)$ and coefficients $\boldsymbol{\alpha}_\Psi^h = (-rp_0, p_F)$ and $b_\Psi^h = p_F f_D \sigma_D$.

The optimal asset demand for investors follows of as

$$K_t^h = \frac{1}{\rho dt}\mathcal{E}_t\left(dR_t^h\right)/\rho\sigma_R^2 dt,$$

with $\sigma_R^2 dt = \mathcal{E}_t(dR_t^h)^2$. Market clearing requires $K_t^h = 1$ and implies for the price coefficients

$$\begin{aligned} p_0 &= -\frac{\rho\sigma_R^2}{r} \\ p_F &= 1. \end{aligned}$$

The same price parameters are obtain for the foreign stock market. The instantaneous volatility of the excess pay-off is given by

$$\sigma_R^2 = \mathcal{E}_t(b_\Psi^h)^2 = \frac{\sigma_D^2}{(\alpha_D + r)^2}.$$

Appendix B: Equilibrium under Complete Risk-Sharing

Proposition 2:

We conjecture a linear price system of the form

$$\begin{aligned} P_t^h &= p_0 + p_F F_t^h \\ P_t^f &= p_0 + p_F F_t^f \\ E_t &= 1. \end{aligned}$$

Let $F_t^h = f_0 + f_D D_t^h$ and $F_t^f = f_0 + f_D D_t^f$.denote again the fundamental values. The home country investor faces excess payoffs R_t^h and R_t^f for home and foreign equity, respectively. The foreign country investor (denoted by $*$) faces excess payoffs (in foreign currency) R_t^{f*} and R_t^{h*} for foreign and home country equity, respectively. Linear approximations allow us to write:

$$\begin{aligned} dR_t^h &= dP_t^h - rP_t^h dt + D_t^h dt \\ dR_t^f &\approx -dE_t\overline{P} + dP_t^f - dE_t dP_t^f - r[P_t^f - \overline{P}(E_t - 1)]dt + [D_t^f - \overline{D}(E_t - 1)]dt \\ dR_t^{f*} &= dP_t^f - rP_t^f dt + D_t^f dt \\ dR_t^{h*} &\approx dE_t\overline{P} + dP_t^h + dE_t dP_t^h - r\left[P_t^h + \overline{P}(E_t - 1)\right]dt + \left[D_t^h + \overline{D}(E_t - 1)\right]dt. \end{aligned}$$

The constant exchange rate ($dE_t = 0$) implies that payoffs in foreign currency terms are equal to home currency payoffs, hence $dR_t^{h*} = dR_t^h$, and $dR_t^{f*} = dR_t^f$. The excess payoffs take on the simple form

$$\begin{aligned} dR_t^j &= \boldsymbol{\alpha}_\Psi^j \boldsymbol{\Psi}_t^j dt + b_\Psi^j dw_t^j \\ dR_t^{j*} &= \boldsymbol{\alpha}_\Psi^{j*} \boldsymbol{\Psi}_t^j dt + b_\Psi^{j*} dw_t^j \end{aligned}$$

where $j = h, f$ denotes the country index, $\boldsymbol{\Psi}_t^j = (1, D_t^j)$ the state variable and $\boldsymbol{\alpha}_\Psi^j = \boldsymbol{\alpha}_\Psi^{j*} = (\alpha_0^j, \alpha_D^j)$, $b_\Psi^j = b_\Psi^{j*}$ coefficients.

Finally, we consider the correlation structure of the payoffs. Let Ωdt denote the covariance matrix of the excess payoffs (dR_t^h, dR_t^f) (in home currency terms) for the home investor and $\Omega^* dt$ the corresponding covariance matrix of the excess payoffs (dR_t^{f*}, dR_t^{h*}) (in foreign currency) for the foreign investor. Symmetry of the two country model implies

$$\Omega = \Omega^* = \begin{pmatrix} \Omega_{11} & \Omega_{21} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}, \qquad \Omega^{-1} = (\Omega^*)^{-1} = \frac{1}{\det \Omega} \begin{pmatrix} \Omega_{22} & -\Omega_{21} \\ -\Omega_{21} & \Omega_{11} \end{pmatrix}$$

with $\det \Omega = \Omega_{11}\Omega_{22} - \Omega_{21}\Omega_{21}$.

For the special case of complete markets with a constant exchange rate, we have $\mathcal{E}_t \left(dE_t dP_t^h \right) = 0$, $\Omega_{21} = 0$, and $\Omega_{11} = \Omega_{22} = \sigma_R^2$. Therefore,

$$\Omega^{-1} = \frac{1}{\sigma_R^4} \begin{pmatrix} \Omega_{11} & 0 \\ 0 & \Omega_{22} \end{pmatrix} = \frac{1}{\sigma_R^2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{\sigma_R^2} \mathbf{1}_{2\times 2}.$$

The first-order condition for the asset demands is given by

$$\begin{pmatrix} K_t^h & K_t^f \\ K_t^{f*} & K_t^{h*} \end{pmatrix} = \frac{1}{\rho dt} \mathcal{E}_t \begin{pmatrix} dR_t^h & dR_t^f \\ dR_t^{f*} & dR_t^{h*} \end{pmatrix} \Omega^{-1}. \tag{6}$$

Market clearing in the two stock markets ($K_t^h + K_t^{h*} = 1$, $K_t^f + K_t^{f*} = 1$) implies the price coefficients

$$\begin{aligned} p_0 &= -\frac{\rho \sigma_R^2}{2r} \\ p_F &= 1. \end{aligned}$$

For the optimal portfolio positions we obtain

$$\begin{pmatrix} K_t^h & K_t^f \\ K_t^{f*} & K_t^{h*} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}.$$

Appendix C: Equilibrium under Incomplete Markets

Proposition 3 (Exchange Rate Dynamics):

We assume that the exchange rate process is of the form $E_t = 1 + e_\Delta \Delta_t + e_\Lambda \Lambda_t$. If equity prices depend linearly on the exchange rate and dividends, then equity prices have a representation

$$\begin{aligned} P_t^h &= p_0 + p_F F_t^h + p_\Delta \Delta_t + p_\Lambda \Lambda_t \\ P_t^f &= p_0 + p_F F_t^f - p_\Delta \Delta_t - p_\Lambda \Lambda_t. \end{aligned}$$

Let $j = h, f$ denote the country index, $\mathbf{\Psi}_t^j = (1, D^j, \Delta_t, \Lambda_t)$ the state variable, $\mathbf{dw}_t^j = (dw_t^j, dw_t) = (dw_t^j, dw_t^h - dw_t^f)$ a (2×1) vector of innovations. For coefficients $\boldsymbol{\alpha}_\Psi^j = (\alpha_0^j, \alpha_D^j, \alpha_\Delta^j, \alpha_\Lambda^j)$, $\boldsymbol{\alpha}_\Psi^{j*} = (\alpha_0^{j*}, \alpha_D^{j*}, \alpha_\Delta^{j*}, \alpha_\Lambda^{j*})$, $\mathbf{b}_\Psi^j = (p_F f_D \sigma_D, b_\Psi^j)$, $\mathbf{b}_\Psi^{j*} = (p_F f_D \sigma_D, b_\Psi^{j*})$ we express excess payoffs as

$$\begin{aligned} dR_t^j &= \boldsymbol{\alpha}_\Psi^j \mathbf{\Psi}_t^j dt + \mathbf{b}_\Psi^j \mathbf{dw}_t^j \\ dR_t^{j*} &= \boldsymbol{\alpha}_\Psi^{j*} \mathbf{\Psi}_t^j dt + \mathbf{b}_\Psi^{f*} \mathbf{dw}_t^j \end{aligned}$$

and the first-order conditions imply for the optimal asset demand

$$\begin{pmatrix} K_t^h & K_t^f \\ K_t^{f*} & K_t^{h*} \end{pmatrix} = \frac{1}{\rho} \begin{pmatrix} \boldsymbol{\alpha}_\Psi^h \mathbf{\Psi}_t^h & \boldsymbol{\alpha}_\Psi^f \mathbf{\Psi}_t^f \\ \boldsymbol{\alpha}_\Psi^{f*} \mathbf{\Psi}_t^f & \boldsymbol{\alpha}_\Psi^{h*} \mathbf{\Psi}_t^h \end{pmatrix} \Omega^{-1}.$$

Market clearing ($K_t^h + K_t^{h*} = 1$, $K_t^{f*} + K_t^f = 1$) gives

$$\begin{aligned} K_t^{h*} - K_t^f &= \frac{1}{\rho} \left[m_\Delta \Delta_t + m_\Lambda \Lambda_t \right] \\ dK_t^{h*} - dK_t^f &= \frac{1}{\rho} \left[-\alpha_D m_\Delta \Delta_t dt + z m_\Lambda \Lambda_t dt \right] + \frac{1}{\rho} \left[m_\Delta \sigma_D + m_\Lambda \sigma_\Lambda \right] dw_t \end{aligned}$$

where we define coefficients

$$\begin{aligned} m_\alpha &= (\alpha_D + r)\overline{P} - \overline{D} \\ m_z &= (-z + r)\overline{P} - \overline{D} \\ m_\Delta &= 2p_\Delta(\alpha_D + r)(\Omega_{12}^{-1} - \Omega_{22}^{-1}) - 2m_\alpha e_\Delta \Omega_{22}^{-1} \\ m_\Lambda &= 2p_\Lambda(-z + r)(\Omega_{12}^{-1} - \Omega_{22}^{-1}) - 2m_z e_\Lambda \Omega_{22}^{-1}. \end{aligned}$$

Finally, we substitute

$$\Lambda_t = \frac{1}{e_\Lambda}(E_t - \overline{E}) - \frac{e_\Delta}{e_\Lambda} \Delta_t$$

and find that the term $(K_t^{h*} - K_t^f)\overline{D}dt + (dK_t^f - dK_t^{h*})\overline{P}$ in the forex order flow constraint (4) is linear in $E_t - \overline{E}$, Δ_t and dw_t. Hence there exists a representation

$$dE_t = k_1 \Delta_t + k_2(E_t - \overline{E}) + k_3 dw_t.$$

Proposition 4 (Existence and Uniqueness of the Incomplete Market Equilibrium):

Market clearing in the two stock markets ($K_t^h + K_t^{h*} = 1$, $K_t^f + K_t^{f*} = 1$) implies 4 parameter constraints (one for each element in $\mathbf{\Psi}_t^j = (1, D^j, \Delta_t, \Lambda_t)$) given by

$$p_0 = \frac{-\rho \det \Omega - \mathcal{E}_t(dE_t dP_t^f)(-\Omega_{12} + \Omega_{11})}{r(\Omega_{11} - 2\Omega_{12} + \Omega_{22})} \tag{7}$$

$$p_F = 1 \tag{8}$$

$$p_\Delta = -e_\Delta \frac{m_\alpha(\Omega_{21} + \Omega_{11})}{(\alpha_D + r)\overline{\Omega}} \tag{9}$$

$$p_\Lambda = -e_\Lambda \frac{m_z(\Omega_{21} + \Omega_{11})}{(-z + r)\overline{\Omega}} \tag{10}$$

where we define $\overline{\Omega} = \Omega_{11} + 2\Omega_{21} + \Omega_{22}$. The forex order flow constraint (4) implies an additional 3 constraints (for $\Delta_t, \Lambda_t, dw_t$) given by

$$e_\Delta \left(\overline{KD} - \kappa\alpha_D\right) + m_\Delta \frac{1}{\rho}\left(\overline{D} + \alpha_D \overline{P}\right) = -\overline{K} \tag{11}$$

$$e_\Lambda \left(\overline{KD} + \kappa z\right) + m_\Lambda \frac{1}{\rho}\left(\overline{D} - z\overline{P}\right) = 0 \tag{12}$$

$$e_\Delta \kappa\sigma_D + e_\Lambda \kappa - m_\Delta \frac{1}{\rho}\overline{P}\sigma_D - m_\Lambda \frac{1}{\rho}\overline{P} = 0. \tag{13}$$

These 7 equations determine the 7 parameters $p_0, p_F, p_\Delta, p_\Lambda, e_\Delta, e_\Lambda, z$ as a function of $\overline{P}$, $\overline{\Lambda}$ and $\overline{K}$ as well as the parameters of the dividend process $(\alpha_D, \overline{D}, \sigma_D)$, the elasticity of the forex liquidity supply, κ, and the investor risk aversion ρ. Moreover, for steady state levels $\overline{P} > 0$, $\overline{\Lambda}$ and $0 < \overline{K} < 1$ can be approximated by

$$\begin{aligned}
\overline{P} &= p_0 + \frac{\overline{D}}{r} + p_\Lambda \overline{\Lambda} = p_0 + \frac{\overline{D}}{r} \\
\overline{K} &= \frac{\rho\left[\Omega_{11} - \Omega_{21}\right] - \mathcal{E}_t(dE_t dP_t^f)}{\rho\left(\Omega_{11} - 2\Omega_{21} + \Omega_{22}\right)} \\
\overline{\Lambda} &= 0.
\end{aligned}$$

The covariances are given by

$$\begin{aligned}
\Omega_{11} &= (f_D\sigma_D)^2 + 2[p_\Delta\sigma_D + p_\Lambda]^2 + 2f_D\sigma_D[p_\Delta\sigma_D + p_\Lambda] \\
\Omega_{12} &= -2(p_\Delta\sigma_D + p_\Lambda)^2 - \left[2(p_\Delta\sigma_D + p_\Lambda) + f_D\sigma_D\right]\overline{P}\left(e_\Delta\sigma_D + e_\Lambda\right) - 2(p_\Delta\sigma_D + p_\Lambda)f_D\sigma_D \\
\Omega_{22} &= (f_D\sigma_D)^2 + 2[\overline{P}\left(e_\Delta\sigma_D + e_\Lambda\right) + p_\Delta\sigma_D + p_\Lambda]^2 + 2f_D\sigma_D[\overline{P}\left(e_\Delta\sigma_D + e_\Lambda\right) + p_\Delta\sigma_D + p_\Lambda]
\end{aligned}$$

and furthermore

$$\overline{\Omega} = 2\left(f_D\sigma_D\right)^2 + 2[\overline{P}\left(e_\Delta\sigma_D + e_\Lambda\right)]^2, \tag{14}$$

where $\overline{\Omega}(z) = \Omega_{11} + 2\Omega_{21} + \Omega_{22} > 0$ represents the instantaneous excess pay-off variance of the total market portfolio of all domestic and foreign equity.

We proceed in two steps to show the existence and uniqueness of a equilibrium. First, we determine the existence and uniqueness of the characteristic root z. Second, we show that the other parameters $p_0, p_\Delta, p_\Lambda, e_\Delta, e_\Lambda$ exist and are uniquely determined.

1) Existence and Uniqueness of Parameter z

Combining equations (12), (10) and the definition of $\overline{\Omega}(z)$ we obtain an expression which characterizes the root z of the system as

$$\frac{\rho}{2}\left(\overline{KD} + \kappa z\right)\overline{\Omega} = f(z) \tag{15}$$

for a function $f(z) = [(-z + r)\overline{P} - \overline{D}]\left(\overline{D} - z\overline{P}\right)$.

The function $f(z)$ represents a convex parabola. It has two intersects with the x-axes at $z_1 = -\overline{D}/\overline{P} + r \leq 0$ and $z_2 = \overline{D}/\overline{P} \geq 0$. Since $\frac{\rho}{2}\left(\overline{KD} + \kappa z\right)\overline{\Omega}$ is upward sloping ($d\overline{\Omega}/dz > 0$), and positive for $z = 0$, it intersects the parabola at least once and at most twice. The first intersection is negative and the second one, if it exists, is positive (and therefore discarded for stability reasons). Hence there exists a unique value $z < 0$ such that the solution is stable. We now have to prove that for such a solution z, the parameters $p_0, p_\Delta, p_\Lambda, e_\Delta, e_\Lambda$ exist and are uniquely defined. To do that we derive some intermediate results on e_Δ and p_Δ.

Assume that the forex supply is sufficiently price elastic with $\kappa > \overline{\kappa} = \overline{KDP}/(\overline{D} - r\overline{P}) = \overline{KDP}/(-rp_0)$. Then $\frac{\rho}{2}\left(\overline{KD} + \kappa z\right)\overline{\Omega}(z)$ intersects the x-axis to the right of $z_1 = -\overline{D}/\overline{P} + r$ and the root z is confined to the interval $z \in [-\overline{D}/\overline{P} + r, -\overline{KD}/\kappa]$. This implies $(-z + r)\overline{P} - \overline{D} < 0$. Moreover, we require that $-\alpha_D < -\overline{D}/\overline{P} + r$ or $(\alpha_D + r)\overline{P} - \overline{D} > 0$. The latter condition can be rewritten as $\alpha_D \overline{P} > -rp_0$, where p_0 represents the risk discount on the asset price. We can make p_0 sufficiently close to zero by setting an upper threshold value for the investor risk aversion, hence requiring $\overline{\rho}_1 > \rho$. It is then easy to show that for any $\rho < \overline{\rho}_1$ and $\kappa > \overline{\kappa}$, we have $e_\Delta < 0$ and $p_\Delta > 0$. It also follows using equation (11) that $m_\Delta < 0$ for a large $\kappa > \overline{\kappa}$.

2) Existence and Uniqueness of the Equilibrium

Equation (15) can be rewritten as:

$$\overline{\Omega} = 2\left(f_D \sigma_D\right)^2 + 2[\overline{P}\left(e_\Delta \sigma_D + e_\Lambda\right)]^2 = \frac{[(-z+r)\overline{P} - \overline{D}]\left(\overline{D} - z\overline{P}\right)}{\frac{\rho}{2}\left(\overline{KD} + \kappa z\right)} > 0$$

A necessary condition for the existence of a real solution for $\overline{e} \equiv e_\Delta \sigma_D + e_\Lambda$ is

$$V(\rho, \kappa) = \frac{[(-z+r)\overline{P} - \overline{D}]\left(\overline{D} - z\overline{P}\right)}{\rho\left(\overline{KD} + \kappa z\right)} - \left(f_D \sigma_D\right)^2 \geq 0.$$

This condition is satisfied only if $\rho\left(f_D \sigma_D\right)^2$ is sufficiently small or risk aversion is below a certain threshold $\rho < \overline{\rho}_2$. We now take $\rho < \min(\overline{\rho}_1, \overline{\rho}_2) = \overline{\rho}$. It is possible to show that $\overline{e} \equiv e_\Delta \sigma_D + e_\Lambda < 0$ (see Appendix D, corollary 2). Therefore we can rewrite equation (15) in linear form as

$$e_\Delta \sigma_D + e_\Lambda = -\frac{1}{\overline{P}}\sqrt{V(\rho, \kappa)}. \tag{16}$$

We define a vector $\mathbf{e} = (e_\Delta, e_\Lambda, m_\Delta, m_\Lambda)$ and matrices

$$\mathbf{A} = \begin{pmatrix} \sigma_D & 1 & 0 & 0 \\ \left(\overline{KD} - \kappa\alpha_D\right) & 0 & \frac{1}{\rho}\left(\overline{D} + \alpha_D \overline{P}\right) & 0 \\ 0 & \left(\overline{KD} + \kappa z\right) & 0 & \frac{1}{\rho}\left(\overline{D} - z\overline{P}\right) \\ \kappa\sigma_D & \kappa & -\frac{1}{\rho}\overline{P}\sigma_D & -\frac{1}{\rho}\overline{P} \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} -\frac{1}{\overline{P}}\sqrt{V(\rho,\kappa)} \\ -\overline{K} \\ 0 \\ 0 \end{pmatrix}$$

so that the linear system $\mathbf{Ae} = \mathbf{b}$ summarizes the 4 equations (11), (12), (13) and (16). We check that $\det(\mathbf{A}) \neq 0$ in general. Hence there exists a unique solution for $\mathbf{e}$ given by $\mathbf{e} = \mathbf{A}^{-1}\mathbf{b}$.

We show that this implies also a unique solution for the price coefficients $\mathbf{p} = (p_\Delta, p_\Lambda)$. Note that

$$\Omega_{11} + \Omega_{12} = \left(f_D \sigma_D\right)^2 - [2(p_\Delta \sigma_D + p_\Lambda \sigma_\Lambda) + f_D \sigma_D]\,\overline{P}\left(e_\Delta \sigma_D + e_\Lambda\right)$$

is linear in $\mathbf{p}$ for a fixed vector $\mathbf{e}$. The equations (9) and (10) are therefore of the form $\mathbf{Cp} = \mathbf{d}$, where we define

$$\mathbf{C} = 2\overline{P}\left(e_\Delta \sigma_D + e_\Lambda\right)\begin{pmatrix} c_\Delta \sigma_D & c_\Delta \\ c_\Lambda \sigma_D & c_\Lambda \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \mathbf{d} = d_0 \begin{pmatrix} c_\Delta \\ c_\Lambda \end{pmatrix}$$

with $d_0 = \left(f_D \sigma_D\right)^2 - f_D \sigma_D \overline{P}\left(e_\Delta \sigma_D + e_\Lambda\right)$, and additional parameters

$$c_\Delta = \frac{e_\Delta[(\alpha_D + r)\overline{P} - \overline{D}]}{(\alpha_D + r)\overline{\Omega}}, \qquad c_\Lambda = \frac{e_\Lambda[(-z + r)\overline{P} - \overline{D}]}{(-z + r)\overline{\Omega}}.$$

We check that $\det(\mathbf{C}) \neq 0$ in general. We can therefore invert $\mathbf{C}$ and obtain a unique solution for $\mathbf{p}$. Finally, the coefficient $p_0 < 0$ is uniquely determined by equation (7). Hence the uniqueness of the incomplete market equilibrium for all $\rho < \overline{\rho}$ and $\kappa > \overline{\kappa}$.

Proposition 5:

For a completely price elastic currency supply ($\kappa \to \infty$), the exchange rate is constant at $\overline{E} = 1$. This requires that $e_\Delta = e_\Lambda = 0$. It follows that $c_\Delta = c_\Lambda = 0$ and $\mathbf{Cp} = \mathbf{d}$ implies $p_\Delta = p_\Lambda = 0$. The latter implies $m_\Delta = m_\Lambda = 0$. Moreover, since $\mathcal{E}_t(dE_t dP_t^f) = 0$, and $\Omega_{21} = 0$, we obtain steady state equity holdings

$$\overline{K} = \frac{[\Omega_{11}]}{(\Omega_{11} + \Omega_{22})} = \frac{1}{2},$$

which correspond to full equity risk sharing as in proposition 2.

Appendix D:

Corollary 1:

Symmetry of the model implies $\mathcal{E}_t(dE_t dR_t^h) = -\mathcal{E}_t(dE_t dR_t^{f*})$. Furthermore,

$$\mathcal{E}_t(dE_t dR_t^h)dt = (e_\Delta \sigma_D + e_\Lambda)\left[f_D \sigma_D + 2\left(p_\Delta \sigma_D + p_\Lambda\right)\right] < 0$$

amounts to showing that $\overline{e} \equiv e_\Delta \sigma_D + e_\Lambda < 0$ and $f_D \sigma_D + 2\left(p_\Delta \sigma_D + p_\Lambda\right) > 0$. We note that the latter follows for some $\kappa > \overline{\kappa}$ since $p_\Delta \sigma_D + p_\Lambda \to 0$ for $\kappa \to \infty$ (see proposition 5) and $f_D \sigma_D > 0$. To simplify notations we define

$$m = \frac{(\overline{KD} - \alpha_D \kappa)\overline{P}}{(\overline{D} + \alpha_D \overline{P})}, \qquad n = \frac{(\overline{KD} + z\kappa)\overline{P}}{(\overline{D} - z\overline{P})}.$$

Clearly, $m < 0$ and $n < 0$ under the parameter constraints of proposition 4. Moreover, $m - n < 0$, because (for $\alpha_D > -z$) we find

$$(\overline{D} - z\overline{P})(\overline{KD} - \alpha_D \kappa) - (\overline{D} + \alpha_D \overline{P})(\overline{KD} + z\kappa) = -(\alpha_D + z)\left[\overline{D}\kappa + \overline{PKD}\right] < 0.$$

Substituting equations (11) and (12) into (13) implies

$$e_\Delta \sigma_D \left[\kappa + m\right] + e_\Lambda \left[\kappa + n\right] = \frac{-\overline{KP}\sigma_D}{(\overline{D} + \alpha_D \overline{P})} < 0.$$

Subtracting the term $e_\Delta \sigma_D (m - n) > 0$ (because $e_\Delta < 0$) from the left hand side implies $e_\Delta \sigma_D \left[\kappa + n\right] + e_\Lambda \left[\kappa + n\right] < 0$. Therefore

$$e_\Delta \sigma_D + e_\Lambda < 0,$$

since $\kappa + n > 0$ is trivially fulfilled (for $\kappa > \overline{\kappa}, \overline{K} > 0, \overline{D} > 0, \overline{P} > 0$).

Appendix E: Correlation Structure

Corollary 2:

The home and foreign excess pay-off processes (in local currency) and the exchange rate returns are

$$\begin{aligned}
dR_t^h &= \alpha_0^h dt + \alpha_D^h D_t^h dt + \alpha_\Delta^h \Delta_t dt + \alpha_\Lambda^h \Lambda_t dt + p_F f_D \sigma_D dw_t^h + (p_\Delta \sigma_D + p_\Lambda) dw_t \\
dR_t^{f*} &= \alpha_0^{f*} dt + \alpha_D^{f*} D_t^f dt + \alpha_\Delta^{f*} \Delta_t dt + \alpha_\Lambda^{f*} \Lambda_t dt + p_F f_D \sigma_D dw_t^f - (p_\Delta \sigma_D + p_\Lambda) dw_t \\
dE_t &= -e_\Delta \alpha_D \Delta_t dt - e_\Lambda \alpha_D \Lambda_t dt + (e_\Delta \sigma_D + e_\Lambda) dw_t
\end{aligned}$$

and the relative payoffs follows as

$$dR_t^{f*} - dR_t^h = 2p_\Delta(\alpha_D + r)\Delta_t dt + 2p_\Lambda(-z + r)\Lambda_t dt - [f_D\sigma_D + 2p_\Delta\sigma_D + 2p_\Lambda]\, dw_t.$$

Hence, we obtain a perfect negative return correlation,

$$-Corr\left[dE_t, (dR_t^{f*} - dR_t^h)/\overline{P}\right] = -1 < 0.$$

Corollary 3:

The net foreign equity inflows are given by

$$dK_t^f - dK_t^{h*} = \frac{1}{\rho}\left[\alpha_D m_\Delta \Delta_t dt - z m_\Lambda \Lambda_t dt\right] - \frac{1}{\rho}\left[m_\Delta\sigma_D + m_\Lambda\sigma_\Lambda\right] dw_t.$$

$$\mathcal{E}_t(dE_t, dK_t^f - dK_t^{h*}) = -\frac{2\kappa}{\overline{P}}(e_\Delta\sigma_D + e_\Lambda)^2 dt$$

Hence

$$-Corr\left[dE_t, (dK_t^f - dK_t^{h*})\right] = 1 > 0.$$

Appendix F: Binding Short Sale Constraint

Define $\mathbf{x}_t = (x^h, x^f, x^b)$ as the (1×3) vector of holdings in home equity, foreign equity, and foreign bonds, respectively. Denote by $\mathbf{dR}_t = (dR^h, dR^f, dR^b)$ the corresponding (1×3) excess pay-off vector. $\mathcal{E}(dR_t^h)$, $\mathcal{E}(dR_t^f)$ are given in Appendix C and $\mathcal{E}(dR_t^b) \approx -\overline{P}dE_t$. We call $\mathbf{\Sigma} = \mathcal{E}(\mathbf{dR}_t^T \mathbf{dR}_t)$ the (3×3) covariance matrix of the excess payoffs. We show that the unconstrained maximization produces an interior solution with $x^b < 0$. This implies that the short sale constraint on foreign riskless bonds is always binding in equilibrium.

$$\mathbf{\Sigma} = \mathcal{E}(\mathbf{dR}_t^T \mathbf{dR}_t) = \begin{pmatrix} \sigma_{pp} & -\overline{P}\sigma_{pe} + \sigma_{fh} & -\overline{P}\sigma_{pe} \\ -\overline{P}\sigma_{pe} + \sigma_{fh} & \overline{P}^2\sigma_{ee} + 2\overline{P}\sigma_{pe} + \sigma_{pp} & \overline{P}^2\sigma_{ee} + \overline{P}\sigma_{pe} \\ -\overline{P}\sigma_{pe} & \overline{P}^2\sigma_{ee} + \overline{P}\sigma_{pe} & \overline{P}^2\sigma_{ee} \end{pmatrix} dt,$$

where we define $\mathcal{E}(dR_t^h dR_t^h) = \sigma_{pp}dt$, $\mathcal{E}(dE_t dR_t^h) = \sigma_{pe}dt$, $\mathcal{E}(dR_t^f dR_t^h) = \sigma_{fh}dt$ and $\mathcal{E}(dE_t dE_t) = \sigma_{ee}dt$. Inverting the symmetric covariance matrix allows us to compute the optimal unconstrained portfolio holdings $\mathbf{x}_t = (x^h, x^f, x^b)$.

In particular for symmetric steady state holdings $\mathbf{x}_t = (1 - \overline{K}, \overline{K}, \overline{x}^b)$ we obtain

$$\begin{aligned} \rho \det \mathbf{\Sigma} &= [-\overline{P}^2\sigma_{ee}\sigma_{fh} - 2\overline{P}^2\sigma_{pe}^2 + \overline{P}^2\sigma_{ee}\sigma_{pp}](\overline{D} - r\overline{P})dt + [\overline{P}^2\sigma_{ee}\sigma_{pp} - \overline{P}^2\sigma_{pe}^2]\overline{P}\sigma_{pe}dt \\ \rho\overline{x}^b \det \mathbf{\Sigma} &= [\overline{P}^2\sigma_{fh}\sigma_{ee} + 2\overline{P}^2\sigma_{pe}^2 - \overline{P}^2\sigma_{pp}\sigma_{ee}](\overline{D} - r\overline{P})dt \\ &\quad + [-\overline{P}^2\sigma_{pp}\sigma_{ee} - \overline{P}\sigma_{pe}\sigma_{pp} + \overline{P}^2\sigma_{pe}^2 - \overline{P}\sigma_{fh}\sigma_{pe}]\overline{P}\sigma_{pe}dt. \end{aligned}$$

and taking the sum $(\sigma_{pp} + \sigma_{fh} > 0)$ implies :

$$\rho(\overline{K} + \overline{x}^b) \det \mathbf{\Sigma} = -(\sigma_{pp} + \sigma_{fh})\overline{P}^2\sigma_{pe}^2 dt < 0.$$

Since $\mathbf{\Sigma}$ is positive semi-definite, $\rho\overline{K} \det \mathbf{\Sigma} \geq 0$. It follows that $\overline{x}^b < 0$. Hence, the constraint $\overline{x}^b \geq 0$ is in fact binding and investors hold zero foreign bonds in the steady state.

Table 1:
Volatility Ratios of Exchange Rate and Stock Market Index Returns

Reported are volatility ratios of daily (log) exchange rate returns and daily (log) foreign stock market index returns (in local currency) for various sample periods. The exchange rate E_t is expressed in foreign currency per dollar ($dE_t > 0$ corresponds to a dollar appreciation), and the index f represents one of the 17 foreign OECD countries. The last row provides the result for the pooled data.

	$\sqrt{\frac{Var(dE_t)}{Var(dR_t^{f*}/\overline{P})}}$		
	1/1/80-12/31/01	1/1/90-12/31/01	1/1/95-12/31/01
	(a)	(b)	(c)
Australia	0.5850	0.6494	0.7070
Austria	0.8205	0.6272	0.6270
Belgium-Luxembourg	0.8386	0.8053	0.6770
Denmark	0.6951	0.6351	0.5540
Finland	0.3690	0.3388	0.2472
France	0.6081	0.5450	0.4785
Germany	0.6211	0.5537	0.4721
Ireland	0.5968	0.5921	0.5361
Italy	0.4901	0.4688	0.4198
Japan	0.6279	0.5444	0.5855
Netherlands	0.6555	0.6553	0.5196
Norway	0.4517	0.4937	0.5023
Portugal	0.6423	0.6530	0.5980
Spain	0.5920	0.5156	0.4478
Sweden	0.4664	0.4424	0.3766
Switzerland	0.8450	0.7241	0.6441
U.K.	0.6599	0.6037	0.4747
Mean	0.6215	0.5793	0.5216
Std. Dev.	0.1328	0.1111	0.1159
Pooled Data	0.4780	0.4754	0.4116

Table 2:
Daily Correlations of Exchange Rate and Foreign Stock Market Excess Returns

Reported are correlations of daily (log) exchange rate returns, $-dE_t$, and the daily (log) foreign stock market index returns (in local currency) relative to the U.S. market index return (in dollars), $(dR_t^{f*} - dR_t^h)/\overline{P}$, for various sample periods. The exchange rate E_t is expressed in foreign currency per dollar ($dE_t > 0$ corresponds to a dollar appreciation). The index f represents one of 17 OECD countries and h the U.S. market. The model predicts $-Corr(dE_t, (dR_t^{f*} - dR_t^h)/\overline{P}) < 0$. We test if the correlation is significantly different from zero using robust standard errors and denote by *,** and *** significance at a 10, 5 and 1 percent level, respectively. The last row provides the result for the pooled data.

	$-Corr\left[dE_t, (dR_t^{f*} - dR_t^h)/\overline{P}\right]$		
	1/1/80-12/31/01	1/1/90-12/31/01	1/1/95-12/31/01
	(a)	(b)	(c)
Australia	0.0558^{***}	0.0304^{*}	0.0242
Austria	-0.0186	-0.0291	-0.0201
Belgium-Luxembourg	-0.0438^{***}	-0.0388^{**}	-0.0226
Denmark	-0.0368^{***}	-0.0495^{***}	-0.0452^{*}
Finland	-0.0954^{***}	-0.1263^{***}	-0.1847^{***}
France	-0.1026^{***}	-0.1638^{***}	-0.1760^{***}
Germany	-0.0805^{***}	-0.1021^{***}	-0.1448^{***}
Ireland	-0.1003^{***}	-0.0883^{***}	-0.0739^{***}
Italy	-0.0385^{***}	-0.0353^{**}	-0.0539^{**}
Japan	0.0636^{***}	0.0723^{***}	0.0587^{**}
Netherlands	-0.1674^{***}	-0.2194^{***}	-0.2052^{***}
Norway	-0.0629^{***}	-0.0956^{***}	-0.0128
Portugal	-0.0253	-0.0339^{*}	-0.0140
Spain	-0.0645^{***}	-0.1301^{***}	-0.1116^{***}
Sweden	-0.0677^{***}	-0.0510^{***}	-0.0163
Switzerland	-0.1240^{***}	-0.1632^{***}	-0.1655^{***}
U.K.	-0.0173	-0.1024^{***}	-0.1042^{***}
Mean	-0.0545	-0.0780	-0.0746
Std. Dev.	0.0586	0.0728	0.0792
Pooled Data	-0.0530^{***}	-0.0761^{***}	-0.0735^{***}

Table 3:
Monthly Correlations of Exchange Rate and Foreign Stock Market Excess Returns

Reported are correlations of monthly (log) exchange rate returns, $-dE_t$, and the daily (log) foreign stock market index returns (in local currency) relative to the U.S. market index return (in dollars), $(dR_t^{f*} - dR_t^h)/\overline{P}$, for various sample periods. The exchange rate E_t is expressed in foreign currency per dollar ($dE_t > 0$ corresponds to a dollar appreciation). The index f represents one of 17 foreign OECD countries and h the U.S. market. The model predicts $-Corr[dE_t, (dR_t^{f*} - dR_t^h)/\overline{P}] < 0$. We test if the correlation is significantly different from zero using robust standard errors and denote by *,** and *** significance at a 10, 5 and 1 percent level, respectively. The last row provides the result for the pooled data.

	$-Corr\left[dE_t, (dR_t^{f*} - dR_t^h)/\overline{P}\right]$		
	1/80-12/01	1/90-12/01	1/95-12/01
	(a)	(b)	(c)
Australia	0.1796^{***}	0.0102	-0.1415
Austria	-0.1020	-0.1998^{**}	-0.1507
Belgium-Luxembourg	-0.2508^{***}	-0.2569^{***}	-0.1352
Denmark	-0.2179^{***}	-0.2934^{***}	-0.3358^{***}
Finland	-0.1580^{**}	-0.2570^{***}	-0.1794^{**}
France	-0.1230^{**}	-0.3473^{***}	-0.3118^{***}
Germany	-0.1409^{**}	-0.2871^{***}	-0.3679^{***}
Ireland	-0.2710^{***}	-0.2805^{***}	-0.2996^{***}
Italy	-0.1308^{**}	-0.1312	-0.1755^{**}
Japan	0.6590	-0.0276	-0.2810^{***}
Netherlands	-0.3403^{***}	-0.3689^{***}	-0.3059^{***}
Norway	-0.0936	-0.1787^{**}	-0.0264
Portugal	-0.0763	-0.1341^{*}	-0.0669
Spain	-0.1250^{**}	-0.2183^{***}	-0.2090^{**}
Sweden	-0.2287^{***}	-0.2862^{***}	-0.0930
Switzerland	-0.1761^{***}	-0.2318^{***}	-0.1376
U.K.	-0.1187^{*}	-0.2778^{***}	-0.2530^{***}
Mean	-0.1009	-0.2169	-0.2041
Std. Dev.	0.2248	0.1059	0.1012
Pooled Data	-0.1232^{***}	-0.2119^{***}	-0.1901^{***}

Table 4:
Regressions of Quarterly Exchange Rate on Foreign Stock Market Excess Returns

Reported are regressions of quarterly (log) exchange rate returns on the quarterly (log) foreign stock market excess return (in local currency) relative to the U.S. stock market return (in dollars) for the period 1990-2002:

$$-dE_t = \alpha + \beta \times (dR_t^{f*} - dR_t^h)/\overline{P} + \epsilon_t.$$

The exchange rate E_t is expressed in foreign currency per dollar ($dE_t > 0$ corresponds to a dollar appreciation). The index f represents one of 17 foreign OECD countries and h the U.S. market. The model predicts $\beta < 0$. We denote by *,** and *** significance at a 10, 5 and 1 percent level, respectively. Newey West adjusted standard errors are reported in parentheses. The last row provides the result for the pooled data.

	Coefficients (1/90-12/01)				
	α		β		Adj R^2
Australia	−0.0092	(0.0064)	−0.1124	(0.1336)	0.0215
Austria	−0.0113	(0.0078)	-0.2046^{**}	(0.0825)	0.1535
Belgium-Luxembourg	−0.0078	(0.0073)	-0.2613^{*}	(0.1430)	0.0659
Denmark	−0.0065	(0.0067)	-0.3266^{***}	(0.0780)	0.2294
Finland	−0.0089	(0.0087)	−0.0734	(0.0451)	0.0340
France	−0.0068	(0.0065)	-0.3999^{***}	(0.0783)	0.2447
Germany	−0.0080	(0.0068)	-0.3385^{***}	(0.0656)	0.2467
Ireland	−0.0090	(0.0069)	-0.3372^{***}	(0.0604)	0.2646
Italy	−0.0126	(0.0081)	−0.1423	(0.0972)	0.0462
Japan	−0.0004	(0.0119)	−0.0530	(0.1483)	0.0151
Netherlands	−0.0052	(0.0066)	-0.5273^{***}	(0.1121)	0.2677
Norway	−0.0091	(0.0070)	-0.1646^{***}	(0.0562)	0.0998
Portugal	−0.0116	(0.0069)	-0.1831^{**}	(0.0740)	0.1226
Spain	−0.0113	(0.0063)	-0.2847^{***}	(0.0659)	0.3029
Sweden	−0.0095	(0.0082)	-0.2698^{***}	(0.0969)	0.2809
Switzerland	−0.0009	(0.0083)	-0.3368^{**}	(0.1377)	0.1305
U.K.	−0.0045	(0.0073)	-0.2738^{**}	(0.1348)	0.0587
Mean	−0.0083	(0.0075)	−0.2523	(0.0947)	0.1477
Std. Dev.	0.0030	(0.0014)	0.1237	(0.0339)	0.1096
Pooled Data	−0.0081	(0.0057)	-0.2083^{***}	(0.0426)	0.1261

Table 5:
Correlation of Exchange Rate Returns and Net Foreign Equity Inflows

Reported are correlations of the exchange rate return, $-dE_t$, and net foreign stock ownership increase by U.S. residents, $dK_t^f - dK_t^{h*}$, for various sample periods. Net foreign stock ownership increase (or net foreign inflow) is defined as net U.S. purchases of foreign equities minus net foreign purchases of U.S. equities, and normalized as a proportion of the average absolute level of net foreign stock ownership increase by U.S. residents over the previous twelve months. The theory predicts $-Corr(dE_t, dK_t^f - dK_t^{h*}) > 0$. We test if the correlation is significantly different from zero using robust standard errors and denote by *,** and *** significance at a 10, 5 and 1 percent level, respectively. The last row provides the result for the pooled data.

	$-Corr(dE_t, dK_t^f - dK_t^{h*})$		
	1/80-12/01	1/90-12/01	1/95-12/01
	(a)	(b)	(c)
Australia	0.0112	-0.0478	-0.0010
Austria	-0.1155^{***}	0.2051^{***}	0.2740^{***}
Belgium-Luxembourg	0.1208	0.2541^{***}	0.3846^{***}
Denmark	-0.0938^{**}	-0.0174	-0.0295
Finland	-0.0002	-0.0194	0.0473
France	0.1400^{**}	0.1539^{**}	0.1814^{**}
Germany	-0.0872	-0.0412	0.1210
Ireland	0.0445	0.1461	0.0769
Italy	-0.0071	0.0824	0.1936^{**}
Japan	0.0382	0.0292	-0.0620
Netherlands	-0.0745	-0.0265	-0.0279
Norway	-0.0162	0.0033	-0.0125
Portugal	0.1844^{***}	0.1971^{***}	0.1582^{***}
Spain	0.0586	0.1521^{***}	0.1939^{***}
Sweden	0.0235	0.0701	0.3620^{***}
Switzerland	0.1061^{*}	0.1608^{*}	0.3052^{***}
U.K.	0.0775	-0.0197	0.0716
Mean	0.0274	0.0754	0.1316
Std. Dev.	0.0824	0.1004	0.1413
Pooled Data	-0.0026	0.0665^{***}	0.1145^{***}

Table 6

Correlation Structure and Stock Market Development

Reported are the panel regressions of the quarterly realized correlations ($QRCorr_{it}$) between foreign stock market excess returns and exchange rate returns on two alternative measures of stock market development and fixed time effects D_t for each quarter of the sample period 1990-2002:

$$\begin{array}{lll} \text{I:} & QRCorr_{it} = \alpha + \beta \times \log(MCap_{it}/GDP_{it}) & +\gamma D_t + \epsilon_{it} \\ \text{II:} & QRCorr_{it} = \alpha + \beta \times \log(TVol_{it}/GDP_{it}) & +\gamma D_t + \epsilon_{it} \end{array}$$

Quarterly realized correlations are calculated based on daily equity market excess returns for 17 OECD countries ($i = 1, 2, ...17$) relative to the U.S. equity market return and daily exchange rate returns of the respective dollar exchange rate. Market development is alternatively measured by the ratio of quarterly capital market capitalization ($MCap_{it}$) to GDP or the ratio of quarterly cross border equity trading volume with the U.S. ($TVol_{it}$) to GDP. We report in parenthesis robust standard errors (Newey-West) and allow for first order serial autocorrelation of the error. Fixed effects are not reported.

	Coefficients ($n = 724$)				
Specification	α		β		Adj R^2
I:	-0.0080	(0.0384)	-0.0715^{***}	(0.0231)	0.292
II:	0.1046^{***}	(0.0403)	-0.0199^{***}	(0.0051)	0.295

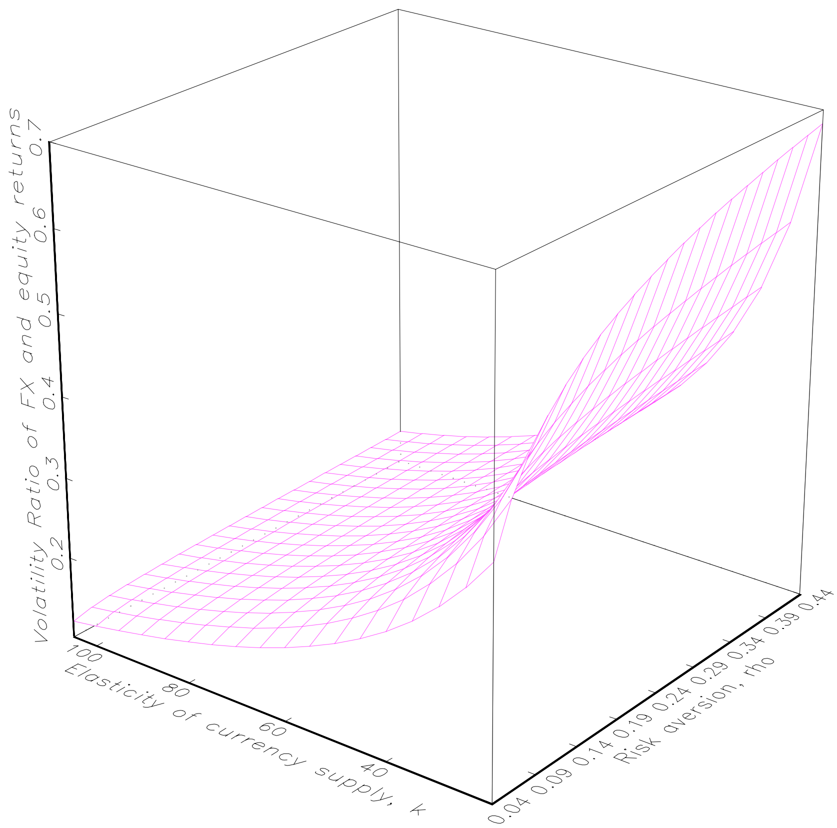

Figure 1: The volatility ratio of forex returns and equity returns is plotted for investor risk aversion parameters 0.04 to 0.44 and an elasticity of forex liquidity supply ranging from 20 to 100. The riskless rate is $r = 0.05$ and the parameters of the dividend process are $\overline{D} = 1$, $\alpha_D = .25$ and $\sigma_D = 0.1$.

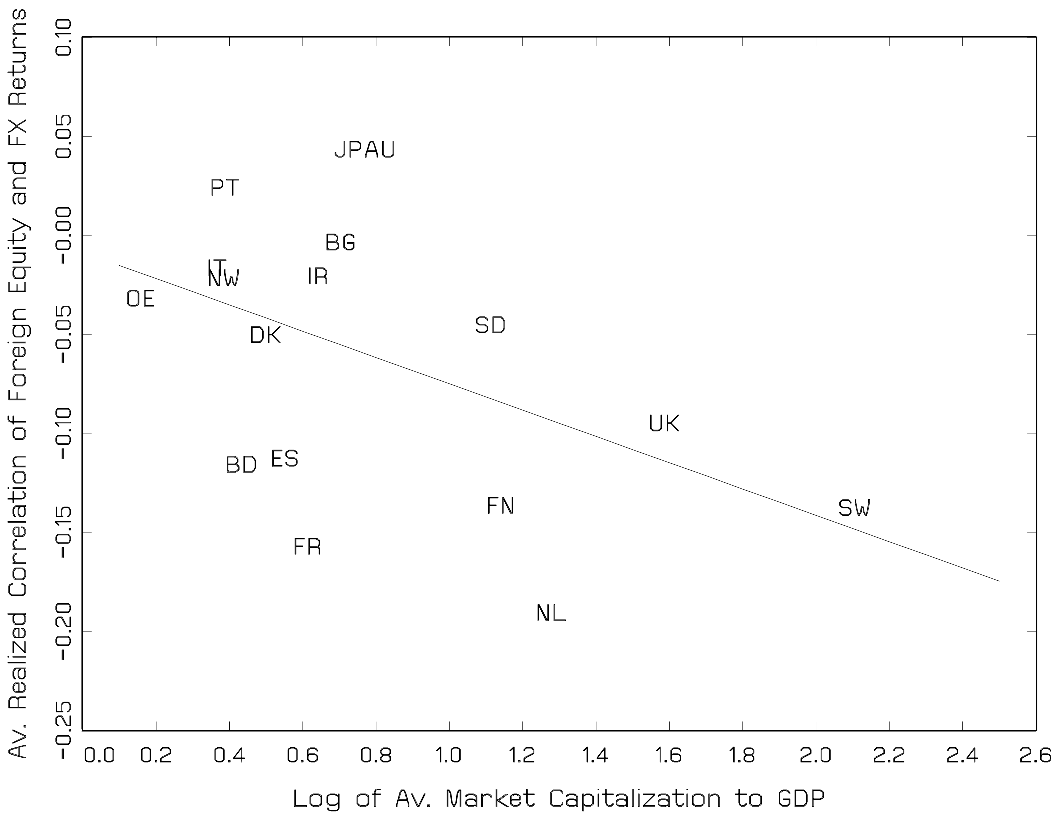

Figure 2: Plotted are the average monthly realized correlation of foreign equity returns (in local currency) and the corresponding foreign exchange return (in dollar terms) as a funtion of the average log market capitalization to GDP ratio for 17 OECD countries.