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Making a Market in Foreign Exchange

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Abstract

In a foreign exchange market there may be no informed traders who have superior information about the market as a whole. Even so, a risk-neutral market maker is confronted with a form of adverse selection costs. A stylized model allows an analytical separation of adverse selection from inventory management costs. A third component of costs of market making arises when the equilibrium price cannot be observed after each trade. This is another aspect of adverse selection. The model also provides a link between the aggressiveness of inventory control and the magnitude of the negative autocorrelation in the change in the mid-point market price, often found in high frequency FX data.

Key words: Foreign exchange market, Dealers, Bid-ask spread, Inventory control, Adverse selection.

JEL classification: F31, G1

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1. Introduction

Widely-dispersed traders in currency markets have a variety of motives for exchanging one currency for another. As in any over-the-counter market, orders come to market-making dealers who stand ready to buy a foreign currency at a bid rate and to sell the currency at an ask rate. Dealers then adjust their bid and ask prices in response to these orders. In addition, an interdealer market usually develops to help multiple dealers match offsetting positions.

There are two general strands in the literature that develop models of dealer markets. In inventory-based approaches, a market-making dealer typically faces unchanging stochastic buy and sell orders that cause changes in inventory positions and these changes influence the dealer's decisions about how to set prices. Some of the early literature includes papers by Demsetz (1968), Garman (1976), Amihud and Mendelsohn (1980), Stoll 1978), Ho and Stoll (1981), and O'Hara and Oldfield (1986). O'Hara (1995) devotes Chapter 2 of her book to this literature.

After the mid 1980s, emphasis shifted to information-based models with the appearance of papers by Copeland and Galai (1983), Glosten and Milgrim (1985), and Kyle (1985). In these models, a market maker is confronted with some traders who have superior information about the true price of the asset being traded. This gives rise to adverse selection and a dealer needs a bid-ask spread to cover the expected losses to those with the superior information. Much of O'Hara's book is devoted to this literature.

In stock markets there presumably is an underlying value for a firm and some people may have superior information about that value. In foreign exchange markets, there is more of question about what constitutes underlying value of one currency in terms of another. Individual agents have their own reasons for wanting to trade currencies, whether for trade-related needs or for desired portfolio shifts. These disparate decisions result in orders coming to the market.

The market then processes these diverse and unsynchronized bits of information. For example, if domestic income is rising relative to foreign income, there will likely be more orders to buy than to sell foreign currency. Even though aggregate income statistics will not be available for weeks or months, an immediate increase in the probability that the next order will be a dealer sale will tend, as shown more formally below, to result in a higher price. Long before the income statistics are common knowledge, other idiosyncratic influences on market supply and demand will occur. As a result, it is often difficult to sort out statistically what is driving the market.

Even if no individual has a reliable signal that the price will rise, the diverse decisions can be viewed as if some traders are acting on the basis of an expected higher price. In the colorful phrase by Surowiecki (2004) it is the wisdom of crowds. The market serves as an aggregator of information that no single agent may actually know. This perspective has been well documented in experiments. See, for example, Plott (2000) on markets as information gathering tools.

These considerations suggest that an inventory-based model can be reconciled with information-based approaches. The myriad dynamic influences on agents' needs or desires to exchange currencies calls for the introduction of a changing stochastic flow of orders into the models of inventory management. The market maker then makes pricing decisions on the basis of the signals received and can expect to lose money without a bid-ask spread..

In what follows, we modify the inventory-management model by specifying that the arrival of buy and sell orders coming to a market maker undergoes both transitory and permanent shifts. A linearized framework and stylized inventory-control mechanism allows for neat results that enable us to indicate analytically separate influences of adverse selection and inventory-management costs and to explore influences on the observed phenomenon of a negative autocorrelation in changes in the mid-point market price of foreign currency.

2. Market Model

Garman (1976) in a paper that introduces the phrase "market microstructure" lays out clearly a set of assumptions that characterize his analysis of a dealer market. The key assumptions are that arrivals of buy and sell orders are Poisson distributed in time and that all transactions are with a single market maker who is a price setter. The analysis was formalized further in an impressive paper by Amihud and Mendelson (1980). They assume, as Garman did, that unchanging stochastic demand and supply are price-dependent Poisson processes. They also assume that the dealer will never let inventories rise above an upper bound or fall below a lower bound. They then prove how it is optimal for a risk-neutral market maker to adjust bid and ask prices in response to inventory positions.

In our model of a single market-making dealer, the key assumption is that the probability of the next customer order being a customer purchase (and dealer sale) depends on the dealer's bid price, ask price, and a changing market equilibrium price. Time will be measured in transactions. A wider bid-ask spread may slow down the arrival of customer orders in calendar time, but the important point is that there will be a sequence of orders, and the mix between buy and sell orders can be influenced by relative prices.

Let ρ_t be the probability that the next customer order at transaction t will result in a dealer sale of one unit. In terms of Poisson-distributed arrival rates, Amihud and Mendelson note that this probability is the price dependent ratio of the arrival rate of buy orders (D) over the sum of the arrival rates of buy and sell orders (D+S). In a somewhat different notation, this probability can be written.

$$\rho_t = D(P_t+b)/[D(P_t+b)+S(P_t-b)] \tag{1}$$

where P_t is the mid-point <u>market price</u> set by the dealer, $P_t + b$ is the ask price, $P_t - b$ is the bid price, and 2b is the bid-ask spread.

To allow for variations over time, we introduce a shift parameter P_t^* into the D and S functions

$$\rho_t = D(P_t+b, P_t^*)/[D(P_t+b, P_t^*)+S(P_t-b, P_t^*)] \tag{1'}$$

with the hypotheses that a higher market price set by the dealer decreases the rate at which buy orders arrive ($D_1 < 0$) and increases the rate at which sell orders arrive ($S_1 > 0$). A higher P_t^* is assumed to increase the rate at which buy orders arrive ($D_2 > 0$) and decrease the rate at which sell orders arrive ($S_2 < 0$).

The idea behind this formulation is that the dispersed customers have their own private reasons for needing to buy or sell the foreign currency. These customers do not trade with each other. They trade with a market maker, who can rarely be sure whether the next order will result in a sale or purchase but can influence the probabilities by price adjustments. P_t* summarizes the net effect of exogenous influences on the flow of orders to the market maker but it is not observed by the market maker before P_t is set. Assume that P_t* is scaled so that if $P_t = P_t^*$, then it is equally likely that the next order is a buy or a sell, i.e., $\rho_t = 1/2$. In this sense P_t* can be called the equilibrium price.

Given the hypotheses about influences on the order arrival rates, the probability that the next order is a customer buy and dealer sale is influenced as follows.

$$\partial\rho_t/\partial P_t = (SD_1 - DS_1)/(D+S)^2 < 0$$
$$\partial\rho_t/\partial P_t^* = (SD_2 - DS_2)/(D+S)^2 > 0 \qquad (2)$$

In other words, ceteris paribus, a higher market price lowers the probability of a dealer sale and a higher equilibrium price raises the probability.

Some neat analytical results are possible, if (1') is approximated by a linear function within a range:

$$\rho_t = 1 \qquad \text{if } c < P_t^* - P_t$$
$$\rho_t = (P_t^* - P_t + c)/2c \qquad \text{if } -c < P_t^* - P_t \leq c \qquad (3)$$
$$\rho_t = 0 \qquad \text{if } P_t^* - P_t \leq -c$$

If the equilibrium price is far enough above the mid-point market price, the dealer's next trade is a sale with probability one. If the equilibrium price is far enough below the mid-point price, the dealer's next trade is a purchase with probability one. In between, the probability of a sale is a decreasing function of P_t and an increasing function of P_t*. A sale means that the dealer's inventory goes down by one unit and a purchase means that the inventory goes up by one unit. Think of c as being three or more standard deviations of the change in P_t* so that in an overwhelming proportion of the time the probability is greater than 0 and less than 1. Furthermore, since P_t^* is not observed when P_t is set, the dealer does not know the probability precisely, so there is always uncertainty about whether a choice of P_t will result in a sale or a purchase by the market maker.

The dealer's expected profit when setting P_t depends on the hypotheses embedded in (3) and the stochastic process for P_t^*. Let the change in P_t^* be subject to both permanent and transitory shocks. This can be represented by an integrated moving-average process:

$$P_t^* = P_{t-1}^* + z_t - \gamma z_{t-1} \qquad 0 < \gamma < 1 \tag{4}$$

where z is an independent random variable with mean zero and variance σ_z^2. The parameter γ represents how much of the random term z is transitory and $(1-\gamma)$ is a measure of how much is permanent. The transitory shocks to the equilibrium price are not a matter of bid-ask bounce or inventory effects. Rather, they can be thought of as the result of offsetting trade flows or portfolio shifts that do not occur at exactly the same time, while the permanent shocks arise from non-offsetting changes in the rates of trade or capital flows.

Given the process in (4), the t-1 expected value of P_t^* is:

$$E_{t-1} P_t^* = P_{t-1}^* - \gamma z_{t-1} \tag{5}$$

It follows that the deviation in the equilibrium price from its rationally expected value is a random variable:

$$P_t^* - E_{t-1} P_t^* = z_t \tag{6}$$

and rational expectations are formed adaptively:

$$E_t P_{t+1}^* = (1-\gamma) P_t^* + \gamma E_{t-1} P_t^* \tag{7}$$

assuming that P_t^* is observable after transaction t.

Now consider a market maker's expected profit when a purchased or sold unit of foreign exchange at transaction t is valued at the new expected equilibrium price $E_tP_{t+1}^*$. In this case with a market price P_t, the dealer's expected profit from the next transaction with information prior to t can be written:

$$E_{t-1}\,\pi_t = E_{t-1}\,[\rho_t\,(P_t + b - E_tP_{t+1}^*) + (1-\rho_t)(E_tP_{t+1}^* - P_t + b)] \tag{8}$$

The first expression in the bracketed term in (8) is the probability ρ_t of a dealer sale times the perceived value of the sale $(P_t + b - E_t\,P_{t+1}^*)$, which is the selling price less the expected value of the unit of inventory sold. The second term is the probability of a dealer purchase $(1-\rho_t)$ times the perceived value of a purchase, $(E_t\,P_{t+1}^* - P_t + b)$, which is the expected value of the additional unit of inventory minus the purchase price.

Substitute in (8) for ρ_t as defined by (3), and manipulate to get

$$E_{t-1}\,\pi_t = -\,E_{t-1}[(P_t^* - P_t)(E_t\,P_{t+1}^* - P_t)/c] + b \tag{9}$$

Equation (9) can be rewritten using (7)

$$E_{t-1}\pi_t = -E_{t-1}[P_t^* - E_{t-1}P_t^* + E_{t-1}P_t^* - P_t][(1-\gamma)P_t^* + \gamma\,E_{t-1}P_t^* - P_t]/c + b$$

$$= -E_{t-1}[z_t + E_{t-1}P_t^* - P_t][(1-\gamma)z_t + E_{t-1}P_t^* - P_t]/c + b$$

The second line makes use of (6). Since P_t is set before z_t occurs and since z_t has zero expected value and is independent of $(E_{t-1}P_t^* - P_t)$, it follows that:

$$E_{t-1}\pi_t = -\,[(1-\gamma)\,\sigma_z^2 + (E_{t-1}P_t^* - P_t)^2]/c + b \tag{10}$$

This is a key analytical result.

3. Influences on the Bid-Ask Spread

The negative terms in (10) represent the expected costs of being a market maker. In order to generate non-negative expected gains from market making, b needs to equal or exceed the expected costs. Recall that b is one-half of the bid-ask spread. Two predictions follow from (10) if expected profits are to be non-negative.

1. The model predicts that a larger variance of the shifts in the equilibrium price attributable to permanent shocks, $(1-\gamma)\ \sigma_z^2$, calls for more of a bid-ask spread. This can be explained intuitively. Suppose the market price is initially equal to the expected equilibrium price. A positive z makes the probability of a dealer sale greater than one half and at the same time (since γ is less than 1) raises the expected equilibrium price at which a sold unit is valued, resulting in an expected loss if b is zero. Similarly, a negative z raises the probability of a dealer purchase above one half and lowers the expected equilibrium value of a purchased unit below the purchase price if b is zero. The bigger these shifts the higher the expected costs.

This variance term is not the result of risk aversion by the dealer. It is a cost that even a risk neutral market maker will incur.

We will call this a cost of adverse selection but need to be clear how we are using the term. Adverse selection usually refers to situations of asymmetric information in which agents with superior information can take advantage of that information. In our case, the superior information is possessed by an aggregation of agents. Suppose an increase in P_t* represents the net effect of changing influences on individual customers in the foreign exchange market with the result that the probability that the next order is a dealer sale rises above one-half. Those individual agents contemplating purchases presumably do not know this any better than those contemplating sales, so there may be no identifiable group with superior information. The superior information is possessed by the market itself facing the market maker.

2. Additional costs will be incurred if $P_t \neq E_{t-1}P_t^*$. This will happen when the dealer makes price adjustments to influence inventory positions and/or when the equilibrium price is not observed directly after each transaction.

Awareness that a shifting equilibrium price is critical to the risk faced by a dealer holding an inventory position can be found at least as early as Tinic (1972), who discusses different types of costs of providing liquidity. He writes (p. 82): "The primary source of risk results from the possibility of a change in the equilibrium price of the security during a period when the specialist possesses long or short positions in the issue." In equation (10), an expected loss arises from the shift in the equilibrium while the dealer's bid and ask prices are fixed. If any net inventory position after the transaction is then valued at the expected equilibrium price, there are no further <u>expected</u> net gains or losses. There is, however, added risk in the sense of variability in the value of non-zero inventory positions when the equilibrium price can wander over time. A risk-averse dealer can control this wandering by shading the market price relative to the expected equilibrium price. This shading involves a cost in the form of expected loss of revenue, as captured by the second term in (10).

Bjonnes and Rime (2004) have commented: "Empirically, the challenge is to disentangle inventory holding costs from adverse selection. Unfortunately, there is no theoretical model based on first principles that incorporates both effects." Equation (10) addresses that concern. The term involving the variance of the permanent shocks to the equilibrium price can be interpreted as the costs attributable to adverse selection. Instead of inventory holding costs per se, the second term in (10) can be interpreted as a cost of inventory management. A risk-averse dealer will choose to forgo some expected profit in order to assure a mean reversion of inventory holdings. Note that for the market as a whole inventories cannot be adjusted by laying off positions to other dealers.

4. Negative Autocorrelation in Price Changes

Another prediction of the model is that when the market maker sets the price equal to the rationally expected equilibrium price, the market transforms a negatively autocorrelated change in the equilibrium price into a pattern in which there is no autocorrelation in the change in the mid-point market price.

To see this, first note that equation (4) has the property that successive changes in the equilibrium price have a negative covariance:

$$cov(P_{t+1}^* - P_t^*, P_t^* - P_{t-1}^*) = -\gamma\sigma_z^2$$

The variance of the change in the equilibrium price is $(1+\gamma^2)\sigma_z^2$. Therefore the first order autocorrelation of the change in the equilibrium price is $-\gamma/(1+\gamma^2)$. This can range from 0 when all the shocks are permanent ($\gamma = 0$) to -0.5 when all the shocks are transitory ($\gamma = 1$).

On the assumption that market price is set always equal to the expected equilibrium price, the covariance of successive changes in the market price is:

$$cov(P_{t+1} - P_t, P_t - P_{t-1}) = cov(E_tP_{t+1}^* - E_{t-1}P_t^*, E_{t-1}P_t^* - E_{t-2}P_{t-1}^*) \tag{11}$$

However, equations (7) and (6) imply

$$E_t P_{t+1}^* - E_{t-1} P_t^* = (1-\gamma)(P_t^* - E_{t-1} P_t^*) = (1-\gamma)z_t \tag{12}$$

and one trade earlier

$$E_{t-1} P_t^* - E_{t-2} P_{t-1}^* = (1-\gamma)(P_{t-1}^* - E_{t-2} P_{t-1}^*) = (1-\gamma)z_{t-1} \tag{13}$$

Substituting (12) and (13) into (11):

$$\text{cov}(P_{t+1} - P_t, P_t - P_{t-1}) = (1-\gamma)^2\text{cov}(z_t,z_{t-1}) = 0 \qquad (14)$$

Therefore, when price is always set equal to the rationally expected equilibrium price, there will be no expected autocorrelation in the change in the mid-point market price.

A number of studies have looked at high frequency exchange-rate data. Ito and Roley (1986), Goodhart and Figliouli (1991), Goodhart and Giugale (1993), Zhou (1996), Ghosh (1997), Danielson and Payne (2002), and Evans (2002) find that a jump in the exchange rate tends to be followed by a movement in the opposite direction. To quote Ito and Roley (1986, p. 16): "there is evidence that 'profit-taking' occurred after a large jump. For a positive jump of 3 yen, for example, the results indicate that during the next segment of the day the yen/dollar rate falls by 0.5 yen. In an efficient market, it is difficult to rationalize such behavior." Similar patterns have been found with a variety of high frequency data.

In some cases the negative autocorrelation is because of a bid-ask bounce using transaction prices, but in other cases a negative autocorrelation was found using the mid-point price. Many of the earlier studies were based on Reuters' EFX data, which show dealers' indicative bid and ask quotes. This means that the quotes are not firm and as Danielsson and Payne (2002) argue, a dealer wanting to buy will shade up both the bid and ask quotes but has little interest in selling at the ask quote. Therefore calculation of the mid-point price with the EFX data has an inherent bid-ask bounce. They report that, using 20-second intervals, returns based on EFX mid-point quotes yield a first order autocorrelation of -0.33. By contrast, the autocorrelation of 20-second returns using the mid-point of the best bid and best ask price in the D2000-2 data, which are firm quotes in an electronic market, are one-third to one-half the value from the corresponding EFX data. A question remainss why the D2000-2 data still exhibit a negative, albeit smaller, autocorrelation of changes in the mid-point market price.

As noted earlier, a single risk-averse market maker will often let P_t differ from $E_{t-1} P_t^*$ to limit the variations of inventories and profits. When inventories are positive, the risk averse dealer will set the market price below the expected equilibrium to raise the probability above one-half that the next transaction is a dealer sale (and customer purchase). When inventories are negative, the dealer will set the market price above expected equilibrium to make it more likely that the next transaction will be a dealer purchase (and customer sale).

More formally, take conditional expected values of equation (3) for the probability of a sale:

$$(E_{t-1}P_t^* - P_t + c)/2c = E_{t-1}\rho_t$$

It follows that, for a target probability ρ_t of making a sale, the price will be set in accordance with:

$$P_t = E_{t-1}P_t^* - (2\rho_t - 1)\, c \tag{15}$$

For a given ρ, the expected change in inventory per trade is $-\rho + (1-\rho) = 1-2\rho$. For example if $\rho = 0.6$, then the expected change in inventories per trade is -0.2 and it would take 5 trades on average to reduce inventory by 1 unit. A more risk-averse trader will use a higher ρ_t when holding positive inventory and a lower ρ_t with negative inventory, since this will result in a smaller expected variance in profits and, given the bid-ask spread, smaller expected profits.

There will be a mapping from the market maker's inventory position to the bid and ask prices. Amihud and Mendelsohn (1980) derive an optimal pattern that assures inventories never exceed an upper bound nor fall below a lower bound. We adopt a simpler mapping here that facilitates an analysis of a tradeoff between mean and variance of the costs of making a market. Suppose the following symmetrical policy is followed. With a value of $\rho > 0.5$ for positive inventory, a target of $1-\rho$ will be set with a negative inventory position, and $\rho = 0.5$ with zero inventory. With this particular inventory policy, the expected profit for the market maker can be rewritten by substitution from (15) into (10):

$$E_{t-1}\pi_t = - (1-\gamma)\, \sigma_z^2 /c - xc(2\rho - 1)^2 + b \tag{16}$$

where x is the fraction of the time inventory is non-zero. A more aggressive inventory-control policy (higher ρ) that is not fully offset by a lower x raises the expected costs of market making and calls for a larger bid-ask spread to assure positive expected profits.

With price often different from the expected equilibrium price, a non-zero autocorrelation in price changes is now possible.

5. A Simulation Experiment

To illustrate these effects, we ran a simulation using Gauss software. The base case has the following parameters: $\gamma = 0.4$, $\sigma_z = 0.1$, and, to make the range of uncertainty about sale or purchase equal to plus or minus three standard deviations of the shock to the equilibrium price, $c = 0.3$. For 1000 observations of an equilibrium price, the values of the variable z were drawn from a normal distribution with a mean of zero and a standard deviation of 0.1. Draws from a uniform distribution from 0 to 1 were used to determine whether the market price relative to the equilibrium price reflected in equation (3) calls for a dealer sale or purchase.

For a given value of ρ, there were 1000 iterations and each iteration consisted of 1000 transactions with a new set of random numbers. After each iteration, we calculated the first-order autocorrelation in the change in the market price. At the end of the 1000 iterations, means and standard deviations for several statistics were calculated.

Table 1. Simulation results (with $c = 0.3$, $\sigma_z = 0.1$, $\gamma = 0.4$) when equilibrium price is observed after each transaction.

ρ	Average cost	Average % zero inventory	Average ending inventory	Market price change autocorrelation
.50	17.0 [20.0] (41.9)	2.4 (1.8)	0.29 (31.1) [31.6]	-0.001 (0.032)
.55	22.1 [22.7] (12.7)	9.4 [9.1] (2.6)	0.51 (7.27)	-0.015 (0.032)
.60	29.5 [30.0] (6.4)	16.7 [16.7] (2.3)	0.07 (3.50)	-0.074 (0.034)
.65	40.1 [40.7] (4.1)	22.9 [23.3] (2.0)	0.01 (2.35)	-0.155 (0.034)
.70	53.3 [54.4] (3.3)	28.3 [28.6] (1.7)	-0.03 (1.71)	-0.225 (0.032)
.75	68.2 [70.0] (2.8)	32.9 [33.3] (1.5)	-0.01 (1.36)	-0.281 (0.030)

(Standard deviations are in parentheses.)
[Predicted values in brackets]

Table 1 shows the results for different values of ρ. The column headed "Average cost" shows the average loss from being the market maker in the absence of a bid-ask spread. For $\rho = 0.50$, this average cost was 17.0, with a standard deviation of 41.9. For expected net revenue to be non-negative when $\rho = 0.5$, equation (10) calls for $b \geq (1-\gamma) \sigma_z^2/c = 0.02$. With 1000 transactions, expected revenue from half of the bid ask spread equal to 0.02 would total 20, as reported in brackets. Given the high standard deviation of average costs, 17 is relatively close to 20.

With $\rho = 0.5$, ending inventory is the cumulative sum of n observations that can each be 1 with probability 1/2 or -1 with probability 1/2. Starting with zero inventory, the mean for ending inventory is 0 and the variance is n. With n = 1000, the variance is therefore 1000, for a predicted standard deviation of 31.6. This is close to the sample standard deviation of ending inventory of 31.1. Because inventories can wander so much when $\rho = 0.5$, potentially large capital gains or losses from positions carried forward generate the large standard deviation of average costs.

The last column indicates that there was, as predicted, an insignificant first-order autocorrelation coefficient for the change in market price when $\rho = 0.5$.

A risk-averse market maker, however, will choose $\rho > 0.5$ when holding a positive inventory position. Starting with zero inventory and $P_t = E_{t-1} P_t^*$, the dealer's position after the next transaction will be 1 or -1 with equal probability. In either case, for $\rho > 0.5$ (or $1- \rho > 0.5$ with negative inventory), the expected time to return to zero inventory is $1/(2\rho -1)$. This implies that on average, inventory will be zero every $1 + 1/(2\rho -1) = 2\rho/(2\rho -1)$ transactions. The inverse, $(2\rho-1)/2\rho$, gives the predicted fraction of the transactions that the dealer starts with zero inventory, recorded in brackets in the third column of Table 1.

The predicted fraction of the time that inventories are not zero is then $x = 1/2\rho$. From equation (16), after substitution for this predicted value of x, the expected average cost per transaction is $(1-\gamma)\, \sigma_z^2 /c + c(2\rho - 1)^2/2\rho$, which is an increasing function of ρ for $\rho > 0.5$. This is the formula used to obtain predicted values for average cost shown in brackets in the second column of Table 1 for different values of ρ. The actual average values are quite close to those predicted.

The last five rows of Table 1 show the results with different values of ρ. Each row used the same random variables for the z's and for the uniform probabilities to determine whether the next transaction is a sale. As predicted, average costs rise with higher values of ρ as does the percentage of the time that inventory is zero. The standard deviations of ending inventory and of average cost also fall with higher values of ρ.

Note finally the last column. Changes in market price clearly become negatively autocorrelated for moderate degrees of inventory control (e.g., $\rho \geq 0.6$) and more so for higher values. The reason is that a more aggressive inventory management policy (higher ρ) results in more frequent passes through zero inventories and a stronger probability of reversal after acquiring the first unit of positive or negative inventory.

6. No Direct Observation of the Equilibrium Price

So far the market maker has been assumed to observe the equilibrium price after each trade. A more plausible scenario is that a dealer never observes the equilibrium price but must infer it approximately from customer responses to market prices. A run of sales is indicative that the dealer's price is below equilibrium and a run of purchases indicates that price is above equilibrium. Price adjustments can be made on the basis of such inferences.

An important implication of the lack of direct observation of the equilibrium price is that it introduces an additional cost of market making. On average the market price will be expected to depart by more from the equilibrium price than would be the case in which the equilibrium price can be observed after each transaction. Let $E_{t-1}^{d} P_t^*$ denote the dealers guess about the equilibrium price. This can differ from the mathematical expected value. If the mean difference between $E_{t-1}^{d} P_t^*$ and $E_{t-1}P_t^*$ is zero, then equation (10) becomes:

$$E_{t-1}\pi_t = - [(1-\gamma)\,\sigma_z^2 + (E_{t-1}^{d} P_t^* - P_t)^2 + (E_{t-1}P_t^* - E_{t-1}^{d} P_t^*)^2]/c + b \qquad (17)$$

The second term inside the brackets is the cost associated with planned inventory control.

The last term inside the brackets in (17) is an unavoidable cost associated with non-observation of the equilibrium price. This is another aspect of adverse selection. The market maker has less information about the equilibrium price than the overall market and consequently incurs extra costs from making a market.

7. The Interdealer Market

Those who have looked closely at the link between an interdealer market and the corresponding customer market conclude that most of the profits for dealers comes from the customer market. According to Braas and Bralver (1990): “sales (and not speculative trading) is the only reliable source of stable revenues for most trading rooms.” In their examination of over forty trading desks around the world, they find that customer business represents between 60 and 150 percent of total revenues. Yao (1998) examines the trades of a large trader in the DM/$ market over a 25 day period in 1995 and finds that customers account for only 14 percent of trade volume but 75 percent of the dealer’s gross profit.

If dealing room profits come largely from the bid-ask spread in the customer market, then interactions in the interdealer market determine how much of this spread is shared with other dealers. The evidence here is of strong mean reversion in dealer inventories and that this inventory control is accomplished primarily by laying off positions with other dealers rather than by price shading.

Bjonnes and Rime (2004) examine the behavior of four different FX traders in a Norwegian bank. They find strong evidence of mean reversion for all four dealers. They also find that adverse selection is responsible for a large proportion of the effective spread and no evidence of inventory control through dealers' own prices. Similar results are found by Mende, Menkhoff and Osler (2004) in a study of the entire deal record of a relatively small German bank in USD/EUR over four months in 2001. This dealer's quoted spreads are not affected by his inventory level and he tends to manage his inventory through outgoing interbank deals.

It makes sense that a dealer who has private information about a large customer order can best profit from that information by rapidly laying off positions before changing quotes in the interdealer market. For example, if there is a large customer purchase, the dealer is short in foreign exchange and can expect losses from having to buy back inventories at a higher price, assuming this customer order signals a permanent increase in the market equilibrium price. Buying back at the current best offer prices in the interdealer market assures a profit from the customer transaction because the bid-ask spread is narrower in the interdealer market than in the customer market.

If the dealer also has reason to expect the best bid price to rise above the current best offer price in the interdealer market, there is an incentive to take a speculative position. The action taken depends on whether the expected gain outweighs the risk from a speculative position. For example, the trader studied by Yao (1998) engaged in limited speculation (5 percent of volume) but did profit from short-term informational advantage from customer orders (28.5 % of total profits) with wide variations.

To reconcile the evidence of individual dealers controlling inventories by laying off positions and market-level inventory control through price adjustments, bear in mind that it has been well established that quotes are adjusted in response to order flow. See for example, Lyons (1997) and Evans and Lyons (2002). Dealers who receive customer orders that change their inventory position also acquire private information about likely price changes. These dealers lay off their inventory positions and perhaps take speculative position via order flow. Other dealers observe the order flow and make price adjustments, so the market itself with multiple dealers does result in price shading in response to customer orders.

8. Concluding Remarks

We have developed a model of the customer market for foreign exchange in which the probability that the next customer order is a purchase is a function of the deviation of market price from an equilibrium price. When the equilibrium price is subject to both permanent and transitory shifts, the bid-ask spread necessary for viable market making has been derived analytically. This spread is an increasing function of the variance of the permanent shocks to the equilibrium price, the variance of the errors in assessing the expected equilibrium price, and of the aggressiveness of the inventory control policy.

If market price is always set equal to the rationally expected equilibrium price, then there will be no predicted autocorrelation in changes in the market price even though the equilibrium price changes are negatively autocorrelated. However, a shading of price to keep inventories under control does introduce a negative autocorrelation in changes in the market price, a pattern often observed in high frequency foreign exchange trading.

Subsequent research can address the effects of a number of alternative market structures within this framework. A natural extension is to introduce multiple dealers who use the private information in their customer orders to take speculative positions in the interdealer market in an attempt to increase their share of the income generated by the bid-ask spread in the customer market. Since dealers will have different inventory positions and information about customer orders, they are unlikely to set the same bid and ask prices and will differ in decisions about market orders in the interdealer market.

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