Informed Speculation and the Choice of Exchange Rate Regime

Mark Aguiar[*]

University of Chicago Graduate School of Business

March, 2002

Abstract

This paper explores the classic question regarding the costs and benefits of limiting exchange rate volatility starting from the premise that asset markets play an important role in aggregating information. I find that government trading, despite being completely observed, alters the ability of an asset market to reveal information. The main insight of the paper is that managing the exchange rate reduces the correlation and informational content of the exchange rate regarding persistent fundamentals and increases the correlation with short-run shocks. The result arises from the impact of government intervention on private speculation. Currency speculators have an incentive to differentiate shocks based on capital gains potential (which is related to persistence) and speculative trading therefore determines how exogenous shocks are reflected in the equilibrium exchange rate. By smoothing the exchange rate, the government reduces the speculative incentive to separate shocks. The net result of government intervention is an exchange rate with lower total variance, but a larger percentage of high frequency volatility. In terms of quantities, movements in government reserves also become less informative as intervention is increased.

1 Introduction

This paper studies the effect of government intervention on an asset market's ability to aggregate and reveal information. In particular, we introduce a government that trades transparently and according to a known rule into a currency market composed of informed speculators and noise traders. Agents in the economy are faced with a filtering problem to extract trades based on "fundamentals" from trades driven by "noise". We find that government intervention designed to dampen currency fluctuations, although perfectly observed, distorts the quality of the market's signal regarding the underlying fundamental. Specifically, smoothing the exchange rate produces a regime that is more informative about mean-reverting shocks and less about long run fundamentals.

The intuition of the result hinges on the fact that speculators are motivated by capital gains. As a result, speculators use their information to determine whether a shock is temporary or permanent. For example, suppose on average speculators believe (based on private and public signals) that a given shock is transitory. If the shock implies a temporary depreciation, then speculators foresee an appreciation and buy the currency. The positive speculative response to a temporary depreciation off-sets the exchange rate's response to the fundamental shock. Conversely, a permanent shock to a fundamental offers little capital gains potential and therefore induces limited trading by speculators. Thus speculators actively trade against mean-reverting shocks while ignoring (or perhaps amplifying) more persistent innovations. By limiting price changes, the government reduces the incentive for speculators to distinguish between shocks. The result of less speculation is an exchange rate relatively more responsive to transitory shocks. Although the managed exchange rate has less overall variance, a larger *proportion* of its variance consists of high-frequency fluctuations – and it is the relative proportions of fundamental and noise that determines a price's informational content.

The motivation for this paper starts with the premise that markets play a crucial role in aggregating and revealing information (classic references include Hayek (1945), Lucas (1972, 1973), and Grossman (1989)). In regard to currency markets, there is consider-

able anecdotal evidence that investors rely on exchange rate movements (or their quantity counterpart, capital flows) as useful reflections of the "consensus view" on fundamentals.[1] More formally, a recent paper by Evans and Lyons (2002) uses order flow data to argue that currency markets are characterized by extensive informational asymmetries.

This paper pushes the literature in a new direction by focusing on government intervention. We focus on currency markets given the important role, both in practice and in policy analysis, of exchange rate management.[2] However, the model's results extend to intervention in any asset market. For example, the framework could be used in evaluating commodity price stabilization schemes or the recent debate regarding the benefits of limiting large movements in stock prices.

The paper is organized as follows. Section two introduces the model. Section three solves for the equilibrium and derives the main informational results. Section four discusses optimal government intervention as well as additional comparative statics and section four concludes.

2 The Model

This section introduces a stylized model to motivate the informational results derived in the next section. Despite a number of simplifying assumptions, the essential elements of the model are fairly robust to alternative approaches, as will be pointed out as the model is introduced. The model involves a small open-economy (home) and the rest of the world

---

[1]See, for example, WSJ, June 15, 1998, "The Outlook: If the yen is any guide, Asia's crisis isn't over," p. A1, and WSJ, November 20, 1998, "Early Warnings: How to tell where the global economy may be headed next," p. A1.

[2]In regard to the informational content of the exchange rate, in the mid 1980s several authors noted that a completely fixed price (e.g. fixed exchange rates or targeted interest rates) could have adverse informational consequences (e.g. Kimbrough (1984), Flood and Hodrick(1985) and Dotsey and King (1986)). In these models, fixing the exchange rate amounts to eliminating a signal. However, in principle the signal can be reclaimed by observing capital flows or money supply. This paper allows for complete observability of prices and quantities. The effect is not one of signal elimination, but rather the endogenous nature of the signal's informativeness.

(foreign). There are two assets, domestic and foreign currency. We let ε represent the number of foreign currency units that exchanges for one unit of domestic currency (note that an increase in ε is an appreciation of the domestic currency). There is one good which has price one in foreign currency (and $\frac{1}{\varepsilon}$ in local currency). The small open economy assumption is that any developments in the home country do not affect the world price of the consumption good. There are three types of agents: domestic producers, rational speculators and noise traders. The paper's main results can be made most clearly in an essentially static model. The only dynamic element is that the history of exchange rates is available for inference at any point in time. The implications of an infinite horizon are discussed in Appendix C.

The underlying "fundamental" will be the productivity of domestic workers. Speculators will have an interest in the "purchasing power" of the currency, which is related to domestic productivity. Speculative trading on the basis of information regarding productivity as well as the extent of noise trading will produce an exchange rate that is a noisy signal of the productivity parameter. The quality of this signal in the presence of government intervention will be the focus of the next section.

1. Domestic producers

There exists a continuum of measure L of domestic producers. Workers active in period t begin the period by setting a foreign currency (real) wage w_t in exchange for which they will provide $F_t + \xi_t$ units of output. The first element of worker productivity, F_t, is a random variable that follows a stochastic process to be discussed below in detail. An important point of the analysis will be how well do workers know this "fundamental". In particular, we will study how effectively the currency market reveals the underlying productivity parameter F_t to uninformed or partially informed agents. The second term, ξ_t, is an additional source of risk that is pure white noise with variance σ_ξ^2. This additional shock ensures that there is no discontinuity in the equilibrium as we eliminate uncertainty regarding the future exchange rate or the fundamental F_t.

4

Domestic producers enjoy exponential utility over consumption at the end of the period. Specifically,

$$U = - \exp(-w_t). \tag{1}$$

Domestic agents behave competitively and drive the wage down until they are indifferent between supplying their labor to the market in exchange for w_t or consuming their own output. That is,

$$- \exp(-w_t) = E\{-\exp(-(F_t + \xi_t^i) \mid \Omega_t^L\}, \tag{2}$$

where the information set of domestic labor is given by Ω_t^L. Therefore, w_t is the certainty equivalent of home production.[3] The information set consists of the history of exchange rates up through period t, i.e. $\Omega_t^L = \{\varepsilon_s\}_{s=0}^{s=t}$.[4] It would not change the main thrust of the analysis if workers had some additional source of information, as long as it was incomplete. The key point is that they rely on the observed path of currency prices to update their priors regarding F_t. As will be shown below, the distribution of F_t conditional on the history of exchange rates is normally distributed. We denote the mean and variance of this conditional distribution as $\hat{F}_t$ and V_t, respectively. Therefore, the certainty equivalent of F_t is

$$w_t = \hat{F}_t - \frac{1}{2}(V_t + \sigma_\xi^2). \tag{3}$$

2. Informed Speculators

The model focuses on speculators who trade between foreign and domestic currency. A key assumption is that the speculators have some information regarding the underlying fundamentals that drive exchange rate movements. It is unrealistic to assume that speculators have all relevant information. However, it is reasonable to suppose that the private sector

---

[3]I am assuming that there is a sufficient number of domestic producers that the marginal producer is engaged in home production, driving the wage to the reservation level.

[4]Formally, the information set is the $\sigma-$algebra generated by $\{\varepsilon_s : s \leq t\}$.

as a group holds information in disaggregated form that is unavailable to any one individual (including the government). For expositional purposes, we start by assuming speculators have full information, but then relax this assumption in Appendix B.[5] As made clear in the appendix, all results extend to the more relevant case where each speculator receives an idiosyncratically noisy signal of the true fundamental which is aggregated through the market. Moreover, in no sense do the results rely on the assumption that individual speculators have "better" information than producers. The important point is that the producer has incomplete information about future productivity and uses asset prices (which aggregate information held by speculators) to augment her priors. With this in mind, let Ω_t^S denote the information set of informed speculators at time t such that

$$\Omega_t^S = \{\varepsilon_s, F_s : s \leq t\}. \tag{4}$$

Each period, a measure-one continuum of speculators solve a one-period portfolio problem that allocates their financial wealth between foreign and domestic currency to maximize end of period wealth. Foreign currency pays a risk-free interest rate of r. There are two motivations for holding domestic currency. The first is a speculative motive that depends on the expected capital gain or loss that arises from the home currency's appreciation or depreciation: $\Delta\varepsilon_{t+1} = \varepsilon_{t+1} - \varepsilon_t$. The second motivation is the use of domestic currency for transactions. The ability to purchase the output of domestic labor pins down the transactions payoff as $F_t + \xi_t - w_t$. In particular, one unit[6] of local currency allows the speculator to make the following contract with domestic residents: the speculator will provide w_t units of the consumption good as wages in exchange for $F_t + \xi_t$ units at the end of the period.[7]

---

[5]A natural approach to model this is that domestic agents produce F_t units of output in period t, take their wages and then save/speculate for consumption in period $t+1$. Having already realized their labor output, these agents have private information. We make the distinction between domestic producers and speculators to simplify the analysis, particularly of welfare.

[6]One unit is, of course, arbitrary. The important point is that holding the currency facilitates the purchase of domestic output.

[7]There may arise a case in which the transactions payoff is negative. The probability of this event could be limited by an appropriate choice of the other parameters (e.g. a high mean value for F and/or a large

We now formally state the speculators' problem. Speculators begin period t with wealth W_t (calculated in foreign currency) and solve the following portfolio problem:

$$\max_{S_t} E\left\{-e^{-\psi W_{t+1}}|\Omega_t^S\right\} \tag{5}$$

$$s.t.\ W_{t+1} = (1+r)W_t + \left(\varepsilon_{t+1} - (1+r)\varepsilon_t + F_t + \xi_t - w_t\right)S_t, \tag{6}$$

where S_t represents the quantity of domestic currency held at the end of period t and Ω_t^S is the information set of rational speculators at time t. The solution to this problem is[8]

$$S_t = \frac{E\left(\Delta\varepsilon_{t+1} - r\varepsilon_t + F_t + \xi_t - w_t|\Omega_t^S\right)}{\psi\sigma_\varepsilon^2}, \tag{7}$$

where σ_ε^2 is the conditional variance of the numerator in equation (7). Note that the white noise output term ξ_t ensures that σ_ε^2 is bounded away from zero even under a pegged exchange rate. Substituting in the equilibrium wage, we have

$$S_t = \frac{E\left(\Delta\varepsilon_{t+1} - r\varepsilon_t + F_t + \xi_t - \hat{F}_t + \frac{1}{2}(V_t + \sigma_\xi^2)|\Omega_t^S\right)}{\psi\sigma_\varepsilon^2}. \tag{8}$$

While speculative demand (8) has been derived in a rather special framework, the essential element is that demand is sensitive to capital gains as well as the underlying fundamental. Note that if the asset under study were a stock, $\Delta\varepsilon_{t+1}$ would be replaced by the capital gain and $F_t + \xi_t - \hat{F}_t + \frac{1}{2}(V_t + \sigma_\xi^2)$ would be replaced by the dividend. I have modeled the fundamental as purchasing power (which depends in turn on domestic productivity), but in a more general framework this could be replaced by other commonly used exchange rate fundamentals, such as aggregate (unobserved) price shocks as in Lucas (1972) or money demand shocks.

3. Noise Traders

In addition to our rational speculators, there also exists a continuum of measure λ of noise traders. These traders behave in a manner similar to those found in DeLong, Shleifer,

---

variance in noise trading). The possibility of a (transactions) loss from holding domestic currency could also be easily motivated by a money-in-the-utility assumption, in which utility of foreign currency is greater than that derived from local currency.

[8]The results of the next section show that W_{t+1} is normally distributed.

Summers and Waldmann (1990). In particular, noise traders believe the expected return on domestic currency is N_t, rather than the quantity in the numerator of equation (7). Specifically, noise traders demand[9]

$$X_t^N = \frac{N_t}{\psi \sigma_\varepsilon^2}. \tag{9}$$

Total private sector demand for the domestic currency is thus

$$\frac{E\left(\Delta \varepsilon_{t+1} - r\varepsilon_t + F_t + \xi_t - \hat{F}_t + \frac{\psi}{2} V_t | \Omega_t^S\right)}{\psi \sigma_\varepsilon^2} + \frac{\lambda N_t}{\psi \sigma_\varepsilon^2}. \tag{10}$$

4. Government Demand

The other participant in the currency market is the government. Let G_t represent the government's holding of domestic currency at time t. If Q_t is the total amount of domestic currency, then $Q_t - G_t$ is the supply of currency held by the private sector. Without loss of generality, we let $Q = 0$ and note that market clearing requires

$$S_t + \lambda X_t^N + G_t = 0. \tag{11}$$

In order to maintain a linear equilibrium exchange rate, we restrict government demand to be linear in the exchange rate. Specifically, the government chooses constants $\tilde{\gamma}$, $\tilde{\eta}$ to form a trading rule

$$G_t = \tilde{\gamma} - \tilde{\eta}\varepsilon_t. \tag{12}$$

We could expand the government's rule to include its best estimate of F_t, so that it stabilizes the exchange rate around the fundamental: $G = \tilde{\gamma} + \tilde{\eta}(\hat{F}_t - \varepsilon_t)$. This would not change

---

[9]Note that noise traders share the same risk aversion parameter ψ as rational speculators and divide by the rational variance σ_ε^2. If either of these were greater for noise traders, than the "trading" presence of noise traders would be less. The relative size of noise traders (superscript N) to speculators (superscript S) can be interpreted as $\lambda \frac{\psi^S}{\psi^N} \frac{\sigma^{2,S}}{\sigma^{2,N}}$. For simplicity, I capture this entire term in λ. This simplification only matters for the analysis if government intervention somehow affected the risk terms of noise traders differently than speculators. Comparative statics regarding λ also indicate how the results are sensitive to extensions that endogenize the measure of noise traders, as in Jeanne and Rose (2002).

the main implications for the informational content of government intervention but does complicate the derivations, so we restrict ourselves to the form (12).

We assume that this demand curve is known to all investors, so that the extent and behavior of government trading is public knowledge. Moreover, the government commits to the parameters $\tilde{\gamma}$ and $\tilde{\eta}$, which are constant over time, so we abstract from problems of time inconsistency. The slope $\tilde{\eta}$ represents government's sensitivity to the asset's price and can be interpreted as the intensity with which the government responds to movements in the exchange rate. This coefficient parameterizes the government's exchange rate policy, with a larger $\tilde{\eta}$ implying a more interventionist regime. In order to simplify notation, we will normalize $\tilde{\eta}$ and $\tilde{\gamma}$ by $\psi\sigma_\varepsilon^2$, such that

$$
\begin{aligned}
\eta &\equiv \psi\sigma_\varepsilon^2 \tilde{\eta} \\
\gamma &\equiv \psi\sigma_\varepsilon^2 \tilde{\gamma}.
\end{aligned}
\tag{13}
$$

Note that σ_ε^2 is bounded away from zero due to the white-noise shock ξ.

We will ultimately solve for the optimal linear trading rule such that the government chooses γ and η to maximize the welfare of domestic producers. The government's budget constraint is that its trading rule is expected to at least break even in expectation. That is, if the government has resources W_t^G in terms of foreign currency at the beginning of period t and decides to purchase G_t units of domestic currency at time t, it will have $W_{t+1}^G = W_t^G + G_t \Delta\varepsilon_t$.[10] We require that $E_0(\Delta W_t^G) \geq 0$, which is equivalent to $E_0(G_t \Delta\varepsilon_t) \geq 0$. Substituting in for G_t, we find that $E_0(G_t \Delta\varepsilon_t) = -\eta E_0(\varepsilon_t \Delta\varepsilon_t)$, where we have used $E_0\{\Delta\varepsilon_t\} = 0$ as $t \to \infty$ (see the equilibrium in the next section). This, plus the presence of a mean-reverting component in ε (which will be the case in equilibrium), implies

$$
\eta \geq 0.
\tag{14}
$$

There is no restriction on γ, a constant quantity of domestic currency held by the government. The parameter γ will only affect the average level of the exchange rate and have no

---

[10]Note that we have assumed that the government does not earn interest on its foreign reserves. If it did earn interest, then the government could subsidize losses in domestic currency up to the extent it is earning interest on its foreign currency. We would still have a lower bound on η.

other role in the analysis. Note that G is a "stock" demand, while changes in this stock are determined by η and movements in ε.

A stronger budget constraint would be that $W_t^G \geq 0$ at every point in time. We relax this by assuming that the foreign central bank engages in a contract at period 0 in which it is willing to support any intervention that is expected to at least break even.[11] That is, it is willing to "purchase" at price zero a stream of loans/payoffs to the domestic central bank that have zero expected value. The domestic central bank may in fact be expected to turn a profit in that it is trading against noise traders. Given that the model is agnostic about the origins and thus citizenship of noise traders, we do not include these transfers from the noise traders to the central bank in the government's objective function.

Before we solve the government's optimal trading rule, we will treat η as an arbitrary (nonnegative) parameter and explore how the choice of η influences the ability of the currency market to reveal information. This will be the focus of the next section and constitute the main results of the paper.

3 Government trading and the informational content of the exchange rate

This section contains the main results of the paper. I first characterize the linear equilibrium.[12] I then show that the informational content of the exchange rate depends explicitly on the government's trading rule.

First I define the equilibrium concept:

An equilibrium exchange rate is a linear function of the state variables

$$\varepsilon_t = \beta_{0,t} + \beta_{F,t}(F_t - \hat{F}_t) + \beta_{N,t} N_t \tag{15}$$

that satisfies the market clearing condition (11) as well as individual demand equations (7)

---

[11]For example, the European Monetary System had a provision for reserve sharing to assist central banks in defending the EMS target zones.

[12]For tractability of the filtering problem, I restrict attention to linear equilibria.

and (9), such that $\sup_t |\beta_{i,t}| < \infty$, $i = 0, F, N$.

Note that the coefficients $\beta_{i,t}$ may depend on time, but not on the realized stochastic variables.[13] The fact that $F_t - \hat{F}_t$ enters as a single state variable rather than two separate variables is not a constraint on the equilibrium, but stems from the fact that the transactions payoff only depends on the difference. To simplify future expressions, I introduce a single notation for this difference:

$$\tilde{F}_t \equiv F_t - \hat{F}_t. \tag{16}$$

To characterize the equilibrium I proceed in three steps: (i) I posit that an equilibrium of the form (15) with arbitrary coefficients exists and solve the implied filtering problem. This yields the stochastic process for $\hat{F}_t$, which I use in step (ii) to derive the speculators' demand function. (iii) Finally, with speculative demand defined, I solve for the coefficients in (15) that satisfy the market clearing condition.

3.1   The Filtering Problem

In order to simplify the filtering problem and subsequent expressions, we allow the length of our time periods Δt become arbitrarily small, i.e. $\Delta t \to dt$, and use the continuous time Kalman Filter. Our underlying stochastic processes are

$$dX_t = AX_t + BdZ \tag{17}$$

$$X_t = \begin{pmatrix} F_t \\ N_t \\ \xi_t \end{pmatrix}, A = \begin{pmatrix} -\mu_F & 0 & 0 \\ 0 & -\mu_N & 0 \\ 0 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} \sigma_f & 0 & 0 \\ 0 & \sigma_n & 0 \\ 0 & 0 & \sigma_\xi \end{pmatrix}$$

$$\mu_F > 0, \; \mu_N \geq 0. \tag{18}$$

where $Z = (Z_1, Z_2, Z_3)'$ is a vector of three independent standard Brownian motions. The processes for F_t and N_t are mean reverting (N may be a random walk if $\mu_N = 0$, and the

---

[13]We will see below that V_t is a deterministic function of time, so that $\beta_{i,t}$ may be functions of V_t.

possibility that N may be explosive is discussed in footnote 14). I restrict μ_F to be strictly positive to ensure the existence of a steady state V when $\mu_N = 0$. A larger μ_i implies that process i is more mean reverting. The relative size of μ_F to μ_N will play an important role in the analysis below.

Note that X_t is not observable to wage setters in the economy. They can, however, deduce a linear function of X_t by observing ε_t:

$$Y_t \;\equiv\; \varepsilon_t - \beta_{0,t} - \beta_{F,t}\hat{F}_t = \beta_t' X_t \tag{19}$$

$$\beta \;=\; \begin{pmatrix} \beta_{F,t} \\ \beta_{N,t} \\ 0 \end{pmatrix}$$

That is $\beta_t' X_t \in \Omega_t^L$. Under a pegged exchange rate, we still have the signal derived from quantities (i.e. the G_t that equilibrates transaction demand and noise trading at the target ε). Formally, our filtering problem can be stated as the underlying state process X_t defined in (17) and the observation process Y_t defined in (19). The dynamics of our observation process Y_t can be derived from Ito's lemma:

$$dY_t \;=\; \overset{\circ}{\beta}{}' X_t dt + \beta' dX_t, \tag{20}$$

where a dot represents partial differentiation with respect to time. The solution to this problem is discussed in Kallianpur (1980). In our case, $\hat{F}_t$ is characterized by a stochastic differential equation

$$\begin{aligned} d\hat{F}_t \;&=\; -\mu_F \hat{F}_t dt \\ &\quad + L_t \left\{ \left(\overset{\circ}{\beta}_{F,t} - \overset{\circ}{\beta}_{N,t}\, \frac{\beta_{F,t}}{\beta_{N,t}} + (\mu_N - \mu_F)\beta_{F,t} \right)\tilde{F}_t dt + \beta' B dZ \right\}, \end{aligned} \tag{21}$$

$$L_t \;\equiv\; \frac{\overset{\circ}{\beta}_{F,t} - \overset{\circ}{\beta}_{N,t}\,\frac{\beta_{F,t}}{\beta_{N,t}} + \beta_{F,t}\left((\mu_N - \mu_F)V_t + \sigma_f^2 \right)}{\beta_{F,t}^2 \sigma_f^2 + \beta_{N,t}^2 \sigma_n^2}.$$

Letting $\tilde{X}_t = (\tilde{F}_t,\, N_t,\, \xi_t)$, we can use (21) to express

$$d\tilde{X}_t = \tilde{A}_t \tilde{F} dt + \tilde{B} dZ, \tag{22}$$

12

where $\tilde{A} = A$ except in the first element, which is

$$\tilde{A}_{t[1,1]} = -\{\mu_F + L_t(\overset{\circ}{\beta}_{F,t} - \overset{\circ}{\beta}_{N,t} \frac{\beta_{F,t}}{\beta_{N,t}} + (\mu_N - \mu_F)\beta_{F,t})\},$$

and $\tilde{B} = B$ except in the first row, which is

$$\tilde{B}_{[1,\cdot]} = (\sigma_f, 0, 0) - L_t \beta' B.$$

The variance of our estimate is given by a nonstochastic differential equation

$$\overset{\circ}{V}_t = -2\mu_F V_t + \sigma_f^2 - \frac{\left[(\overset{\circ}{\beta}_{F,t} - \overset{\circ}{\beta}_{N,t} \frac{\beta_{F,t}}{\beta_{N,t}} + (\mu_N - \mu_F)\beta_{F,t})V_t + \sigma_f^2\right]^2}{\beta_{F,t}^2 \sigma_f^2 + \beta_{N,t}^2 \sigma_n^2}. \tag{23}$$

We have solved out any reference to $\hat{N}_t$ using the fact that $Y_t = \beta' X_t$, which implies

$$\hat{N}_t = E\{N_t | \Omega_t^L\} = \frac{1}{\beta_{N,t}}\{Y_t - \beta_{F,t} \hat{F}_t\}. \tag{24}$$

3.2   Speculative Demand

We now solve for speculative demand. Let dQ represent the payoff to a unit of domestic currency net of opportunity cost (which is capital gains plus transactions payoff minus $r\varepsilon_t$):

$$dQ_t = d\varepsilon_t + \left(\tilde{F}_t + \frac{1}{2}(V_t + \sigma_\xi^2)\right) dt + d\xi_t - r\varepsilon_t dt. \tag{25}$$

The solution to the optimal portfolio problem implies

$$\psi \sigma_\varepsilon^2 S_t dt = E\left\{dQ_t \mid \Omega_t^S = \{\varepsilon_t, F_t\}\right\}. \tag{26}$$

Our candidate linear equilibrium (15), equation (22), and Ito's lemma imply

$$E\{dQ_t \mid \varepsilon_t, F_t\} = \left(\tilde{F}_t + \frac{1}{2}(V_t + \sigma_\xi^2)\right) dt - r\varepsilon_t dt + (\overset{\circ}{\beta}_t + \beta_t' \tilde{A}_t)\tilde{X} dt \tag{27}$$

3.3   Equilibrium

We now impose the market clearing condition and solve for the equilibrium. Market clearing requires

$$\psi \sigma_\varepsilon^2 S_t + \lambda N_t + \gamma - \eta \varepsilon_t = 0. \tag{28}$$

13

Using equations (26) and(27) for $\psi\sigma_\varepsilon^2 S_t$ and (15) for ε_t, market clearing implies the following relationships

$$\beta_{F,t} = \frac{1}{r+\eta+\mu_F}\left\{\overset{\circ}{\beta}_{F,t} - \beta_{F,t}L_t + 1\right\} \tag{29}$$

$$\beta_{N,t} = \frac{\lambda}{r+\eta+\mu_N}$$

$$\beta_{0,t} = \frac{\gamma + \frac{1}{2}\left(V_t + \sigma_\xi^2\right)}{r+\eta}$$

We will see below that the important variable in terms of information is the ratio of β_F to

β_N. I thus define a new variable

$$\rho_t \equiv \frac{\beta_{F,t}}{\beta_N}, \tag{30}$$

where I have dropped the time subscript on β_N as by (29) it is constant over time. As L_t depends on V_t as well as $\overset{\circ}{\rho}_t$, equation (29) defines an implicit expression for $\overset{\circ}{\rho}$ as a function of ρ and V :

$$\overset{\circ}{\rho}_t = g(\overset{\circ}{\rho}_t, \rho_t, V_t)\left\{\begin{array}{c} (\mu_N + r + \eta)(\sigma_f^2\rho^2 + \sigma_n^2)(\lambda\rho - 1) \\ -\lambda\rho(\mu_N - \mu_F)\left(\sigma_n^2 - (\mu_N - \mu_F)\rho^2 V\right) \end{array}\right\}. \tag{31}$$

The function g is expressed explicitly in equation (37) in Appendix A and is strictly positive at the steady state. Moreover, at the steady state $\sigma_n^2 > -\mu_N\rho^2 V$. Thus at the steady state $\rho > \frac{1}{\lambda}$ iff $\mu_N > \mu_F$. The other variable that is a function of time is V_t given by (23) which we rewrite here (as a function of ρ):

$$\overset{\circ}{V}_t = -2\mu_F V_t + \sigma_f^2 - \frac{\left[(\frac{\mu_N+r+\eta}{\lambda\rho_t}\overset{\circ}{\rho}_t + (\mu_N - \mu_F))V_t + \sigma_f^2\right]^2}{\sigma_f^2 + \rho_t^{-2}\sigma_n^2}. \tag{32}$$

We have a two variable differential system that satisfies market clearing and individual demand functions. A solution to this system will satisfy our criteria for a linear equilibrium. The system of differential equations (31) and (32) can be represented in the phase plane depicted in figure 1. At the saddle point steady state, the $\overset{\circ}{V} = 0$ line is downward sloping, while the $\overset{\circ}{\rho} = 0$ line has a shallower slope that is nonpositive. The (linearized) dynamics across the phase plane are also depicted in figure 1.

Figure 1: Phase Plane
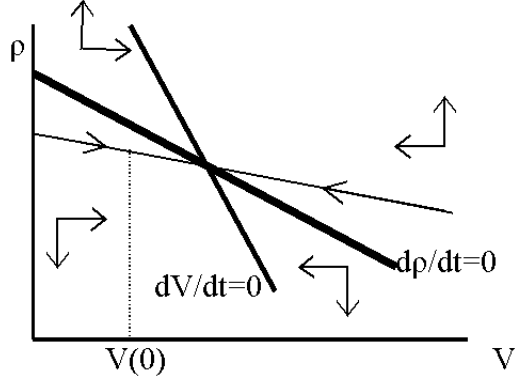
Proposition 1 *There exists a saddle-point stable steady state equilibrium.*

Proof: Appendix A.■

Note that at the steady state, our filter variance satisfies

$$0 = -2\mu_F V + \sigma_f^2 - \frac{\left[(\mu_N - \mu_F)V + \sigma_f^2\right]^2}{\sigma_f^2 + \rho^{-2}\sigma_n^2}. \tag{33}$$

Implicit differentiation indicates that the $\overset{\circ}{V} = 0$ line slopes down, which I restate as:

Lemma 2 *The steady state variance of the filter estimate is decreasing in ρ.*

This makes intuitive sense as the larger the ρ, the more sensitivity the exchange rate displays toward F, and thus the better ε reveals movements in the fundamental. Note that it is the relative sensitivity that matters and not the absolute levels of β_F and β_N – scaling both up or down by a constant proportion does not change the value of the exchange rate as a signal. Note that equation (33) implies that the number of noise traders λ does not enter the term for variance directly. We will see that λ influences the steady state variance only

15

through ρ, while the variance of noise σ_n^2 increases V directly as well as indirectly through a decrease in ρ.

3.4 Government intervention and information

We now turn to the impact of government intervention on the informational content of the exchange rate. The following is a major result of the paper:

Proposition 3 *Government intervention increases the informational content of the exchange rate if the fundamental is more mean reverting than the noise and decreases the informational content if the fundamental is more persistent than the noise. That is, $\frac{dV}{d\eta} \gtreqless 0$ if and only if $\mu_N \gtreqless \mu_F$, where V is the steady state variance.*

Proof: Lemma 2 indicates that our variance is decreasing in ρ, so we need to explore the sign of $\frac{d\rho}{d\eta}$ at the steady state. Note that our $\overset{\circ}{V}= 0$ line is not dependent on η, so we only need to look at the behavior of the $\overset{\circ}{\rho}= 0$ line. From (31), the steady state ρ satisfies

$$(\sigma_f^2\rho^2 + \sigma_n^2)(\lambda\rho - 1)(\mu_N + r + \eta) - \lambda(\mu_N - \mu_F)\rho\left(\sigma_n^2 - (\mu_N - \mu_F)\rho^2 V\right) = 0. \qquad (34)$$

Implicit differentiation yields

$$\frac{d\rho}{d\eta} = \frac{(\sigma_f^2\rho^2 + \sigma_n^2)(1 - \lambda\rho)}{\partial F/\partial \rho}. \qquad (35)$$

Saddle-point stability implies the denominator is positive (see Appendix A), so $sign(\frac{d\rho}{d\eta}) = sign(1 - \lambda\rho)$. From our discussion above regarding equation (31), we have $\rho > \frac{1}{\lambda}$ iff $\mu_N > \mu_F$. Thus $sign(1 - \lambda\rho) = sign(\mu_F - \mu_N)$. This implies that in response to an increase in η, the $\overset{\circ}{\rho}= 0$ line shifts up if $\mu_F > \mu_N$, down if $\mu_F < \mu_N$, and remains invariant if $\mu_F = \mu_N$. Given that the steady state variance is increasing as we move down the $\overset{\circ}{V}= 0$ line, this proves the result.∎

Remark on a completely pegged exchange rate: Note that as $\eta \to \infty$, $\rho \to \frac{1}{\lambda}$. At a fixed exchange rate, government demand is our signal and equals $\psi\sigma_\varepsilon^2 G_t = \tilde{F}_t + \lambda N_t + c_0$, where c_0 is a constant. Thus, the relevant ratio for signal extraction is $\frac{1}{\lambda}$, indicating there is no discontinuity in the result as the government completely pegs the exchange rate.

16

3.4.1 Intuition

Mathematically, we see the result by taking the limit of ρ as $\eta \to \infty$. As the government intervenes more aggressively, both β_F and β_N approach zero. However, the *ratio* of the two is what determines the informational content of the exchange rate. As noted above, if $\mu_F > \mu_N$, then ρ is less than $\frac{1}{\lambda}$ in the absence of government trading, reflecting that speculators trade against F relatively intensively. Conversely, ρ is greater than $\frac{1}{\lambda}$ if the fundamental is relatively persistent. Equation (34) implies that as $\eta \to \infty$, $\rho \to \frac{1}{\lambda}$. Thus ρ (and therefore information regarding F) is increasing in η if and only if our fundamental is relatively mean reverting.

The economic intuition for this result hinges on how speculators respond to government intervention. Speculators use their information to distinguish between the two shocks in their pursuit of capital gains. Suppose that $\mu_F > \mu_N$ so that the fundamental is more mean reverting than the noise. In this case, a positive shock to F_t will be quickly reversed and so implies a depreciating exchange rate. Speculators, on observing the shock to F_t, will have a capital gains incentive to sell the currency, dampening the impact on the current spot rate. Given that $\mu_F > \mu_N$, this dampening is greater for fundamental shocks than for noise shocks (take the extreme of the noise as a random walk, and we see that there is no off-setting speculation for a shock to N).[14] Speculators therefore trade against the direct impact of a shock to F_t to a greater extent than a noise shock, reducing the relative sensitivity of the exchange rate to the fundamental. As the government intervenes, it smooths all shocks

---

[14]In a similar sense, if a noise shock is likely to be followed by an even larger noise shock (say from momentum trading), then the intuition would be that speculators would amplify the noise trades. The more the noise traders exhibit explosive dynamics, the smaller (more negative) μ_N. The analysis is essentially the same as presented, i.e. reducing μ_N implies more noise relative to signal and a smaller informational cost of government intervention. Allowing speculators to amplify noise shocks is problematic in the current set-up in that (absent government intervention) speculators and noise traders are the only two class of actors in the market and thus cannot both be simultaneously buying or selling. Separating the speculative and transaction demands for currency would introduce an additional class of market participants and support an equilibrium with $\mu_N < 0$.

indiscriminately, crowding out speculative opportunities. That is, the government displaces targeted trading (recall speculators in this case trade against F shocks relatively intensively) with its uniform trading (the government cannot distinguish between F and N as well as the market). So we move from a situation in which F had relatively less impact than N, to a situation where the two shocks share more equal impact. The reverse is true if $\mu_F < \mu_N$: In this case, speculators trade relatively more against the mean reverting noise, accentuating the information content of the exchange rate.

Note that if $\mu_F = \mu_N$, the two processes have the same mean-reversion and thus a shock to F_t or N_t has identical implications for capital gains. In this case, speculators have no reason to differentiate between the two in terms of capital gains (although it matters for transaction demand). This is also the case in which government intervention has no effect on informational content. The fact that government trading has no affect in this special case underscores that government trading is not introducing noise directly or hiding the signal. The mechanism through which government trading impacts information revelation is directly related to the pursuit of capital gains.

4 Optimal Government Intervention and Additional Comparative Statics

We now turn to the question of optimal government intervention. I assume the objective of the government is to maximize the steady state welfare of domestic residents, which is equivalent to maximizing their real wage. It is a direct result from (3), that the objective is equivalent to minimizing the steady state variance, V. Proposition 3 states that the variance is a declining function of η if $\mu_N < \mu_F$ and vice versa. Therefore, the government will chose to peg if F is relatively mean reverting. If F is relatively persistent, on the other hand, the government wants to decrease η. The budget constraint requires that $\eta \geq 0$, so that the optimal policy if productivity is relatively persistent is $\eta = 0$, implying a free float.

Of course, the model is extremely parsimonious in order to highlight the informational

consequences of government trading. The only friction in the model is one of information aggregation and the government's optimal policy is therefore one of maximizing information revelation. We have found that this implies either a hard peg or a free float, depending on the relative persistence of the underlying random variables. While this highlights the main contribution of the paper, there are in practice other motivations for reducing exchange rate volatility, such as limiting the impact of monetary shocks in the presence of nominal rigidities. Introducing such additional considerations may warrant intervention even if it reduces the quality of the exchange rate as a signal. In particular, η may be an interior optimum.

In assessing such potential trade-offs, we need to explore the significance of government intervention as we vary other parameters of the model. First, it is clear from Proposition 3 that for the government to have any effect on information aggregation, there must be a difference in persistence between the fundamental and noise. An another important ingredient is the magnitude of noise trading. We have two different parameters that measure the presence of noise trading. The number of noise traders is captured by λ, while the variance of noise trading is captured by σ_n^2. It can be shown that ρ is decreasing in λ and σ_n^2 so that (as we would expect) more noise traders (in number or in variance) reduces the relative importance of the fundamental in exchange rate movements. Moreover, as the relative importance of speculators decreases, the impact of government intervention is less relevant to information aggregation. In particular, as $\lambda \to \infty$, we have $\frac{dV}{d\eta} \to 0$ and, similarly, as $\sigma_n^2 \to \infty$, $\frac{dV}{d\eta} \to 0$. Recall that the mechanism works through the crowding out of speculators – the less important speculators are to begin with, the less impact government intervention has on the informational content of the asset price.

5 Conclusion

This paper highlights a key interaction between a government that smooths an asset price and informed speculators that trade for capital gains. In particular, we have seen that government intervention reduces the incentive for speculators to use their private informa-

tion to differentiate between shocks. In the context of exchange rates, we have found that the policy implications depend on the relative capital gains potential of particular types of shocks. If we think of noise traders as generating high frequency fluctuations in the exchange rate, for example, the results favor less intervention. Conversely, if our "noise" shocks are relatively persistent, then the tendency should be to manage the exchange rate.

The paper's main results extend beyond exchange rates to any asset that is traded for capital gains. Governments have historically engaged in commodity price stabilizations through the use of buffer stocks. There has also been a recent rise in the advocacy of government intervention in equity markets (e.g. Japan in 2001, Hong Kong in 1998 and Fed Chair Greenspan's famous "irrational exuberance" speech of 1996). This paper points out that even with complete transparency, government trading may have adverse consequences on a market's ability to aggregate information.

6 Appendix

6.1 A. Proof of Proposition 1

We start by repeating equation (29):

$$\beta_{F,t} = \frac{1}{r + \eta + \mu_F} \left\{ \overset{\circ}{\beta}_{F,t} - \beta_{F,t} L_t + 1 \right\}. \tag{36}$$

Substituting in $\rho_t = \frac{\beta_{F,t}}{\beta_N}$ and the expression for L_t from (21) implies

$$\overset{\circ}{\rho}_t = g(\overset{\circ}{\rho}_t, \rho_t, V_t) \left\{ \begin{array}{c} (\mu_N + r + \eta)(\sigma_f^2 \rho^2 + \sigma_n^2)(\lambda\rho - 1) \\ -\lambda\rho(\mu_N - \mu_F)\left(\sigma_n^2 - (\mu_N - \mu_F)\rho^2 V\right) \end{array} \right\} \tag{37}$$

$$g(\overset{\circ}{\rho}_t, \rho_t, V_t) = \lambda(\sigma_n^2 - 2(\mu_N - \mu_F)V_t\rho_t^2 - \rho_t V_t \overset{\circ}{\rho}_t).$$

The other differential equation of the system is

$$\overset{\circ}{V}_t = -2\mu_F V_t + \sigma_f^2 - \frac{\left[(\frac{\mu_N + r + \eta}{\lambda\rho_t})\overset{\circ}{\rho}_t + (\mu_N - \mu_F))V_t + \sigma_f^2 \right]^2}{\sigma_f^2 + \rho_t^{-2}\sigma_n^2}. \tag{38}$$

It is possible to show that at the steady state, $\sigma_n^2 - 2\mu_N V_t \rho_t^2 > 0$,[15] so that $g(\overset{\circ}{\rho}_t, \rho_t, V_t) > 0$ at the steady state.

I first show the existence of a steady state and then prove it is saddle point stable (for completeness, I also show the phase plane is as drawn in the text). At the steady state, from (37) and (38) we have

$$(\mu_N + r + \eta)(\sigma_f^2 \rho^2 + \sigma_n^2)(\lambda \rho - 1) - \lambda \rho(\mu_N - \mu_F)\left(\sigma_n^2 - (\mu_N - \mu_F)\rho^2 V_\rho\right) = 0, \qquad (39)$$

where V_ρ solves

$$(\mu_N - \mu_F)^2 \rho^2 V^2 + 2(\mu_N \rho^2 \sigma_f^2 + \mu_F \sigma_n^2)V - \sigma_n^2 \sigma_f^2 = 0. \qquad (40)$$

We are looking for a $\rho > 0$ that solves (39). Note that (40) implies that V_ρ at $\rho = 0$ is finite. Thus, the left hand side of (39) is negative at $\rho = 0$. Moreover, differentiation of (40) implies that $\frac{d\rho^2 V_\rho}{d\rho} > 0$. As $\rho \to \infty$, the higher order coefficients of ρ in (39) dominate. As they are positive, the left hand side of (39) becomes positive as $\rho \to \infty$. Thus, there exists a steady state ρ and V at which $\rho > 0$. Now for stability.

In a $\rho \times V$ phase plane, equation 39 maps out a "$\overset{\circ}{\rho} = 0$" line while equation (40) traces a "$\overset{\circ}{V} = 0$" curve. In a small neighborhood around a steady state the dynamics of the linear approximation characterize the dynamics of the system. That is,

$$\frac{d}{dt}\begin{pmatrix} \rho \\ V \end{pmatrix} = \begin{pmatrix} \frac{\partial \overset{\circ}{\rho}}{\partial \rho} & \frac{\partial \overset{\circ}{\rho}}{\partial V} \\ \frac{\partial \overset{\circ}{V}}{\partial \rho} & \frac{\partial \overset{\circ}{V}}{\partial V} \end{pmatrix}\begin{pmatrix} \rho \\ V \end{pmatrix}, \qquad (41)$$

where the derivatives are evaluated at the steady state. Saddle point stability requires that the two eigenvalues of the Jacobian have opposite signs. This is equivalent to

$$\frac{\partial \overset{\circ}{\rho}}{\partial \rho}\frac{\partial \overset{\circ}{V}}{\partial V} - \frac{\partial \overset{\circ}{\rho}}{\partial V}\frac{\partial \overset{\circ}{V}}{\partial \rho} < 0, \qquad (42)$$

---

[15]Note that $\rho^2 V$ is the variance of our estimate of N as $E(N - \hat{N}) = \rho E(F - \hat{F})$. The unconditional steady state variance of N is $\frac{\sigma_n^2}{2\mu_N}$. This must be greater than the conditional variance as long as $\rho > 0$, implying $\frac{\sigma_n^2}{2\mu_N} > \rho^2 V$.

which in turn is the same as

$$\frac{d\rho}{dV}\Big|_{\overset{\circ}{\rho}=0} > \frac{d\rho}{dV}\Big|_{\overset{\circ}{V}=0},\tag{43}$$

so that at the stable steady state, the $\overset{\circ}{\rho}=0$ line crosses the $\overset{\circ}{V}=0$ from below. Straight forward differentiation of (40) and (39) indicates that $\frac{\partial \overset{\circ}{V}}{\partial V}<0$, $\frac{\partial \overset{\circ}{V}}{\partial \rho}<0$ and $\frac{\partial \overset{\circ}{\rho}}{\partial V}>0$ ($\frac{\partial \overset{\circ}{\rho}}{\partial V}=0$ when $\mu_F = \mu_N$) Thus the $\overset{\circ}{V}=0$ line is downward sloping. Differentiating (39) with respect to ρ yields

$$\begin{aligned}\frac{\partial \overset{\circ}{\rho}}{\partial \rho} &= (\mu_N + r + \eta)(3\lambda\sigma_f^2\rho^2 - 2\sigma_f^2\rho + \lambda\sigma_n^2) \\ &\quad -\lambda(\mu_N - \mu_F)\left(\sigma_n^2 - 3(\mu_N - \mu_F)\rho^2 V\right).\end{aligned}\tag{44}$$

If $\mu_N > \mu_F$, then equation (39) implies that $\rho > \frac{1}{\lambda}$ at the steady state. Therefore, $3\lambda\sigma_f^2\rho^2 - 2\sigma_f^2\rho > 0$. The only other negative term is $-\lambda\mu_N\sigma_n^2$ which cancels with itself, implying $\frac{\partial \overset{\circ}{\rho}}{\partial \rho} > 0$. Now for the case $\mu_F > \mu_N$. Equation (39) implies

$$\begin{aligned}&(\mu_N + r + \eta)(\lambda\sigma_f^2\rho^2 - \sigma_f^2\rho) \\ &= (\rho^{-1} - \lambda)\sigma_n^2(\mu_N + r + \eta) + \lambda(\mu_N - \mu_F)\left(\sigma_n^2 - (\mu_N - \mu_F)\rho^2 V\right).\end{aligned}\tag{45}$$

Substituting into (44) and canceling terms implies

$$\frac{\partial \overset{\circ}{\rho}}{\partial \rho} > 0.\tag{46}$$

Therefore,

$$\frac{d\rho}{dV}\Big|_{\overset{\circ}{\rho}=0} = -\frac{\frac{\partial \overset{\circ}{\rho}}{\partial V}}{\frac{\partial \overset{\circ}{\rho}}{\partial \rho}} \leq 0,\tag{47}$$

so that the $\overset{\circ}{\rho}=0$ line is downward sloping when $\mu_N \neq \mu_F$ and is horizontal when $\mu_N = \mu_F$. The partial derivatives also indicate the dynamics depicted in figure (1).

Note that as $V \to 0$, the $\overset{\circ}{V}=0$ line implies $\rho \to \infty$, while the $\overset{\circ}{\rho}=0$ implies ρ is finite. Thus at V close to zero, the $\overset{\circ}{V}=0$ line lies "above" the $\overset{\circ}{\rho}=0$ line, implying the latter crosses from below as V increases. That is, there exists a steady state at which $\frac{d\rho}{dV}\Big|_{\overset{\circ}{\rho}=0} > \frac{d\rho}{dV}\Big|_{\overset{\circ}{V}=0} \implies \frac{\partial \overset{\circ}{\rho}}{\partial \rho}\frac{\partial \overset{\circ}{V}}{\partial V} - \frac{\partial \overset{\circ}{\rho}}{\partial V}\frac{\partial \overset{\circ}{V}}{\partial \rho} < 0.$ ∎

6.2   B. Disaggregated Information

In the text we assumed that speculators had perfect information regarding the fundamental, F. More generally, let speculator j have the information set $\Omega_t^{S_j} = \left\{ s_t^j, \{\varepsilon_\tau\}_{\tau=0}^{\tau=t} \right\}$ where $s_t^j = F_t + \nu_t^j$ and

$$\nu_t^j \quad \sim \quad N(F_t, \sigma_\nu^2), \; iid \text{ across } j \tag{48}$$

$$\int_{j \in J} \nu_t^j dj \quad = \quad F_t.$$

That is, each speculator has a noisy signal of the fundamental, but the market as a whole has perfect information. The ability of the market to aggregate and reveal this information is captured by V.

Given that s_t^j and $\hat{F}_t$ are independent and jointly normal, least squares projection implies

$$E\left\{ F_t | \Omega_t^{S_j} \right\} \quad = \quad a_t S_t + (1 - a_t)\hat{F}_t \tag{49}$$

$$a_t \quad \equiv \quad \frac{V_t}{V_t + \sigma_\nu^2}.$$

Aggregating over speculators and using $\int_{j \in J} \nu_t^j dj = F_t$ implies

$$\int_{j \in J} E\left\{ F_t | \Omega_t^{S_j} \right\} dj \quad = \quad a_t F_t + (1 - a_t)\hat{F}_t, \text{ and so} \tag{50}$$

$$\int_{j \in J} E\left\{ \tilde{F}_t | \Omega_t^{S_j} \right\} dj \quad = \quad a_t \tilde{F}_t. \tag{51}$$

The aggregate speculative demand is the same as (26), but with $\tilde{F}_t$ replaced by $a_t \tilde{F}_t$. Solving for market clearing as before, we have the following differential equation for ρ :

$$\overset{\circ}{\rho}_t \quad = \quad \tilde{g}(\overset{\circ}{\rho}_t, \rho_t, V_t) \left\{ \begin{array}{c} (\mu_N + r + \eta)(\sigma_f^2 \rho^2 + \sigma_n^2)(\lambda\rho - a) \\ -a\lambda\rho(\mu_N - \mu_F)\left(\sigma_n^2 - (\mu_N - \mu_F)\rho^2 V\right) \end{array} \right\} \tag{52}$$

$$\tilde{g}(\overset{\circ}{\rho}_t, \rho_t, V_t) \quad = \quad \lambda(\sigma_n^2 + (1-a)\sigma_f^2 \rho^2 - 2a(\mu_N - \mu_F)V_t \rho_t^2 - a\rho_t V_t \overset{\circ}{\rho}_t).$$

Note that (52) and (44) are the same if $a = 0$ (i.e. when private signals are perfectly informative). Moreover, as in the full information case, $\rho > \frac{a}{\lambda}$ if $\mu_N > \mu_F$, and $\rho < \frac{a}{\lambda}$ if $\mu_N < \mu_F$. A similar argument as presented in Appendix A proves the existence of a saddle path stable steady state. We also have:

Proposition 4 *At a saddle-point stable steady state, government intervention increases the informational content of the exchange rate if the fundamental is more mean reverting than the noise and decreases the informational content if the fundamental is more persistent than the noise. That is, $\frac{dV}{d\eta} \gtreqless 0$ if and only if $\mu_N \gtreqless \mu_F$.*

Proof: Recall that $sign(\frac{dV}{d\eta}) = sign(-\frac{d\rho}{d\eta})$. As saddle path stability requires $\frac{\partial \mathring{\rho}}{\partial \rho}|_{\mathring{\rho}=0} > 0$, the implicit function theorem implies that $sign(-\frac{d\rho}{d\eta}|_{\mathring{\rho}=0}) = sign(\frac{\partial \mathring{\rho}}{\partial \eta}|_{\mathring{\rho}=0})$. Differentiating equation (52) at the steady state implies

$$\frac{\partial \mathring{\rho}}{\partial \eta}|_{\mathring{\rho}=0} = (\sigma_f^2 \rho^2 + \sigma_n^2)(\lambda\rho - a). \tag{53}$$

As $sign(\lambda\rho - a) = sign(\mu_N - \mu_F)$, we have $sign(\frac{dV}{d\eta}) = sign(-\frac{d\rho}{d\eta}|_{\mathring{\rho}=0}) = sign(\mu_N - \mu_F)$.∎

6.3 C. Infinite Horizon

In this appendix, I show how the results of the paper extend to the case where speculators have infinite horizons. Specifically, speculators chose consumption, C_t, and portfolio policies to maximize

$$E\left[\int_{\tau=t}^{\infty} -\exp(-\delta\tau - C_\tau)d\tau \mid \Omega_t^S, \right] \tag{54}$$

where $\Omega_t^S = \left\{F_t, \{\varepsilon_s\}_{s=0}^{s=t}\right\}$. All other aspects of the model in the text remain the same. The speculator's state variables are $\tilde{F}$, N, and wealth, W. The stochastic behavior of $\tilde{F}$, and N are described in the text. Wealth follows

$$dW_t = (rW_t - C_t)dt + S_t dQ_t. \tag{55}$$

This is a standard portfolio problem and solved in a very similar setup by Wang (1993). I therefore omit the details of the solution. The optimization produces speculative demand S_t that can be expressed as

$$S_t = -(J_{WW}\sigma_Q^2)^{-1}\left(E_t dQ + J_{W\tilde{F}}\sigma_{Q,\tilde{F}} + J_{WN}\sigma_{Q,N}\right), \tag{56}$$

24

where $\sigma_{i,j}$ is the covariance between state variable i and j and $J(W, \tilde{F}, N)$ is the speculator's value function. In particular, J_{Wi} is the change in the marginal value of an extra dollar of wealth in response to an increase in the state variable $i = \tilde{F}, N, W$. Note that the first term $E_t dQ$ is expected capital gains and was the focus of our myopic model in the body of the paper. We also have the additional components that make up hedging demand: $-(J_{WW})^{-1}(J_{W\tilde{F}}\sigma_{Q,\tilde{F}} + J_{WN}\sigma_{Q,N})$.[16]

Figure 2 explores the informational effect of government intervention through its impact on ρ. Panel A shows that the basic intuition of the one-period model extends to the infinite horizon case. That is, an increase in government intervention η leads to a decline in ρ (and therefore information) if $\mu_N > \mu_F$ and an increase in ρ if $\mu_N < \mu_F$. However, we can see in Panel B (in which σ_n^2 is relatively large) that we may still lose information even if the fundamental is more mean reverting than noise. This deviation from Proposition 3 is due to hedging demand.

The intuition as to why hedging demand influences the result is as follows. First, concavity implies $J_{WW} < 0$, so the larger $J_{W\tilde{F}}\sigma_{Q,\tilde{F}} + J_{WN}\sigma_{Q,N}$, the greater the hedging demand for domestic currency. All else equal, $-(J_{WW})^{-1}J_{W\tilde{F}}$ is decreasing in $\tilde{F}$ and (less sensitively) increasing in N while the opposite is true for J_{WN}. In panel A, $\sigma_{Q,\tilde{F}}$ and $\sigma_{Q,N}$ are positive and an increase in $\tilde{F}$ or N leads to reduced hedging demand. That is hedging demand supports the stabilizing nature of myopic demand.

However, it may be the case that $\sigma_{Q,\tilde{F}}$ is negative. The ambiguity of $\sigma_{Q,\tilde{F}}$ arises because dQ is a function of $d\tilde{F}$ and dN. While $d\tilde{F}$ is of course positively correlated with itself, it is negatively correlated with dN (a shock to N causes $\hat{F}$ to increase (uninformed observers are "fooled" by the increase in ε) but not F itself, causing a fall in $\tilde{F} = F - \hat{F}$). If this negative correlation dominates (which it will when σ_n^2 is large), the correlation of $d\tilde{F}$ with dQ may also be negative. This corresponds to panel B. In this case, an increase in $\tilde{F}$ leads to greater hedging demand, working opposite to myopic demand. Hedging demand therefore may result in speculators buying when the price is relatively high, magnifying the exchange

---

[16]See Ingersoll (1987) for a discussion of hedging demand.

Figure 2: The impact of government intervention in an infinite horizon model
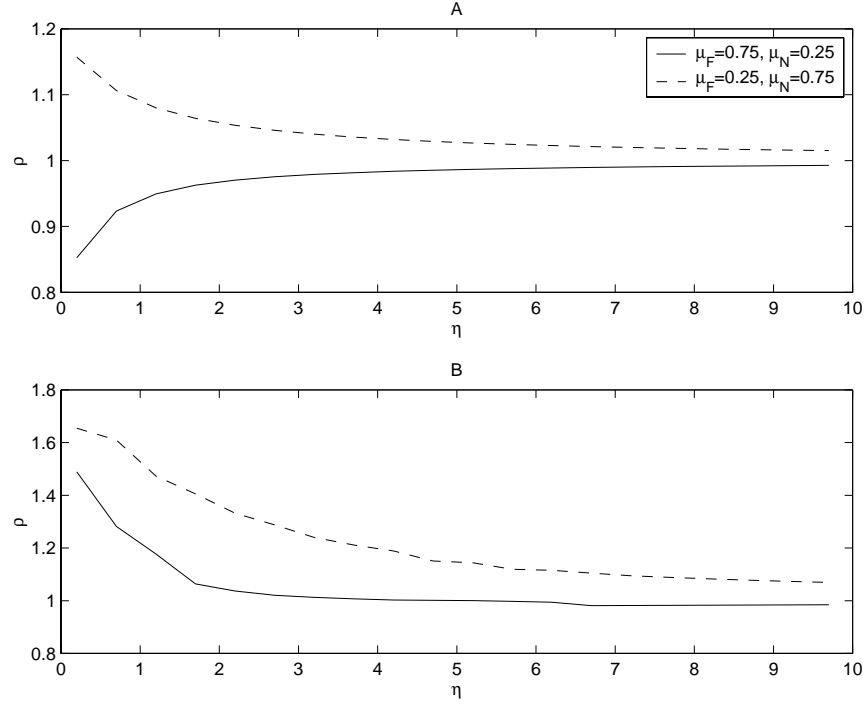
Panel A: $\sigma_n = 1$, Panel B: $\sigma_n = 3$. In both panels: $\sigma_f = 1, \sigma_\xi = 2, r = 0.05, \psi = 1, \lambda = 1$.

rate's response to F. This implies that the exchange rate may be more sensitive to F in the presence of hedging demand. As the government intervenes, $\sigma_{Q,\tilde{F}} \to 0$ driving hedging demand to zero, and so we lose the information generated through hedging demand.

In short, when $\mu_N > \mu_F$, both myopic and hedging demand provide information regarding F and government intervention negatively impacts information revelation. However, if $\mu_N < \mu_F$, hedging and myopic demand may work at cross purposes, yielding an adverse informational impact of intervention even when the fundamental is relatively mean reverting.

26

References

[1] De Long, J.B., A. Shleifer, L.H. Summers, and R.J. Waldmann (1990), "Noise Trader Risk in Financial Markets," Journal of Political Economy, Vol. 98 No. 4., 703-738.

[2] Dotsey, M. and R.G. King (1986), "Informational Implications of Interest Rate Rules," American Economic Review, Vol. 76 No. 1, 33-42.

[3] Evans, M. and R.K. Lyons (2002), "Order Flow and Exchange Rate Dynamics," Journal of Political Economy, Vol. 110, No. 1, 170-180.

[4] Flood, R.P. and R.J. Hodrick (1985), "Central Bank Intervention in a Rational Open Economy: A Model with Asymmetric Information," in J. Bhandari (ed.), *Exchange Rate Management under Uncertainty,* MIT Press.

[5] Grossman, S.J. (1989), *The Informational Role of Prices*, MIT Press.

[6] Hayek, F.A. (1945), "The Use of Knowledge in Society," American Economic Review, Vol. 35 No. 4, 519-530.

[7] Helpman, E. (1981), "An Exploration in the Theory of Exchange-Rate Regimes," Journal of Political Economy, Vol. 89 No. 51, 865-890.

[8] Ingersoll, Jonathan E. (1987), *Theory of Financial Decision Making*, Rowman & Littlefield.

[9] Kallianpur, Gopinath (1980), *Stochastic Filtering Theory,* Springer-Verlag.

[10] Kimbrough, K.P. (1984), "Aggregate Information and the Role of Monetary Policy in an Open Economy," Journal of Political Economy, Vol. 92 No.2, 268-85.

[11] Lucas, R.E. Jr. (1972), "Expectations and the Neutrality of Money," Journal of Economic Theory, Vol. 4 No. 2, 103-24.

[12] ——————— (1973), "Some International Evidence on Output-Inflation Tradeoffs," American Economic Review, Vol. 63, 326-34.

[13] Jeanne, Olivier and Andrew K. Rose, "Noise Trading and Exchange Rate Regimes," Quarterly Journal of Economics, Vol. 117 No. 2, 537-570.

[14] Stock, James H. and Mark W. Watson ed. (1993), Business cycles, indicators, and forecasting, Chicago : University of Chicago Press, 1993.

[15] Wang, Jiang (1993), "A Model of Intertemporal Asset Prices Under Asymmetric Information," Review of Economic Studies, Vol. 60, 249-282.