MLE/GMM Problem Set   Prof Evans

1)   Using the data supplied on my Web page, calculate the maximum likelihood estimates for the following model.

$$x_t = \alpha_o + \alpha_1 \, \sigma_t^2 + \varepsilon_t \qquad\qquad\qquad \varepsilon_t \sim N(0 \, \sigma_t^2)$$

$$\sigma_t^2 = \beta_o + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \varepsilon_{t-2}^2 + \beta_3 \varepsilon_{t-3}^2 + \beta_4 \varepsilon_{t-4}^2 \qquad \beta_i > 0$$

For the purpose of you calculations you may assume that $\varepsilon_{0-i} = 0$ for all $i \geq 0$. Your answer should include a mathematical description of how you computed the sample likelihood function

2)   Describe how the model in question 2 could be estimated by GMM if instead of assuming that $\varepsilon_t \sim N(0, \sigma_t^2)$ we assume $E_t \varepsilon_{t+1}^2 = \sigma_t^2$ (i.e. drop the normality assumption). What instruments can you use? Obtain the GMM estimates of α_i and β_i in an exactly identified model. Compare the MLE estimates and the GMM estimates.

Notes: You will have to use the simplex algorithm. Download the code for the procedure from my webpage. To use the simplex procedure you need to write an objective function procedure ff(b), that is passed to simplex as:

    {bend,vff,cnv} =simplex(&ff,b0,prt)

where

    b0: starting values
    ff: procedure returning value of function to be minimized
    prt: printing pointer,
            if prt = 1, intermediate steps will be printed
            if prt = 2, intermediate steps will be printed
                    with parameter estimates and gradients
    bend: best parameter vector found
    vff: value of function at bend
    cnv: if converged = 1