Econometrics II Mid-Term Exam 2006

October 23, 2009

INSTRUCTIONS: You should attempt all of the questions. You have 1hour 40 minutes for Part A of the exam and there are 100 points so you should apportion your time accordingly. You may not use any notes, your computer, refer to any books, or confer with anyone while the exam is in progress. Please insure that you explain all the steps in your answer legibly.

Part A

1. *Show that the variance of the random variable $X_1 - E[X_1|X_2]$ cannot be greater than the variance of X_1, and that the two variances are equal if X_1 and X_2 are independent. You may assume that $E[X_1] = 0$. [10 points]*

2. *Consider two linear regressions, one restricted and the other unrestricted:*

$$\begin{aligned} y &= X\beta + u, \\ y &= X\beta + Z\gamma + u. \end{aligned}$$

   Show that, in the case of mutually orthogonal regressors, with $X'Z = 0$, the estimates of β from the two regressions are identical. [10 points]

3. *Consider the linear regression model*

$$y_i = \beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} + u_i.$$

   Explain how you could estimate this model subject to the restriction that $\beta_2 + \beta_3 = 1$ by running a regression that imposes the restriction. Also, explain how you could estimate the unrestricted model in such a way that the value of one of the coefficients would be zero if the restriction held exactly for your data. [10 points]

4. *Consider the linear regression model*

$$y = X\beta + u, \qquad u \sim N(0, \sigma^2 I), \qquad E[u|X] = 0,$$

   where X is an $n \times k$ matrix. If σ_0 denotes the true value of σ, how is the quantity $(y' M_x y)/\sigma_0^2$ distributed? Explain your reasoning. Use this result to derive a test of the null hypothesis that $\sigma = \sigma_0$. Is this a one-tailed test or a two-tailed test? [15 points]

5. *Resolve the following paradox: the asymptotic approximation to the true distribution of b and s^2 implies that we treat s^2 as a constant when we draw inferences about β but treat s^2 as a normally distributed random variable when we draw inferences about σ^2. (NB I am using Hayashi's notation.) [15 points]*

6. *Consider the time series regression model*

$$y_t = \beta x_t + u_t$$

where x_t is stationary and ergodic and

$$u_t = \rho u_{t-1} + v_t, \qquad |\rho| < 1$$

with v_t white noise. Explain how β and ρ can be estimated by a least squares regression under the null hypothesis that $\beta\rho = 1$. Describe how the parameters can be estimated from another regression without imposing the null hypothesis. How can the estimates from this second approach be used to the test the restriction $\beta\rho = 1$.[20 points]

7. *Let the matrix of stationary and ergodic predetermined variables X be partitioned as $[X_1, X_2]$ where X_1 and X_2 are $n \times d$ matrices with*

$$E[X_i X_i'] \equiv \Sigma_{xx} = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21}' & \Sigma_{22} \end{bmatrix}.$$

We are interested in estimating

$$y_i = x_{i,1}'\beta + \varepsilon_i$$

where $x_{i,1}'$ is the i'th. row of X_1. We can establish that the variables in X_2 are predetermined but not those in X_1. We also know that $g_i \equiv x_{i,2} \cdot \varepsilon_i$ is a martingale difference sequence with $E[g_i g_i'] = S$. Derive the asymptotic distribution of the estimator

$$\tilde{\beta} = [X_2' X_1]^{-1} X_2' Y.$$

Suppose that $\Sigma_{12} = 0$. What will the estimates $\tilde{\beta}$ tell us about the values of β? [20 points]